

# Maxwell's Equations Test III

# Boundary Conditions for Electromagnetics

	Differential Form	Integral Form
Gauss's Law	$\nabla \cdot \vec{D} = \rho_v$	$\oint_S \vec{D} \cdot d\vec{S} = Q$
Faraday's Law	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_C \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$
Gauss's Law for Magnetism	$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot d\vec{S} = 0$
Ampere's Law	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\oint_C \vec{H} \cdot d\vec{l} = \int_S \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$

Field Components	General Form	Medium 1: Dielectric Medium 2: Dielectric	Medium 1: Dielectric Medium 2: Conductor
Tangential E	$\hat{n}_2 \times (\vec{E}_1 - \vec{E}_2) = 0$	$E_{1t} = E_{2t}$	$E_{1t} = E_{2t} = 0$
Normal D	$\hat{n}_2 \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$	$D_{1n} - D_{2n} = \rho_s$	$D_{1n} = \rho_s$ $D_{2n} = 0$
Tangential H	$\hat{n}_2 \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$	$H_{1t} = H_{2t}$	$H_{1t} = J_s$ $H_{2t} = 0$
Normal B	$\hat{n}_2 \cdot (\vec{B}_1 - \vec{B}_2) = 0$	$B_{1n} = B_{2n}$	$B_{1n} = B_{2n} = 0$

$$\vec{D} = \epsilon \vec{E} = \epsilon_0 \epsilon_r \vec{E} = \epsilon_0 (1 + \chi_e) \vec{E} \quad \vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H} = \mu_0 (1 + \chi_m) \vec{H} \quad \vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\epsilon} \rho_v = 0 \quad \rho_v(t) = \rho_{v0} e^{-(\sigma/\epsilon)t} = \rho_{v0} e^{-t/\tau}$$

$$V(\vec{R}, t) = \frac{1}{4\pi\epsilon} \int_{v'} \frac{\rho_v(\vec{R}', t - R'/u_p)}{R'} dv'$$

$$\vec{B} = \hat{\phi} \frac{\mu_0 I}{2\pi r}$$

$$\vec{A}(\vec{R}, t) = \frac{\mu}{4\pi} \int_{v'} \frac{\vec{J}(\vec{R}', t - R'/u_p)}{R'} dv'$$

$$Z_{in} = \left( \frac{N_1}{N_2} \right)^2 Z_L$$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \quad \eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\epsilon}} = \frac{\eta_0}{\sqrt{\epsilon_r}} \Rightarrow \eta_0 \approx 120\pi$$

$$\vec{B} = \nabla \times \vec{A} \quad \vec{u} = \hat{\phi} \omega r$$

$$\frac{d^2 \vec{E}_x}{dz^2} + k^2 \vec{E}_x = 0$$

$$u_p = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\frac{d^2 \vec{E}_y}{dz^2} + k^2 \vec{E}_y = 0$$

$$\lambda = \frac{2\pi}{k} = \frac{u_p}{f} \quad S_{av}^i = \frac{|E_0^i|^2}{2\eta_1} \hat{z} \quad \vec{E}_z = 0$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\vec{H} = \frac{1}{\eta} \hat{k} \times \vec{E} \quad S_{av}^r = |\Gamma|^2 S_{av}^i (-\hat{z}) \quad \frac{d^2 \vec{H}_x}{dz^2} + k^2 \vec{H}_x = 0$$

$$\vec{E} = -\eta \hat{k} \times \vec{H} \quad S_{av}^t = |\tau|^2 \frac{|E_0^i|^2}{2\eta_2} \hat{z} \quad \frac{d^2 \vec{H}_y}{dz^2} + k^2 \vec{H}_y = 0$$

$$\vec{H}_z = 0 \quad \lambda = \frac{\lambda_0}{\sqrt{\epsilon_0}} = \frac{c}{f\sqrt{\epsilon_r}} \quad \tau_{\parallel} = (1 + \Gamma_{\parallel}) \frac{\cos \theta_i}{\cos \theta_t}$$

$$E(z, t) = \Re[\vec{E}(z) e^{j\omega t}] = \hat{x} a_x \cos(\omega t - kz) + \hat{y} a_y \cos(\omega t - kz + \delta)$$

