

1. Solution:

$$\Sigma F_x = 0$$

$$\sigma_x \frac{dy}{dp} = 100 \frac{dx}{dp} + 121.4 \cos \theta$$

$$\Sigma F_y = 0$$

$$100 \frac{dy}{dp} - 80 \frac{dx}{dp} + 121.4 \sin \theta = 0$$

$$\text{Since } \cos \theta = \frac{dy}{dp}, \quad \sin \theta = \frac{dx}{dp}$$

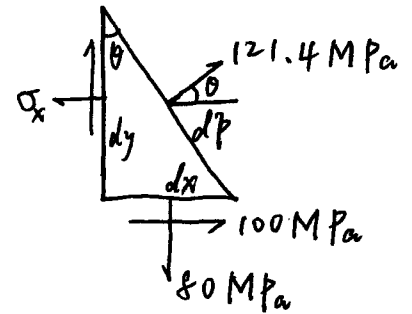
The above 2 eqns become

$$\begin{cases} \sigma_x \cos \theta = 100 \sin \theta + 121.4 \cos \theta & \text{----- } \textcircled{A} \\ 100 \cos \theta - 80 \sin \theta + 121.4 \sin \theta = 0 \end{cases}$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{-100}{41.4} = \tan \theta$$

$$\Rightarrow \theta = -67.5^\circ$$

$$\begin{aligned} \text{From } \textcircled{A} \text{ We have } \sigma_x &= 100 \tan \theta + 121.4 \\ &= 100 \tan(-67.5^\circ) + 121.4 \\ &= -120.1 \text{ (MPa)} \end{aligned}$$



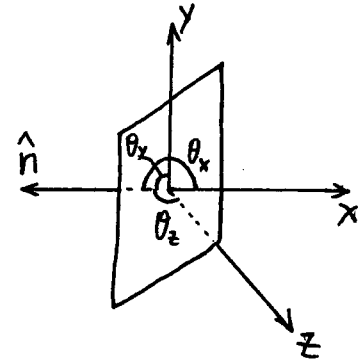
2. Solution:

For face at $x = -a$:

$$\cos(n, x) = \cos(180^\circ) = -1$$

$$\cos(n, y) = \cos(90^\circ) = 0$$

$$\cos(n, z) = \cos(90^\circ) = 0$$

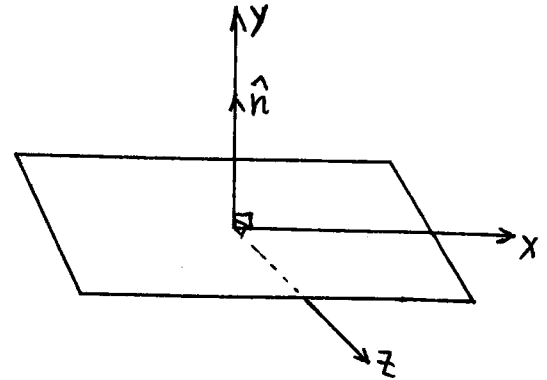


For face at $y = +b$:

$$\cos(n, x) = \cos(90^\circ) = 0$$

$$\cos(n, y) = \cos(0^\circ) = 1$$

$$\cos(n, z) = \cos(90^\circ) = 0$$

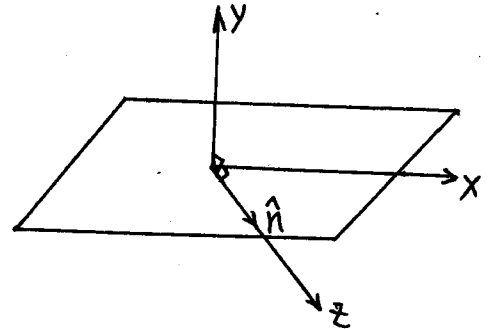


For face at $z = +c$:

