SOLUTION HW # 11

Problem 7.19 Ignoring reflection at the air–soil boundary, if the amplitude of a 3-GHz incident wave is 10 V/m at the surface of a wet soil medium, at what depth will it be down to 1 mV/m? Wet soil is characterized by $\mu_r = 1$, $\varepsilon_r = 9$, and $\sigma = 5 \times 10^{-4}$ S/m.

Solution:

$$E(z) = E_0 e^{-\alpha z} = 10 e^{-\alpha z},$$

$$\frac{\sigma}{\omega\varepsilon} = \frac{5 \times 10^{-4} \times 36\pi}{2\pi \times 3 \times 10^9 \times 10^{-9} \times 9} = 3.32 \times 10^{-4}.$$

Hence, medium is a low-loss dielectric.

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\sigma}{2} \cdot \frac{120\pi}{\sqrt{\varepsilon_r}} = \frac{5 \times 10^{-4} \times 120\pi}{2 \times \sqrt{9}} = 0.032 \quad \text{(Np/m)},$$

$$10^{-3} = 10 e^{-0.032z}, \quad \ln 10^{-4} = -0.032z,$$

$$z = 287.82 \text{ m}.$$

Problem 7.21 Based on wave attenuation and reflection measurements conducted at 1 MHz, it was determined that the intrinsic impedance of a certain medium is $28.1\angle 45^\circ$ (Ω) and the skin depth is 2 m. Determine the following:

(a) The conductivity of the material.

(b) The wavelength in the medium.

(c) The phase velocity.

Solution:

(a) Since the phase angle of η_c is 45°, the material is a good conductor. Hence,

$$\eta_c = (1+j)\frac{\alpha}{\sigma} = 28.1 e^{j45^\circ} = 28.1 \cos 45^\circ + j28.1 \sin 45^\circ,$$

or

$$\frac{\alpha}{\sigma} = 28.1 \cos 45^\circ = 19.87.$$

Since $\alpha = 1/\delta_s = 1/2 = 0.5$ Np/m,

$$\sigma = \frac{\alpha}{19.87} = \frac{0.5}{19.87} = 2.52 \times 10^{-2} \text{ S/m}.$$

(b) Since $\alpha = \beta$ for a good conductor, and $\alpha = 0.5$, it follows that $\beta = 0.5$. Therefore,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.5} = 4\pi = 12.57 \text{ m}.$$

(c) $u_p = f\lambda = 10^6 \times 12.57 = 1.26 \times 10^7$ m/s.

Problem 7.22 The electric field of a plane wave propagating in a nonmagnetic medium is given by

$$\mathbf{E} = \hat{\mathbf{z}}25e^{-30x}\cos(2\pi\times 10^9 t - 40x) \quad \text{(V/m)}$$

Obtain the corresponding expression for $\mathbf{H}$.

Solution: From the given expression for $\mathbf{E}$,

$$\omega = 2\pi\times 10^9 \quad \text{(rad/s)},$$
$$\alpha = 30 \quad \text{(Np/m)},$$
$$\beta = 40 \quad \text{(rad/m)}.$$

From (7.65a) and (7.65b),

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon' = -\omega^2\mu_0\varepsilon_0\varepsilon_r' = -\frac{\omega^2}{c^2}\,\varepsilon_r',$$
$$2\alpha\beta = \omega^2\mu\varepsilon'' = \frac{\omega^2}{c^2}\,\varepsilon_r''.$$

Using the above values for ω, α, and β, we obtain the following:

$$\varepsilon_r' = 1.6,$$
$$\varepsilon_r'' = 5.47.$$
$$\eta_c = \sqrt{\frac{\mu}{\varepsilon'}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$$
$$= \frac{\eta_0}{\sqrt{\varepsilon_r'}}\left(1 - j\frac{\varepsilon_r''}{\varepsilon_r'}\right)^{-1/2} = \frac{377}{\sqrt{1.6}}\left(1 - j\frac{5.47}{1.6}\right)^{-1/2} = 157.9\,e^{j36.85^\circ} \quad (\Omega).$$
$$\widetilde{\mathbf{E}} = \hat{\mathbf{z}}25e^{-30x}e^{-j40x},$$
$$\widetilde{\mathbf{H}} = \frac{1}{\eta_c}\hat{\mathbf{k}}\times\widetilde{\mathbf{E}} = \frac{1}{157.9\,e^{j36.85^\circ}}\,\hat{\mathbf{x}}\times\hat{\mathbf{z}}25e^{-30x}e^{-j40x} = -\hat{\mathbf{y}}\,0.16\,e^{-30x}e^{-40x}e^{-j36.85^\circ},$$
$$\mathbf{H} = \mathfrak{Re}\{\widetilde{\mathbf{H}}e^{j\omega t}\} = -\hat{\mathbf{y}}\,0.16\,e^{-30x}\cos(2\pi\times 10^9 t - 40x - 36.85^\circ) \quad \text{(A/m)}.$$

