SOLUTION # 12

Problem 8.9 The three regions shown in Fig. P8.9 contain perfect dielectrics. For a wave in medium 1, incident normally upon the boundary at $z = -d$, what combination of ε_{r_2} and d produces no reflection? Express your answers in terms of ε_{r_1}, ε_{r_3} and the oscillation frequency of the wave, f.

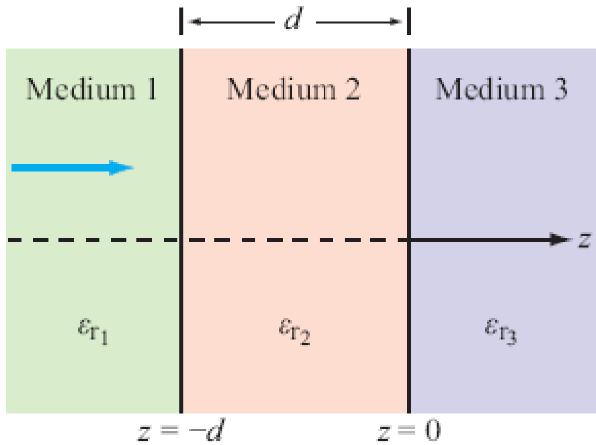

Figure P8.9: Dielectric layers for Problems 8.9 to 8.11.

Solution: By analogy with the transmission-line case, there will be no reflection at $z = -d$ if medium 2 acts as a quarter-wave transformer, which requires that

$$d = \frac{\lambda_2}{4}$$

and

$$\eta_2 = \sqrt{\eta_1 \eta_3} \,.$$

The second condition may be rewritten as

$$\frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \left[\frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} \, \frac{\eta_0}{\sqrt{\varepsilon_{r_3}}}\right]^{1/2}, \qquad \text{or} \quad \varepsilon_{r_2} = \sqrt{\varepsilon_{r_1}\varepsilon_{r_3}} \,,$$

$$\lambda_2 = \frac{\lambda_0}{\sqrt{\varepsilon_{r_2}}} = \frac{c}{f\sqrt{\varepsilon_{r_2}}} = \frac{c}{f(\varepsilon_{r_1}\varepsilon_{r_3})^{1/4}} \,,$$

and

$$d = \frac{c}{4 f(\varepsilon_{r_1}\varepsilon_{r_3})^{1/4}} \,.$$

Problem 8.27 A plane wave in air with

$$\widetilde{\mathbf{E}}^{\mathrm{i}} = \hat{\mathbf{y}}\, 20 e^{-j(3x+4z)} \quad \text{(V/m)}$$

is incident upon the planar surface of a dielectric material, with $\varepsilon_{\mathrm{r}} = 4$, occupying the half-space $z \geq 0$. Determine:

(a) The polarization of the incident wave.

(b) The angle of incidence.

(c) The time-domain expressions for the reflected electric and magnetic fields.

(d) The time-domain expressions for the transmitted electric and magnetic fields.

(e) The average power density carried by the wave in the dielectric medium.

Solution:

(a) $\widetilde{\mathbf{E}}^{\mathrm{i}} = \hat{\mathbf{y}}\, 20 e^{-j(3x+4z)}$ V/m.

Since $\mathbf{E}^{\mathrm{i}}$ is along $\hat{\mathbf{y}}$, which is perpendicular to the plane of incidence, the wave is perpendicularly polarized.

(b) From Eq. (8.48a), the argument of the exponential is

$$-jk_1(x \sin\theta_{\mathrm{i}} + z \cos\theta_{\mathrm{i}}) = -j(3x + 4z).$$

Hence,

$$k_1 \sin\theta_{\mathrm{i}} = 3, \qquad k_1 \cos\theta_{\mathrm{i}} = 4,$$

from which we determine that

$$\tan\theta_{\mathrm{i}} = \frac{3}{4} \qquad \text{or} \qquad \theta_{\mathrm{i}} = 36.87^\circ,$$

and

$$k_1 = \sqrt{3^2 + 4^2} = 5 \quad \text{(rad/m)}.$$

Also,

$$\omega = u_{\mathrm{p}} k = ck = 3 \times 10^8 \times 5 = 1.5 \times 10^9 \quad \text{(rad/s)}.$$

