

Framework for analytical quantification of disaster resilience

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ABSTRACT

The concepts of disaster resilience and its quantitative evaluation are presented and a unified terminology for a common reference framework is proposed and implemented for evaluation of health care facilities subjected to earthquakes. The evaluation of disaster resilience is based on dimensionless analytical functions related to the variation of functionality during a period of interest, including the losses in the disaster and the recovery path. This evolution in time including recovery differentiates the resilience approach from the other approaches addressing the loss estimation and their momentary effects. The recovery process usually depends on available technical and human resources, societal preparedness, public policies and may take different forms, which can be estimated using simplified recovery functions or using more complex organizational and socio-political models. Losses are described as functions of fragility of systems that are determined using multidimensional performance limit thresholds. The proposed framework is formulated and exemplified for a typical Californian Hospital building using a simplified recovery model, considering direct and indirect losses in its physical system and in the population served by the system. A hospital network is also analyzed to exemplify the resilience framework. Resilience function captures the effect of the disaster, but also the results of response and recovery, the effects of restoration and preparedness. Therefore, such a function becomes an important tool in the decision process for both the policy makers and the engineering professionals.

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1. Introduction

Over the past years the natural and man-made disasters with which the human society had to cope with had stressed the necessity to be prepared and to be able to recover in a short time from a sudden and unexpected change in the community's technical, organizational, social and economical condition. The concepts of 'risk reduction', 'vulnerability', 'recovery' and 'resilience' have become keywords when dealing with hazardous events, but there is a need to go beyond the intuitive definition and provide a quantitative evaluation of them. When a disaster strikes, the community affected requires immediate help to survive, resources, and efforts to recover in a short time. In other words, the community needs to be "prepared" and less "vulnerable", in order to achieve a high 'resilience'.

The outcomes of the 2005 World Conference on Disaster Reduction (WCDR) confirmed the importance of the entrance of the term resilience into disaster discourse and gave birth to a

new culture of disaster response. Resilience, according to the dictionary, means "the ability to recover from (or to resist being affected by) some shock, insult or disturbance" and the root of the term has to be found in the Latin word 'resilio' that literary means 'to jump back'. Manyena [1], evaluating all the possible definitions provided from the 90' to nowadays, suggests that Resilience could be viewed as the "intrinsic capacity of a system, community or society predisposed to a shock or stress to adapt and survive by changing its non-essential attributes and rebuilding itself". As regards its relationship with the concept of vulnerability, it can be accepted that the latter is closely associated to the level of resilience, but it is a complementary aspect of the community preparedness.

Emphasizing the concept of *resilience* means to focus on the quality of life of the people at risk and to develop opportunities to enhance a better outcome. In contrast, the *vulnerability* approach places stress on the production of nature [2] to resist the natural hazard. Engineers, guided by legislation, play a guiding role in the quantification of vulnerability. In order to understand better the relationship between vulnerability and fragility, it is convenient to focus on the field of seismic engineering. Given a certain control parameter (e.g. the shaking intensity), *vulnerability* defines the loss while *fragility* gives the probability of some undesirable event (e.g. collapse). Thus, fragility functions may assess the probability

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that a building will collapse, as well as that a factory may release hazardous materials into the atmosphere, given a certain seismic intensity. On the other side, vulnerability functions would provide as a function of the same control parameter the damage factor for the building (e.g. valuated as the repair cost divided by the replacement cost) or the quantity of hazardous materials released. In the last years, as the idea of the necessity of building disaster-resilient communities gains acceptance, new methods have been proposed to quantify resilience beyond estimating losses. Because of the vastness of the definition, resilience necessarily has to take into account its entire complex and multiple dimensions, which includes technical, organizational, social, and economic facets. Bruneau et al. [3,4] offered a very broad definition of resilience to cover all actions that reduce losses from hazard, including effects of mitigation and rapid recovery. However, Bruneau et al. [3,4] defined a fundamental framework for evaluating community resilience without a detailed quantification and definition.

After the general framework provided by Bruneau et al. [3] various studies have been carried out, with the goal of practically evaluating the concept of resilience and identifying the main units of measurement of it.

Miles and Chang [5] present a comprehensive conceptual model of recovery, which establishes the relationships among a community's household business, lifeline networks, and neighborhoods. The primary aim is to discuss issues of community recovery and to attempt to operationalize it. Even if a measure of resilience is not provided in their work, the paper points out the necessity to correlate the concept of recovery to real factors, such as the household object, whose attributes are the income, the year the building of residence was built, and the possible existence of any retrofit building.

Davidson and Cagnan [6] developed a model of the post-earthquake restoration processes for an electric power system. A discrete event simulation model based on available data was built, with the goal of improving the quantitative estimates of restoration times that are required to evaluate economic losses, and identify ways to improve the restoration processes in future earthquakes.

Chang and Shinozuka [7] contribute to the literature on disaster resilience discussing a quantitative measure of resilience based on the case study of the Memphis water system. They explored the extent to which loss estimation models can be used to measure resilience.

Cimellaro et al. [8], attempted to formulate the first framework to quantify resilience, however only the uncertainties of the intensity measure I were considered, whereas in the framework proposed in this work all other uncertainties are involved.

Bruneau and Reinhorn [4] for the first time relate probability functions, fragilities and resilience in a single integrated approach for acute care facilities. After having defined the main properties and concepts of resilience, two different options to quantify the disaster resilience of acute care facilities are exposed as the percentage of healthy population and as the number of patients/day that can receive service.

While this literature survey is by no mean comprehensive, it is presented here to highlight several distinct techniques, and set the stage for future developments in this work.

The goal of this paper has been to provide a framework for quantitative definition of resilience using an analytical function that may fit both technical and organizational issues showing two applications to health care facilities of the methodology.

2. Definitions and formulations

To establish a common framework for resilience, a unified terminology is proposed, while the fundamental concepts are analyzed and presented in this paper.

Definition 1. Resilience (R) is defined as a function indicating the capability to sustain a level of functionality or performance for a given building, bridge, lifeline networks, or community, over a period defined as the control time (T_{LC}) that is usually decided by owners, or society (usually is the life cycle, life span of the system etc.).

Definition 2. The recovery time (T_{RE}) is the period necessary to restore the functionality of a structure, an infrastructure system (water supply, electric power, hospital building, etc., or a community), to a desired level that can operate or function the same, close to, or better than the original one.

The recovery time T_{RE} is a random variable with high uncertainties that includes the construction recovery time and the business interruption time and it is usually smaller than the control time T_{LC} . It typically depends on the earthquake intensities and on the location of the system with its given resources such as capital, materials and labor, following the major seismic event. For these reasons, this recovery time is the most difficult quantity to predict in the resilience function. Porter et al. [9] attempted to make a distinction between downtime (recovery time) and repair time, and tried to quantify the latter. In that work, damage states were combined with repair duration, and with probability distributions to estimate assembly repair durations. While the previous definitions apply to structures, infrastructure, or societal organizations, a more general application of such definitions is for "disaster resilient communities".

Definition 3. Disaster resilient community is a community that can withstand an extreme event, natural or man made, with a tolerable level of losses, and is able to take mitigation actions consistent with achieving that level of protection [10].

Using MCEER (Multidisciplinary Center of Earthquake Engineering to Extreme Event) terminology, the seismic performance of the system is measured through a unique decision variable (DV) defined as "Resilience" that combines other variables (economic losses, casualties, recovery time etc.) which are usually employed to judge seismic performance. This Resilience is defined graphically as the normalized shaded area underneath the functionality function of a system, defined as $Q(t)$. $Q(t)$ is a non-stationary stochastic process and each ensemble is a piecewise continuous function as the one shown in Fig. 2(a), where the functionality $Q(t)$ is measured as a dimensionless (percentage) function of time. For a single event, Resilience is given by the following equation [4,8,11]

$$R = \int_{t_{OE}}^{t_{OE}+T_{LC}} Q(t)/T_{LC} dt \quad (1)$$

where

$$Q(t) = [1 - L(I, T_{RE})][H(t - t_{OE}) - H(t - (t_{OE} + T_{RE}))] \times f_{REC}(t, t_{OE}, T_{RE}) \quad (2)$$

where $L(I, T_{RE})$ is the loss function; $f_{REC}(t, t_{OE}, T_{RE})$ is the recovery function; $H()$ is the Heaviside step function, T_{LC} is the control time of the system, T_{RE} is the recovery time from event E and; t_{OE} is the time of occurrence of event E .

