Seismic behavior of bidirectional-resistant ductile end diaphragms with buckling restrained braces in straight steel bridges

Oguz C. Celik, Michel Bruneau

Abstract

The ductile end diaphragm concept developed for regular (i.e., straight) slab-on-girder or deck-truss steel bridge superstructures is expanded to make it applicable to bidirectional earthquake excitation. Buckling restrained braces (BRBs) are used as the ductile fuses. Two retrofit schemes (Retrofit Scheme-1 and Retrofit Scheme-2) are investigated to seek the best geometrical layout to maximize the dissipated hysteretic energy of the ductile diaphragms with BRB end diaphragms. Closed form solutions are presented for practical design purposes. Behavioral characteristics (strength, stiffness, and drift) of the proposed retrofit schemes for end diaphragms are quantified with an emphasis on hysteretic energy dissipation. Results from selected numerical examples show that the generic bridge geometry, bidirectional loading, and the loading ratio (or the assumed combination rule), have a pronounced effect on the end diaphragm’s inelastic behavior. Volumetric hysteretic energy dissipation is used to compare the effectiveness of the proposed retrofit schemes under several loading cases. These indicate that, in most cases, Retrofit Scheme-1 is superior to Retrofit Scheme-2 and may exhibit better seismic response.

1. Introduction

Many slab-on-girder and deck truss steel bridges in North America are located in seismic regions. Since most of them were built without seismic design considerations, they may suffer damage in future earthquakes. The end diaphragms in these bridges generally do not have ductile details (members and connections). Recent earthquake reconnaissance investigations have reported damage in bridge end diaphragms due to transverse earthquake effects [Bruneau et al. [1], Itani et al. [2]]. Seismic retrofit research strategies are needed for this problem. One approach that has been proposed (Zahrai and Bruneau [3,4]; Bruneau et al. [5], Carden et al. [6,7]) suggests that special ductile end diaphragms could provide an appropriate retrofit solution. This concept requires replacing existing end diaphragms with specially detailed diaphragms that can act as "seismic fuses," i.e., which could yield prior to other substructure and superstructure elements. This concept has been experimentally verified using specially designed ductile end diaphragms that have one of the following: shear panel systems (SPS), steel triangular plate added damping and stiffness devices (TADAS), eccentrically braced end diaphragms (EBF), or buckling restrained braces (BRBs) also known as unbonded braces. However, in all cases considered to date, the ductile diaphragm concepts were limited to the retrofit of bridges against earthquake excitation in the transverse direction, and had to be combined with another retrofit solution to achieve resistance in the longitudinal direction.

BRBs have recently been implemented in buildings as energy dissipation members, mostly in Japan and in the United States. Because of their stable, unpinched hysteretic characteristics and ease of design, the rate of implementation in building applications is increasing. BRBs have also been used to retrofit the Minato bridge in Japan (Kanaji et al. [8]; Kanaji et al. [9]), the world’s third longest truss bridge, using a concept similar to the one developed by Sarraf and Bruneau [10,11].

In light of the superior hysteretic behavior of BRBs over “traditional” braces and other energy dissipation devices which were shown to be effective in bridge end diaphragms to improve seismic response in the transverse direction, this paper investigates the possibility of extending the ductile end diaphragm concept using BRBs to resist bidirectional earthquake effects in straight bridge superstructures.
2. Proposed retrofit schemes

For seismic retrofit purposes, two types of bracing configurations acting as ductile fuses in bridge end diaphragms are considered:

- **Retrofit Scheme-1**: Two pairs of BRBs are installed at each end of a span, in a configuration that coincides with the transverse and longitudinal directions (Fig. 1a). In other words, one pair of BRBs are oriented parallel to the longitudinal axis of the bridge, connecting the abutment to the underside of the bridge deck (or other component of the bridge if preferred), and another pair is in the conventional diaphragm configuration, and is thus perpendicular to the axis of the bridge in straight bridges.

- **Retrofit Scheme-2**: A single pair of BRBs are installed at each end of a span, in a configuration that does not coincide with the bridge longitudinal and transverse directions (Fig. 1b). Instead, the BRBs connect the abutment to the underside of the bridge deck at a certain distance and an angle that allow them to resist lateral loads applied in all horizontal directions.

