1. Solution:

Homogeneous and uniform beam $\Rightarrow I_{yz} = 0$

Hence, no cross coupling : $v(x) = 0$

Also, no axial loads : $u(x) = 0$

GDE : $[E_0 I_{yy}^* w''(x)]'' + [E_0 I_{yz}^* v''(x)]'' - N w''(x) = f_z^c(x) + M_y'(x)$

Simplify : $E_0 I_{yy} w^{iv}(x) = f_0 \left(\frac{x}{L} - 1\right)$

$$\Rightarrow E_0 I_{yy} w'''(x) = f_0 \left(\frac{x^2}{2L} - x\right) + C_1$$

$$E_0 I_{yy} w''(x) = f_0 \left(\frac{x^3}{6L} - \frac{x^2}{2}\right) + C_1 x + C_2$$

$$E_0 I_{yy} w'(x) = f_0 \left(\frac{x^4}{24L} - \frac{x^3}{6}\right) + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$E_0 I_{yy} w(x) = f_0 \left(\frac{x^5}{120L} - \frac{x^4}{24}\right) + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

Determine $C_1 \sim C_4$ using B.C.'s :

① $w(0) = 0 \quad \Rightarrow C_4 = 0$

② $w'(0) = 0 \quad \Rightarrow C_3 = 0$

③ $w(L) = 0 \quad \Rightarrow f_0 \left(-\frac{L^4}{30}\right) + \frac{C_1 L^3}{6} + \frac{C_2 L^2}{2} = 0 \qquad (\Delta)$

④ $E_0 I_{yz}^* v''(L) + E_0 I_{yy}^* w''(L) - M_{yT}(L) = M_y(L)$

$$\Rightarrow E_0 I_{yy} w''(L) = -M_0$$

$$\Rightarrow -M_0 = f_0 \left(-\frac{L^2}{3}\right) + C_1 L + C_2$$

$$\Rightarrow C_2 = -M_0 + \frac{f_0 L^2}{3} - C_1 L$$

$$(\Delta) \Rightarrow \quad 0 = -\frac{f_0}{30}L^4 + \frac{L^3}{6}c_1 + \frac{L^2}{2}\left(-M_0 + \frac{L^2}{3}f_0 - c_1 L\right)$$

$$0 = \frac{2}{15}f_0 L^4 - \frac{M_0 L^2}{2} - \frac{c_1 L^3}{3}$$

$$\therefore \quad \underline{c_1 = \frac{2}{5}f_0 L - \frac{3M_0}{2L}}$$

$$c_2 = -M_0 + \frac{f_0 L^2}{3} - \frac{2}{5}f_0 L^2 + \frac{3}{2}M_0$$

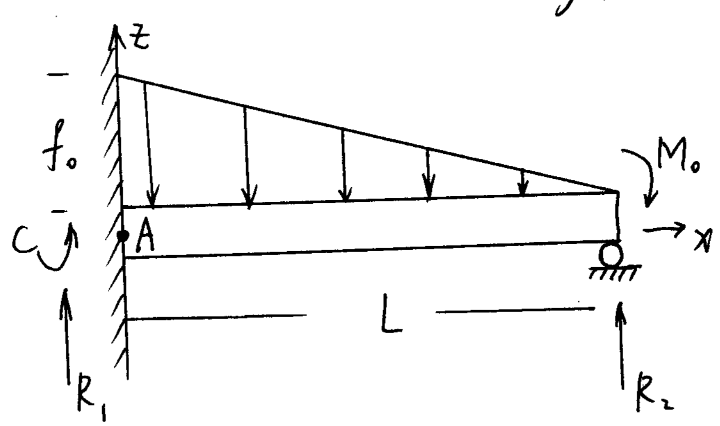
$$\Rightarrow \quad \underline{c_2 = \frac{M_0}{2} - \frac{f_0 L^2}{15}}$$

$$\therefore \quad E_0 I_{yy} w(x) = f_0\left(\frac{x^5}{120L} - \frac{x^4}{24}\right) + \frac{x^3}{6}\left(\frac{2}{5}f_0 L - \frac{3M_0}{2L}\right) + \frac{x^2}{2}\left(\frac{M_0}{2} - \frac{f_0 L^2}{15}\right).$$

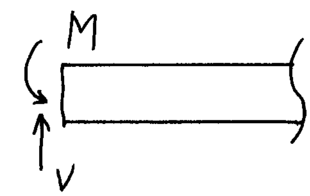
2. Solution,

$u(x) = 0$ since no axial load.