2.1. The four dimensions of resilience

While defining Resilience is clearly challenging, identifying the features of organizations and other social units that make them resilient is even more difficult. Resilience is an important concept for disaster managements of complex systems. Researchers at the

MCEER [3,8,11] have identified four dimensions along which resilience can be improved. These are *robustness*, *resourcefulness*, *redundancy*, and *rapidity*. These dimensions can be better understood by looking at the functionality curve shown in Fig. 2.

Rapidity is the “capacity to meet priorities and achieve goals in a timely manner in order to contain losses and avoid future disruption” [3]. Mathematically it represents the slope of the functionality curve (Fig. 2(a)) during the recovery-time and it can be expressed by the following Eq. (3)

$$\text{Rapidity} = \frac{dQ(t)}{dt}; \quad \text{for } t_{0E} \leq t \leq t_{0E} + T_{RE} \quad (3)$$

where d/dt is the differential operator; $Q(t)$ is the functionality of the system. An average estimation of rapidity can be defined by knowing the total losses and the total recovery time to reach again 100% of functionality, as follows

$$\text{Rapidity} = \frac{L}{T_{RE}} \quad (\text{average recovery rate in percentage/time}) \quad (4)$$

where L is the loss, or drop of functionality, right after the extreme event.

Robustness referring to engineering systems is, “the ability of elements, systems or other units of analysis to withstand a given level of stress, or demand without suffering degradation or loss of function” [3]. It is therefore the residual functionality right after the extreme event (Fig. 2(b)) and can be represented by the following relation

$$\text{Robustness} = 1 - \tilde{L}(m_L, \sigma_L); \quad (\%) \quad (5)$$

where \tilde{L} is a random variable expressed as function of the mean m_L and the standard deviation σ_L . A more explicit definition of robustness is obtained when the dispersion of the losses is expressed directly as follows

$$\text{Robustness} = 1 - \tilde{L}(m_L, +a\sigma_L); \quad (\%) \quad (6)$$

where a is a multiplier of the standard deviation corresponding to a specific level of losses. A possible way to decrease uncertainty in robustness of the system is to reduce the dispersion in the losses represented by σ_L . In this definition, robustness reliability is therefore also the capacity of keeping the variability of losses within a narrow band, independently of the event itself (Fig. 2(b)). Two examples of systems with and without robustness are respectively the Emergency Operation Center (EOC) and the Office of Emergency Management (OEM) organization during the World Trade Center disaster in 2001 [12]. The EOC facility, part of OEM, was not sufficiently robust to survive the September 11, attack (being located in the 23rd floor of the 7 World Trade Center). However, on the strength of its resourcefulness, OEM exhibited considerable robustness as an organization, demonstrating an ability to continue to function even after losing its WTC facility and a great part of its communications and information technology infrastructure. When the latter was restored, it contributed to the resilience of the OEM as a functional and effective organizational network.

According to the earthquake engineering field, *Redundancy* is “the quality of having alternative paths in the structure by which the lateral forces can be transferred, which allows the structure to remain stable following the failure of any single element” [13]. In other words, it describes the availability of alternative resources in the recovery process of a system. Redundancy is “the extent to which elements, systems, or other units of analysis exist that are substitutable, i.e. capable [of] satisfying functional requirements in the event of disruption, degradation, or loss of functionality” [3]. Simply, it describes the availability of alternative resources in the loss or recovery process.

Redundancy is a very important attribute of resilience, since it represents the capability to use alternative resources, when the principal ones are either insufficient or missing. If the system is resilient, there will always be at least one scenario allowing recovery, irrespective of the extreme event. If this condition is not met by the system, then changes to the system can be made, such as duplicating components to provide alternatives in case of failure.

An example of a system without redundancy is well illustrated in the World Trade Center terrorist attack mentioned above, where the EOC facility was destroyed and there was no other office, which could immediately, or instantaneously, replace the main facility. Redundancy should be developed in the system in advance and it should exist in a latent form as a set of possibilities to be enacted through the creative efforts of responders as indicated later.

Resourcefulness is “the capacity to identify problems, establish priorities, and mobilize resources when condition exist that threaten to disrupt some element, system, or other unit of analysis” [3]. This is a property difficult to quantify since it mainly depends on human skills and improvisation during the extreme event.

Resourcefulness and Redundancy are strongly interrelated. For example, resources, and resourcefulness, can create redundancies that did not exist previously. In fact, one of the major concerns with the increasingly intensive use of technology in emergency management, is the tendency to over-rely on these tools, so that if technology fails, or it is destroyed, the response falters. To forestall this possibility, many planners advocate Redundancy. Changes in Resourcefulness and Redundancy will affect the shape and the slope of the recovery curve and the recovery time T_{RE} . It also affects Rapidity and Robustness. It is through Redundancy and Resourcefulness (as means of resilience) that the Rapidity and Robustness (the ends of resilience) of an entire system can be improved.

2.2. Loss function

Earthquake losses are by their very nature highly uncertain, and are different for every specific scenario considered. However, some common parameters affecting these losses can be identified. In fact the loss function $L(I, T_{RE})$ is expressed as a function of earthquake intensity I and recovery time T_{RE} . The total losses can be divided in two types: Direct losses (L_D) which occur “instantaneously” during the disaster, and Indirect losses (L_I) which have also temporal dependencies. Within these two groups, they can be distinguished two subcategories: Economic losses (L_E) and Casualties losses (L_C). Therefore, losses L consist of four contributions: (i) Direct economic losses L_{DE} (or Contents losses); (ii) Direct Casualties losses L_{DC} ; (iii) Indirect economic losses L_{IE} (or Business interruption losses); (iv) Indirect Casualties losses L_{IC} all function of recovery period T_{RE} . For simplicity of presentation, L_{DE} , L_{DC} , L_{IE} and L_{IC} are described considering a health care facility, so that the direct economic losses that are mainly physical structural and non-structural losses can be expressed as ratios of building repair and replacement costs as follows

$$L_{DE}(I) = \sum_{j=1}^n \left[\frac{C_{s,j}}{I_s} \cdot \prod_{i=1}^{T_i} \frac{(1 + \delta_i)}{(1 + r_i)} \right] \cdot P_j \left\{ \bigcup_{i=1}^n (R_i \geq r_{limi}) / I \right\} \quad (7)$$

where P_j is the probability of exceeding a performance limit state j conditional an extreme event of intensity I occurs, also known as the fragility function; $C_{s,j}$ are the building repair costs associate with a j damage state; I_s are the replacement building costs; r_i is the annual discount rate; t_i is the time range in years between the initial investments and the occurrence time of the extreme event; δ_i is the annual depreciation rate. Eq. (7) assumes that the initial value of the building is affected by the discount rate, but the

value also decreases with time according to the depreciation rate δ_i , which may vary with time.

Direct economic losses $L_{DE}(I)$ are obtained for every structural and non-structural component k using the formulation in Eq. (7). In essential facilities like hospitals, research laboratories or some highly specialized manufacturing plants the non-structural losses can be much larger than the structural losses. Then, the direct economic losses are obtained using a weighted average expressed as

$$L_{DE}(I) = \left(\sum_{k=1}^N w_k \cdot L_{DE,k}(I) \right) / N \quad (8)$$

where $L_{DE,k}(I)$ is the direct economic loss associated with component k , N is the total number of structural and non-structural components in the system and w_k is a weight factor associated with each structural/non-structural component in the building. Non-structural components include ceilings, elevators, mechanical and electrical equipments, piping, partitions, glass etc. Direct casualties losses L_{DC} are measured as a ratio of the (instantaneous) number of injured or dead N_{in}^1 and the total number of occupants N_{tot}

$$L_{DC}(I) = \frac{N_{in}}{N_{tot}} \quad (9)$$

The number of injured patients N_{in} depends on multiple factors such as, the time of day of earthquake occurrence, the age of the population and the number and proximity of available health care facilities. The time at which the earthquake occurs determines the number of patients exposed to injury, so the probability of having a large number of injured patients varies during the day. Moreover, the age of population is also very important as indicated by Peek-Asa et al. [14] who found that during the 1994 Northridge earthquake the predominant number of injured patients was elderly.

