

1. Solution:

i. a.

$$e_a = e_x = 0.0002$$

$$e_c = e_y = -0.0001$$

$$e_b = e_{x^*x} = 0.0004$$

$$\alpha = 45^\circ; \quad C_{x^*x} = \cos(-45^\circ) = 0.707$$

$$C_{x^*y} = \cos(45^\circ) = 0.707$$

$$\epsilon_{x^*x} = 0.0004 = 0.0002(0.707)^2 - 0.0001(0.707)^2 + \gamma_{xy}(0.707)(0.707)$$

$$\Rightarrow \gamma_{xy} = 0.007 = 2 \epsilon_{xy}$$

b. Principal strains:

$$\tan(2\theta) = \frac{2 \epsilon_{xy}}{\epsilon_{xx} - \epsilon_{yy}} = \frac{0.0007}{0.0002 + 0.0001} = \frac{7}{3}$$

$$\Rightarrow 2\theta = 66.81^\circ; \quad \theta = 33.405^\circ$$

$$\begin{aligned} \epsilon_{I,II} &= \frac{\epsilon_{xx} + \epsilon_{yy}}{2} \pm \sqrt{\left(\frac{\epsilon_{xx} - \epsilon_{yy}}{2}\right)^2 + \epsilon_{xy}^2} \\ &= \frac{0.0002 - 0.0001}{2} \pm \sqrt{\left(\frac{0.0002 + 0.0001}{2}\right)^2 + (0.00035)^2} \\ &= 0.00005 \pm 0.00038 \\ &= \underline{0.000431}, \quad \underline{-0.000331} \end{aligned}$$

ii. a.

$$e_a = 0.001 = e_x$$

$$e_b = 0.003; \theta = 60^\circ; C_{x^*x} = \cos(-60^\circ) = 0.5$$

$$C_{x^*y} = \cos(30^\circ) = 0.866$$

$$e_c = -0.002; \theta = 120^\circ; C_{x^*x} = \cos(-120^\circ) = -0.5$$

$$C_{x^*y} = \cos(-30^\circ) = 0.866$$

$$\epsilon_{x^*x} = 0.003 = 0.001 \cdot (0.5)^2 + \epsilon_{yy} (0.866)^2 + \gamma_{xy} (0.5)(0.866)$$

$$\Rightarrow 0.00275 = 0.75 \epsilon_{yy} + 0.433 \gamma_{xy} \quad \text{--- --- --- } \textcircled{1}$$

$$\epsilon_{x^*x} = -0.002 = 0.001(-0.5)^2 + \epsilon_{yy} (0.866)^2 + \gamma_{xy} (-0.5)(0.866)$$

$$\Rightarrow -0.00175 = 0.75 \epsilon_{yy} - 0.433 \gamma_{xy} \quad \text{--- --- --- } \textcircled{2}$$

$$\therefore \underline{\epsilon_{yy} = 0.000666}, \quad \underline{\gamma_{xy} = 0.00520 = 2\epsilon_{xy}}$$

b. Principal strain:

$$\tan 2\theta = \frac{2\epsilon_{xy}}{\epsilon_{xx} - \epsilon_{yy}} = \frac{0.00520}{0.001 - 0.000666} = 15.568$$

$$\Rightarrow 2\theta = 86.341^\circ, \quad \theta = 43.1705^\circ$$

$$\epsilon_{I,II} = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} \pm \sqrt{\left(\frac{\epsilon_{xx} - \epsilon_{yy}}{2}\right)^2 + \epsilon_{xy}^2}$$

$$= \frac{0.001 + 0.000666}{2} \pm \sqrt{\left(\frac{0.001 - 0.000666}{2}\right)^2 + 0.00260^2}$$

$$= 0.000833 \pm 0.000261 = \underline{0.003438}, \quad \underline{-0.00177}$$

iii. a.

$$e_a = \underline{0.002 = \epsilon_{xx}}$$

$$\theta = 30^\circ; \quad C_{x^*x} = \cos(-30^\circ) = 0.866; \quad C_{x^*y} = \cos 60^\circ = 0.5$$

$$e_b = 0.001;$$

$$\theta = 150^\circ; \quad C_{x^*x} = \cos(-150^\circ) = -0.866; \quad C_{x^*y} = \cos(-60^\circ) = 0.5$$

$$e_c = -0.003 = -\epsilon_{yy} \quad \Rightarrow \quad \underline{\epsilon_{yy} = 0.003}$$

$$\epsilon_{x^*x} = 0.002 = \epsilon_{xx} (0.866)^2 + 0.003 (0.5)^2 + \gamma_{xy} (0.866)(0.5)$$

$$0.00125 = 0.75 \epsilon_{xx} + 0.433 \gamma_{xy} \quad \dots \textcircled{1}$$

$$\epsilon_{x^*x} = 0.001 = \epsilon_{xx} (-0.866)^2 + 0.003 (0.5)^2 + \gamma_{xy} (-0.866)(0.5)$$