(c)

$$\eta_1 = \eta_0 = 377\ \Omega,$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{\mathrm{r}_2}}} = \frac{\eta_0}{2} = 188.5\ \Omega,$$

$$\theta_{\mathrm{t}} = \sin^{-1}\left[\frac{\sin\theta_{\mathrm{i}}}{\sqrt{\varepsilon_{\mathrm{r}_2}}}\right] = \sin^{-1}\left[\frac{\sin 36.87^\circ}{\sqrt{4}}\right] = 17.46^\circ,$$

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_{\mathrm{i}} - \eta_1 \cos\theta_{\mathrm{t}}}{\eta_2 \cos\theta_{\mathrm{i}} + \eta_1 \cos\theta_{\mathrm{t}}} = -0.41,$$

$$\tau_\perp = 1 + \Gamma_\perp = 0.59.$$

In accordance with Eq. (8.49a), and using the relation $E_0^{\rm r} = \Gamma_\perp E_0^{\rm i}$,

$$\widetilde{\mathbf{E}}^{\rm r} = -\hat{\mathbf{y}}\, 8.2\, e^{-j(3x-4z)},$$
$$\widetilde{\mathbf{H}}^{\rm r} = -(\hat{\mathbf{x}} \cos\theta_{\rm i} + \hat{\mathbf{z}} \sin\theta_{\rm i})\, \frac{8.2}{\eta_0}\, e^{-j(3x-4z)},$$

where we used the fact that $\theta_{\rm i} = \theta_{\rm r}$ and the z-direction has been reversed.

$$\mathbf{E}^{\rm r} = \mathfrak{Re}[\widetilde{\mathbf{E}}^{\rm r} e^{j\omega t}] = -\hat{\mathbf{y}}\, 8.2 \cos(1.5\times10^9 t - 3x + 4z) \quad \text{(V/m)},$$
$$\mathbf{H}^{\rm r} = -(\hat{\mathbf{x}}\, 17.4 + \hat{\mathbf{z}}\, 13.06) \cos(1.5\times10^9 t - 3x + 4z) \quad \text{(mA/m)}.$$

(d) In medium 2,

$$k_2 = k_1 \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = 5\sqrt{4} = 20 \quad \text{(rad/m)},$$

and

$$\theta_{\rm t} = \sin^{-1}\left[\sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sin\theta_{\rm i}\right] = \sin^{-1}\left[\frac{1}{2}\sin 36.87^\circ\right] = 17.46^\circ$$

and the exponent of $\mathbf{E}^{\rm t}$ and $\mathbf{H}^{\rm t}$ is

$$-jk_2(x\sin\theta_{\rm t} + z\cos\theta_{\rm t}) = -j10(x\sin 17.46^\circ + z\cos 17.46^\circ) = -j(3x + 9.54\,z).$$

Hence,

$$\widetilde{\mathbf{E}}^{\rm t} = \hat{\mathbf{y}}\, 20\times 0.59\, e^{-j(3x+9.54z)},$$
$$\widetilde{\mathbf{H}}^{\rm t} = (-\hat{\mathbf{x}}\cos\theta_{\rm t} + \hat{\mathbf{z}}\sin\theta_{\rm t})\, \frac{20\times0.59}{\eta_2}\, e^{-j(3x+9.54z)}.$$
$$\mathbf{E}^{\rm t} = \mathfrak{Re}[\widetilde{\mathbf{E}}^{\rm t} e^{j\omega t}] = \hat{\mathbf{y}}\, 11.8 \cos(1.5\times10^9 t - 3x - 9.54z) \quad \text{(V/m)},$$
$$\mathbf{H}^{\rm t} = (-\hat{\mathbf{x}}\cos 17.46^\circ + \hat{\mathbf{z}}\sin 17.46^\circ)\, \frac{11.8}{188.5} \cos(1.5\times10^9 t - 3x - 9.54z)$$
$$= (-\hat{\mathbf{x}}\, 59.72 + \hat{\mathbf{z}}\, 18.78) \cos(1.5\times10^9 t - 3x - 9.54z) \quad \text{(mA/m)}.$$

