1. Solution:

Create table to determine A^*, $\bar{x}'^*$, $\bar{y}'^*$:

i	A_i	E_i/E_o	$\frac{E_i}{E_o}A_i$	$\bar{x}_i'$	$\bar{y}_i'$	$\frac{E_i}{E_o}\bar{x}_i' A_i$	$\frac{E_i}{E_o}\bar{y}_i' A_i$
1	$32a^2$	$3/5$	$(96/5)a^2$	0	$14a$	0	$1344/5\ a^3$
2	$12a^2$	$4/5$	$(48/5)a^2$	0	$7a$	0	$336/5\ a^3$
3	$32a^2$	1	$32a^2$	0	0	0	0

$$A^* = \sum_i \frac{E_i}{E_o} A_i = \frac{304}{5} a^2 , \qquad \bar{x}'^* = 0$$

$$\bar{y}'^* = \frac{1680/5}{304/5} a = \frac{1680}{304} a$$

Hence, M.W.C @ $x', y' = (0, \frac{1680}{304} a) = (0, 5.5263 a)$

Creat table to find $I_{xx'}^*$, $I_{yy'}^*$, $I_{xy'}^*$:

i	A_i	$\frac{E_i}{E_o}$	$\bar{x}_i'$	$\bar{y}_i'$	I_{xx_o}	I_{yy_o}	I_{xy_o}	$\frac{E_i}{E_o}(I_{x_{oi}}+A_i\bar{y}'^2)$	$\frac{E_i}{E_o}(I_{y_{oi}}+A_i\bar{x}'^2)$
1	$32a^2$	$3/5$	0	$14a$	$\frac{32}{3}a^4$	$\frac{2048}{3}a^4$	0	$3769\frac{3}{5}a^4$	$409\frac{3}{5}a^4$
2	$12a^2$	$4/5$	0	$7a$	$144a^4$	a^4	0	$585\frac{3}{5}a^4$	$\frac{4}{5}a^4$
3	$32a^2$	1	0	0	$\frac{32}{3}a^4$	$\frac{2048}{3}a^4$	0	$32/3\ a^4$	$2048/3\ a^4$

And $\frac{E_i}{E_o}(I_{xy_{oi}} + A_i \bar{x}'\bar{y}') = 0$, $i = 1, 2, 3$

$$I_{xx'}^* = 4365\tfrac{13}{15} a^4 , \quad I_{yy'}^* = 1093\tfrac{1}{15} a^4 , \quad I_{xy'}^* = 0$$

To find modulus weighted properties, use transformations:

$$I_{xx}^* = I_{xx'}^* - A^*(\bar{y}'^*)^2 = 4365\tfrac{13}{15}a^4 - \left(\tfrac{304}{5}a^2\right)\left(\tfrac{1680}{304}a\right)^2 = \underline{2509\, a^4}$$

$$I_{yy}^* = I_{yy'}^* - A^*(\bar{x}'^*)^2 = 1093\tfrac{1}{15}a^4 - \left(\tfrac{304}{5}a^2\right)\cdot 0^2 = \underline{1093.1\, a^4}$$

$$I_{xy}^* = I_{xy'}^* - A^* \bar{x}'^* \bar{y}'^* = \underline{0}$$

Example 11.5

A cantilever beam is shown in Fig. 11.10 loaded on two of its faces. What is the maximum compressive normal stress $(\tau_{xx})_{max}$?

It should be clear that the maximum compressive stress τ_{xx} will occur somewhere on the cross section of the beam at its base at the wall. We will consider each loading

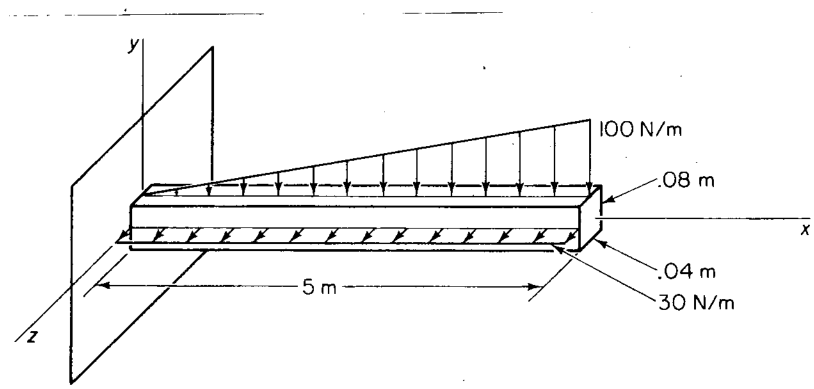


Figure 11.10 Cantilever beam loaded on two faces.

separately, and then, using our imagination, we will find where the superposition yields the largest compressive stress on the cross section at the wall.