Problem 7.26 The inner and outer conductors of a coaxial cable have radii of 0.5 cm and 1 cm, respectively. The conductors are made of copper with $\varepsilon_r = 1$, $\mu_r = 1$, and $\sigma = 5.8\times 10^7$ S/m, and the outer conductor is 0.5 mm thick. At 10 MHz:

(a) Are the conductors thick enough to be considered infinitely thick as far as the flow of current through them is concerned?

(b) Determine the surface resistance R_s.

(c) Determine the ac resistance per unit length of the cable.

Solution:

(a) From Eqs. (7.72) and (7.77b),

$$\delta_s = [\pi f\mu\sigma]^{-1/2} = [\pi\times 10^7\times 4\pi\times 10^{-7}\times 5.8\times 10^7]^{-1/2} = 0.021 \text{ mm}.$$

Hence,

$$\frac{d}{\delta_s} = \frac{0.5 \text{ mm}}{0.021 \text{ mm}} \approx 25.$$

Hence, conductor is plenty thick.

(b) From Eq. (7.92a),

$$R_s = \frac{1}{\sigma\delta_s} = \frac{1}{5.8\times 10^7\times 2.1\times 10^{-5}} = 8.2\times 10^{-4}\ \Omega.$$

(c) From Eq. (7.96),

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{8.2\times 10^{-4}}{2\pi}\left(\frac{1}{5\times 10^{-3}} + \frac{1}{10^{-2}}\right) = 0.039 \quad (\Omega/\text{m}).$$

Problem 7.29 The electric-field phasor of a uniform plane wave traveling downward in water is given by

$$\widetilde{\mathbf{E}} = \hat{\mathbf{x}}\, 5e^{-0.2z}e^{-j0.2z} \quad \text{(V/m)}$$

where $\hat{\mathbf{z}}$ is the downward direction and $z = 0$ is the water surface. If $\sigma = 4$ S/m,

(a) Obtain an expression for the average power density.

(b) Determine the attenuation rate.

(c) Determine the depth at which the power density has been reduced by 40 dB.

Solution:

(a) Since $\alpha = \beta = 0.2$, the medium is a good conductor.

$$\eta_c = (1+j)\frac{\alpha}{\sigma} = (1+j)\frac{0.2}{4} = (1+j)0.05 = 0.0707e^{j45^\circ} \quad (\Omega).$$

From Eq. (7.109),

$$\mathbf{S}_{av} = \hat{\mathbf{z}}\frac{|E_0|^2}{2|\eta_c|}e^{-2\alpha z}\cos\theta_\eta = \hat{\mathbf{z}}\frac{25}{2\times 0.0707}e^{-0.4z}\cos 45^\circ = \hat{\mathbf{z}}125e^{-0.4z} \quad (\text{W/m}^2).$$

(b) $A = -8.68\alpha z = -8.68 \times 0.2z = -1.74z$ (dB).

(c) 40 dB is equivalent to 10^{-4}. Hence,

$$10^{-4} = e^{-2\alpha z} = e^{-0.4z}, \qquad \ln(10^{-4}) = -0.4z,$$

or $z = 23.03$ m.

Problem 8.2 A plane wave traveling in medium 1 with $\varepsilon_{r1} = 2.25$ is normally incident upon medium 2 with $\varepsilon_{r2} = 4$. Both media are made of nonmagnetic, non-conducting materials. If the electric field of the incident wave is given by

$$\mathbf{E}^i = \hat{\mathbf{y}} 8 \cos(6\pi \times 10^9 t - 30\pi x) \quad \text{(V/m)}.$$

(a) Obtain time-domain expressions for the electric and magnetic fields in each of the two media.

(b) Determine the average power densities of the incident, reflected and transmitted waves.