(e)

$$S^{\rm t}_{\rm av} = \frac{|E_0^{\rm t}|^2}{2\eta_2} = \frac{(11.8)^2}{2\times188.5} = 0.36 \quad \text{(W/m}^2\text{)}.$$

Problem 8.32 A perpendicularly polarized wave in air is obliquely incident upon a planar glass–air interface at an incidence angle of 30°. The wave frequency is 600 THz (1 THz $= 10^{12}$ Hz), which corresponds to green light, and the index of refraction of the glass is 1.6. If the electric field amplitude of the incident wave is 50 V/m, determine the following:

(a) The reflection and transmission coefficients.

(b) The instantaneous expressions for **E** and **H** in the glass medium.

Solution:

(a) For nonmagnetic materials, $(\varepsilon_2/\varepsilon_1) = (n_2/n_1)^2$. Using this relation in Eq. (8.60) gives

$$\Gamma_\perp = \frac{\cos\theta_i - \sqrt{(n_2/n_1)^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{(n_2/n_1)^2 - \sin^2\theta_i}} = \frac{\cos 30^\circ - \sqrt{(1.6)^2 - \sin^2 30^\circ}}{\cos 30^\circ + \sqrt{(1.6)^2 - \sin^2 30^\circ}} = -0.27,$$

$$\tau_\perp = 1 + \Gamma_\perp = 1 - 0.27 = 0.73.$$

(b) In the glass medium,

$$\sin\theta_t = \frac{\sin\theta_i}{n_2} = \frac{\sin 30^\circ}{1.6} = 0.31,$$

or $\theta_t = 18.21^\circ$.

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{\eta_0}{n_2} = \frac{120\pi}{1.6} = 75\pi = 235.62 \quad (\Omega),$$

$$k_2 = \frac{\omega}{u_p} = \frac{2\pi f}{c/n} = \frac{2\pi f n}{c} = \frac{2\pi \times 600 \times 10^{12} \times 1.6}{3 \times 10^8} = 6.4\pi \times 10^6 \text{ rad/m},$$

$$E_0^t = \tau_\perp E_0^i = 0.73 \times 50 = 36.5 \text{ V/m}.$$

From Eqs. (8.49c) and (8.49d),

$$\widetilde{\mathbf{E}}_\perp^t = \hat{\mathbf{y}} E_0^t e^{-jk_2(x\sin\theta_t + z\cos\theta_t)},$$

$$\widetilde{\mathbf{H}}_\perp^t = (-\hat{\mathbf{x}}\cos\theta_t + \hat{\mathbf{z}}\sin\theta_t)\frac{E_0^t}{\eta_2} e^{-jk_2(x\sin\theta_t + z\cos\theta_t)},$$

and the corresponding instantaneous expressions are:

$$\mathbf{E}_\perp^t(x,z,t) = \hat{\mathbf{y}}36.5\cos(\omega t - k_2 x\sin\theta_t - k_2 z\cos\theta_t) \quad \text{(V/m)},$$

$$\mathbf{H}_\perp^t(x,z,t) = (-\hat{\mathbf{x}}\cos\theta_t - \hat{\mathbf{z}}\cos\theta_t)0.16\cos(\omega t - k_2 x\sin\theta_t - k_2 z\cos\theta_t) \quad \text{(A/m)},$$

with $\omega = 2\pi \times 10^{15}$ rad/s and $k_2 = 6.4\pi \times 10^6$ rad/m.