$$\cos(n, x) = \cos(90^\circ) = 0$$

$$\cos(n, y) = \cos(90^\circ) = 0$$

$$\cos(n, z) = \cos(0^\circ) = 1$$



3. SOLUTION:

$$\textcircled{1} \text{ a) } c_1 = (80 - 120) / 2 = -20$$

$$r_1 = (100^2 + 100^2)^{1/2} = 141.4 \text{ MPa}$$

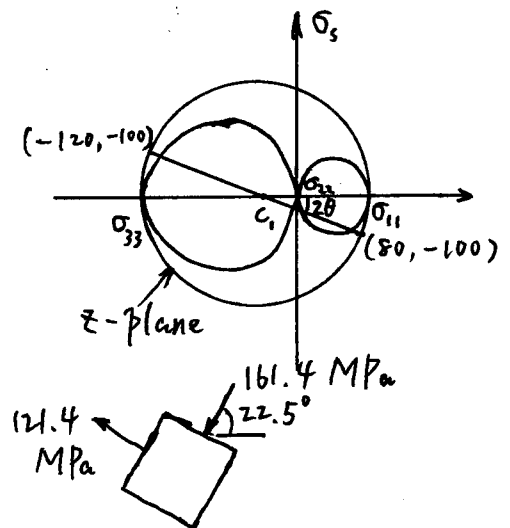
$$\sigma_{11} = c_1 + r_1 = 121.4 \text{ MPa}$$

$$\sigma_{22} = 0$$

$$\sigma_{33} = c_1 - r_1 = -161.4 \text{ MPa}$$

$$\text{c) } \sigma_{s,z} = \sigma_{s,\max} = r_1 = 141.4 \text{ MPa}$$

$$\text{b) } \theta = \frac{1}{2} \tan^{-1}(-1) = -22.5^\circ$$



$$\textcircled{2} \text{ a) } c_1 = \frac{6 - 18}{2} = -6$$

$$r_1 = \sqrt{9^2 + 12^2} = 15$$

$$\sigma_{11} = c_1 + r_1 = 9 \text{ ksi}$$

$$\sigma_{22} = 0$$

$$\sigma_{33} = c_1 - r_1 = -6 - 15 = -21 \text{ ksi}$$

$$c) \sigma_{s,z} = \sigma_{s,max} = r_1 = 15 \text{ ksi}$$

$$b) \theta = \frac{1}{2} \tan^{-1} \left(\frac{9}{12} \right) = 18.4^\circ$$

$$\textcircled{3} a) c_1 = 24/2 = 12$$

$$r_1 = \sqrt{5^2 + 12^2} = 13 = \sigma_{s,z}$$

$$\sigma_{11} = c_1 + r_1 = 25 \text{ ksi}$$

$$\sigma_{22} = 10 \text{ ksi}$$

$$\sigma_{33} = c_1 - r_1 = -1 \text{ ksi}$$

$$c) \sigma_{s,z} = \sigma_{s,max} = r_1 = 13 \text{ ksi}$$

$$b) \theta = \frac{1}{2} \tan^{-1} \left(\frac{5}{12} \right) = 11.3^\circ$$

$$\textcircled{4} a) c_1 = 10/2 = 5$$

$$r_1 = 10$$

$$\sigma_{11} = c_1 + r_1 = 15 \text{ ksi}$$

$$\sigma_{22} = -5 \text{ ksi}$$

$$\sigma_{33} = -8 \text{ ksi} \text{ since } \sigma_{xz} = \sigma_{yz} = 0$$

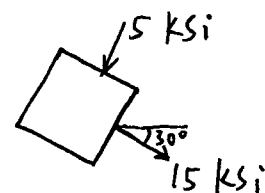
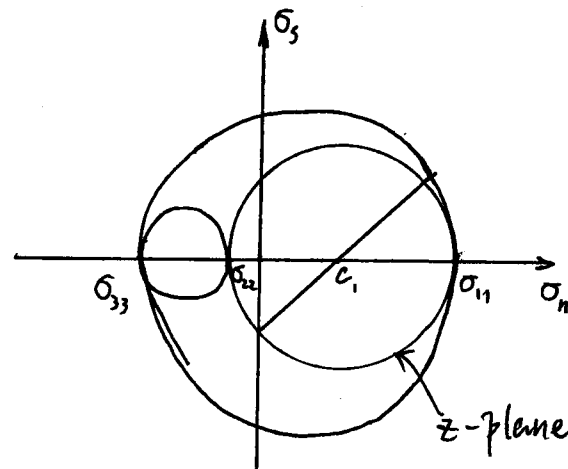
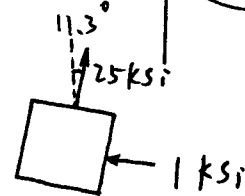
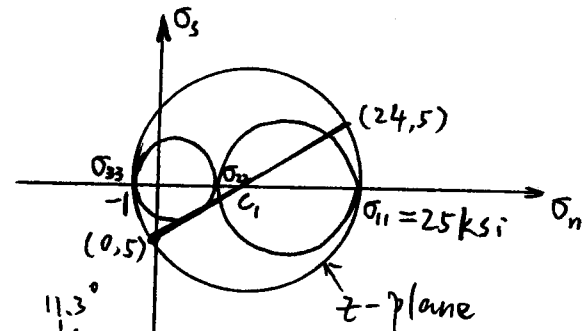
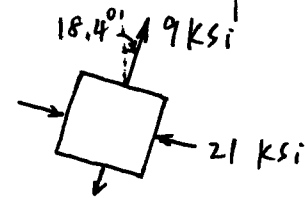
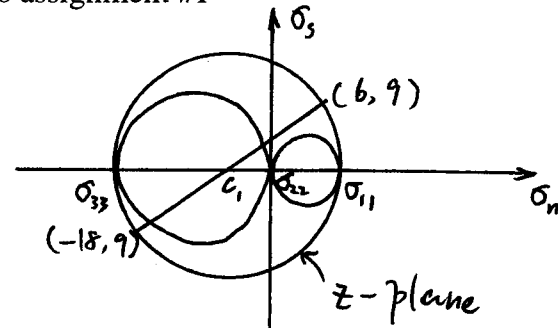
$$c) \sigma_{s,z} = r_1 = 10 \text{ ksi}$$