First consider the triangular load. It has a bending moment at the wall which equals $(5)(100)(\frac{1}{2})(\frac{2}{3}5) = 833.3$ N-m in magnitude. Hence, the maximum compressive stress will be at the bottom line AB (see Fig. 11.11) of the cross section, given as

$$|(\tau_{xx})_{max}|_1 = \frac{(833.3)(.04)}{(\frac{1}{12})(.04)(.08)^3} = 1.953 \times 10^7 \text{ Pa}$$

As for the uniform loading, it has a bending moment at the base about the y axis, which is $(5)(30)(\frac{5}{2}) = 375$ N-m in magnitude. A moment's thought should reveal that the maximum compressive, stress will be along line AC of the base cross section and have a magnitude given as

$$|(\tau_{xx})_{max}|_2 = \frac{(375)(.02)}{(\frac{1}{12})(.08)(.04)^3} = 1.758 \times 10^7 \text{ Pa}$$

The maximum compressive stress from the combined loading must occur at point A, where we have the coincidence of maxima compressive stresses from the two loads.

$$\therefore (\tau_{xx})_{max} = -1.953 \times 10^7 - 1.758 \times 10^7 = -3.711 \times 10^7 \text{ Pa}$$

Notice in this problem that we used physical feel and common sense freely rather than more formal approaches. You are urged to do likewise when it is possible, as was the case in this problem.

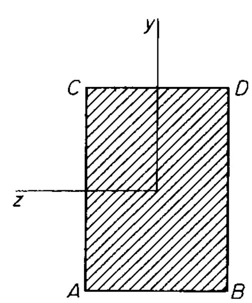


Figure 11.11 Cross section of cantilever at the base.

HW5 - problem # 2:

b|c :

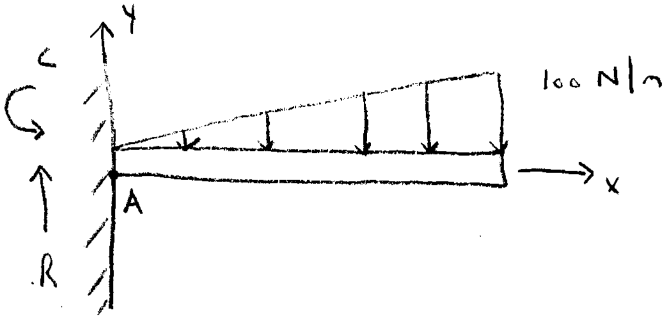
$I_{zz} = 1/12\, bh^3 = 1/12\, (.04)(.08)^3 = 1.7066\text{e}-6\ m^4$

$I_{yy} = 1/12\, bh^3 = 1/12\, (.08)(.04)^3 = 4.266\text{e}-7\ m^4$

$I_{yz} = 0$ (symmetry)

Triangular load : (M_z induces y (v) deflection):

$\frac{w(x)}{100} = \frac{x}{5}$; $w(x) = 20x$ N/m



Find values of C, R: Use statics!

$\Sigma M_A = 0: \quad C - (100)(5)(1/2)(\frac{10}{3}) = 0$

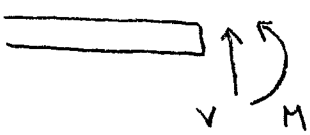
$C = 833.33$ N-m

$\Sigma F_y = 0: \quad R - (100)(5)(1/2) = 0$

$R = 250$ N-m

Now, find $M_z(x)$:

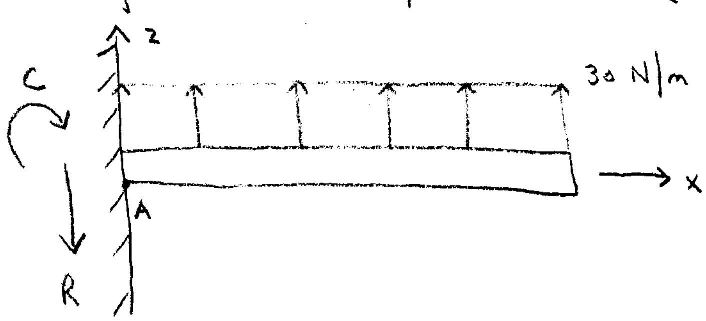
Convention:



