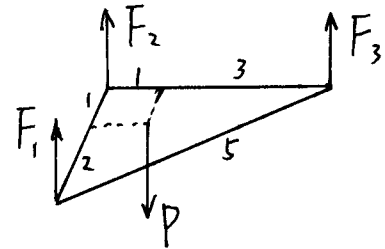


1. Solution:

From geometry, we can find that $a=1$, so load applied at $x=1, y=1$.



$$\sum F_z = 0 \Rightarrow F_1 + F_2 + F_3 = P \quad \text{--- (A)}$$

$$\sum M_x = 0 \Rightarrow 4F_3 = P \Rightarrow F_3 = P/4$$

$$\sum M_y = 0 \Rightarrow 3F_1 = P \Rightarrow F_1 = P/3$$

$$\text{So (A)} \Rightarrow F_2 = P - P(\frac{1}{3} + \frac{1}{4}) = \frac{5}{12}P$$

$$\text{T.C.E.} \Rightarrow C = \sum_{i=1}^3 \int_0^{F_i} \frac{F_i L}{AE} dF_i - P\Delta$$

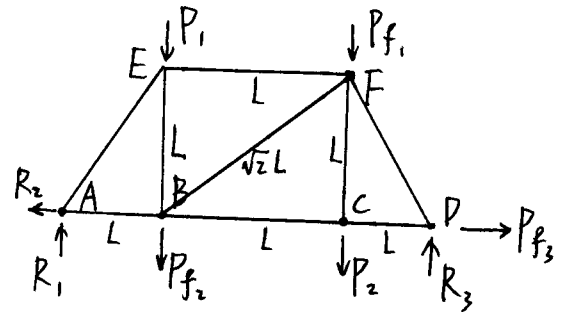
$$\text{Stationary value} \quad \frac{\partial C}{\partial P} = 0 \Rightarrow \sum_{i=1}^3 \frac{F_i L}{AE} \frac{\partial F_i}{\partial P} = \Delta$$

Member	F_i	$\partial F_i / \partial P$
1	$P/3$	$1/3$
2	$5P/12$	$5/12$
3	$P/4$	$1/4$

$$\text{So, } \Delta = \frac{PL}{AE} \left[\left(\frac{1}{3}\right)^2 + \left(\frac{5}{12}\right)^2 + \left(\frac{1}{4}\right)^2 \right]$$

$$= \underline{\underline{0.3248 \text{ mm}}}$$

2. Apply fictitious loads at B, F, D in direction of Δ 's.



$$\sum F_x = 0 \Rightarrow \underline{R_2 = P_{f3}}$$

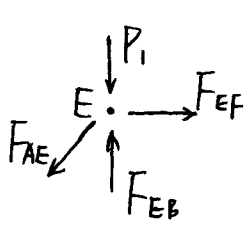
$$\sum M_A = 0 \Rightarrow L(P_{f2} + 2P_2 - 3R_3 + P_1 + 2P_{f1}) = 0 \Rightarrow \underline{R_3 = (P_1 + 2P_2 + 2P_{f1} + P_{f2})/3}$$

$$\sum F_y = 0 \Rightarrow R_1 = P_1 + P_{f1} + P_{f2} + P_2 - R_3 \Rightarrow \underline{R_1 = \frac{2}{3}P_1 + \frac{1}{3}P_2 + \frac{1}{3}P_{f1} + \frac{2}{3}P_{f2}}$$

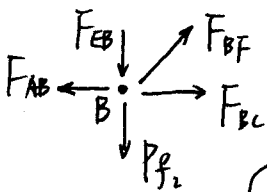
$$\begin{cases} \sum F_x = 0 \Rightarrow \frac{\sqrt{2}}{2} F_{AE} + F_{AB} = P_{f3} \\ \sum F_y = 0 \Rightarrow \frac{1}{3}(2P_1 + P_2 + P_{f1} + 2P_{f2}) + \frac{\sqrt{2}}{2} F_{AE} = 0 \end{cases}$$

$$F_{AE} = -\frac{\sqrt{2}}{3} (2P_1 + P_2 + P_{f_1} + 2P_{f_2})$$

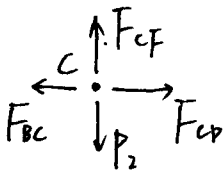
$$F_{AB} = P_{f_3} + \frac{1}{3} (2P_1 + P_2 + P_{f_1} + 2P_{f_2})$$