Problem 8.34 Show that for nonmagnetic media, the reflection coefficient $\Gamma_\parallel$ can be written in the following form:

$$\Gamma_\parallel = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} .$$

Solution: From Eq. (8.66a), $\Gamma_\parallel$ is given by

$$\Gamma_\parallel = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \frac{(\eta_2/\eta_1)\cos\theta_t - \cos\theta_i}{(\eta_2/\eta_1)\cos\theta_t + \cos\theta_i} .$$

For nonmagnetic media, $\mu_1 = \mu_2 = \mu_0$ and

$$\frac{\eta_2}{\eta_1} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = \frac{n_1}{n_2} .$$

Snell's law of refraction is

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} .$$

Hence,

$$\Gamma_\parallel = \frac{\dfrac{\sin\theta_t}{\sin\theta_i}\cos\theta_t - \cos\theta_i}{\dfrac{\sin\theta_t}{\sin\theta_i}\cos\theta_t + \cos\theta_i} = \frac{\sin\theta_t \cos\theta_t - \sin\theta_i \cos\theta_i}{\sin\theta_t \cos\theta_t + \sin\theta_i \cos\theta_i} .$$

To show that the expression for $\Gamma_\parallel$ is the same as

$$\Gamma_\parallel = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} ,$$

we shall proceed with the latter and show that it is equal to the former.

$$\frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} = \frac{\sin(\theta_t - \theta_i)\cos(\theta_t + \theta_i)}{\cos(\theta_t - \theta_i)\sin(\theta_t + \theta_i)} .$$

Using the identities (from Appendix C):

$$2\sin x \cos y = \sin(x+y) + \sin(x-y),$$

and if we let $x = \theta_t - \theta_i$ and $y = \theta_t + \theta_i$ in the numerator, while letting $x = \theta_t + \theta_i$ and $y = \theta_t - \theta_i$ in the denominator, then

$$\frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} = \frac{\sin(2\theta_t) + \sin(-2\theta_i)}{\sin(2\theta_t) + \sin(2\theta_i)} .$$

But $\sin 2\theta = 2\sin\theta\cos\theta$, and $\sin(-\theta) = -\sin\theta$, hence,

$$\frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} = \frac{\sin\theta_t \cos\theta_t - \sin\theta_i \cos\theta_i}{\sin\theta_t \cos\theta_t + \sin\theta_i \cos\theta_i} ,$$

which is the intended result.

Problem 8.35 A parallel-polarized beam of light with an electric field amplitude of 10 (V/m) is incident in air on polystyrene with $\mu_r = 1$ and $\varepsilon_r = 2.6$. If the incidence angle at the air–polystyrene planar boundary is 50°, determine the following:

(a) The reflectivity and transmissivity.

(b) The power carried by the incident, reflected, and transmitted beams if the spot on the boundary illuminated by the incident beam is 1 m^2 in area.

Solution:

(a) From Eq. (8.68),

$$\begin{aligned}
\Gamma_{\parallel} &= \frac{-(\varepsilon_2/\varepsilon_1)\cos\theta_i + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_i}}{(\varepsilon_2/\varepsilon_1)\cos\theta_i + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_i}} \\
&= \frac{-2.6\cos 50^\circ + \sqrt{2.6 - \sin^2 50^\circ}}{2.6\cos 50^\circ + \sqrt{2.6 - \sin^2 50^\circ}} = -0.08, \\
R_{\parallel} &= |\Gamma_{\parallel}|^2 = (0.08)^2 = 6.4 \times 10^{-3}, \\
T_{\parallel} &= 1 - R_{\parallel} = 0.9936.
\end{aligned}$$

(b)

$$\begin{aligned}
P^i_{\parallel} &= \frac{|E^i_{\parallel 0}|^2}{2\eta_1} A\cos\theta_i = \frac{(10)^2}{2 \times 120\pi} \times \cos 50^\circ = 85 \text{ mW}, \\
P^r_{\parallel} &= R_{\parallel} P^i_{\parallel} = (6.4 \times 10^{-3}) \times 0.085 = 0.55 \text{ mW}, \\
P^t_{\parallel} &= T_{\parallel} P^i_{\parallel} = 0.9936 \times 0.085 = 84.45 \text{ mW}.
\end{aligned}$$