$M(x) + 833.33 - 250x + \left[\underbrace{\frac{1}{2}x}_{\text{"area"}} \cdot \underbrace{20x}_{w(x)} \cdot \underbrace{\frac{x}{3}}_{\text{"centroid"}} \right] = 0$

$M(x) = -833.33 + 250x - \frac{20}{6}x^3$ N-m

Rectangular load: (M_y induces z (w) deflection): $w(x) = 30$ N



$\Sigma M_A = 0 \quad -C + (30)(5)(5/2) = 0$

$C = 375$ N-m

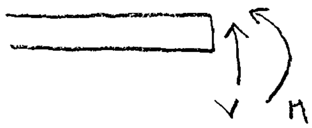
$\Sigma F_z = 0: \quad -R + (30)(5) = 0$

$R = 150$ N

Now, find $M_y(x)$:

$$M(x) - 375 + 150x - (30x)\left(\frac{x}{2}\right) = 0$$

Convention:



$$M(x) = 375 - 150x + 15x^2 \quad \text{N-m}$$

y-deflection: $$v'' = \frac{I_{yy} M_z + \cancel{I_{zy} M_y}}{E(I_{zz} I_{yy} - \cancel{I_{zy}^2})}$$ (worry about signs later)

$$v'' = \frac{(4.266e-7)(250x - 833.33 - 3.33x^3)}{E\,(4.266e-7)(1.7066e-6)} = \frac{1.464e8\,x - 4.882e8 - 1.951e6\,x^3}{E}$$

$$v' = \frac{1}{E}\left(0.732e8\,x^2 - 4.882e8\,x - 0.4878e6\,x^4\right) + C_1$$

$$v = \frac{1}{E}\left(0.244e8\,x^3 - 2.441e8\,x^2 - 0.09775e6\,x^5\right) + C_1 x + C_2$$

BC's: $v(0) = v'(0) = 0 \longrightarrow C_1 = C_2 = 0$

$$v = \frac{1}{E}\left(2.44e7\,x^3 - 2.441e8\,x^2 - 9.775e4\,x^5\right)\text{m} \quad (\text{dir: } -y)$$

z-deflection: $$w'' = \frac{\cancel{M_z I_{zy}} + M_y I_{zz}}{E(I_{zz} I_{yy} - \cancel{I_{zy}^2})}$$ (worry about signs later)

$$w'' = \frac{(1.7066e-6)(375 - 150x + 15x^2)}{E\,(4.266e-7)(1.7066e-6)} = \frac{8.790e8 - 3.516e8\,x + 3.516e7\,x^2}{E}$$

$$w' = \frac{1}{E}\left(8.790e8\,x - 1.758e8\,x^2 + 1.172e7\,x^3\right) + C_1$$

$$W = \frac{1}{E}\left(4.395e8\,x^2 - 0.586e8\,x^3 + 0.293e7\,x^4\right) + C_1 x + C_2$$

BC's : $w(0) = w'(0) = 0 \longrightarrow C_1 = C_2 = 0$

$$W = \frac{1}{E}\left(4.395e8\,x^2 - 5.86e7\,x^3 + 2.93e6\,x^4\right)\ m \qquad (\text{dir: } +z)$$

At tip : $x = 5$; $V = \dfrac{-3.357e9}{E}\ m$, $z = \dfrac{5.493e9}{E}\ m$

Assume $E = E(\text{1040 steel}) = 2.07 \times 10^{11}\ Pa) \longrightarrow V = -0.016\ m = 16\ mm$

$z = 0.0265\ m = 26.5\ mm$

Total deflection. $\delta_{tip} = \sqrt{V_{tip}^2 + W_{tip}^2} = \dfrac{6.437e9}{E}\ m$

Orientation : $\tan\alpha = \dfrac{V_{tip}}{W_{tip}}$; $\alpha = \tan^{-1}\left(\dfrac{V_{tip}}{W_{tip}}\right) = -31.43°$