Note that in Retrofit Scheme-1 (in Fig. 1a), the pair of braces oriented in the transverse direction can be connected either to the abutment or between web stiffeners of the bridge girders (the latter choice is common for steel bridges). The pair of longitudinal braces is a new concept and need to be connected at the abutment, either at the bearing level (on the horizontal side) or on the vertical side. The braces connecting to the abutment need to be in series with lock-up devices that allow thermal expansion under normal conditions, but engage the braces during earthquakes. Detailing decisions depend on the existing boundary conditions of the girders. For the deck level connection, specially designed cross beams are required to elastically resist forces from the BRB, unless connection to the existing interior cross frames or girders is developed without damaging any internal component (capacity design).

3. Modeling bridge end diaphragms and assumptions

3.1. Bearings

Neoprene bearings, bidirectional sliding bearings, and other bearings with negligible strength to horizontal deformations (and to some degree, even bearings damaged by an earthquake that could still slide in a stable manner) are considered in this work. This case is further called the “floating span.” Floating span type bridges have no resistance to lateral earthquake loading, and therefore need to be restrained laterally by devices to limit their horizontal displacements. In this study, the BRBs serve this purpose.

3.2. Buckling restrained braces (BRBs)

Black et al. [12] characterized the hysteretic behavior of BRBs. The results suggested that bilinear approximation could be used with confidence, since a good agreement in seismic response was observed between results obtained from the bilinear models for a set of earthquake data. Moreover, Sabelli et al. [13] modeled BRBs as simple truss elements with ideal bilinear hysteretic behavior, that does not exhibit stiffness or strength degradation. A bilinear hysteretic model for BRBs is therefore assumed in this study. Although some studies (Carden et al. [7], Black et al. [12]) suggest that BRBs may have compressive strength up to 10%–15% greater than their tensile strength, this effect is neglected here for simplicity. Additionally, post-yield stiffness of braces is set to zero, assuming an elastic-perfectly plastic axial force–displacement relationship although the post-yield stiffness ratio may reach 0.025 in BRBs (Carden et al. [7]). Since closed form expressions are sought for practical design purposes, these approximations help reduce the complexity of these expressions.
3.3. System idealization

Previous research on the behavior of steel slab-on-girder bridges demonstrates that seismic demand under lateral load concentrates at the end diaphragms (Zahrai and Bruneau [14]). Furthermore, the presence of intermediate cross braces does not impact the seismic behavior of these bridges and can be neglected. This leads to the development of a simplified structural model to simulate the system behavior. For Retrofit Scheme-1, the steps followed to idealize a typical bridge with end diaphragms into a simpler model are given in Fig. 2. A similar process of idealization is followed for Retrofit Scheme-2.

First, the steel girders and concrete deck are considered rigid in their own plane. The concrete deck and the steel girders are continuously connected, but are assumed to be fully flexible about their connection axis (parallel to the bridge axis); i.e., the angle between the plane of the concrete deck and the plane of the steel beam can change without developing out-of-plane flexural moments. Second, by removing the steel girders, the only restraint of the concrete deck against horizontal lateral loads are provided by the BRBs. The boundary conditions can therefore be modeled as fixed pin support to which the BRBs can connect. Finally, the interior segment of the bridge is eliminated and the two end segments of the bridge are brought together. Furthermore, since the deck is rigid, it is possible to superimpose the two segments on top of each other, as shown at the bottom of Fig. 2.

These assumptions, along with the assumed pinned end connections for BRBs, lead to an idealized and relatively simple system model which is actually a three-dimensional truss supporting a rigid deck. This simplified model captures the actual behavior of slab-on-girder bridges with the various configurations of BRB diaphragms considered when subjected to lateral loading applied at deck level. Another assumption is that the BRBs are not active under gravity loading.

Closed form solutions are obtained for bidirectional earthquake excitations for the two different diaphragm bracing configurations considered. These formulas can be used to investigate load–displacement behavior for the proposed retrofit systems. The analytical models account for general system geometric dimensions, girder spacing (s), end diaphragm depth (d) and length to internal diaphragm anchor point (a), as well as bidirectional earthquake effects. Of all of these values, the spacing of girders and the girder depth are already known if the bridge is an existing structure. The value of “a” could be eventually chosen to be a function of the girder spacing and would be selected based on engineering judgment (the outcome of this study could help in selecting an appropriate value for this parameter). Cross sectional areas of BRBs are taken to be the same for each of the two end diaphragms used in each specific bridge.

4. Closed form solutions for Retrofit Scheme-1

Currently, many structural analysis software programs (including SAP2000 [15]) enable users to perform static pushover analysis.