$$0.00025 = 0.75 \epsilon_{xx} - 0.433 \gamma_{xy} \quad \dots \textcircled{2}$$

$$\text{Solve } \textcircled{1} \text{ \& } \textcircled{2} \Rightarrow \underline{\epsilon_{xx} = 0.0001}, \quad \underline{\gamma_{xy} = 0.001154 = 2 \epsilon_{xy}}$$

b. Principal strains:

$$\tan 2\theta = \frac{2\epsilon_{xy}}{\epsilon_{xx} - \epsilon_{yy}} = \frac{0.001154}{0.0001 - 0.003} = -0.3979$$

$$\Rightarrow 2\theta = -21.703^\circ \Rightarrow \underline{\theta = -10.851^\circ}$$

$$\epsilon_{I,II} = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} \pm \sqrt{\left(\frac{\epsilon_{xx} - \epsilon_{yy}}{2}\right)^2 + \epsilon_{xy}^2}$$

$$= \frac{0.0001 + 0.003}{2} \pm \sqrt{\left(\frac{0.0001 - 0.003}{2}\right)^2 + 0.000577^2}$$

$$= 0.00155 \pm 0.001561$$

$$= \underline{0.00311}, \quad \underline{-0.0000105}$$

2.

a.

$$\gamma_{yz} = 2 \epsilon_{yz} = 2 \times 0.0025 = 0.005$$

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu} = 1.0 \times 10^7 \text{ psi}$$

$$\nu = \frac{\lambda}{2(\lambda + \mu)} = 0.3$$

$$G = \frac{E}{2(1 + \nu)} = 3.846 \times 10^6 \text{ psi}$$

$$\sigma_{xy} = G \gamma_{xy} \Rightarrow 0.75 \times 10^5 = 3.846 \times 10^6 \gamma_{xy}$$

$$\Rightarrow \gamma_{xy} = 0.0195$$

$$\sigma_{xz} = G \gamma_{xz} \Rightarrow 0.5 \times 10^5 = 3.846 \times 10^6 \gamma_{xz}$$

$$\Rightarrow \gamma_{xz} = 0.0130$$

$$\sigma_{yz} = G \gamma_{yz} \Rightarrow \sigma_{yz} = 3.846 \times 10^6 (0.005) = 19230 \text{ psi}$$

$$\begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \end{bmatrix} = \begin{bmatrix} 1/E & -\nu/E & -\nu/E \\ -\nu/E & 1/E & -\nu/E \\ -\nu/E & -\nu/E & 1/E \end{bmatrix} \cdot \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \end{bmatrix} + \Delta T \begin{bmatrix} \alpha \\ \alpha \\ \alpha \end{bmatrix}$$

$$\begin{bmatrix} 0.001 \\ \epsilon_{yy} \\ 0.0055 \end{bmatrix} = \begin{bmatrix} 10^{-7} & -3 \times 10^{-8} & -3 \times 10^{-8} \\ -3 \times 10^{-8} & 10^{-7} & -3 \times 10^{-8} \\ -3 \times 10^{-8} & -3 \times 10^{-8} & 10^{-7} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{xx} \\ 2 \times 10^5 \\ \sigma_{zz} \end{bmatrix} + 2.5 \begin{bmatrix} 5 \times 10^{-5} \\ 5 \times 10^{-5} \\ 5 \times 10^{-5} \end{bmatrix}$$

$$\Rightarrow \sigma_{xx} = 1.130 \times 10^5 \text{ psi} \quad , \quad \sigma_{zz} = 1.476 \times 10^5 \text{ psi}$$

$$\epsilon_{yy} = 0.0123$$

$$\begin{aligned} \text{b. } \epsilon_{zz} (\text{new}) &= \epsilon_{zz} (\text{old}) + 0.001 \\ &= 0.0065 \end{aligned}$$

$$\begin{aligned} 0.0065 &= -3e-8 (1.130e5) - 0.006 + e-7 (1.476e5) \\ &\quad + 5e-5 (\Delta T) \end{aligned}$$

$$\Rightarrow \Delta T = 22.6^\circ \text{C}$$

c.  $\gamma_{yz}$ ,  $\gamma_{xz}$ ,  $\gamma_{xy}$  : unchanged.

$$\epsilon_{zz} = 0.0065 \quad (\text{part b})$$

$$\begin{aligned} \epsilon_{xx} &= e-7 (1.130e5) - 0.006 - 3e-8 (1.476e5) \\ &\quad + 5e-5 (22.6) = 0.002 \end{aligned}$$

$$\begin{aligned} \epsilon_{yy} &= -3e-8 (1.130e5) + 0.02 - 3e-8 (1.476e5) \\ &\quad + 5e-5 (22.6) = 0.0133 \end{aligned}$$

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