The *indirect economic losses* $L_{IE}(I, T_{RE})$ are time dependent compared to all the previous losses considered. Among the post-earthquake losses these are the most difficult to quantify, because of the different forms they can take. They mainly consist of *business interruptions, relocation expenses, rental income losses, etc.* Losses of revenue, either permanent or temporary, can be caused by damage to structures and contents, and this is most important for manufacturing and retail facilities, and to lifelines. Damage to the former could mean less ability to deliver resources and services, like electricity, water, natural gas, or transportation. For example, structural damage such as collapse of a bridge span in a major highway generates direct losses, and indirect losses due to the loss of revenues from impact on the traffic to businesses served. In other cases, even if structural damage and loss of contents are minimal, there may be some indirect losses due to the disruption of services such as water and power. These losses can be more significant than the direct losses. Therefore, Indirect economic losses L_{IE} due to business interruption should be modeled considering both the structural and non-structural losses L_{DE} , and the time necessary to repair the structure T_{RE} [5,15,16]. These two quantities are not independent, but are related because the recovery time T_{RE} increases with the extent of structural damage. The fourth losses considered in the formulation are the indirect casualties losses (L_{IC}) that describe the number of patients that are injured or die because of hospital dysfunction, for example. For a hospital, L_{IC} is expressed in a form similar to Eq. (9) as the ratio

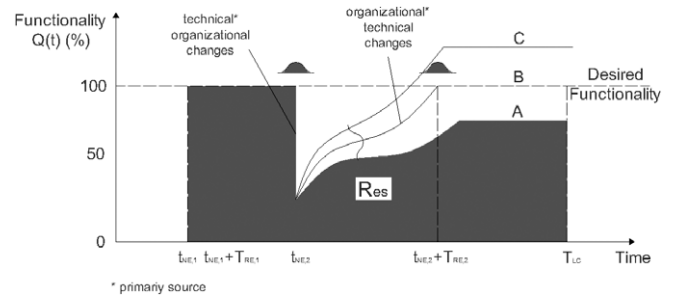


Fig. 1. Schematic representation of disaster resilience.

between the number of injured persons N_{in} due to dysfunction and the total population N_{tot} served

$$L_{IC}(I) = \frac{N_{in}}{N_{tot}} \quad (10)$$

Finally, the total losses L can be expressed as a combination of Direct L_D and Indirect L_I losses as follow

$$L(I, T_{RE}) = L_D(I) + \alpha_I L_I(I, T_{RE}) \quad (11)$$

where α_I = the weighting factor related to indirect losses (i.e. importance of the facilities for the community, influence of the facilities versus other system, etc.).

Additionally, L_D and L_I are given by

$$\begin{aligned} L_D &= L_{DE}^{\alpha_{DE}} \cdot (1 + \alpha_{DC} L_{DC}) \\ L_I &= L_{IE}^{\alpha_{IE}} \cdot (1 + \alpha_{IC} L_{IC}) \end{aligned} \quad (12)$$

where α_{DE} is a weighting factor related to construction losses in economic terms; α_{IE} is a weighting factor related to business interruption, relocation expenses, rental income losses, etc.; α_{DC} , α_{IC} are the weighting factors related to the nature of occupancy (i.e. schools, critical facilities, density of population). These weighting factors are determined based on socio-political criteria (cost benefit analyses, emergency functions, social factors, etc.). Engineers, economists, and social scientists usually address this subject jointly. It should be noted that the two casualties' losses in Eqs. (9) and (10) do not appear as loss functions, but as penalty functions in Eq. (12).

2.3. Simplified recovery function models

Most of the models available in the literature, including the "PEER equation framework" [17] developed at the Pacific Earthquake Engineering Research Center (PEER), are loss estimation models that focus on initial losses caused by disaster, where losses are measured relative to pre-disaster conditions. The temporal dimension of post-disaster loss recovery is not part of that formulation. As indicated in Fig. 1 the *recovery time* T_{RE} and the *recovery path* are essential for evaluating resilience, so they should be estimated accurately. Unfortunately, most common loss models, such as HAZUS [18] evaluate the recovery time in crude terms and assume that within one year, everything returns to normal. However, as shown in Fig. 1 the system considered may not necessary return to the pre-disaster baseline performance. It may exceed the initial performance (Fig. 1-curve C), when the recovery process ends, in particular when the system (e.g. community, essential facility, etc.) may use the opportunity to fix pre-existing problems inside the system itself. On the other hand, the system may suffer permanent losses and equilibrate below the baseline performance (Fig. 1-curve A).

A clear example of the condition shown in Fig. 1-curve A is represented by Kobe earthquake that clearly demonstrates that certain kinds of long-term impacts losses do occur, at least in

¹ The two groups can be considered separately, but in this formulation are grouped for simplicity.

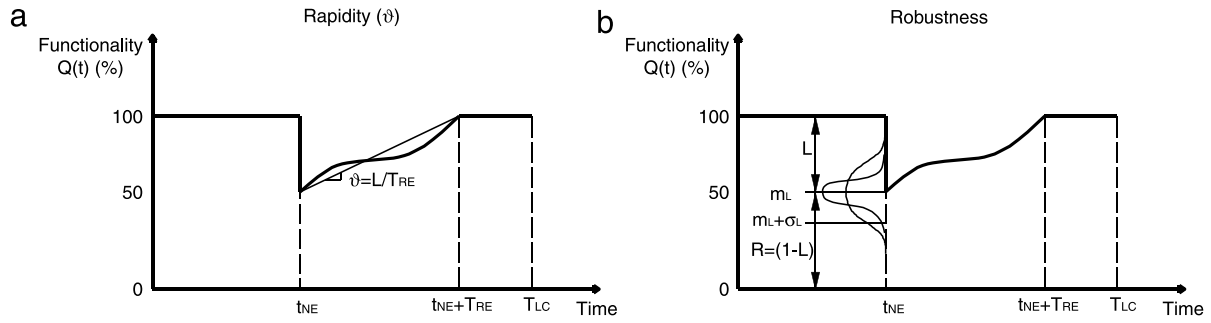


Fig. 2. Dimensions of resilience: Rapidity (a) and Robustness (b).

catastrophic disasters. In 1994, prior to the earthquake, the Port of Kobe was the world's sixth largest container port in terms of cargo throughput; in 1997, after repairs had been completed, it ranked seventeenth [19]. In fact, performance and recovery of transportation systems often requires longer repair times than other lifeline systems and in the case of Kobe port, it appeared to play a major role in the development of long-term impacts. Transportation losses served to accentuate existing social and economic conditions of vulnerability, and they led to permanent loss in business and therefore the port never came back to its pre-earthquake ranking.

These considerations show that the recovery process is complex and it is influenced by time dimensions, spatial dimensions (e.g., different neighborhood may have different recovery paths) and by interdependencies between different economic sectors that are interested in the recovery process. Therefore, different critical facilities (e.g. hospitals) that belong to the same community, but are located in different neighborhoods, have different recovery paths and in some areas (mainly poor areas), these essential facilities may experience long term or permanent losses [20]. In summary, the recovery process shows disparities among different geographic regions in the same community, showing different rates and quality of recovery. Modeling recovery of a single critical facility or of an entire community is a complex subject. These two processes cannot be assumed to be independent.

Information on comprehensive models that describe the recovery process is very limited. Miles and Chang [5] set out the foundations for developing models of community recovery presenting a comprehensive conceptual model and discussing some related issues. Once these complex recovery models are available, it is possible to describe relationships across different scales-socioeconomic agents, neighborhood and community, and to study the effects of different policies and management plans in an accurate way. In this chapter, the recovery process is oversimplified using recovery functions that can fit the more accurate results obtained with the Miles and Chang [5] model or with the recovery model proposed in next section that is valid for health care facility systems.