Please cite this article in press as: Celik OC, Bruneau M. Seismic behavior of bidirectional-resistant ductile end diaphragms with buckling restrained braces in straight steel bridges. Engineering Structures (2008), doi:10.1016/j.engstruct.2008.08.013
However, when the bridge system is complex, data generation may be cumbersome, and the numerical results obtained usually need to be checked using simple models for reliability purposes. Thus, analytical closed form solutions can be powerful tools in simplified analysis and for preliminary design purposes. Fig. 2 (as explained previously) shows the selected configuration of BRBs for Retrofit Scheme-1.

In that model, all braces and other members representing the existing bridge elements are assumed to be pin connected. The bridge deck is idealized by truss elements with infinite axial stiffness. Equal proportions of the total lateral load in a given direction are applied at each corner of the deck. $P_L$ and $P_T$ are the lateral earthquake loads acting at the deck level on one diaphragm in the longitudinal and transverse directions, respectively.

Nonlinear pushover analysis is adopted in this paper. BRBs are treated as axially yielding members with cyclic symmetric elastic-plastic behavior. The ratio of $P_T/P_L$ (or $P_T/P_L$) is typically set constant in pushover analyses. As the system yields in one direction, the forces cannot increase anymore, and the displacement path “bifurcates” toward the yielding direction. Pushover stops when the prescribed axial displacement ductility level (i.e. $\mu$) of BRBs is reached. Also, dissipated energy is calculated at maximum BRB ductility reached.

4.1. Brace axial force ratio (elastic behavior)

Total base shear forces in the elastic range are equal to $V_L = 2P_L$ and $V_T = 2P_T$, since there are two end diaphragms considered in this model. With reference to the three-dimensional idealized truss system given in Fig. 2, static equilibrium gives the following brace axial force ratio under bidirectional loading:

$$\frac{C_T}{C_L} = \frac{T_T}{T_L} = \frac{P_T}{P_L} \sqrt{\frac{1 + (d/s)^2}{1 + (d/a)^2}}$$

(1)

where $P_T/P_L$, $C_T$, $T_T$, $C_L$, $T_L$ are the ratio of bidirectional loads (transverse to longitudinal), axial compression and tension forces in longitudinal BRBs, axial compression and tension forces in transverse BRBs, respectively. The possible limits of this ratio and the corresponding meaning are further described below:

If $\frac{C_T}{C_L} = \frac{T_T}{T_L} > 1$ then the transverse braces yield first

If $\frac{C_T}{C_L} = \frac{T_T}{T_L} < 1$ then the longitudinal braces yield first

If $\frac{C_T}{C_L} = \frac{T_T}{T_L} = 1$ then all the braces yield at the same time.
After surveying a bridge inventory in North America, it is recognized that practical numerical values for \(d/s\) fall in the range of 0.25, 0.50, 1.00, 1.25, and 1.50, covering most short and medium span slab-on-girder and deck-truss steel bridges. Also, \(d/a\) can be set equal to 0.20, 0.40, 0.60, 0.80, and 1.00 (Celik and Bruneau [16]). Using these values, variations of the brace axial force ratio with respect to end diaphragm geometric relationships are given in Fig. 3. Since many bridge standards and regulations basically rely on two simplified combination rules to account for bidirectional earthquake effects in seismic design, the 30% rule as per AASHTO [17] and the 40% rule as per ATC-32 [18] are selected to show the impact of this value on the brace force ratio.

Since the bridge behavior is bidirectional due to bidirectional loading and there are two possible yielding mechanisms, both transverse and longitudinal responses are investigated separately for both transverse and longitudinal braces yielding cases.

### 4.2. Behavior when transverse braces yield

#### 4.2.1. Transverse response

Fig. 4 shows a typical hysteretic curve of the system both in the transverse and longitudinal directions. When \(C_T > C_L, T_L\), only the transverse braces yield, and base shear strength \(V_{yT}\), yield displacement \(\Delta_{yT}\) and corresponding drift \(\Delta_{yT}/d\) at yield, global ductility \(\mu_{GT}\), and the stiffness of the system \(K_T\) in the transverse direction are obtained depending on the bridge geometry and BRB properties as follows:

\[
V_{yT} = \frac{n_T}{\sqrt{1 + (d/s)^2}} (F_y A) 
\]

\[
\Delta_{yT} = \frac{(s^2 + d^2)}{s} \left( \frac{F_y}{E} \right) 
\]

\[
\Delta_{yT}/d = \frac{1}{d/s} \left( \frac{F_y}{E} \right) 
\]

\[
\mu_{GT} = \mu 
\]

\[
K_T = \frac{n_T (d/s)}{\left[1 + (d/s)^2\right]^{3/2}} \left( \frac{EA}{d} \right) 
\]

\(n_T\) and \(n_L\) (as shown later) are the number of braces placed in the transverse and longitudinal directions, respectively. An equal number of braces in both directions are used in this study.