$$|\vec{E}(z, t)| = [a_x^2 \cos^2(\omega t - kz) + a_y^2 \cos^2(\omega t - kz + \delta)]^{1/2} \quad \psi(z, t) = \tan^{-1} \left( \frac{E_y(z, t)}{E_x(z, t)} \right)$$

$$\Gamma_{\perp} = \frac{E_{\perp 0}^r}{E_{\perp 0}^i} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$$

$$S_{av} = \hat{z} \frac{1}{2\eta} (|E_{x0}|^2 + |E_{y0}|^2) = \hat{z} \frac{|\vec{E}|^2}{2\eta} \quad (W/m^2)$$

$$\tau_{\perp} = \frac{E_{\perp 0}^t}{E_{\perp 0}^i} = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$$

$$\Gamma_{\parallel} = \frac{-(\epsilon_2/\epsilon_1) \cos \theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2 \theta_i}}{(\epsilon_2/\epsilon_1) \cos \theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2 \theta_i}}$$

$$\tau_{\perp} = 1 + \Gamma_{\perp} \quad R_{\parallel} = |\Gamma_{\parallel}|^2$$

$$\tau_{\parallel} = 1 - R_{\parallel} \quad T_{\parallel} = 1 - R_{\parallel}$$

$$V_{emf} = V_{emf}^{motional} + V_{emf}^{transformer} = -N \frac{d\Phi}{dt} = -N \frac{d}{dt} \int_S \vec{B} \cdot d\vec{S}$$

$$V_{emf}^m = \oint_C (\vec{u} \times \vec{B}) \cdot d\vec{l} \quad \nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\nabla \times \vec{H} = j\omega \epsilon \vec{E} \quad \text{or} \quad \vec{E} = \frac{1}{j\omega \epsilon} \nabla \times \vec{H}$$

$$\nabla \times \vec{E} = -j\omega \mu \vec{H} \quad \text{or} \quad \vec{H} = -\frac{1}{j\omega \mu} \nabla \times \vec{E}$$

$$V_{emf} = V_{emf}^{tr} + \vec{V}_{emf}^m = \oint_C \vec{E} \cdot d\vec{l} \quad R = \frac{d}{\sigma A}$$

$$= -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} + \oint_C (\vec{u} \times \vec{B}) \cdot d\vec{l}$$

$$E(z, t) = \Re[\vec{E}(z) e^{j\omega t}] = \hat{x} |E_{x0}^+| \cos(\omega t - kz + \Phi^+)$$

$$H(z, t) = \Re[\vec{H}(z) e^{j\omega t}] = \hat{y} \frac{|E_{x0}^+|}{\eta} \cos(\omega t - kz + \Phi^+)$$

$$\Gamma_{\parallel} = \frac{E_{\parallel 0}^r}{E_{\parallel 0}^i} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}$$

$$\tau_{\parallel} = \frac{E_{\parallel 0}^t}{E_{\parallel 0}^i} = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}$$

$$\tau_{\parallel} = (1 + \Gamma_{\parallel}) \frac{\cos \theta_i}{\cos \theta_t}$$

$$c = 3 \times 10^8 \quad m/s \quad R_{dc} = \frac{l}{\sigma A}$$

$$\eta_0 = 120\pi \quad \Omega \quad R_{ac} = \frac{l}{\sigma \omega \delta_s}$$

$$\epsilon_r = \left( \frac{c}{u_p} \right)^2 \quad k_2 = \sqrt{\frac{\epsilon_2}{\epsilon_1}} k_1$$

$$\eta = \eta_0 / \sqrt{\epsilon_r} \quad E^r = \Gamma E^i$$

$$\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega \epsilon} = \frac{\sigma}{2\pi f \epsilon_r \epsilon_0}$$

$$\begin{aligned}\tilde{E}_{\perp}^i &= \hat{y}E_{\perp 0}^i e^{-jk_1(x\sin\theta_i+z\cos\theta_i)} \\ \tilde{H}_{\perp}^i &= (-\hat{x}\cos\theta_i + \hat{z}\sin\theta_i) \frac{E_{\perp 0}^i}{\eta_1} e^{-jk_1(x\sin\theta_i+z\cos\theta_i)}\end{aligned}\quad \begin{aligned}\tan 2\gamma &= (\tan 2\psi_0)\cos\delta \quad -\frac{\pi}{2} \leq \gamma \leq \frac{\pi}{2} \\ \sin 2\chi &= (\sin 2\psi_0)\sin\delta \quad -\pi/4 \leq \chi \leq \pi/4\end{aligned}$$

