A wave traveling along a string is given by
\[ y(x,t) = 2 \sin(4\pi t + 10\pi x) \text{ (cm)}, \]
where \( x \) is the distance along the string in meters and \( y \) is the vertical displacement. Determine: (a) the direction of wave travel, (b) the reference phase \( \phi_0 \), (c) the frequency, (d) the wavelength, and (e) the phase velocity.

**Solution:**

(a) We start by converting the given expression into a cosine function of the form given by (1.17):
\[ y(x,t) = 2 \cos \left( 4\pi t + 10\pi x \right) \text{ (cm)}. \]
Since the coefficients of \( t \) and \( x \) both have the same sign, the wave is traveling in the negative \( x \)-direction.

(b) From the cosine expression, \( \phi_0 = -\pi/2 \).

(c) \( \omega = 2\pi f = 4\pi \),
\[ f = 4\pi/2\pi = 2 \text{ Hz}. \]

(d) \( 2\pi/\lambda = 10\pi \),
\[ \lambda = 2\pi/10\pi = 0.2 \text{ m}. \]

(e) \( u_\phi = f\lambda = 2 \times 0.2 = 0.4 \text{ (m/s)}. \)
A wave traveling along a string in the +x-direction is given by
\[ y_1(x, t) = A \cos(\omega t - \beta x), \]
where \( x = 0 \) is the end of the string, which is tied rigidly to a wall, as shown in Fig. P1.7. When wave \( y_1(x, t) \) arrives at the wall, a reflected wave \( y_2(x, t) \) is generated. Hence, at any location on the string, the vertical displacement \( y_s \) is the sum of the incident and reflected waves:
\[ y_s(x, t) = y_1(x, t) + y_2(x, t). \]

(a) Write an expression for \( y_2(x, t) \), keeping in mind its direction of travel and the fact that the end of the string cannot move.

(b) Generate plots of \( y_1(x, t) \), \( y_2(x, t) \) and \( y_s(x, t) \) versus \( x \) over the range \(-2\lambda \leq x \leq 0\) at \( \omega t = \pi/4 \) and at \( \omega t = \pi/2 \).

**Solution:**

(a) Since wave \( y_2(x, t) \) was caused by wave \( y_1(x, t) \), the two waves must have the same angular frequency \( \omega \), and since \( y_2(x, t) \) is traveling on the same string as \( y_1(x, t) \), the two waves must have the same phase constant \( \beta \). Hence, with its direction being in the negative x-direction, \( y_2(x, t) \) is given by the general form
\[ y_2(x, t) = B \cos(\omega t + \beta x + \phi_0), \quad (1) \]
where \( B \) and \( \phi_0 \) are yet-to-be-determined constants. The total displacement is
\[ y_s(x, t) = y_1(x, t) + y_2(x, t) = A \cos(\omega t - \beta x) + B \cos(\omega t + \beta x + \phi_0). \]

Since the string cannot move at \( x = 0 \), the point at which it is attached to the wall, \( y_s(0, t) = 0 \) for all \( t \). Thus,
\[ y_s(0, t) = A \cos \omega t + B \cos(\omega t + \phi_0) = 0. \quad (2) \]
(i) Easy Solution: The physics of the problem suggests that a possible solution for (2) is \( B = -A \) and \( \phi_0 = 0 \), in which case we have
\[
y_2(x,t) = -A \cos(\omega t + \beta x).
\] (3)

(ii) Rigorous Solution: By expanding the second term in (2), we have
\[
A \cos \omega t + B(\cos \omega t \cos \phi_0 - \sin \omega t \sin \phi_0) = 0,
\]
or
\[
(A + B \cos \phi_0) \cos \omega t - (B \sin \phi_0) \sin \omega t = 0.
\] (4)
This equation has to be satisfied for all values of \( t \). At \( t = 0 \), it gives
\[
A + B \cos \phi_0 = 0,
\] (5)
and at \( \omega t = \pi/2 \), (4) gives
\[
B \sin \phi_0 = 0.
\] (6)
Equations (5) and (6) can be satisfied simultaneously only if
\[
A = B = 0
\] (7)
or
\[
A = -B \quad \text{and} \quad \phi_0 = 0.
\] (8)
Clearly (7) is not an acceptable solution because it means that \( y_1(x,t) = 0 \), which is contrary to the statement of the problem. The solution given by (8) leads to (3).