Please cite this article in press as: Celik OC, Bruneau M. Seismic behavior of bidirectional-resistant ductile end diaphragms with buckling restrained braces in straight steel bridges. Engineering Structures (2008), doi:10.1016/j.engstruct.2008.08.013
Non-dimensional expressions have also been generated to generalize these equations, yet, a single specific value of the yield strength, \( F_y \), is used. Recent investigations and implementations in buildings with BRBs suggest that unbonded steel core materials with a grade ranging from low yield strength (100–235 MPa) up to high yield strength 415 MPa (60 ksi) could be used successfully. For bridge retrofit design purposes, a 345 MPa (50 ksi) grade steel with yield strength 415 MPa (60 ksi) could be used successfully. For grade ranging from low yield strength (100–235 MPa) up to high yield strength (415 MPa), the variation in drift is relatively insignificant, suggesting that the appropriate value for the d/s ratio increases as the d/s ratio increases due to larger direction angles resulting in smaller horizontal force components.

4.2.2. Longitudinal response

Since yielding only occurs transversely for this Retrofit Scheme, members in the longitudinal direction remain elastic. The longitudinal base shear is written as:

\[
V_L = \frac{n_T (P_L/P_T)}{\sqrt{1 + (d/s)^2}} (F_{yA}).
\]  

(7)

The longitudinal displacement when transverse braces yield is:

\[
\Delta_L = \frac{s (P_L/P_T)}{a^2 \sqrt{2 + d^2}} \left( \frac{F_y}{E} \right).
\]  

(8)

and the corresponding drift is written as:

\[
\frac{\Delta}{d} = \frac{(P_L/P_T)}{(d/a) \sqrt{1 + (d/s)^2}} \left( \frac{F_y}{E} \right).
\]  

(9)

The longitudinal stiffness in terms of non-dimensional geometric ratios is:

\[
K_L = \frac{n_T (d/a)}{\left[ 1 + (d/a)^2 \right]^{3/2}} \left( \frac{EA}{d} \right).
\]  

(10)

Non-dimensional base shear and drift are also dependent on the previously defined bidirectional load ratio \( P_L/P_T \) or \( P_T/P_L \), Two code defined values of \( P_L/P_T \) are assumed, namely 0.30 and 0.40, to explore the impact of the combination rules on system behavior. However, the widely accepted 30% rule is used in developing the forthcoming diagrams in this paper. Further details of these derivations and the diagrams obtained for the 40% rule can be found in Celik and Bruneau [16]. Fig. 6a shows the variation of non-dimensional base shear in the longitudinal direction as a function of end diaphragm geometric ratios when transverse braces yield. There is a decrease in this value as the d/s ratio increases due to larger direction angles resulting in smaller horizontal force components.

The variation of longitudinal drift as a function of end diaphragm geometric ratios and the \( P_L/P_T \) value are shown in Fig. 6b. For a constant d/s, these curves reveal that longitudinal drift becomes minimum at \( d/a = 0.707 \). However, as seen on the same figure, when the ratio of d/a is greater than 0.5, the variation in drift is relatively insignificant, suggesting that appropriate d/a ratios could be selected between 0.5 and 1.0, if the intent is to minimize drift. Again, Fig. 6c shows that maximum non-dimensional longitudinal stiffness is reached at \( d/a = 0.707 \).

4.2.3. Dissipated energy per BRB volume

Generally, the total hysteretic energy dissipated in one cycle is the sum of the areas under the global hysteretic curves in both directions (i.e., summation of the areas under Fig. 4a and b), or simply equal to the energy dissipated by the yielding braces. Both are equivalent (per the conservation of energy principle), but the former gives the energy dissipation in terms of global ductility, while the latter gives results in terms of the member ductility \( (\mu) \), which seems more convenient to obtain simpler formulas. The following expression gives the volumetric energy dissipation for the system considered:

\[
\frac{E_H}{\text{Vol.}} = \frac{4(\mu - 1)}{1 + (d/s)^2} \left[ E \right] \left( \frac{F_y^2}{P_T} \right).
\]  

(11)