Different types of recovery functions can be selected depending on the system and society preparedness response. Three possible recovery functions are shown in Eq. (13): (i) *linear*, (ii) *exponential* [21] and (iii) *trigonometric* [22]

$$\text{linear: } f_{\text{rec}}(t) = a \left(\frac{t - t_{0E}}{T_{\text{RE}}} \right) + b;$$

$$\text{exponential: } f_{\text{rec}}(t) = a \exp[-b(t - t_{0E})/T_{\text{RE}}]; \quad (13)$$

$$\text{trigonometric: } f_{\text{rec}}(t) = a/2 \{1 + \cos[\pi b(t - t_{0E})/T_{\text{RE}}]\};$$

where a , b , are constant values that are calculated using curve fitting to available data sources, while t_{0E} is the instant of time when the extreme event strikes and T_{RE} is the recovery time necessary to go back to pre-disaster condition evaluated starting from t_{0E} .

It is important to mention that the constants in the model can be continuously updated as soon as more data are available using for example a Bayesian approach.

The simplest form is a *linear recovery function* that is generally used when there is no information regarding the preparedness, resources available and societal response (Fig. 3(a)). The *exponential recovery function* can be used where the societal response is driven by an initial inflow of resources, but then the rapidity of recovery decreases as the process nears its end (Fig. 3(b)). *Trigonometric recovery function* can be used when the societal response and the recovery are driven by lack or limited organization and/or resources. As soon as the community organizes itself, with the help of other communities (for example), then the recovery system starts operating and the rapidity of recovery increases (Fig. 3(c)). For example, such a recovery occurred after Nisqually Earthquake [23].

2.4. Mechanical analogy

The functionality of a system (e.g. structural, organizational, etc.) can be described by nonlinear differential equations similar to the one that applies to the fundamental laws of mechanical systems. The equation of motion for a linearly damped harmonic oscillator is given by

$$m\ddot{Q}(t) + c\dot{Q}(t) + kQ(t) = 0 \quad (14)$$

where m , c and k are parameters describing the model and Q is the functionality of the system. The solution of Eq. (14) has different outcomes, depending on the value of the factor $\zeta = c/2m\omega$, equivalent to the damping factor, and the factor $\omega = 2\pi/T$ where T is the period of the system. The recovery process after an extreme event goes back to its initial condition without oscillations, therefore only the two cases for $\zeta = 1$ and $\zeta > 1$, will be considered:

For *over-damped* systems, $\zeta > 1$, the general solution is

$$Q(t) = 1 - e^{-\alpha t} (Ae^{\beta t} + Be^{-\beta t}) \quad (15)$$

with $\alpha = \omega\zeta$ and $\beta = \omega\sqrt{(\zeta^2 - 1)}$. Placing the initial condition $Q(0) = 1 - L(I, T_{\text{RE}})$ and $\dot{Q}(0) = 0$ where $L(I, T_{\text{RE}})$ are the total losses given in Eq. (11), the solution is given by

$$Q(t) = 1 - e^{-\alpha t} \left[\left(\frac{\alpha + \beta}{2\beta} \right) e^{\beta t} + \left(\frac{\beta - \alpha}{2\beta} \right) e^{-\beta t} \right]. \quad (16)$$

For *critically-damped* systems ($\zeta = 1$), placing the same initial condition $Q(0) = 1 - L_0$ and $\dot{Q}(0) = 0$, the solution is given by

$$Q(t) = 1 - L_0 e^{-\omega t} (1 + \omega t). \quad (17)$$

Therefore, the expression of functionality is described by three parameters ω , ζ and L_0 for the case when the system is over-damped (Eq. (16)) and by two parameters model (ω and L_0) for the case when the system is critically damped (Eq. (17)). L_0 is related to the robustness dimension, while ω and ζ are related to the rapidity dimension. Furthermore, rapidity of recovery increases when either ω increases and ζ reduces as shown in Fig. 4(a)–(b).

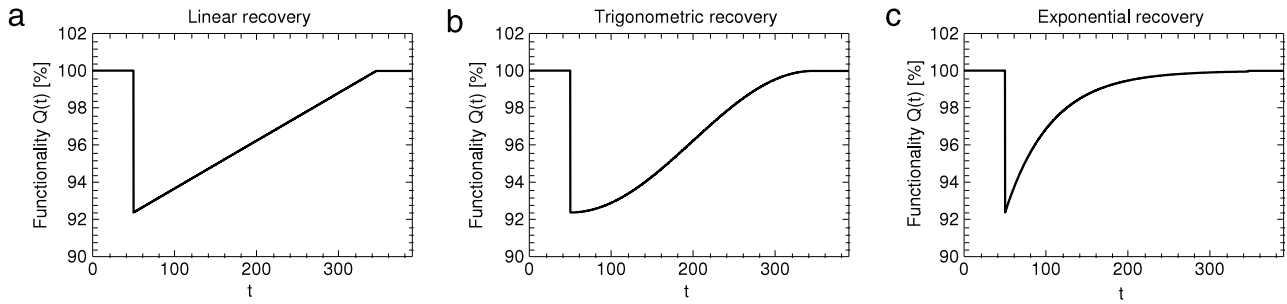


Fig. 3. Functionality curves (a) average prepared community, (b) not well prepared community, (c) well prepared community.

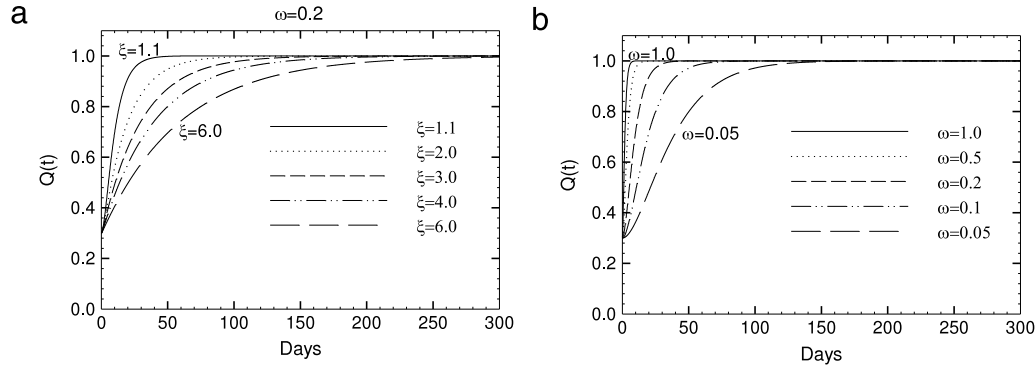


Fig. 4. Functionality curves (a) Three parameters model in Eq. (16); (b) Two parameters model in Eq. (17).

2.5. Fragility function

In the calculation of disaster resilience through functionality losses (Eq. (7)), fragility functions are used. Such fragility functions represent the probability that the maximum response $\mathbf{R}(\mathbf{x}, I, t) = \{R_1, \dots, R_n\}$ of a specific structure, (or family of structures), exceeds a given performance threshold $\mathbf{R}_{LS}(\mathbf{x}, I) = \{R_{LS1}, \dots, R_{LSn}\}$, associated with a desired limit state, conditional to the occurrence of an earthquake of intensity I .

The response, \mathbf{R} , and response threshold, \mathbf{R}_{LS} , are functions of the structural properties of the system \mathbf{x} , the ground motion intensity I and the time t . However, in the formulation it is assumed that the response threshold $\mathbf{R}_{LS}(\mathbf{x})$ does not depend on the ground motion history and so does not depend on time, while the demand $R_j(\mathbf{x}, I, t)$ of the generic j th component is replaced by its maximum value over the duration of the response history $R_j(\mathbf{x}, I)$. The dependence of the response $\mathbf{R}(\mathbf{x}, I)$ on \mathbf{x} and I , and the dependence of the response threshold $\mathbf{R}_{LS}(\mathbf{x})$ on \mathbf{x} will be omitted in the following for sake of simplicity. With these assumptions, the general definition of fragility $F_{r_{LS}}$ based on *Earthquake Intensity I* can be written as [24,25]

$$F_{r_{LS}}(i) = P(R_j \geq R_{LSj} | I = i) \quad (18)$$

where R_j is the response parameter related to a certain measure (deformation, force, velocity, etc.); $R_{LS,j}$ is the response threshold parameter correlated with the performance level; I is the *Earthquake Intensity* measure (*Pga*, *Pgv*, *Modified Mercalli Intensity*, etc.); i is a given earthquake intensity value.