(b) At \( \omega t = \pi/4 \),
\[
y_1(x,t) = A \cos(\pi/4 - \beta x) = A \cos \left( \frac{\pi}{4} - \frac{2\pi x}{\lambda} \right),
\]
\[
y_2(x,t) = -A \cos(\omega t + \beta x) = -A \cos \left( \frac{\pi}{4} + \frac{2\pi x}{\lambda} \right).
\]
Plots of \( y_1, y_2 \), and \( y_3 \) are shown in Fig. P1.7(b).
At $\omega t = \pi/2$,

\[
y_1(x, t) = A \cos(\pi/2 - \beta x) = A \sin \beta x = A \sin \frac{2\pi x}{\lambda},
\]
\[
y_2(x, t) = -A \cos(\pi/2 + \beta x) = A \sin \beta x = A \sin \frac{2\pi x}{\lambda}.
\]

Plots of $y_1$, $y_2$, and $y_3$ are shown in Fig. P1.7(c).
Figure P1.7: (c) Plots of $y_1$, $y_2$, and $y_s$ versus $x$ at $\omega t = \pi/2$. 
Problem 3  An oscillator that generates a sinusoidal wave on a string completes 20 vibrations in 50 s. The wave peak is observed to travel a distance of 2.8 m along the string in 5 s. What is the wavelength?

Solution:

\[ T = \frac{50}{20} = 2.5 \text{ s}, \quad u_p = \frac{2.8}{5} = 0.56 \text{ m/s}, \]
\[ \lambda = u_p T = 0.56 \times 2.5 = 1.4 \text{ m}. \]
Problem 1.15

A laser beam traveling through fog was observed to have an intensity of 1 (\(\mu\text{W/m}^2\)) at a distance of 2 m from the laser gun and an intensity of 0.2 (\(\mu\text{W/m}^2\)) at a distance of 3 m. Given that the intensity of an electromagnetic wave is proportional to the square of its electric-field amplitude, find the attenuation constant \(\alpha\) of fog.

Solution: If the electric field is of the form

\[
E(x,t) = E_0 e^{-\alpha x} \cos(\omega t - \beta x),
\]

then the intensity must have a form

\[
I(x,t) \approx |E_0 e^{-\alpha x} \cos(\omega t - \beta x)|^2 \\
\approx E_0^2 e^{-2\alpha x} \cos^2(\omega t - \beta x)
\]

or

\[
I(x,t) = I_0 e^{-2\alpha x} \cos^2(\omega t - \beta x)
\]

where we define \(I_0 \approx E_0^2\). We observe that the magnitude of the intensity varies as \(I_0 e^{-2\alpha x}\). Hence,

at \(x = 2\) m, \(I_0 e^{-4\alpha} = 1 \times 10^{-6}\) (W/m\(^2\)),

at \(x = 3\) m, \(I_0 e^{-6\alpha} = 0.2 \times 10^{-6}\) (W/m\(^2\)).

\[
\frac{I_0 e^{-4\alpha}}{I_0 e^{-6\alpha}} = \frac{10^{-6}}{0.2 \times 10^{-6}} = 5
\]

\[e^{-4\alpha} \cdot e^{6\alpha} = e^{2\alpha} = 5\]

\[\alpha = 0.8\] (NP/m).
Problem 1.17

Complex numbers $z_1$ and $z_2$ are given by

$$z_1 = 3 - j2$$
$$z_2 = -4 + j3$$

(a) Express $z_1$ and $z_2$ in polar form.

(b) Find $|z_1|$ by first applying Eq. (1.41) and then by applying Eq. (1.43).

(c) Determine the product $z_1z_2$ in polar form.

(d) Determine the ratio $z_1/z_2$ in polar form.

(e) Determine $z_3^1$ in polar form.

Solution:

(a) Using Eq. (1.41),

$$z_1 = 3 - j2 = 3.6e^{-j33.7^\circ}$$
$$z_2 = -4 + j3 = 5e^{j43.1^\circ}.$$ 

(b) By Eq. (1.41) and Eq. (1.43), respectively,

$$|z_1| = |3 - j2| = \sqrt{3^2 + (-2)^2} = \sqrt{13} = 3.60,$$
$$|z_1| = \sqrt{(3 - j2)(3 + j2)} = \sqrt{13} = 3.60.$$ 

(c) By applying Eq. (1.47b) to the results of part (a),

$$z_1z_2 = 3.6e^{-j33.7^\circ} \times 5e^{j43.1^\circ} = 18e^{j109.4^\circ}.$$ 

(d) By applying Eq. (1.48b) to the results of part (a),

$$\frac{z_1}{z_2} = \frac{3.6e^{-j33.7^\circ}}{5e^{j43.1^\circ}} = 0.72e^{-j176.8^\circ}.$$ 