$$\begin{aligned}\tilde{E}_{\perp}^r &= \hat{y}E_{\perp 0}^r e^{-jk_1x_r} = \hat{y}E_{\perp 0}^r e^{-jk_1(x\sin\theta_r-z\cos\theta_r)} \\ \tilde{H}_{\perp}^r &= \hat{y}_r \frac{E_{\perp 0}^r}{\eta_1} e^{-jk_1x_r} = (\hat{x}\cos\theta_r + \hat{z}\sin\theta_r) \frac{E_{\perp 0}^r}{\eta_1} e^{-jk_1(x\sin\theta_r-z\cos\theta_r)}\end{aligned}\quad \tan\psi_0 = \frac{a_y}{a_x} \quad 0 \leq \psi_0 \leq \frac{\pi}{2}$$

$$\begin{aligned}\tilde{E}_{\perp}^t &= \hat{y}E_{\perp 0}^t e^{-jk_2x_t} = \hat{y}E_{\perp 0}^t e^{-jk_2(x\sin\theta_t+z\cos\theta_t)} \\ \tilde{H}_{\perp}^t &= \hat{y}_t \frac{E_{\perp 0}^t}{\eta_2} e^{-jk_2x_t} = (-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t) \frac{E_{\perp 0}^t}{\eta_2} e^{-jk_2(x\sin\theta_t+z\cos\theta_t)} \\ E(z,t) &= \Re[\tilde{E}(z)e^{j\omega t}] = \eta_2 \hat{x} a_x \cos(\omega t - kz) + \hat{y} a_y \cos(\omega t - kz + \delta)\end{aligned}$$

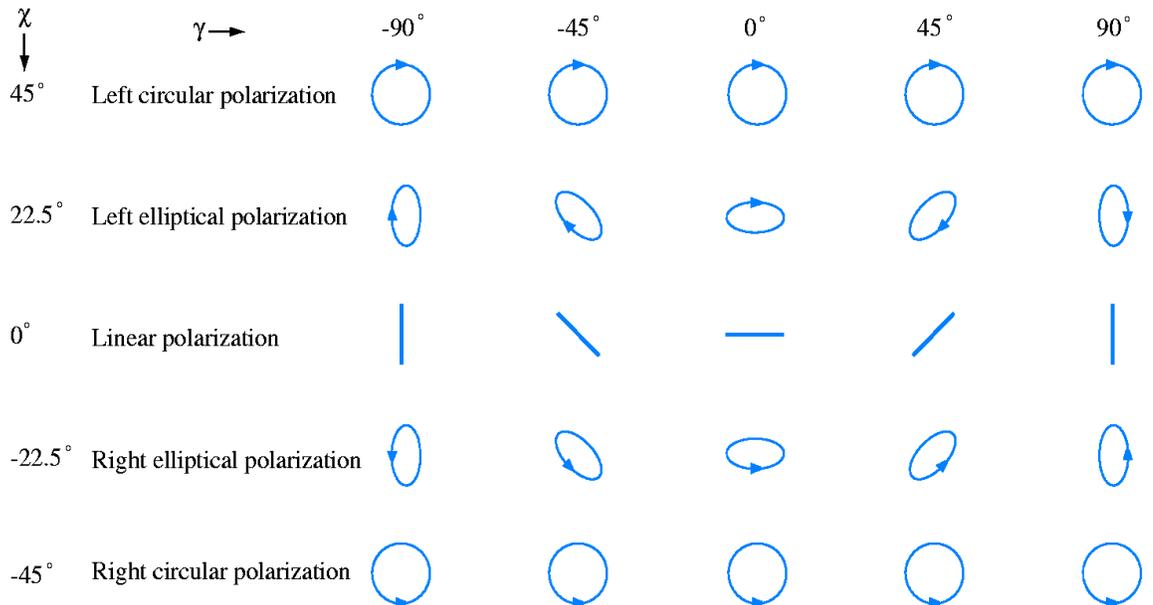
$$\begin{aligned}\tilde{E}_{\parallel}^i &= \hat{y}_i E_{\parallel 0}^i e^{-jk_1x_i} = (\hat{x}\cos\theta_i - \hat{z}\sin\theta_i) E_{\parallel 0}^i e^{-jk_1(x\sin\theta_i+z\cos\theta_i)} \\ \tilde{H}_{\parallel}^i &= \hat{y} \frac{E_{\parallel 0}^i}{\eta_1} e^{-jk_1x_i} = \hat{y} \frac{E_{\parallel 0}^i}{\eta_1} e^{-jk_1(x\sin\theta_i+z\cos\theta_i)}\end{aligned}\quad R' = R'_1 + R'_2 = \frac{R_s}{2\pi} \left( \frac{1}{a} + \frac{1}{b} \right)$$