However, another definition of fragility functions based on *earthquake hazard* can be given using the return period of the design earthquake. In order to find the expression of fragility curves as function of the earthquake hazard two assumptions are necessary: (i) the structural response are lognormally distributed under earthquake ground motions corresponding at the same probability of exceedance; (ii) the seismic hazard curves of the

structural responses are described by the following expression [26]

$$\lambda = H(r_{LS})_{1 \text{ yr}} = P(R \geq r_{LS})_{1 \text{ yr}} = 1/T_r = K_0 \cdot r_{LS}^{-K_1} \quad (19)$$

where λ = average annual frequency of the exceedance of a given response threshold; $H(\cdot)$ = seismic hazard curve function; T_r = return period between two exceeded response thresholds, K_0 and K_1 = parameters representing the seismic hazard curve. The estimates of λ are function of the geometry and material properties of the specific structure and therefore it needs to be estimated for each specific building.

The definition of fragility based on *earthquake hazard H* after some mathematical manipulations [25] is given by the following integral

$$F_{r_{LS}}(t_r) = P(R_i \geq r_{LSi} | T_r = t_r) = \int_{r_{LS}}^{\infty} f_R(r, t_r) dr \quad (20)$$

where the hazard is given by the return period t_r of a given earthquake event; $f_r(r, t_r)$ is the *pdf* of the maximum structural responses corresponding to a given annual frequency λ or return period t_r [25]. It is important to mention that there is not a *one-to-one correspondence* between *Earthquake Intensity I* and *Earthquake Hazard H*, in fact, different values of earthquake intensities I (*Pga*, *Pgv*, S_a etc.) can correspond to a unique earthquake hazard (e.g. T_r , the annual frequency of exceedance λ etc.). The advantage of the second formulation in Eq. (20) with respect to Eq. (18) is that it takes into account directly the uncertainties of occurrence in estimating the *Earthquake Intensity* parameters I at the site. Therefore, in professional practice, where buildings are designed according to a given return period t_r , (a measure of hazard), it is possible to use directly the expression of fragility curve given in Eq. (20) for evaluating directly the probability of functionality, or damage, of the system. When the number of response parameters to be checked is n the definition of fragility given in Eq. (20) can be written in the following form

$$F_{r_{LS}}(t_r) = P\left(\bigcup_{i=1}^n (R_i \geq r_{LSi}) \mid T_r = t_r\right) = \int_{r_{LS}}^{\infty} f_R(r, t_r) dr \quad (21)$$

where the first right term of Eq. (21) is the conditional probability of the multi-component response exceeding multi-dimensional limit state.

The definition of fragility in Eq. (18) requires implicitly the definition of the performance limit states, \mathbf{R}_{LS} , which is given using the multidimensional performance limit state function (MLS), that allows considering multiple limit states related to different quantities in the same formulation [24,25].

The MLS function $g(\mathbf{R}, \mathbf{R}_{LS})$ for the n -dimensional case, when n different types of limit states are considered simultaneously, is given by

$$g(\mathbf{R}, \mathbf{R}_{LS}) = \sum_{i=1}^n \left(\frac{R_i}{R_{LS,i}} \right)^{N_i} - 1 \quad (22)$$

where R_i is the dependent response threshold parameter (deformation, force, velocity, etc.), that is correlated with damage; $R_{LS,i}$ is the independent capacity threshold parameter and N_i are the interaction factors determining the shape of n -dimensional surface. Further details about the multidimensional performance limit state function can be found in Cimellaro and Reinhorn [25].

3. Numerical examples

Two case studies are illustrated in this section to show the implementation of the procedure for evaluating disaster resilience. The first case is a loss estimation study of a specific hospital; it is aimed to provide a more accurate evaluation of economic losses for buildings located at specific sites. In this case, an accurate analysis was performed using nonlinear dynamic analysis with an adequate description of limit state thresholds and their variability.

The second case is a regional loss estimation study aimed to evaluate the economic losses of a hospital network within a geographical region, such as a city (in this case Memphis, Tennessee). The responses of the buildings were estimated using an equivalent linearization spectral capacity method as presented by Reinhorn et al. [27] similar to the procedure described in HAZUS [28]. The limit states were expressed in terms of median and log-standard deviation chosen according to the building type and the design code [28].

3.1. Demonstration case study: Hospital building

The methodology described above has been applied to a hospital, an essential facility in the San Fernando Valley in Southern California, chosen as a typical case study for MCEER demonstration project. The hospital was constructed in the early 1970s to meet the seismic requirements of the 1970 Uniform Building Code.

It was selected since it is a complex structure with impact and implications related to various levels of functionality of services and structural safety [29].

The structure is a four-story steel framed building with plan dimensions of 83.90×17.25 m (275×56.5 ft). It is rectangular in plan, with one small penthouse in the central part of the building. The height of the building is 15.54 m (51 ft). The lateral force resisting system is comprised of four moment-resisting frames in the North-South direction and two perimeter moment-resisting frames in the East-West direction. Further details of the hospital can be found in Cimellaro et al. [24].

A MDOF numerical model was developed in IDARC2D [30] and used to perform the nonlinear time history analysis of the hospital. The ultimate curvature of the structural elements was set to 50 times the yield curvature, and the post-elastic stiffness was set equal to 1% to the elastic stiffness. A spread plasticity model has been assumed for the inelastic strain distribution.

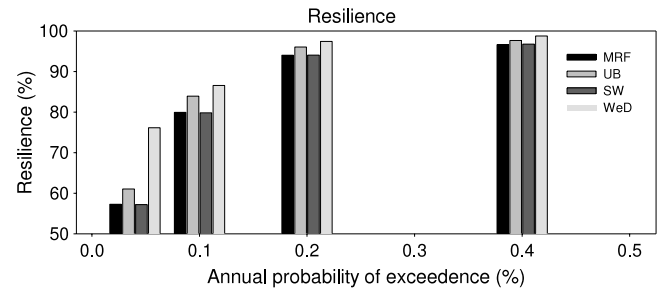


Fig. 5. Comparison of different rehabilitation strategies in term of disaster resilience.

The plasticized length is determined by the ratio between the maximum bending moment value in the element and the yield one of the element itself, and the inelastic stress has assumed to have a linear distribution inside the plastic regions. The assumed hysteretic model does not assume any degradation in stiffness or in strength, but only a reduction in the hysteretic energy dissipated in each cycle at the developing of the cyclic excitation of the system. The building is modeled as a series of plane frames linked by a rigid horizontal diaphragm, where each frame is in the same vertical frame, and no torsional effects are considered. It is a two dimensional model where all moment resisting frames are modeled with rigid beam-column connections and other beam-column connections of all the non-moment resisting frames (MRF) were assumed to be pinned.

A series of 100 synthetic near fault ground motions, described as the “MCEER series” [31] corresponding to different return periods (250, 500, 1000 and 2500 years) has been used to determine the fragility curves of the building [29] using the procedure described by Cimellaro et al. [24]. Losses have been determined according to HAZUS [28]. In this case study, a discount annual rate of 4% and a depreciation annual rate of 1% are assumed.

Resilience was calculated from the control time T_{LC} equal to the maximum recovery period T_{RE} , or 297 days in this example. Since the recovery period T_{RE} for each hazard is different, the resilience function changes little, implying that the structure has consistent design for various levels of hazards. When combining resilience associated with different hazard levels, a final value of 83.1% is obtained. Furthermore, four different seismic retrofit schemes to improve the disaster resilience of the hospital were considered for this case study: (a) Moment resisting frames (MRF); (b) Buckling restrained braces (UB); (c) Shear walls (SW) and (d) Weakening and Damping [29]. All retrofit strategies have been optimized with the procedure described in Viti et al. [29] and Cimellaro [32].