$v(x) = 0$ since homogeneous cross-section.



Convention:



$0 \le x \le L$: $M_y(x) - M_0 + (L-x)R_2 - \left[f_0\left(1-\frac{x}{L}\right)\right]\left[\frac{L-x}{2}\right]\left[\frac{L-x}{3}\right] = 0$

$$M_y(x) = M_0 - (L-x)R_2 + \frac{f_0(L-x)^3}{6L} = EI_{yy}w''(x)$$

$$EI_{yy}w'(x) = M_0 x + \frac{(L-x)^2}{2}R_2 - \frac{f_0(L-x)^4}{24L} + C_1$$

$$EI_{yy}w(x) = \frac{M_0 x^2}{2} - \frac{(L-x)^3}{6}R_2 + \frac{f_0(L-x)^5}{120L} + C_1 x + C_2$$

5 unknowns: C, R_1, R_2, C_1, C_2.

5 equations: $\underbrace{① \ \Sigma F_z = 0 \quad ② \ \Sigma M_A = 0}_{\text{Statics}}$ $\left.\begin{matrix} ③ \ w(0)=0 \\ ④ \ w'(0)=0 \\ ⑤ \ w(L)=0 \end{matrix}\right\}$ BC's

$$③ \Rightarrow \quad 0 = -\frac{L^3}{6}R_2 + \frac{f_0 L^4}{120} + C_2$$

$$④ \Rightarrow \quad 0 = \frac{L^2}{2}R_2 - \frac{f_0 L^3}{24} + C_1$$

$$⑤ \Rightarrow \quad 0 = \frac{M_0 L^2}{2} + C_1 L + C_2 \quad \Rightarrow C_2 = -C_1 L - \frac{M_0 L^2}{2}$$

$$\Rightarrow C_1 = -\frac{L^2}{6}R_2 + \frac{f_0 L^3}{120} - \frac{M_0 L}{2}$$

$$R_2 = \frac{f_0 L}{10} + \frac{3M_0}{2L}$$

$$\therefore C_1 = -\frac{f_0 L^3}{120} - \frac{3M_0 L}{4}$$

$$C_2 = \frac{f_0 L^4}{120} + \frac{M_0 L^2}{4}$$

$$\therefore \; EI_{yy}w(x) = \frac{M_o x^2}{2} - \left(\frac{f_o L}{10} + \frac{3M_o}{2L}\right)\frac{(L-x)^3}{6} + \frac{f_o(L-x)^5}{120L}$$

$$+\left(-\frac{f_o L^3}{120} - \frac{3M_o L}{4}\right)x + \left(\frac{f_o L^4}{120} + \frac{M_o L^2}{4}\right)$$

Check BC's :

$$b \;\; x = L : \quad w(L) = \frac{M_o L^2}{2} - \frac{f_o L^4}{120} - \frac{3M_o L^2}{4} + \frac{f_o L^4}{120} + \frac{M_o L^2}{4} = 0 \quad \checkmark$$

$$a \;\; x = 0 : \quad w(0) = -\frac{f_o L^4}{60} - \frac{M_o L^2}{4} + \frac{f_o L^4}{120L} + \frac{f_o L^4}{120} + \frac{M_o L^2}{4} = 0 \quad \checkmark$$

$$a \;\; x = 0 : \quad w'(0) = \frac{L^2}{2}\left(\frac{f_o L}{10} + \frac{3M_o}{2L}\right) - \frac{f_o L^3}{24} - \frac{f_o L^3}{120} - \frac{3M_o L}{4} = 0$$

$\checkmark$