$$\tilde{E}_{\parallel}^r = \hat{y}_r E_{\parallel 0}^r e^{-jk_1x_r} = (\hat{x}\cos\theta_r + \hat{z}\sin\theta_r) E_{\parallel 0}^r e^{-jk_1(x\sin\theta_r-z\cos\theta_r)}$$

$$\tilde{H}_{\parallel}^r = -\hat{y} \frac{E_{\parallel 0}^r}{\eta_1} e^{-jk_1x_r} = -\hat{y} \frac{E_{\parallel 0}^r}{\eta_1} e^{-jk_1(x\sin\theta_r+z\cos\theta_r)}$$

$$\tilde{E}_{\parallel}^t = \hat{y}_t E_{\parallel 0}^t e^{-jk_2x_t} = (\hat{x}\cos\theta_t - \hat{z}\sin\theta_t) E_{\parallel 0}^t e^{-jk_2(x\sin\theta_t+z\cos\theta_t)}$$

$$\tilde{H}_{\parallel}^t = \hat{y} \frac{E_{\parallel 0}^t}{\eta_2} e^{-jk_2x_t} = \hat{y} \frac{E_{\parallel 0}^t}{\eta_2} e^{-jk_2(x\sin\theta_t+z\cos\theta_t)} \quad \delta_s = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$



	Any Medium	Lossless Medium ( $\sigma=0$ )	Low-Loss Medium ( $\epsilon''/\epsilon' \ll 1$ )	Good Conductor ( $\epsilon''/\epsilon' \gg 1$ )	Units
$\alpha =$	$\omega \left\{ \frac{\mu\epsilon'}{2} \left[ \sqrt{1 + \left( \frac{\epsilon''}{\epsilon'} \right)^2} - 1 \right] \right\}^{1/2}$	0	$\frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}}$	$\sqrt{\pi f \mu \sigma}$	(Np/m)
$\beta =$	$\omega \left\{ \frac{\mu\epsilon'}{2} \left[ \sqrt{1 + \left( \frac{\epsilon''}{\epsilon'} \right)^2} + 1 \right] \right\}^{1/2}$	$\omega \sqrt{\mu\epsilon}$	$\omega \sqrt{\mu\epsilon}$	$\sqrt{\pi f \mu \sigma}$	(rad/m)
$\eta_c =$	$\sqrt{\frac{\mu}{\epsilon'}} \left( 1 - j \frac{\epsilon''}{\epsilon'} \right)^{-1/2}$	$\sqrt{\frac{\mu}{\epsilon}}$	$\sqrt{\frac{\mu}{\epsilon}}$	$(1+j) \frac{\alpha}{\sigma}$	( $\Omega$ )
$u_p =$	$\omega / \beta$	$1 / \sqrt{\mu\epsilon}$	$1 / \sqrt{\mu\epsilon}$	$\sqrt{4\pi f / \mu \sigma}$	(m/s)
$\lambda =$	$2\pi / \beta = u_p / f$	$u_p / f$	$u_p / f$	$u_p / f$	(m)

$$\nabla^2 \tilde{\mathbf{E}} - \gamma^2 \tilde{\mathbf{E}} = 0$$

$$\epsilon_{\text{complex}} = \epsilon - j \frac{\sigma}{\omega}$$

$$\gamma^2 = -k^2$$

$$\eta_c = |\eta_c| e^{j\theta_\eta}$$

$$E(z,t) = \Re e \left[ \tilde{E}(z) e^{j\omega t} \right] = \hat{x} \left| E_{x0}^+ \right| \cos(\omega t - kz + \Phi^+)$$