Note that recent experimental investigations on BRBs suggest that these braces can exhibit stable and ductile hysteretic behavior up to member axial displacement ductilities of \( \mu = 20 \) or more. To investigate the possible hysteretic energy dissipation in bridge end diaphragms using BRBs, the variation of hysteretic energy dissipation per brace volume is plotted in Fig. 6d for different bridge geometries (i.e., d/a and d/s ratios) for \( \mu = 20 \). The impact of member ductility ratio on the dissipated energy is obvious and investigated in Celik and Bruneau [16]. Hysteretic energy increases (logically) as member ductility increases. Fig. 6d illustrates that non-dimensional dissipated hysteretic energy increases as d/a increases for constant values of d/s (which could be important in an existing bridge retrofit design), but decreases as d/s increases for constant values of d/a (which could be important in a new bridge design). However, as observed on the relevant diagrams,
the decrease in energy dissipation is relatively less for larger values of d/s. Apparently, there is no optimal hysteretic energy dissipation within the assumed geometric range in this special case. Note that since longitudinal response is elastic, all hysteretic energy is dissipated by the transverse braces. In other words, displacement in the longitudinal direction remains unchanged upon yielding of the transverse braces. During cyclic loading, only elastic loading/unloading develops in the longitudinal braces. For the response in the longitudinal direction, the following relationships can be similarly developed:

\[ V_L = \frac{n_t}{(P_L/P_T) \sqrt{1 + (d/a)^2}} (F_y A) \]  \quad (12)

\[ \Delta_L = \frac{a(s^2 + d^2)^{3/2}}{s^2(P_L/P_T) \sqrt{a^2 + d^2}} (F_y) \]  \quad (13)

\[ \Delta_L / d = \frac{1 + (d/s)^3}{(P_L/P_T)(d/s) \sqrt{1 + (d/a)^2}} (F_y) \]  \quad (14)

\[ \mu_{GL} = 1 \]  \quad (15)

\[ K_t = \frac{n_t(d/s)}{[1 + (d/a)^2]^{3/2}} \left( \frac{EA}{d} \right) \]  \quad (16)

From Fig. 7a, when longitudinal braces yield, non-dimensional transverse base shear force is found to decrease as the d/a ratio increases, due to larger direction angles as previously explained. To evaluate the variation of transverse drift with end diaphragm geometric ratios, similar curves are produced for grade 50 steel, as done before, and are given in Fig. 7b. This figure shows a decrease in transverse drift as the d/a ratio increases, since lateral stiffness of the bridge increases as the d/a ratio increases. The d/s ratio has an important impact on drift, since transverse drift varies significantly depending on the d/a values. Since Eq. (16) is the same as Eq. (6), the variation of transverse stiffness is the same as seen in Fig. 5c, keeping in mind that longitudinal braces yield.

4.3. Behavior when longitudinal braces yield

4.3.1. Transverse response

When the longitudinal braces yield, base shear strength, lateral displacement, global displacement ductility, and the stiffness of the system in the transverse direction are reached as follows:

\[ V_L = \frac{n_t}{(P_L/P_T) \sqrt{1 + (d/a)^2}} (F_y A) \]  \quad (17)

\[ \Delta_L = \frac{a(s^2 + d^2)^{3/2}}{s^2(P_L/P_T) \sqrt{a^2 + d^2}} (F_y) \]  \quad (18)

\[ \Delta_L / d = \frac{1 + (d/s)^3}{(P_L/P_T)(d/s) \sqrt{1 + (d/a)^2}} (F_y) \]  \quad (19)

\[ \mu_{GL} = \mu \]  \quad (20)

\[ K_t = \frac{n_t(d/a)}{[1 + (d/a)^2]^{3/2}} \left( \frac{EA}{d} \right) \]  \quad (21)

Fig. 7c shows non-dimensional longitudinal base shear strength versus d/a ratio. A decrease in the base shear is observed as the d/a ratio increases due to larger direction angles from the horizontal direction. The resulting curve of longitudinal drift versus d/a ratio is illustrated in Fig. 7d. Longitudinal drift decreases as d/a increases. Also seen in Fig. 7d, the rate of decrease in drift is slower for values of d/a = 0.5 or larger, suggesting appropriate values between 0.5 and 1.0.

From Eq. (20), global system ductility in the longitudinal direction is equal to the member ductility, and hysteretic energy...
is only dissipated by the longitudinal braces. Again Eq. (21) is identical to Eq. (10), and the variation of longitudinal stiffness would be identical to Fig. 6c, except in this case, the longitudinal braces yield.

4.3.3. Dissipated energy per BRB volume

As previously stated, hysteretic energy dissipation per volume is given as:

\[ E_H = \frac{4(\mu - 1)}{Vol.} \left( \frac{F_y^2}{E} \right) \left( \frac{d/a}{d/s} \right)^2 \]  

Using \( \mu = 20 \), Fig. 8 demonstrates the variation of volumetric hysteretic energy dissipation with end diaphragm geometric ratios. As opposed to the transverse brace yielding case, dissipated energy decreases as \( d/a \) increases when the longitudinal braces yield. This behavioral difference comes from different base shear versus displacement response of each system. Additionally, smaller \( d/s \) ratios corresponding to fewer girders (or girders with larger spacing) result in less energy dissipation in the system.

