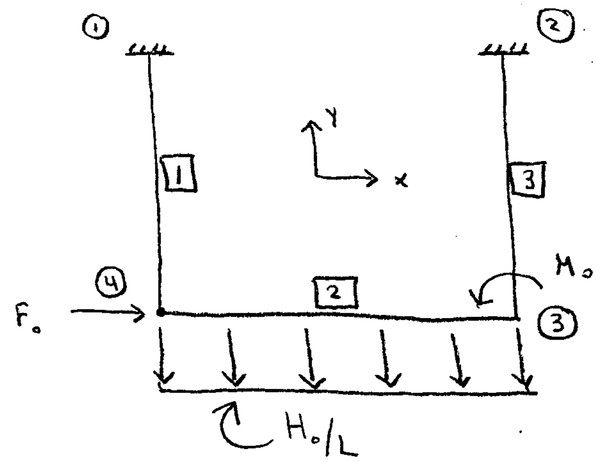
MAE 415 - Analysis of Structures **December 6, 2000**
Bloebaum/Hulme **Pop Quiz #2**

A portal frame beam structure is suspended from the ceiling as shown in the figure below:



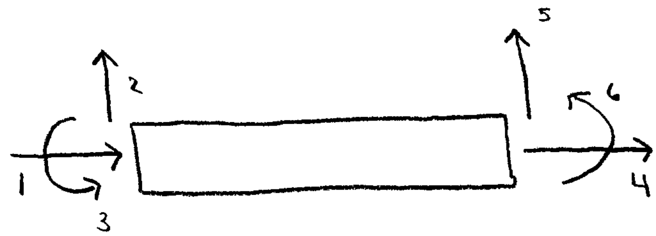
All members:

Length : L

Material : E_o

Geometry : A_o, I_o

The **element** DOF are to be ordered as follows:



The **global** DOF vector is given as follows: $\mathbf{u} = [u_1\ v_1\ \theta_1\ u_2\ v_2\ \theta_2\ u_3\ v_3\ \theta_3\ u_4\ v_4\ \theta_4]^T$

1. Recite the **element** stiffness matrix for element 3.

$$K^{\boxed{3}} = \frac{E_o I_o}{L^3}\begin{bmatrix} \frac{A_oL^2}{I_o} & 0 & 0 & -\frac{A_oL^2}{I_o} & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ -\frac{A_oL^2}{I_o} & 0 & 0 & \frac{A_oL^2}{I_o} & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix}$$

2. To align the element and global DOF for element 3, what will be required:
   a) a "sign grid" change
   b) a re-ordering of the matrix elements
   c) BOTH of the above
   d) NEITHER of the above

3. *List* 2 approaches for handling the distributed load, as discussed in class. For each, recite 1 advantage and 1 disadvantage.

- Point load at center : A: most accurate D: Requires 2 elements
- Point loads at ends : A: can use 1 element D: less accurate approx.

4. How many **total** DOF exist in this structure: 12

5. Considering boundary conditions, how many **free** DOF exist in this structure: 6

6. Recite the **(global)** force vector, *in transposed form*, applying known information.

$$\tilde{F} = \begin{bmatrix} R_{1H} & R_{1V} & R_{1\theta} & R_{2H} & R_{2V} & R_{2\theta} & 0 & -\frac{H_o}{2} & M_o & F_o & -\frac{H_o}{2} & 0 \end{bmatrix}^T$$

7. Will the "sign grid" patterns be equivalent or different for elements 1 and 3? Why?

Same. Low/high DOF (global/element) match with a 90° cw rotation for both elements.

8. Upon analyzing this structure, you find that its structural integrity is not sufficient. You decide to place a truss element connecting nodes 1 and 3. Please recite the **element** stiffness equation for this new element.

$$K^{\boxed{1-3}} = \frac{E_o A_o}{L} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ & s^2 & -cs & -s^2 \\ & & c^2 & cs \\ \text{Sym.} & & & s^2 \end{bmatrix}$$

$\alpha = 135°$

$c^2 = 0.5$

$s^2 = 0.5$

$cs = -0.5$