$$\Gamma = \frac{\eta_{c_2} - \eta_{c_1}}{\eta_{c_2} + \eta_{c_1}} \quad \alpha^2 - \beta^2 = -\omega^2 \mu \epsilon' = -\frac{\omega^2}{c^2} \epsilon_r'$$

$$H(z,t) = \Re e \left[ \tilde{H}(z) e^{j\omega t} \right] = \hat{y} \frac{|E_{x0}^+|}{\eta} \cos(\omega t - kz + \Phi^+)$$

$$\tau = 1 + \Gamma = \frac{2\eta_{c_2}}{\eta_{c_2} + \eta_{c_1}} \quad 2\alpha\beta = \omega^2 \mu \epsilon'' = \frac{\omega^2}{c^2} \epsilon_r''$$

$$\gamma^2 = -\omega^2 \mu \epsilon_{\text{complex}} = -\omega^2 \mu (\epsilon' - j\epsilon'') = -\omega^2 \mu (\epsilon - j\frac{\sigma}{\omega})$$

$$\gamma = \alpha + j\beta \Rightarrow \gamma^2 = (\alpha + j\beta)^2 = (\alpha^2 - \beta^2) + j2\alpha\beta$$

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{u_{p_2}}{u_{p_1}} = \frac{n_1}{n_2} = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}} = \sqrt{\frac{\mu_{r_1} \epsilon_{r_1}}{\mu_{r_2} \epsilon_{r_2}}} = \sqrt{\frac{\epsilon_{r_1}}{\epsilon_{r_2}}} = \frac{\eta_2}{\eta_1}$$

$$\bar{S}_{av} = \frac{1}{2} \Re e \left[ \tilde{E} \times \tilde{H}^* \right] = \frac{\hat{z} \left( |E_{x0}|^2 + |E_{y0}|^2 \right)}{2} e^{-2\alpha z} \Re e \left( \frac{1}{\eta_c^*} \right)$$

$$\Omega_p = \iint_{4\pi} F(\theta, \phi) d\Omega \quad (sr) \quad D = \frac{4\pi}{\Omega_p} \cong \frac{4\pi}{\beta_{xz} \beta_{yz}}$$

$$\tilde{H}_\phi = \frac{I_0 l k^2}{4\pi} e^{-jkR} \left[ \frac{j}{kR} + \frac{1}{(kR)^2} \right] \sin \theta$$

$$\bar{S}_{av}(z) = \hat{z} \frac{|E_0|^2}{2|\eta_c|} e^{-2\alpha z} \cos \Theta_\eta \quad (W/m^2) \quad Z_s = \frac{1+j}{\sigma \delta_s} = R_s + j\omega L_s$$

$$\tilde{E}_R = \frac{2I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[ \frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \cos \theta$$

$$R_s = \frac{1}{\sigma \delta_s} = \sqrt{\frac{\pi f \mu}{\sigma}} \quad \delta_s = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$\tilde{E}_\theta = \frac{I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[ \frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \sin \theta$$

$$L_s = \frac{1}{\omega \sigma \delta_s} = \frac{1}{2} \sqrt{\frac{\mu}{\pi f \sigma}}$$

$$\xi_t A_t = \xi_t D_t \lambda^2 / 4\pi = G_t \lambda^2 / 4\pi$$

$$S_r = G_t S_{iso} = \xi_t D_t S_{iso} = \frac{\xi_t D_t P_t}{4\pi R^2}$$

$$P_{rec} = \xi_r P_{int}$$

$$F(\theta, \phi) = \frac{S(R, \theta, \phi)}{S_{\max}}$$

$$P_{int} = S_r A_r = \frac{\xi_r A_r P_t}{\lambda^2 R^2}$$

$$P_n = KT_{\text{sys}} B$$

$$\frac{P_{rec}}{P_t} = \frac{\xi_t \xi_r A_t A_r}{\lambda^2 R^2} = G_t G_r \left( \frac{\lambda}{4\pi R} \right)^2$$

$$S_r = \frac{\xi_t A_t P_t}{\lambda^2 R^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \quad F/m$$

$$\mu_0 = 4\pi \times 10^{-7} \quad H/m$$

$$\frac{P_{rec}}{P_t} = G_t G_r \left( \frac{\lambda}{4\pi R} \right)^2 F_t(\theta_t, \phi_t) F_r(\theta_r, \phi_r)$$

$$k = 1.38 \times 10^{-23} \quad J/K$$

# Maxwell's Equations

$$\begin{aligned} \nabla \cdot \vec{D} &= \rho \\ \nabla \cdot \vec{E} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \end{aligned}$$