Table 1 shows the values of resilience for the four different retrofit techniques and for different probabilities of exceedence. The resilience values shown in the last row of Table 1 consider the uncertainties of the ground motion parameters. All values of resilience are normalized respect to the control period T_{LC} assumed equal to the largest recovery T_{RE} time among the different retrofit techniques. All values of resilience are comparable because all techniques are equally effective in improving the resilience of the hospital.

The same values of Resilience (y-axis) as function of the annual probability of exceedence (x-axis) are shown in Fig. 5. This shows that the best improvement in terms of resilience is obtained using a retrofit strategy based on weakening and damping (WeD). Although in term of resilience the difference seems small, the loss term (complementary to resilience) shows clearly the advantage of the WeD scheme. This retrofit technique produces both a reduction of displacements and of accelerations [29]. The reduction of accelerations is important for hospitals, because many of building contents (nonstructural components) are acceleration sensitive.

Table 1
Resilience vs. different hazard levels for different Retrofit strategies.

Resilience (%)				
Probability of exceedence in 50 yrs (%)	Moment resisting frames	Buckling restrained braces	Shear walls	Weakening + damping
(1)	(2)	(3)	(4)	(5)
20	96.6	97.6	96.8	98.8
10	94.0	96.1	94.0	97.4
5	79.9	83.9	79.8	86.6
2	57.3	61.1	57.2	76.1
Total hazard	83.1	86.8	83.2	91.03
Loss of resilience	16.9	13.2	16.8	9.0

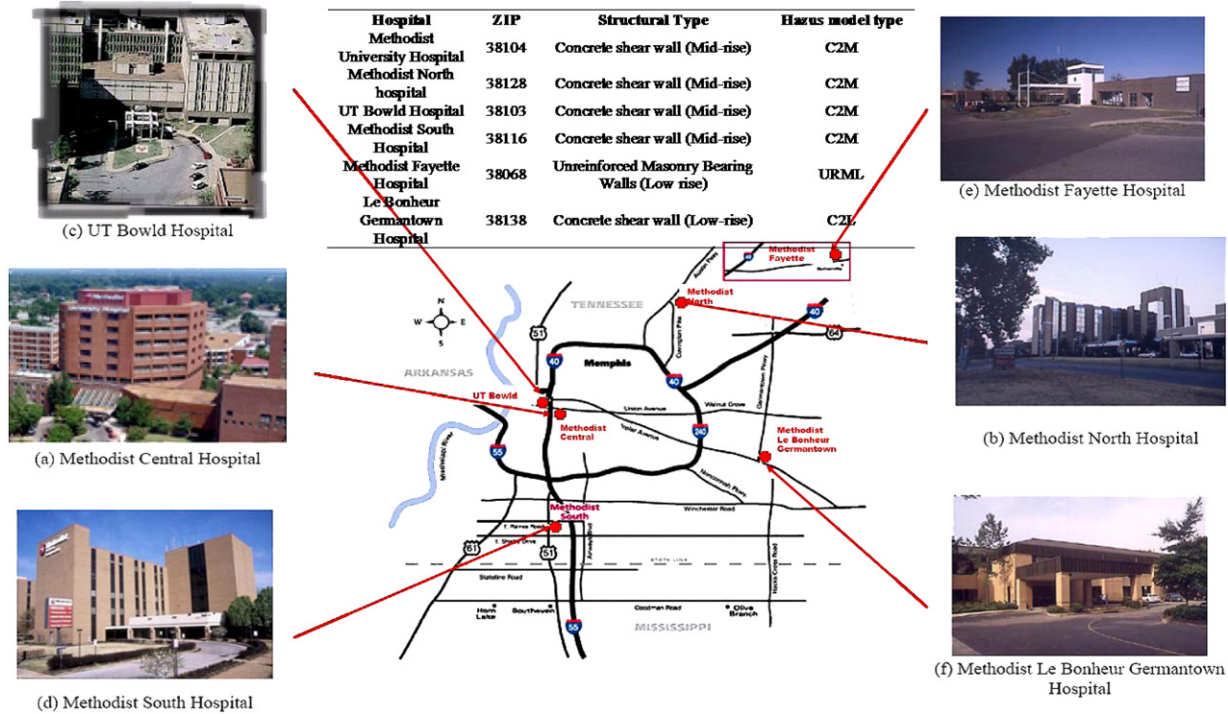


Fig. 6. Hospital network definition.

3.2. Retrofit of a hospital network

An example based on a series of hospital buildings described by Park et al. [33] is chosen to illustrate how to apply the proposed resilience framework to a group of structures. They consist of five concrete shear wall systems and one unreinforced masonry bearing system (Fig. 6).

Alternative retrofit actions are selected as defined in FEMA 276 [34] and directly correlated to the HAZUS code levels. Therefore, the HAZUS code levels are assigned as performance measures (PM) to the retrofit strategies mentioned above with following assumptions: (i) It is assumed that the “No Action” option, corresponds to the “low” code level; (ii) “Retrofit to life safety level” option is assumed to be a “moderate” code level; and (iii) “Retrofit to immediate occupancy level” option is assumed to be a “high” code level. For the “rebuild option”, a special “high” code level is assumed because hospitals are classified as essential facilities. Four hazard levels are considered for generation of the loss-hazard curves taking into account a range of levels of earthquakes in the region. These levels include earthquakes with 2%, 5% 10% and 20% probability of exceedence P in 50 years. The control time for the decision analysis is usually based on the decision maker’s interest in evaluating the retrofit alternatives. A 50 years control period could be chosen for evaluating the

hospital systems, which may be consistent with the period used for calculation of earthquake hazards (e.g. as in 2% probability of exceedence in 50 years). However, a decision maker in charge with financing the retrofit could be interested in a shorter period, more in line with the lifespan of new construction. Generally, seismic losses associated with seismic vulnerable structures increases if longer control periods are considered. For example, retrofit can hardly be justified for a one-year period because the probability of encountering a large earthquake within this period is very low, whereas the probability increases appreciably for a 50-year period, so the retrofit becomes more cost-effective in reducing losses. A decision maker siding with the users’ community could be interested therefore in a longer T_{LC} . In this example, a control period of 30-years is assumed for T_{LC} as the baseline value in line with the lifespan of the structure as mentioned above.

As indicated before, four alternative actions related to retrofit are considered for each structural type: (1) no action; (2) rehabilitation to life safety level; (3) retrofit to the immediate occupancy level; (4) construction of a new building. The retrofit levels are, as defined in FEMA 276 [34], the target performance expected for earthquake rehabilitation. The cost of seismic retrofit for building systems depends on numerous factors, such as building type, earthquake hazard level, desired performance level, occupancy or usage type. These costs generally increase as the