5. Closed form solutions for Retrofit Scheme-2

This section presents the development of analytical expressions that describe the behavior of bridges with Retrofit Scheme-2 implemented in the end diaphragms. An ideal three-dimensional truss system shown in Fig. 9 was used to represent the entire bridge superstructure to analyze the end diaphragm behavior. Again, a cyclic symmetric bilinear hysteretic model for BRBs is used in the analysis of the bridge end diaphragms. The brace lengths become equal to each other as given below:

\[ L = \sqrt{a^2 + s^2 + d^2}. \]  

The parameter \( \beta \) is the angle between the BRBs and the vertical plane for the diagonal braces and equals:

\[ \sin \beta = \frac{\sqrt{a^2 + s^2}}{a^2 + s^2 + d^2}. \]
Fig. 10. Transverse behavior when DT braces yield: (a) variation of brace axial force ratio with bridge geometric relations for \( P_L/P_T = 0.30 \) and \( P_T/P_L = 0.30 \); (b) variation of brace axial force ratio with bridge geometric relations for \( P_L/P_T = 0.40 \) and \( P_T/P_L = 0.40 \); (c) non-dimensional transverse base shear strength versus \( d/a \) ratio for \( P_L/P_T = 0.30 \); (d) transverse drift versus \( d/a \) ratio for \( P_L/P_T = 0.30 \); (e) global transverse ductility ratio versus \( s/a \) ratio and local ductility for \( P_L/P_T = 0.30 \); (f) non-dimensional transverse stiffness versus \( d/a \) and \( s/a \) ratios.

Also, Fig. 9 shows the bidirectional loading and braces’ axial forces under these effects. As was discussed for Retrofit Scheme-1, this model provides a valid representation of the actual retrofit for Retrofit Scheme-2. Similar to Retrofit Scheme-1, equal proportions of the total lateral load in a given direction are applied at each corner of the deck. \( P_L \) and \( P_T \) are the lateral earthquake loads acting at the deck level on one diaphragm in the longitudinal and transverse directions, respectively. The ratio of \( P_L/P_T \) (or \( P_T/P_L \)) is kept constant in pushover analyses. The BRBs are assumed to be active only under earthquake loading and therefore do not carry any gravity loads.

5.1. Brace axial force ratio (elastic behavior)

The ratio of elastic axial compression and tension forces in the BRBs is given below as a function of the ratio of bidirectional earthquake forces \( (P_L/P_T \) or \( P_T/P_L \)) imposed on the system and the geometric properties of the end diaphragm. Note that the ratio of the elastic brace forces becomes the ratio of forces created in opposite diagonal directions. This ratio equals:

\[
\frac{C_T}{C_L} = \frac{T_T}{T_L} = \frac{1 - \left( \frac{P_L}{P_T} \right)}{1 + \left( \frac{P_L}{P_T} \right)}.
\]  

(25)

When the value of axial forces for the BRBs in the DT direction is greater than for the braces in the DL direction, axial yielding in the DT braces occurs. The type of collapse mode can be determined using Eq. (25), which captures the relative magnitudes of braces axial forces.

Based on these explanations, the potential collapse modes are defined as:

If \( \frac{C_T}{C_L} = \frac{T_T}{T_L} > 1 \) then the DT braces yield

If \( \frac{C_T}{C_L} = \frac{T_T}{T_L} < 1 \) then the DL braces yield

If \( \frac{C_T}{C_L} = \frac{T_T}{T_L} = 1 \) then all the braces yield at the same time.

Fig. 10a and b show the variation of axial force ratio with bidirectional loading and \( s/a \) ratios. As before, the curves are generated for \( (P_L/P_T \) or \( P_T/P_L \)) values of 0.30 and 0.40. The practical values of the \( s/a \) ratio are again set to 0.25, 0.50, 0.75, 1.00, 1.25, and 1.50. Since bidirectional response develops under bidirectional loading, again, the behavior is investigated in the transverse and longitudinal directions.
5.2. Behavior when DT braces yield

5.2.1. Transverse response

As discussed in Retrofit Scheme-1, similar hysteretic behavior in the transverse direction develops as shown in Fig. 4a. After determining the yielding braces for a specified system geometry and loading ratio per Eq. (25), the behavioral characteristics of the system such as base shear strength ($V_{yT}$), yield displacement ($\Delta_{yT}$) and the corresponding drift ($\Delta_{yT}/d$), global ductility demand ($\mu_{GT}$) and the initial stiffness of the system ($K_T$) in the transverse direction are obtained as follows. Note that the following equations (especially for base shears and stiffnesses) and the corresponding diagrams are obtained for the end diaphragm systems having four BRBs that are equivalent to the total number of braces used.