## Assumptions:

linear, isotropic, and homogeneous medium:

$$\begin{aligned} \vec{D} &= \epsilon \vec{E} \\ \vec{B} &= \mu \vec{H} \end{aligned}$$

Time variation is a sinusoidal function:

$$\vec{E}(x, y, z, t) = \Re\{e\}[\vec{E}(x, y, z)e^{j\omega t}]$$

$$\begin{aligned} \nabla \cdot \vec{E} &= \tilde{\rho}_v / \epsilon \\ \nabla \times \vec{E} &= -j\omega\mu\vec{H} \\ \nabla \cdot \vec{H} &= 0 \\ \nabla \times \vec{H} &= \vec{J} + j\omega\epsilon\vec{E} \end{aligned}$$

Charge free medium

$$\tilde{\rho}_v = 0$$

$$\begin{aligned} \nabla \cdot \vec{E} &= 0 \\ \nabla \times \vec{E} &= -j\omega\mu\vec{H} \\ \nabla \cdot \vec{H} &= 0 \\ \nabla \times \vec{H} &= \vec{J} + j\omega\epsilon\vec{E} \end{aligned}$$

$$\vec{J} = \sigma \vec{E}$$

$$\epsilon_{\text{complex}} = \epsilon - j\frac{\sigma}{\omega}$$

$$\begin{aligned} \nabla \cdot \vec{E} &= 0 \\ \nabla \times \vec{E} &= -j\omega\mu\vec{H} \\ \nabla \cdot \vec{H} &= 0 \\ \nabla \times \vec{H} &= j\omega\epsilon_{\text{complex}}\vec{E} \end{aligned}$$

$$\begin{aligned} \frac{d^2 \tilde{E}_x}{dz^2} + k^2 \tilde{E}_x &= 0 \\ \frac{d^2 \tilde{E}_y}{dz^2} + k^2 \tilde{E}_y &= 0 \\ \tilde{E}_z &= 0 \\ \frac{d^2 \tilde{H}_x}{dz^2} + k^2 \tilde{H}_x &= 0 \\ \frac{d^2 \tilde{H}_y}{dz^2} + k^2 \tilde{H}_y &= 0 \\ \tilde{H}_z &= 0 \end{aligned}$$

Solution:

$$\vec{E}(z, t) = \Re\{e\}[\vec{E}(z)e^{j\omega t}] = x\tilde{a}_x \cos(\omega t - kz + \delta_x) + y\tilde{a}_y \cos(\omega t - kz + \delta_y)$$

$$\vec{H} = \frac{1}{\eta} \hat{z} \times \vec{E}$$

Intrinsic Impedance

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\frac{\omega}{k} = u_p = \frac{1}{\sqrt{\mu\epsilon}} = \lambda f$$

$$\vec{H}(z, t) = \Re\{e\}[\vec{H}(z)e^{j\omega t}] = x\tilde{b}_x \cos(\omega t - kz + \delta_x) + y\tilde{b}_y \cos(\omega t - kz + \delta_y)$$

Plane Wave  
Cartesian Coordinate  
Propagation in z-direction

## Wave Equation

$$\begin{aligned} \nabla^2 \tilde{E} + k^2 \tilde{E} &= 0 \\ \nabla^2 \tilde{H} + k^2 \tilde{H} &= 0 \end{aligned}$$

lossless medium

$$\sigma = 0$$

$$\epsilon_{\text{complex}} = \epsilon - j\frac{\sigma}{\omega}$$

Wavenumber

$$k^2 = -\gamma^2 = \omega^2 \mu\epsilon$$

$$\begin{aligned} \nabla^2 \tilde{E} - \gamma^2 \tilde{E} &= 0 \\ \nabla^2 \tilde{H} - \gamma^2 \tilde{H} &= 0 \end{aligned}$$

Propagation Constant

$$\gamma^2 = -\omega^2 \mu\epsilon_{\text{complex}}$$