(e) By applying Eq. (1.49) to the results of part (a),

$$z_3^1 = (3.6e^{-j33.7^\circ})^3 = (3.6)^3e^{-j3 \times 33.7^\circ} = 46.66e^{-j101.1^\circ}.$$
Problem 1.26

Find the phasors of the following time functions:

(a) \( \upsilon(t) = 9 \cos(\omega t - \pi/3) \) (V)
(b) \( \upsilon(t) = 12 \sin(\omega t + \pi/4) \) (V)
(c) \( i(x,t) = 5e^{-3x} \sin(\omega t + \pi/6) \) (A)
(d) \( i(t) = -2 \cos(\omega t + 3\pi/4) \) (A)
(e) \( i(t) = 4 \sin(\omega t + \pi/3) + 3 \cos(\omega t - \pi/6) \) (A)

Solution:

(a) \( \tilde{V} = 9e^{-j\pi/3} \) V.
(b) \( \upsilon(t) = 12 \sin(\omega t + \pi/4) = 12 \cos(\pi/2 - (\omega t + \pi/4)) = 12 \cos(\omega t - \pi/4) \) V, \( \tilde{V} = 12e^{-j\pi/4} \) V.
(c)
\[
i(t) = 5e^{-3x} \sin(\omega t + \pi/6) \text{ A} = 5e^{-3x} \cos[\pi/2 - (\omega t + \pi/6)] \text{ A} \\
\quad = 5e^{-3x} \cos(\omega t - \pi/3) \text{ A}, \\
\tilde{i} = 5e^{-3x} e^{-j\pi/3} \text{ A}.
\]
(d)
\[
i(t) = -2 \cos(\omega t + 3\pi/4), \\
\tilde{i} = -2e^{j3\pi/4} = 2e^{-j\pi}e^{j3\pi/4} = 2e^{-j\pi/4} \text{ A}.
\]
(e)
\[
i(t) = 4 \sin(\omega t + \pi/3) + 3 \cos(\omega t - \pi/6) \\
\quad = 4 \cos[\pi/2 - (\omega t + \pi/3)] + 3 \cos(\omega t - \pi/6) \\
\quad = 4 \cos(-\omega t + \pi/6) + 3 \cos(\omega t - \pi/6) \\
\quad = 4 \cos(\omega t - \pi/6) + 3 \cos(\omega t - \pi/6) = 7 \cos(\omega t - \pi/6), \\
\tilde{i} = 7e^{-j\pi/6} \text{ A}.
\]
Problem 1.27

Find the instantaneous time sinusoidal functions corresponding to the following phasors:

(a) \( \tilde{V} = -5e^{j\pi/3} \) (V)
(b) \( \tilde{V} = j6e^{-j\pi/4} \) (V)
(c) \( \tilde{I} = (6 + j8) \) (A)
(d) \( \tilde{I} = -3 + j2 \) (A)
(e) \( \tilde{I} = j \) (A)
(f) \( \tilde{I} = 2e^{j\pi/6} \) (A)

Solution:

(a) \[
\tilde{V} = -5e^{j\pi/3} \ V = 5e^{j(\pi/3-\pi)} \ V = 5e^{-j\pi/3} \ V, \\
v(t) = 5\cos(\omega t - 2\pi/3) \ V.
\]

(b) \[
\tilde{V} = j6e^{-j\pi/4} \ V = 6e^{j(-\pi/4+\pi/2)} \ V = 6e^{j\pi/4} \ V, \\
v(t) = 6\cos(\omega t + \pi/4) \ V.
\]

(c) \( \tilde{I} = (6 + j8) \) A = \( 10e^{j53.1^\circ} \) A, \\
\( i(t) = 10\cos(\omega t + 53.1^\circ) \) A.

(d) \( \tilde{I} = -3 + j2 = 3.61e^{j146.31^\circ}, \\
i(t) = \Re\{3.61e^{j146.31^\circ}e^{j\omega t}\} = 3.61\cos(\omega t + 146.31^\circ) \) A.

(e) \( \tilde{I} = j = e^{j\pi/2}, \\
i(t) = \Re\{e^{j\pi/2}e^{j\omega t}\} = \cos(\omega t + \pi/2) = -\sin \omega t \) A.

(f) \( \tilde{I} = 2e^{j\pi/6}, \\
i(t) = \Re\{2e^{j\pi/6}e^{j\omega t}\} = 2\cos(\omega t + \pi/6) \) A.