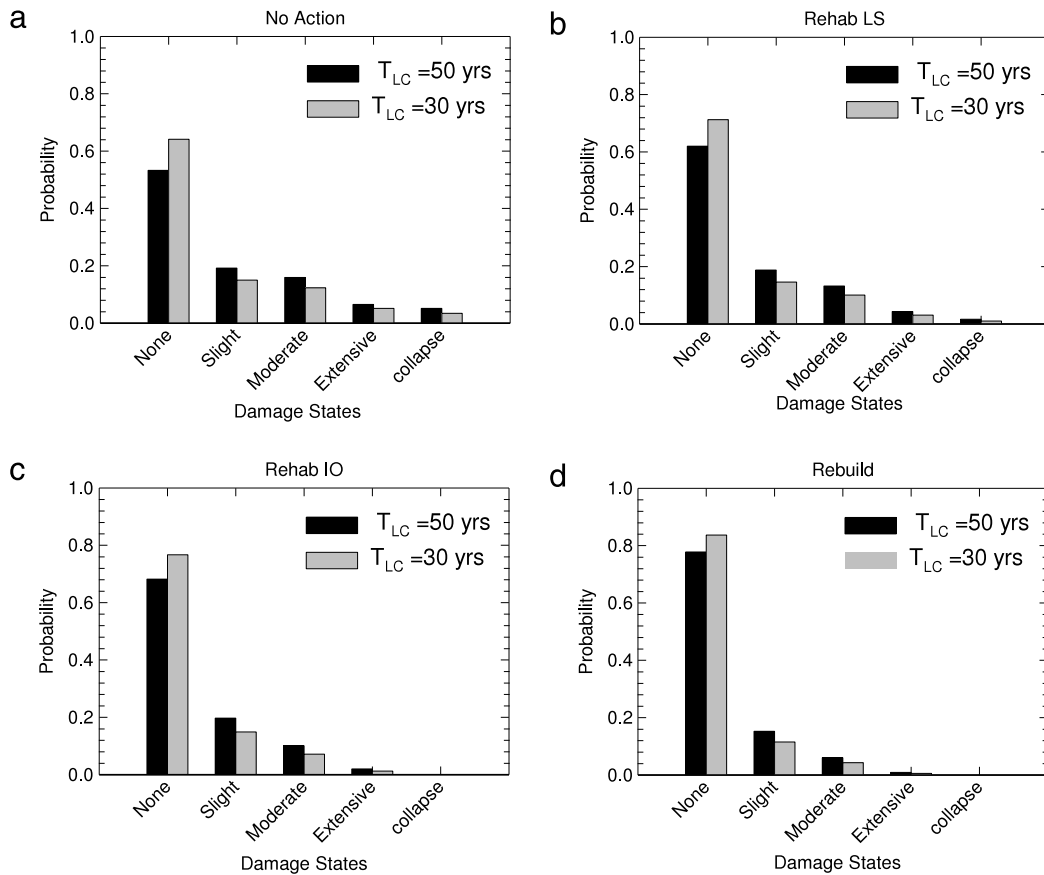


Fig. 7. Structural performance (damage) distributions for different rehabilitation strategies.

target performance level becomes higher. On the contrary, with higher performance levels lower seismic losses are expected. The initial retrofit costs for the options considered here are obtained from FEMA 227 [35] and FEMA 156 [36]. Damage fragility curves are generated for both structural and nonstructural damage, using HAZUS assessment data. The nonstructural damage fragility curves consist of acceleration-sensitive components and drift sensitive components [28]. In this way the structural, the nonstructural acceleration sensitive, and the drift-sensitive damage, can be assessed separately using their respective fragility curves. In this example both structural and nonstructural damage fragility curves for C2L, C2M and URML type structures for different code levels are generated [11].

Fig. 7 shows the performance (damage) distributions for the C2M structures within a 30 year period, compared with a 50 year period. As expected, the probability of having no damage increases with the reduced control period. More details can be found in Cimellaro et al. [11]. Among the large number of seismic losses described in the previous sections, several attributes that are typically considered crucial for hospital systems are selected for this study and are listed in Table 2 along with a brief explanation of each parameter.

Losses in undamaged sectors of the hospital due to business interruption are not considered in this example.

The disaster resilience value is calculated according to Eq. (1). The expected equivalent earthquake losses for each rehabilitation scheme are shown in the third column of Table 3, which are obtained considering the probability of each level of the earthquake, along with the initial rehabilitation costs, followed by the total expected losses considering an observation period T_{LC} of 30 years.

If uncertainties in the seismic input are considered by using four different hazard levels, then resilience can be evaluated using

Eq. (1) for different rehabilitation strategies and compared, as shown in Fig. 8. The initial costs of rehabilitation for different rehabilitation strategies, the expected equivalent earthquake loss and the total costs (including the initial costs of the entire system that is estimated equal to 87.3 million \$) are all reported in Table 3.

The recovery time and resilience values are summarized in Table 3. For this case study, it is shown that the Rebuild option has the largest disaster resilience of 98.7%, when compared with the other three strategies, but it is also the most expensive solution (\$ 92.3 millions). However, if No Action is taken the disaster resilience is still reasonably high (65.0%). As shown in this case study, initial investments and resilience are not linearly related. When the functionality $Q(t)$ is very high, improving it by a small amount requires investing a very large amount compared with the case when the function $Q(t)$ of the system is low. Although this is an obviously expected engineering outcome, the procedure presented here provides a quantification, which may be used by decision makers.

4. Remarks and conclusions

The definition of disaster resilience combines information from technical and organizational fields, from seismology and earthquake engineering to social science and economics. Many assumptions and interpretations have to be made in the study of disaster resilience. However, the final goal is to integrate the information from these different fields into a unique function leading to results that are unbiased by uninformed intuition or preconceived notions of risk. The goal of this paper has been to provide a framework for the quantitative definition of resilience using an analytical function that may fit both technical

Table 2
Losses considered in the hospital network.

Category	Loss	Description
Structural losses (L_S)	Initial cost	Cost of seismic rehabilitation or constructing a new building to improve structural performance
	Structural repair cost	Cost for repairing damage to structural components such as beams, columns, joints, etc.
Nonstructural losses (L_{NS})	Direct economic losses ($L_{NS,DE}$)	Cost for repairing damage to nonstructural components such as architectural, electrical and mechanical items.
	Loss of building contents	Cost equivalent to the loss of building contents such as furniture, equipment (not connected to the structure), computers, etc.
	Indirect economic losses ($L_{NS,IE}$)	Disruption cost and rental cost for using temporary space in case the building must be shut down for repair
	Indirect casualties losses ($L_{NS,IC}$)	Loss of functionality
	Direct casualties losses ($L_{NS,DC}$)	Death Injury

Table 3
Costs, recovery time and resilience of buildings for rehabilitation strategies ($T_{LC} = 65$ days).

Rehabilitation alternatives	Rehabilitation costs \$ million ^a	Expected earthquake loss \$ million ^a	Total costs \$ million	Recovery time T_{RE} (days)	Resilience Res (%)
(1)	(2)	(3)	(4)	(5)	(6)
No action	0.0 (0%)	32.3 (37%)	119.7	65	65.0
Life safety (LS)	32.8 (38%)	18.8 (22%)	138.9	38	87.1
Immediate occupancy (IO)	66.4 (76%)	9.5 (11%)	163.2	10	96.8
Rebuild	92.3 (106%)	5.8 (7%)	185.4	6	98.7

^a Percentage of initial investments.

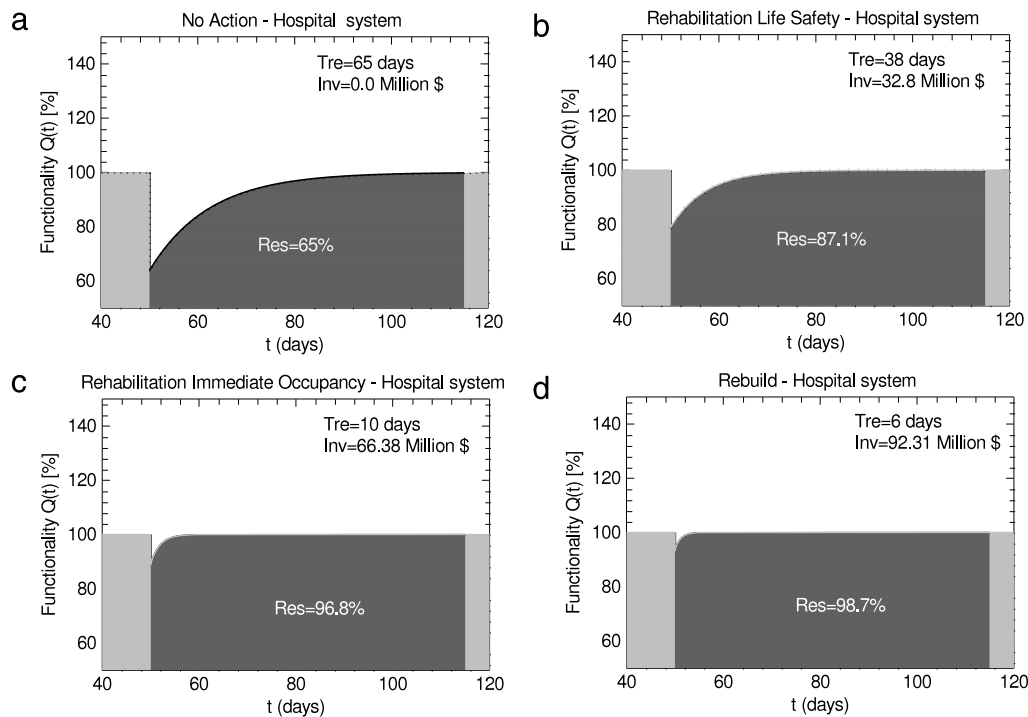


Fig. 8. Functionality curves: (a) No action; (b) Life safety rehabilitation; (c) Immediate occupancy rehabilitation; (d) Rebuild for entire hospital system.

and organizational issues. The fundamental concepts of disaster resilience discussed herein provide a common frame of reference and a unified terminology. Two applications of this methodology to health care facilities are presented in order to show the implementation issues. However, it is important to note that the assumptions made herein are only representative for the cases presented. For other problems, users can focus on those assumptions that are mostly affecting the problem at hand, while using the case study as guidance.