Using the non-dimensional properties, the transverse base shear strength is expressed by:

$$V_{yT} = \left[ \frac{4(s/a)}{\sqrt{1 + (s/a)^2 + (d/a)^2} [1 - (s/a)(P_L/P_T)]} \right] (F_y A).$$  \hspace{1cm} (26)

The lateral drift ($\Delta_{yT}/d$, at a member ductility of $\mu$, in the transverse direction is:

$$\frac{\Delta_{yT}}{d} = \left[ \frac{1 + (s/a)^2 + (d/a)^2}{(s/a)(d/a) [1 - (s/a)(P_L/P_T)]} \right] \left( \frac{F_y}{E} \right).$$

and the corresponding drift at yield is obtained by substituting $\mu = 1$ in Eq. (27) as

$$\frac{\Delta_{yT}}{d} = \left[ \frac{1 + (s/a)^2 + (d/a)^2}{(s/a)(d/a) [1 - (s/a)(P_L/P_T)]} \right] \left( \frac{F_y}{E} \right).$$

The global displacement ductility in the transverse direction ($\mu_{GT}$) for the system is:

$$\mu_{GT} = \left[ \frac{1 - (s/a)(P_L/P_T) + 1 + (s/a)(P_L/P_T)}{2} \right].$$

The initial stiffness of the system in the transverse direction is obtained from equations above, taking $\mu = 1$, as:

$$K_T = \left[ \frac{4(s/a)^2(d/a)}{[1 + (s/a)^2 + (d/a)^2]^{3/2}} \right] \left( \frac{EA}{d} \right).$$

These behavioral characteristics are demonstrated in Fig. 10c–f. It is observed that in Retrofit Scheme-2, when the DT braces yield, the base shear strength decreases as $d/a$ increases for constant values of $s/a$ and decreases as $s/a$ decreases for constant values of $d/a$ (i.e., smaller base shears are obtained at smaller $\beta$ angles). Transverse drift ($\Delta_{yT}/d$) decreases as $d/a$ increases, revealing that BRBs with larger direction angles would be preferable to obtain stiffer diaphragms. For a constant value of $d/a$, the transverse drift decreases as $s/a$ increases, showing that Retrofit Scheme-2 is more effective when sufficient girder spacing exists in the bridge superstructure. Also, the change in drift is less for larger values of $s/a$ ratios. Global transverse ductility ($\mu_{GT}$) decreases as $s/a$ increases. Expectedly, for constant values of $s/a$, the global ductility increases as the local (BRB) ductility ($\mu_i$) increases. Initial stiffness increases as $d/a$ and $s/a$ ratios increase. However, this increase is less after values of $d/a = 0.60$. Similar behavioral tendencies are observed in the longitudinal direction.

5.2.2. Longitudinal response

Similar equations are obtained for the response in the longitudinal direction. The base shear strength in the longitudinal direction is:

$$V_{yL} = \left[ \frac{4(s/a)(P_L/P_T)}{\sqrt{1 + (s/a)^2 + (d/a)^2} [1 + (s/a)(P_L/P_T)]} \right] (F_y A).$$

The longitudinal drift is given by

$$\frac{\Delta_{yL}}{d} = \left[ \frac{1 + (s/a)^2 + (d/a)^2}{(s/a)(d/a) [1 - (s/a)(P_L/P_T)]} \right] \left( \frac{F_y}{E} \right).$$

The global ductility in the longitudinal direction is derived as:

$$\mu_{GT} = \left[ \frac{[(s/a)(P_L/P_T) - 1] + (s/a)(P_L/P_T) + 1}{2(s/a)(P_L/P_T)} \right].$$

The initial stiffness in the longitudinal direction is found to be:

$$K_L = \left[ \frac{4(s/a)^2(d/a)}{[1 + (s/a)^2 + (d/a)^2]^{3/2}} \right] \left( \frac{EA}{d} \right).$$

These behavioral characteristics are shown in Fig. 11a–d.