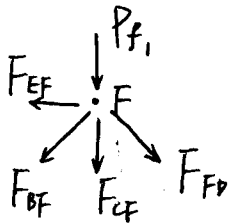
(E) $\Sigma F_x = 0 \Rightarrow F_{EF} = \frac{\sqrt{2}}{2} F_{AE} \Rightarrow F_{EF} = -\frac{1}{3} (2P_1 + P_2 + P_{f_1} + 2P_{f_2})$
 $\Sigma F_y = 0 \Rightarrow F_{EB} = P_1 + \frac{\sqrt{2}}{2} F_{AE} \Rightarrow F_{EB} = \frac{1}{3} (P_1 - P_2 - P_{f_1} - 2P_{f_2})$



(B) $\Sigma F_x = 0 \Rightarrow \frac{\sqrt{2}}{2} F_{BF} + F_{BC} - F_{AB} = 0$
 $\Rightarrow F_{BC} = F_{AB} - \frac{\sqrt{2}}{2} F_{BF} = P_{f_3} + \frac{1}{3} (P_1 + 2P_2 + P_{f_2} + 2P_{f_1})$
 $\Sigma F_y = 0 \Rightarrow \frac{\sqrt{2}}{2} F_{BF} - F_{EB} - P_{f_2} = 0$
 $\Rightarrow F_{BF} = \sqrt{2} (F_{EB} + P_{f_2}) = \frac{\sqrt{2}}{3} (P_1 - P_2 - P_{f_1} + P_{f_2})$



(C) $\Sigma F_x = 0 \Rightarrow F_{CD} = F_{BC} = P_{f_3} + \frac{1}{3} (P_1 + 2P_2 + P_{f_2} + 2P_{f_1})$
 $\Sigma F_y = 0 \Rightarrow F_{CF} = P_2$



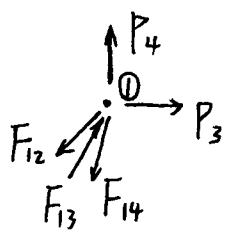
(F) $\Sigma F_x = 0 \Rightarrow \frac{\sqrt{2}}{2} F_{FD} - F_{EF} - \frac{\sqrt{2}}{2} F_{BF} = 0$
 $\Rightarrow F_{FD} = \sqrt{2} F_{EF} + F_{BF} = -\frac{\sqrt{2}}{3} (P_1 + 2P_2 + 2P_{f_1} + P_{f_2})$

Member	F_i	$F_i/10^3$	$\partial F_i / \partial P_{f_1}$	$\partial F_i / \partial P_{f_2}$	$\partial F_i / \partial P_{f_3}$	L_i
AB	$P_{f_3} + \frac{1}{3} (2P_1 + P_2 + P_{f_1} + 2P_{f_2})$	60	1/3	2/3	1	L
AE	$-\frac{\sqrt{2}}{3} (2P_1 + P_2 + P_{f_1} + 2P_{f_2})$	$-60\sqrt{2}$	$-\sqrt{2}/3$	$-2\sqrt{2}/3$	0	$\sqrt{2}L$
EB	$-\frac{1}{3} (P_1 - P_2 - P_{f_1} - 2P_{f_2})$	-20	-1/3	-2/3	0	L
EF	$-\frac{1}{3} (2P_1 + P_2 + P_{f_1} + 2P_{f_2})$	-60	-1/3	-2/3	0	L
BF	$\frac{\sqrt{2}}{3} (P_1 - P_2 - P_{f_1} + P_{f_2})$	$-20\sqrt{2}$	$-\sqrt{2}/3$	$\sqrt{2}/3$	0	$\sqrt{2}L$
BC	$P_{f_3} + \frac{1}{3} (P_1 + 2P_2 + 2P_{f_1} + P_{f_2})$	80	2/3	1/3	1	L
FC	P_2	100	0	0	0	L
CD	$P_{f_3} + \frac{1}{3} (P_1 + 2P_2 + P_{f_2} + 2P_{f_1})$	80	2/3	1/3	1	L
FD	$-\frac{\sqrt{2}}{3} (P_1 + P_{f_2} + 2P_{f_1} + P_{f_2})$	$-80\sqrt{2}$	$-2\sqrt{2}/3$	$-\sqrt{2}/3$	0	$\sqrt{2}L$