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References

- [1] Manyena SB. The concept of resilience revisited. *Disasters* 2006;30:434–50.
- [2] Smith N, O'Keefe P. Geography, man and the concept of nature. In: Agnew J, Livingstone DN, Roger A, editors. *Human geography: an essential anthology*. Blackwell Publishing; 1996. p. 283–95.
- [3] Bruneau M, Chang S, Eguchi R, Lee G, O'Rourke T, Reinhorn AM, et al. A framework to quantitatively assess and enhance the seismic resilience of communities. *Earthq Spectra* 2003;19(4):733–52.
- [4] Bruneau M, Reinhorn AM. Exploring the concept of seismic resilience for acute care facilities. *Earthq Spectra* 2007;23(1):41–62.
- [5] Miles SB, Chang SE. Modeling community recovery from earthquakes. *Earthq Spectra* 2006;22(2):439–58.
- [6] Davidson R, Cagnan Z. Restoration modeling of lifeline systems. 2004.
- [7] Chang SE, Shinozuka M. Measuring improvements in the disaster resilience of communities. *Earthq Spectra* 2004;20(3):739–55.
- [8] Cimellaro GP, Reinhorn AM, Bruneau M. Resilience of a health care facility. In: Proceedings of annual meeting of the Asian Pacific network of centers for earthquake engineering research. ANGER. 2005.
- [9] Porter KA, Kiremidjian AS, LeGrue JS. Assembly-based vulnerability of buildings and its use in performance evaluation. *Earthq Spectra* 2001;(17):291–312.
- [10] Mileti D. *Disasters by design: a reassessment of natural Hazards in the United States*. Washington (DC): Joseph Henry Press; 1999. p. 351.
- [11] Cimellaro GP, Fumo C, Reinhorn AM, Bruneau M. Quantification of seismic resilience of health care facilities. MCEER technical report-MCEER-09-0009. Buffalo (NY): Multidisciplinary center for earthquake engineering research; 2009.
- [12] Kendra J, Wachtendorf T. Elements of resilience after the world trade center disaster: reconstituting New York city's emergency operations centre. *Disasters* 2003;27(1):37–53.
- [13] FEMA 356. Pre-standard and commentary for the seismic rehabilitation of buildings. Washington (DC): Federal Emergency Management Agency and US army corps of engineers 2000; 2000.
- [14] Peek-Asa C, Kraus JF, Bourque LB, Vimalachandra D, Yu J, Abrams J. Fatal and hospitalized injuries resulting from the 1994 northridge earthquake. *Int J Epidemiol* 1998;27(3):459–65.
- [15] Werner SD, Taylor CE, Cho S, Lavoie J-P, Huyck C, Eitzel C, et al. REDARS 2 methodology and software for seismic risk analysis of highway systems. MCEER technical report—MCEER-06-SP08. Buffalo (NY): MCEER Multidisciplinary Center Earthquake Engineering Research; 2006.
- [16] Miles SB, Chang SE. Urban disaster recovery: a framework and simulation model. MCEER technical report—MCEER-03-0005. Buffalo (NY): State University of New York, University at Buffalo; 2003.
- [17] Cornell A, Krawinkler H. Progress and challenges in seismic performance assessment. *Peer News* 2000;3(2).
- [18] Whitman RV, Anagnos T, Kircher CA, Lagorio HJ, Lawson RS, Schneider P. Development of a national earthquake loss estimation methodology. *Earthq Spectra* 1997;13(4):643–62.
- [19] Chang SE, Nojima N. Measuring post-disaster transportation system performance: the 1995 Kobe earthquake in comparative perspective. *Transp Res A* 2001;35(6):475–94.
- [20] Chang SE. Transportation performance, disaster vulnerability, and long-term effects of earthquakes. In: Second euroconference on global change and catastrophe risk management. 2000.
- [21] Kafali C, Grigoriu M. Rehabilitation decision analysis. In: ICOSAR'05: proceedings of the ninth international conference on structural safety and reliability. 2005.
- [22] Chang SE, Shinozuka M. Measuring improvements in the disaster resilience of communities. *Earthq Spectra* 2004;20(3):739–55.
- [23] Filiatrault A, Uang C-M, Folz B, Christopoulos C, Gatto K. Reconnaissance report of the February 28, 2001 Nisqually, Seattle-Olympia, earthquake University of California at San Diego. 2001.
- [24] Cimellaro GP, Reinhorn AM, Bruneau M, Rutenberg A. Multidimensional fragility of structures: formulation and evaluation. MCEER technical report—MCEER-06-0002. Buffalo (NY): University at Buffalo, The State University of New York; 2006. p. 123.
- [25] Cimellaro GP, Reinhorn AM. Multidimensional performance limit state for hazard fragility functions. *J Eng Mech, ASCE*. 2010 in press [doi:10.1061/(ASCE)EM.1943-7889.0000201].
- [26] Cornell A. Calculating building seismic performance reliability: a basis for multi-level design norms. In: Proceedings of the 11th world conference on earthquake engineering. 1996.
- [27] Reinhorn AM, Barron-Corverra R, Ayala AG. Spectral evaluation of seismic fragility of structures. In: Proceedings ICOSAR 2001. 2001.
- [28] FEMA. HAZUS-MH version 1.1. FEMA's software program for estimating potential losses from disasters. Technical manual Federal Emergency Management Agency and US army corps of engineers 2005. Washington (DC); 2005 (Available at: www.fema.gov/hazus).
- [29] Viti S, Cimellaro GP, Reinhorn AM. Retrofit of a hospital through strength reduction and enhanced damping. *Smart Struct Syst Int J* 2006;2(4):339–355.
- [30] Reinhorn AM, Roh H, Sivaselvan M, Kunnath SK, Valles RE, Madan A, et al. IDARC 2D version 7.0: a program for the inelastic damage analysis of structures. MCEER technical report—MCEER-09-0006. University at Buffalo—The State University of New York; 2009.
- [31] Wanitkorkul A, Filiatrault A. Simulation of strong ground motions for seismic fragility evaluation of nonstructural components in hospitals. MCEER technical report—MCEER-05-0005. Buffalo (NY): Multidisciplinary center for earthquake engineering research; 2005.
- [32] Cimellaro GP. Simultaneous stiffness-damping optimization of structures with respect to acceleration, displacement and base shear. *Eng Struct* 2007;29:2853–70.
- [33] Park J, Goodno B, Bostrom A, Craig J. Probabilistic decision analysis for seismic rehabilitation of a regional building system. In: Proceeding of 13th world conference on earthquake engineering. Paper #2254. 2004.
- [34] FEMA. Example applications of the NEHRP guidelines for the seismic rehabilitation of buildings. FEMA 276. Washington (DC): Federal Emergency Management Agency and US army corps of engineers; 1999.
- [35] FEMA. A benefit-cost model for the seismic rehabilitation of buildings. FEMA 227. Federal Emergency Management Agency and US army corps of engineers. 1992.
- [36] FEMA. Typical costs for seismic rehabilitation of existing buildings, vol. 1—summary. FEMA 156. Washington (DC): Federal Emergency Management Agency and US army corps of engineers; 1995.