

(1) Solution:

(a) $V = -P \cdot q$

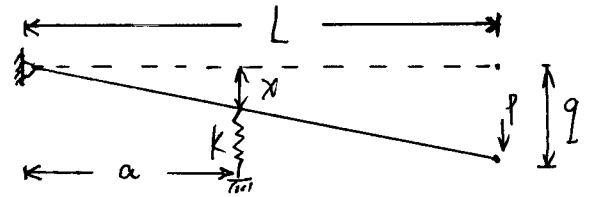
$U = \frac{1}{2} k x^2$

$\left(\frac{x}{q} = \frac{a}{L} \Rightarrow x = \frac{q a}{L} \right)$

$\Pi = U + V = \frac{1}{2} k \left(\frac{q a}{L} \right)^2 - P q$

$\frac{\partial \Pi}{\partial q} = 0 \Rightarrow k q \frac{a^2}{L^2} = P$

$\Rightarrow \underline{q = \frac{P}{k} \cdot \frac{L^2}{a^2}}$

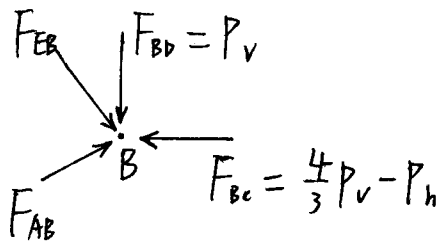


(b) There is 1 degree-of-freedom, which is q in this case.

(2) Solution:

$$\begin{cases} \frac{3}{5} F_{CD} = P_v \Rightarrow F_{CD} = \frac{5}{3} P_v \\ -F_{BC} + \frac{4}{5} F_{CD} = P_h \Rightarrow F_{BC} = \frac{4}{3} P_v - P_h \end{cases}$$

$$\begin{cases} F_{DE} = \frac{4}{5} \cdot F_{CD} = \frac{4}{5} \cdot \frac{5}{3} P_v = \frac{4}{3} P_v \\ F_{BD} = \frac{3}{5} \cdot F_{CD} = \frac{3}{5} \cdot \frac{5}{3} P_v = P_v \end{cases}$$



$$\begin{cases} \frac{4}{5} F_{AB} + \frac{3}{5} F_{EB} = F_{BC} = \frac{4}{3} P_v - P_h \\ \frac{3}{5} F_{AB} - \frac{4}{5} F_{EB} = F_{BD} = P_v \end{cases}$$

$$\Rightarrow \begin{cases} F_{AB} = \frac{5}{3} P_v - \frac{4}{5} P_h \\ F_{EB} = -\frac{3}{5} P_h \end{cases}$$

Member	F_i	$\frac{\partial F_i}{\partial P_h}$	$\frac{\partial F_i}{\partial P_v}$
AB	$\frac{5}{3} P_v - \frac{4}{5} P_h$	$-\frac{4}{5}$	$\frac{5}{3}$
BC	$\frac{4}{3} P_v - P_h$	-1	$\frac{4}{3}$
CD	$\frac{5}{3} P_v$	0	$\frac{5}{3}$
BD	P_v	0	1
ED	$\frac{4}{3} P_v$	0	$\frac{4}{3}$
BE	$-\frac{3}{5} P_h$	$-\frac{3}{5}$	0

T.C.E. : $C = \sum_{i=1}^6 \int_0^{F_i} \frac{F_i L}{AE} dF_i - P_v \Delta_v - P_h \Delta_h$

Applying Principles of stationary value :

$$\frac{\partial C}{\partial P_v} = 0 \Rightarrow \frac{L}{AE} \sum_{i=1}^6 F_i \frac{\partial F_i}{\partial P_v} = \Delta_v$$

$$\therefore \Delta_v = \frac{1}{20} \left[\frac{50}{3} \cdot \frac{5}{3} + \frac{40}{3} \cdot \frac{4}{3} + \frac{50}{3} \cdot \frac{5}{3} + 10 + \frac{40}{3} \cdot \frac{4}{3} \right] \Rightarrow \Delta_v = \underline{5.055 \text{ mm}}$$

$$\frac{\partial C}{\partial P_h} = 0 \Rightarrow \frac{L}{AE} \sum_{i=1}^6 F_i \frac{\partial F_i}{\partial P_h} = \Delta_h$$

$$\therefore \Delta_h = \frac{1}{20} \left[\frac{50}{3} \left(-\frac{4}{5}\right) + \frac{40}{3} (-1) \right] \Rightarrow \Delta_h = \underline{-\frac{4}{3} \text{ mm}}$$

Therefore, the magnitude of Δ will be $\Delta = \left[5.055^2 + \left(-\frac{4}{3}\right)^2 \right]^{\frac{1}{2}}$
 $= \underline{5.228 \text{ mm}}$

Direction will be $\alpha = \tan^{-1} \frac{1.333}{5.055} = \underline{14.775^\circ}$