$\Delta_1 = \sum_{i=1}^9 \frac{F_i L_i}{A_i E_i} \frac{\partial F_i}{\partial P_{f_1}}$, $\Delta_2 = \sum_{i=1}^9 \frac{F_i L_i}{A_i E_i} \frac{\partial F_i}{\partial P_{f_2}}$, $\Delta_3 = \sum_{i=1}^9 \frac{F_i L_i}{A_i E_i} \frac{\partial F_i}{\partial P_{f_3}}$

$$\begin{aligned} S_0, \Delta_1 &= \frac{L}{AE} \left(20 + 40\sqrt{2} + \frac{20}{3} + 20 + \frac{40\sqrt{2}}{3} + \frac{160}{3} + \frac{160}{2} + \frac{320}{3} \right) = 335.413 \frac{L}{AE} \\ \Delta_2 &= \frac{L}{AE} \left(40 + 80\sqrt{2} + \frac{40}{3} + 40 - \frac{40\sqrt{2}}{3} + \frac{80}{3} + \frac{80}{3} + \frac{160}{3}\sqrt{2} \right) = 316.333 \frac{L}{AE} \\ \Delta_3 &= \frac{L}{AE} (60 + 80 + 80) = 220 \frac{L}{AE} \end{aligned}$$

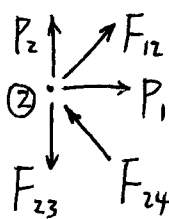
3. Solution: Must find internal member forces



$$\begin{cases} \sum F_x = 0 : P_3 - \frac{5}{13} F_{14} + \frac{9}{15} F_{13} - \frac{1}{\sqrt{2}} F_{12} = 0 \\ \sum F_y = 0 : P_4 - \frac{12}{13} F_{14} + \frac{12}{15} F_{13} - \frac{1}{\sqrt{2}} F_{12} = 0 \end{cases}$$

Subtracting $\Rightarrow F_{13} = \frac{35}{13} F_{14} + 5P_3 - 5P_4$

Therefore: $F_{12} = \sqrt{2} \left(\frac{16}{13} F_{14} + 4P_3 - 3P_4 \right)$



$$\sum F_x = 0 : P_1 + \frac{1}{\sqrt{2}} F_{12} - \frac{4}{5} F_{24} = 0$$

$$\Rightarrow F_{24} = \frac{20}{13} F_{14} + \frac{5}{4} P_1 + 5P_3 - \frac{15}{4} P_4$$

$$\sum F_y = 0 : P_2 + \frac{1}{\sqrt{2}} F_{12} + \frac{3}{5} F_{24} - F_{23} = 0$$

$$\Rightarrow F_{23} = \frac{28}{13} F_{14} + \frac{3}{4} P_1 + P_2 + 7P_3 - \frac{21}{4} P_4$$

Member	F_i	$\partial F_i / \partial F_{14}$	L_i / A_i
14	F_{14}	1	$13a / A$
13	$\frac{35}{13} F_{14} + 5P_3 - 5P_4$	$35/13$	$15a / A$
12	$\sqrt{2} \left[\frac{16}{13} F_{14} + 4P_3 - 3P_4 \right]$	$\sqrt{2} (16)/13$	$9a / A$
24	$\frac{20}{13} F_{14} + \frac{5}{4} P_1 + 5P_3 - \frac{15}{4} P_4$	$20/13$	$5a / A$
23	$\frac{28}{13} F_{14} + \frac{3}{4} P_1 + P_2 + 7P_3 - \frac{21}{4} P_4$	$28/13$	$3a / A$

T.C.E. $\Rightarrow C = \sum_{i=1}^5 \int F_i \frac{F_i L_i}{A_i E_i} dF_i - P_1 \Delta_1 - P_2 \Delta_2 - P_3 \Delta_3 - P_4 \Delta_4$

$\frac{\partial C}{\partial F_{14}} = 0 \Rightarrow \frac{1}{E} \sum_{i=1}^5 \frac{F_i L_i}{A_i} \frac{\partial F_i}{\partial F_{14}} = 0$ from which it can be found that

$$\frac{a}{13AE} \left[\frac{27504}{13} F_{14} + 188P_1 + 4865P_3 + 84P_2 - 4305P_4 \right] = 0$$

$$\Rightarrow 188P_1 + 4865P_3 + 84P_2 - 4305P_4 = -\frac{27504}{13} (0.7) A$$

To find temperature change: $-\Delta_T = -L_{14} d\Delta T$

$$C = \sum_{i=1}^5 \int F_i \frac{F_i L_i}{A_i E_i} dF_i - P_1 \Delta_1 - P_2 \Delta_2 - P_3 \Delta_3 - P_4 \Delta_4 + F_{14} \Delta_T$$

$$\partial C / \partial F_{14} = 0 \Rightarrow -\Delta_T \equiv -L_{14} d\Delta T = \sum_{i=1}^5 \frac{F_i L_i}{A_i E_i} \frac{\partial F_i}{\partial F_{14}}$$

$$\Delta T = \frac{27504 (0.7) A a}{(13^2) A E L_{14} d} = \frac{27504 (0.7) (10^6)}{3 (13^2) (70 \cdot 10^3) (9\sqrt{2})(24)} = \underline{5.33} \text{ } ^\circ\text{C}$$