$$\sigma_{s,max} = \frac{15 + 8}{2} = 11.5 \text{ ksi}$$

$$b) \theta = \frac{1}{2} \tan^{-1} \left(-\frac{-5\sqrt{3}}{5} \right) = +30^\circ$$

$$\textcircled{5} a) c_1 = \frac{200}{2} = 100$$

$$r_1 = 200$$



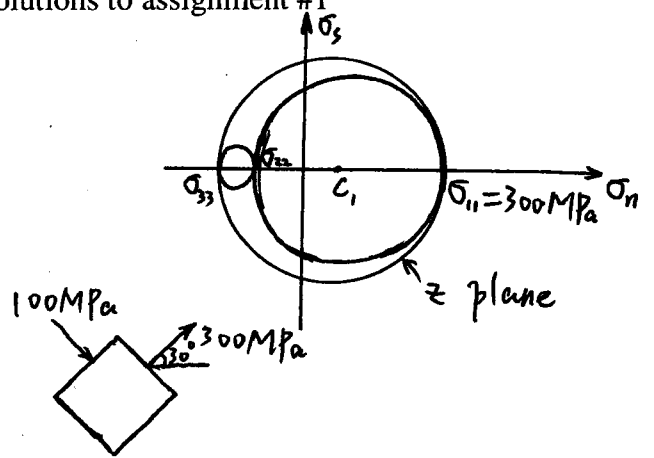
$$\sigma_{11} = 200 + 100 = 300 \text{ MPa}$$

$$\sigma_{22} = -90 \text{ MPa} \text{ since no } \sigma_{xz}, \sigma_{yz}$$

$$\sigma_{33} = c_1 - r_1 = -100 \text{ MPa}$$

$$c) \sigma_{s,z} = \sigma_{s,max} = r_1 = 200 \text{ MPa}$$

$$b) \theta = \frac{1}{2} \tan^{-1}(-\sqrt{3}) = -30^\circ$$



⑥ Since no shear stresses at all planes and all $\sigma_x, \sigma_y, \sigma_z$ are equal, all principal stresses are 70 MPa regardless of orientation.

