Solution:

1. Nomenclature

— Figure 12 & 13

Returning to our three-member truss, figure 12 shows the truss geometry and elements. Elements 1 and 3 have a length L, while element 2 has a length of $L_2 = \sqrt{2}L$. To simplify the calculations, element 2 has a Young's modulus of $E_2 = \sqrt{2}E$ while elements 1 and 2 have a modulus of E.

Figure 12

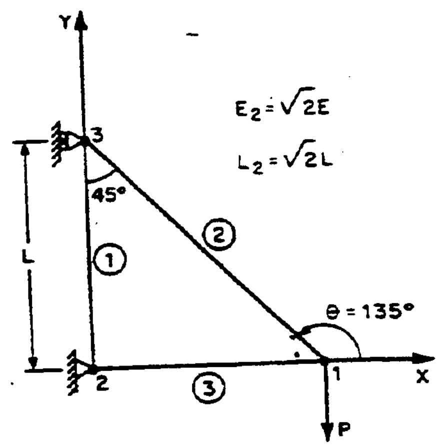

Figure 13

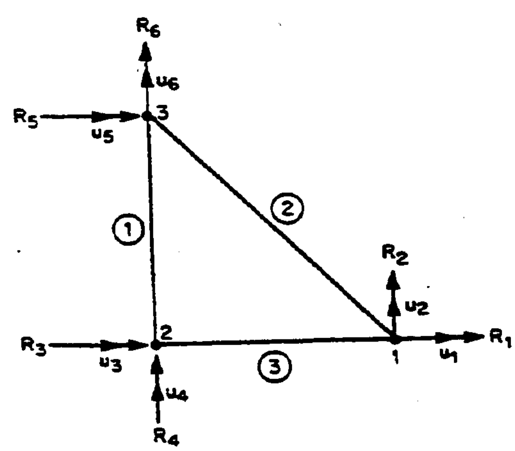

Figure 13 shows the applied force and reactions acting on the truss. Note that these forces are externally applied forces. The forces $\{F\}_i$ that we calculated for the element stiffness matrix are the nodal forces caused by the displacements of the elements. For **equilibrium, the sum of the applied forces and the nodal forces must be zero.** We will solve these equilibrium equations, but first we must calculate the stiffness matrices for each element, applying the equation 11.

For element 1, $\theta = 90°$, $c = 0$ and $s = 1$. Substituting into the equation in figure 10 yields equation 14.

Equation 14

Element 1 →

$$\begin{Bmatrix} F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix}$$

For element 2, $\theta = 135°$, $c = 1/\sqrt{2}$, and $s = -1/\sqrt{2}$. The angle between the X axis and the element is measured counterclockwise, as in the derivation of the equation 11.

The stiffness matrix for element 2 is given in equation 15A.

Equation 15A

Element 2 →

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_5 \\ F_6 \end{Bmatrix} = \frac{AE_2}{2L_2} \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_5 \\ u_6 \end{Bmatrix}$$

Equation 15B

Element 3 →

$$\begin{Bmatrix} F_3 \\ F_4 \\ F_1 \\ F_2 \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \\ u_1 \\ u_2 \end{Bmatrix}$$

For element 3, $\theta = 0°$, and the stiffness matrix reduces to the equation 15B.

We now have expressions for all the external forces in the problem. Combining equations (figure 17 to figure 19), we obtain the global stiffness matrix and load and displacement vectors in equation 20.

Equation 20

b. System matrix:

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} & -1 & 0 & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 & -1 & -\frac{1}{2} & \frac{3}{2} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix}$$

Equation 20 represents the set of linear algebraic equations to be solved for the unknown displacements. Again, since there are three nodes, there are 6 DOF in the problem and therefore six linear equations. As we established earlier, the set of forces R_i are the external forces including the support reactions. In this problem, there are three unknown reactions: R_3, R_4 and R_5. Force R_2 is the external force $-P$ (force is negative due to sign convention). Forces R_1 and R_6 are 0.

c. Node 3 --- roller --- Vertical displacement = 0

Node 2 --- pin connection --- Vertical displacement = 0

Node 2 --- pin connection --- horizontal displacement = 0

Node 1 --- applied load --- $F_v = -P$

Node 1 --- applied load --- $F_H = 0$

Node 3 --- reaction force --- $F_v = 0$

d.

$$u_1 = -\frac{PL}{AE} = -1.0 \times 10^{-6}\ m$$

$$u_2 = -\frac{4PL}{AE} = -4.0 \times 10^{-6}\ m$$

$$u_6 = -\frac{PL}{AE} = -1.0 \times 10^{-6}\ m$$

$$R_3 = \frac{AE}{2L}(-2u_1) = -\frac{AE u_1}{L} = 1000\ N$$

$$R_4 = \frac{AE}{2L}(-2u_6) = -\frac{AE u_6}{L} = 1000\ N$$

$$R_5 = \frac{AE}{2L}(-u_1 + u_2 - u_6) = -1000\ N$$

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