Solution:

(a)

$$\mathbf{E}^i = \hat{\mathbf{y}}\, 8 \cos(6\pi \times 10^9 t - 30\pi x) \quad \text{(V/m)},$$

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} = \frac{\eta_0}{\sqrt{2.25}} = \frac{\eta_0}{1.5} = \frac{377}{1.5} = 251.33\ \Omega,$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \frac{\eta_0}{\sqrt{4}} = \frac{377}{2} = 188.5\ \Omega,$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1/2 - 1/1.5}{1/2 + 1/1.5} = -0.143,$$

$$\tau = 1 + \Gamma = 1 - 0.143 = 0.857,$$

$$\mathbf{E}^r = \Gamma \mathbf{E}^i = -1.14 \hat{\mathbf{y}} \cos(6\pi \times 10^9 t + 30\pi x) \quad \text{(V/m)}.$$

Note that the coefficient of x is positive, denoting the fact that $\mathbf{E}^r$ belongs to a wave traveling in $-x$-direction.

$$\mathbf{E}_1 = \mathbf{E}^i + \mathbf{E}^r = \hat{\mathbf{y}}\,[8\cos(6\pi \times 10^9 t - 30\pi x) - 1.14\cos(6\pi \times 10^9 t + 30\pi x)] \quad \text{(A/m)},$$

$$\mathbf{H}^i = \hat{\mathbf{z}} \frac{8}{\eta_1} \cos(6\pi \times 10^9 t - 30\pi x) = \hat{\mathbf{z}}\, 31.83 \cos(6\pi \times 10^9 t - 30\pi x) \quad \text{(mA/m)},$$

$$\mathbf{H}^r = \hat{\mathbf{z}} \frac{1.14}{\eta_1} \cos(6\pi \times 10^9 t + 30\pi x) = \hat{\mathbf{z}}\, 4.54 \cos(6\pi \times 10^9 t + 30\pi x) \quad \text{(mA/m)},$$

$$\mathbf{H}_1 = \mathbf{H}^i + \mathbf{H}^r$$
$$= \hat{\mathbf{z}}\,[31.83 \cos(6\pi \times 10^9 t - 30\pi x) + 4.54 \cos(6\pi \times 10^9 t + 30\pi x)] \quad \text{(mA/m)}.$$

Since $k_1 = \omega\sqrt{\mu\varepsilon_1}$ and $k_2 = \omega\sqrt{\mu\varepsilon_2}$,

$$k_2 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\, k_1 = \sqrt{\frac{4}{2.25}}\, 30\pi = 40\pi \quad \text{(rad/m)},$$

$$\mathbf{E}_2 = \mathbf{E}^t = \hat{\mathbf{y}}\, 8\tau \cos(6\pi \times 10^9 t - 40\pi x) = \hat{\mathbf{y}}\, 6.86 \cos(6\pi \times 10^9 t - 40\pi x) \quad \text{(V/m)},$$

$$\mathbf{H}_2 = \mathbf{H}^t = \hat{\mathbf{z}} \frac{8\tau}{\eta_2} \cos(6\pi \times 10^9 t - 40\pi x) = \hat{\mathbf{z}}\, 36.38 \cos(6\pi \times 10^9 t - 40\pi x) \quad \text{(mA/m)}.$$

(b)

$$\mathbf{S}^i_{av} = \hat{\mathbf{x}} \frac{8^2}{2\eta_1} = \frac{64}{2 \times 251.33} = \hat{\mathbf{x}}\, 127.3 \quad \text{(mW/m}^2\text{)},$$

$$\mathbf{S}^r_{av} = -|\Gamma|^2 \mathbf{S}^i_{av} = -\hat{\mathbf{x}}\,(0.143)^2 \times 0.127 = -\hat{\mathbf{x}}\, 2.6 \quad \text{(mW/m}^2\text{)},$$

$$\mathbf{S}^t_{av} = \frac{|E^t_0|^2}{2\eta_2}$$
$$= \hat{\mathbf{x}}\, \tau^2 \frac{(8)^2}{2\eta_2} = \hat{\mathbf{x}} \frac{(0.86)^2 64}{2 \times 188.5} = \hat{\mathbf{x}}\, 124.7 \quad \text{(mW/m}^2\text{)}.$$

Within calculation error, $\mathbf{S}^i_{av} + \mathbf{S}^r_{av} = \mathbf{S}^t_{av}$.

Problem 8.4 A 200-MHz, left-hand circularly polarized plane wave with an electric field modulus of 5 V/m is normally incident in air upon a dielectric medium with $\varepsilon_r = 4$, and occupies the region defined by $z \geq 0$.

(a) Write an expression for the electric field phasor of the incident wave, given that the field is a positive maximum at $z = 0$ and $t = 0$.