$$⑦ a) c_1 = 60$$

$$r_1 = 98.5$$

$$\sigma_{11} = 158.5 \text{ MPa}$$

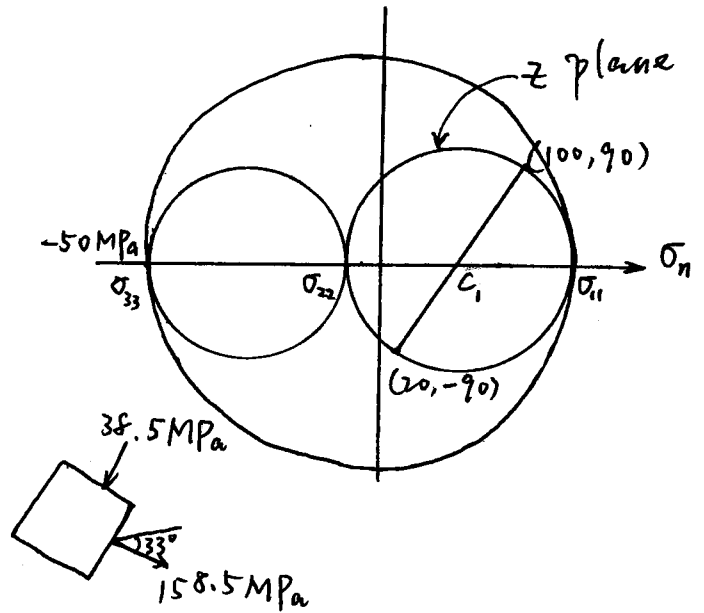
$$\sigma_{22} = -38.5 \text{ MPa}$$

$$\sigma_{33} = -50 \text{ MPa}$$

$$c) \sigma_{s,z} = 98.5 \text{ MPa}$$

$$\sigma_{s,max} = 104.3 \text{ MPa}$$

$$b) \theta = \frac{1}{2} \tan^{-1}\left(\frac{90}{40}\right) = 33^\circ$$



$$⑧ c_1 = 40$$

$$a) r_1 = 45$$

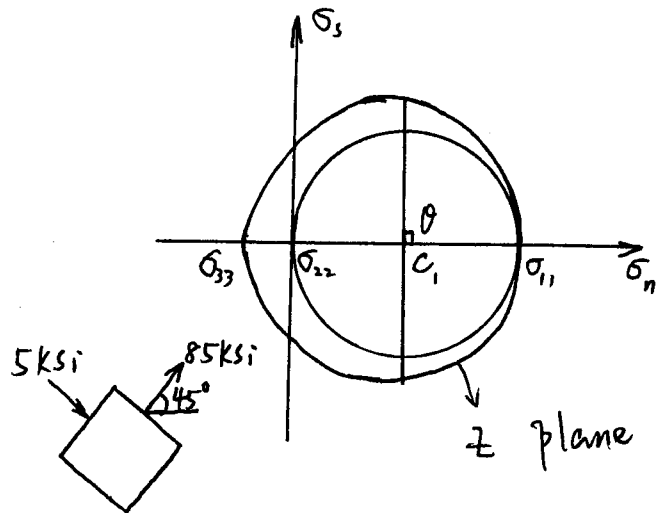
$$\sigma_{11} = 85 \text{ ksi}$$

$$\sigma_{22} = 0$$

$$\sigma_{33} = -5 \text{ ksi}$$

$$c) \sigma_{s,z} = \sigma_{s,max} = 45 \text{ ksi}$$

$$b) \theta = 90^\circ / 2 = 45^\circ$$



⑨ a) $c_1 = \frac{16 - 32}{2} = -8$

$r_1 = (24^2 + 10^2)^{1/2} = 26$

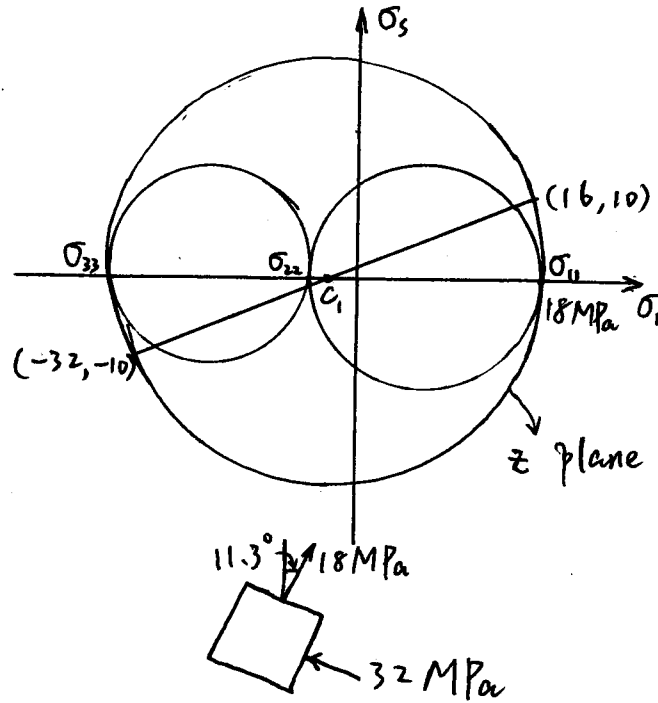
$\sigma_{11} = 18 \text{ MPa}$

$\sigma_{22} = -12 \text{ MPa}$ since no σ_{xz}, σ_{yz}

$\sigma_{33} = c_1 - r_1 = -34 \text{ MPa}$

c) $\sigma_{s,z} = \sigma_{s,max} = 26 \text{ MPa}$

b) $\theta = \frac{1}{2} \tan^{-1} \left(\frac{10}{24} \right) = 11.3^\circ$



□
(09-07-00)