(b) Calculate the reflection and transmission coefficients.

(c) Write expressions for the electric field phasors of the reflected wave, the transmitted wave, and the total field in the region $z \leq 0$.

(d) Determine the percentages of the incident average power reflected by the boundary and transmitted into the second medium.

Solution:

(a)

$$k_1 = \frac{\omega}{c} = \frac{2\pi \times 2 \times 10^8}{3 \times 10^8} = \frac{4\pi}{3} \text{ rad/m},$$
$$k_2 = \frac{\omega}{u_{p_2}} = \frac{\omega}{c}\sqrt{\varepsilon_{r_2}} = \frac{4\pi}{3}\sqrt{4} = \frac{8\pi}{3} \text{ rad/m}.$$

LHC wave:

$$\widetilde{\mathbf{E}}^i = a_0(\hat{\mathbf{x}} + \hat{\mathbf{y}}e^{j\pi/2})e^{-jkz} = a_0(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-jkz},$$
$$\mathbf{E}^i(z,t) = \hat{\mathbf{x}}a_0\cos(\omega t - kz) - \hat{\mathbf{y}}a_0\sin(\omega t - kz),$$
$$|\mathbf{E}^i| = [a_0^2\cos^2(\omega t - kz) + a_0^2\sin^2(\omega t - kz)]^{1/2} = a_0 = 5 \quad \text{(V/m)}.$$

Hence,

$$\widetilde{\mathbf{E}}^i = 5(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-j4\pi z/3} \quad \text{(V/m)}.$$

(b)

$$\eta_1 = \eta_0 = 120\pi \quad (\Omega), \qquad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{2} = 60\pi \quad (\Omega).$$

Equations (8.8a) and (8.9) give

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = \frac{-60}{180} = -\frac{1}{3}, \qquad \tau = 1 + \Gamma = \frac{2}{3}.$$

(c)

$$\widetilde{\mathbf{E}}^r = 5\Gamma(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{jk_1 z} = -\frac{5}{3}(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{j4\pi z/3} \quad \text{(V/m)},$$
$$\widetilde{\mathbf{E}}^t = 5\tau(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-jk_2 z} = \frac{10}{3}(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-j8\pi z/3} \quad \text{(V/m)},$$

$$\widetilde{\mathbf{E}}_1 = \widetilde{\mathbf{E}}^i + \widetilde{\mathbf{E}}^r = 5(\hat{\mathbf{x}} + j\hat{\mathbf{y}})\left[e^{-j4\pi z/3} - \frac{1}{3}e^{j4\pi z/3}\right] \quad \text{(V/m)}.$$

(d)

$$\% \text{ of reflected power} = 100 \times |\Gamma|^2 = \frac{100}{9} = 11.11\%,$$
$$\% \text{ of transmitted power} = 100 \times |\tau|^2 \frac{\eta_1}{\eta_2} = 100 \times \left(\frac{2}{3}\right)^2 \times \frac{120\pi}{60\pi} = 88.89\%.$$

Problem 8.6 A 50-MHz plane wave with electric field amplitude of 50 V/m is normally incident in air onto a semi-infinite, perfect dielectric medium with $\varepsilon_r = 36$. Determine the following:

(a) Γ

(b) The average power densities of the incident and reflected waves.

(c) The distance in the air medium from the boundary to the nearest minimum of the electric field intensity, $|\mathbf{E}|$.

Solution:

(a)

$$\eta_1 = \eta_0 = 120\pi \quad (\Omega), \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{120\pi}{\sqrt{\varepsilon_{r_2}}} = \frac{120\pi}{6} = 20\pi \quad (\Omega),$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{20\pi - 120\pi}{20\pi + 120\pi} = -0.71.$$

Hence, $|\Gamma| = 0.71$ and $\theta_\eta = 180^\circ$.

(b)

$$S^i_{av} = \frac{|E^i_0|^2}{2\eta_1} = \frac{(50)^2}{2 \times 120\pi} = 3.32 \quad (\text{W/m}^2),$$

$$S^r_{av} = |\Gamma|^2 S^i_{av} = (0.71)^2 \times 3.32 = 1.67 \quad (\text{W/m}^2).$$

(c) In medium 1 (air),

$$\lambda_1 = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^7} = 6 \text{ m}.$$

From Eqs. (8.16) and (8.17),

$$l_{max} = \frac{\theta_r \lambda_1}{4\pi} = \frac{\pi \times 6}{4\pi} = 1.5 \text{ m},$$

$$l_{min} = l_{max} - \frac{\lambda_1}{4} = 1.5 - 1.5 = 0 \text{ m (at the boundary)}.$$

Problem # 6.1

Calculate the net reflection coefficient Γ for the plane wave incident upon the stratified media shown in Figure 12-25. Assume that the dielectric constants of the media are $\epsilon_{r_1} = 1$, $\epsilon_{r_2} = 6.25$, $\epsilon_{r_3} = 2.25$, and $\epsilon_{r_4} = 1$. Also, assume that $\ell_2 = \lambda_2/8$ and $\ell = \lambda_3/5$.

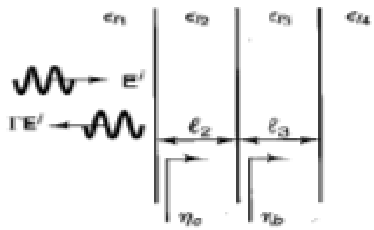

Figure 12-25 Incident and reflected waves at a stack of three interfaces of dissimilar materials.

Solution:

Using the specified dielectric constants, we first calculate the following parameters:

$$\eta_1 = \eta_4 = \frac{377}{\sqrt{1}} = 377 \qquad [\Omega]$$

$$\eta_2 = \frac{377}{\sqrt{6.25}} = 150.8 \qquad [\Omega]$$

$$\eta_3 = \frac{377}{\sqrt{2.25}} = 251.33 \qquad [\Omega]$$

$$\tan(\beta_3\ell_3) = \tan\left[\frac{2\pi}{\lambda_3} \times \frac{\lambda_3}{5}\right] = \tan(2\pi/5) = 3.08$$

$$\tan(\beta_2\ell_2) = \tan(2\pi/8) = 0.785.$$

The effective wave impedance η_b just to the right of the interface between regions 2 and 3 can be found from Equation (12.160):

$$\eta_b = \eta_3 \frac{\eta_4 + j\eta_3 \tan(\beta_3\ell_3)}{\eta_3 + j\eta_4 \tan(\beta_3\ell_3)}$$

$$= 251.33 \frac{377 + j(251.33)(3.08)}{251.33 + j(377)(3.08)} = 176.94 - j43.34.$$

Next, the effective wave impedance η_c just to the right of the interface between regions 1 and 2 is

$$\eta_c = \eta_2 \frac{\eta_b + j\eta_2 \tan(\beta_2\ell_2)}{\eta_2 + j\eta_b \tan(\beta_2\ell_2)}.$$

$$= 150.8 \frac{(176.94 - j43.34) + j(150.8)(0.785)}{150.8 + j(176.94 - j43.34)(0.785)}$$

$$= 121.66 - j30.2 \qquad [\Omega].$$

Finally, the effective reflection coefficient is

$$\Gamma = \frac{\eta_c - \eta_1}{\eta_c + \eta_1} = \frac{121.66 - j30.2 - 377}{121.66 - j30.2 + 377}$$

$$= 0.515\angle -169.8^\circ.$$

This results in a power reflection coefficient of

$$|\Gamma|^2 = 0.265 = 26.5\%.$$

Problem # 6.2

A perpendicularly polarized plane wave is incident from free space onto a lossless dielectric surface at an angle of 30° with respect to the surface normal. If the material parameters are $\epsilon = 4.0\,\epsilon_o$ and $\mu = \mu_o$, find the angle of transmission and the reflection and transmission coefficients.

Solution:

From Equation (12.174), the angle of transmission

$$\theta_t = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_i\right] = \sin^{-1}\left[\frac{1}{\sqrt{4}}\sin(30°)\right] = 14.48°.$$

The intrinsic impedances of the two media are

$$\eta_1 = \sqrt{\frac{\mu_o}{\epsilon_o}} = 377 \quad [\Omega], \; \eta_2 = \sqrt{\frac{\mu_o}{4\epsilon_o}} = 188.5 \quad [\Omega].$$

Substituting these values into Equations (12.177) and (12.178), we find that

$$\Gamma_\perp = \frac{188.5\cos(30°) - 377\cos(14.48°)}{188.5\cos(30°) + 377\cos(14.48°)} = -0.382$$

$$T_\perp = \frac{2 \times 188.5\cos(30°)}{188.5\cos(30°) + 377\cos(14.48°)} = 0.618.$$

Equation List

$$\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \tag{12.177}$$

$$T_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}, \tag{12.178}$$