5.2.3. Dissipated energy per BRB volume

Volumetric hysteretic energy dissipated through a full cycle of displacement is written as follows:

$$E_{H/Vol.} = 2(\mu - 1) \left( \frac{F_y}{E} \right).$$

5.3. Behavior when DL braces yield

5.3.1. Transverse response

Similar equations as obtained above can be reached when the DL braces yield. The base shear strength in the transverse direction is expressed using the non-dimensional system geometric properties as:

$$V_{yT} = \left[ \frac{4(s/a)}{\sqrt{1 + (s/a)^2 + (d/a)^2} [1 + (s/a)(P_L/P_T)]} \right] (F_y A).$$

The lateral drift ($\Delta_{yT}/d$) in the transverse direction is

$$\frac{\Delta_{yT}}{d} = \left[ \frac{1 + (s/a)^2 + (d/a)^2}{(s/a)(d/a) [1 - (s/a)(P_L/P_T)]} \right] \left( \frac{F_y}{E} \right).$$

and the corresponding drift at yield is obtained by substituting $\mu = 1$ as

$$\frac{\Delta_{yT}}{d} = \left[ \frac{1 + (s/a)^2 + (d/a)^2}{(s/a)(d/a) [1 + (s/a)(P_L/P_T)]} \right] \left( \frac{F_y}{E} \right).$$

The global displacement ductility ($\mu_{GT}$) of the system is:

$$\mu_{GT} = \left[ \frac{[(s/a)(P_L/P_T) - 1] + (s/a)(P_L/P_T) + 1}{2(s/a)(P_L/P_T)} \right].$$
5.3.2 Longitudinal response

The initial stiffness of the system in the transverse direction is written as:

$$K_T = \left[ \frac{4(s/a)^2 (d/a)}{1 + (s/a)^2 + (d/a)^2} \right] \left( \frac{EA}{d} \right).$$

These behavioral characteristics are demonstrated in Fig. 12a–c.

5.3.2 Longitudinal response

The base shear strength in the longitudinal direction is:

$$V_{dL} = \left[ \frac{-4(s/a)(P_l/P_T)}{\sqrt{1 + (s/a)^2 + (d/a)^2}[(s/a)(P_l/P_T) + 1]} \right] (F_y A).$$

The lateral drift in the longitudinal direction is:

$$\frac{\Delta_L}{d} = \frac{1 + (s/a)^2 + (d/a)^2}{2/d} \left[ \frac{1 - (s/a)(P_l/P_T)}{(s/a)(P_l/P_T) + 1} \right] \left( \frac{F_y}{E} \right).$$

and the longitudinal drift at brace yielding is:

$$\frac{\Delta_{Ld}}{d} = \left[ \frac{1 + (s/a)^2 + (d/a)^2}{(s/a)(P_l/P_T) [1 + (s/a)(P_l/P_T)]} \right] \left( \frac{F_y}{E} \right).$$

The global ductility in the longitudinal direction is similarly derived as:

$$\mu_{GL} = \left[ \frac{[s/a)(P_l/P_T) + 1] \mu + [s/a)(P_l/P_T) - 1]}{2(s/a)(P_l/P_T)} \right].$$

The initial stiffness in the longitudinal direction becomes

$$K_L = \left[ \frac{-4(d/a)}{1 + (s/a)^2 + (d/a)^2} \right] \left( \frac{EA}{d} \right).$$

These behavioral characteristics are demonstrated in Fig. 13a–c.
ARTICLE IN PRESS

Fig. 13. Longitudinal behavior when DL braces yield: (a) non-dimensional longitudinal base shear strength versus d/a ratio for $P_L/P_T = 0.30$; (b) longitudinal drift versus d/a ratio for $P_L/P_T = 0.30$; (c) global longitudinal ductility ratio versus s/a ratio and local ductility for $P_L/P_T = 0.30$; (d) variation of volumetric energy dissipation versus member (BRB) ductility.

5.3.3. Dissipated energy per BRB volume

Finally, volumetric hysteretic energy dissipated through a full cycle of displacement is written as follows:

$$
\frac{E_H}{\text{Vol.}} = 2(\mu - 1) \left( \frac{F}{F} \right).
$$

Fig. 13d shows the variation of volumetric hysteretic energy dissipation with member (BRB) ductility.

In Retrofit Scheme-2 and when the DL braces yield, the tendency is the same as in the DT brace yielding case, with the exception that the global transverse ductility ratio ($\mu_{GT}$) increases, as s/a increases since the stiffness in the transverse direction decreases upon yielding. Again, for constant values of s/a, the global ductility increases as the local (BRB) ductility ($\mu$) increases. Longitudinal base shear strength decreases as d/a increases due to the lesser horizontal component of the BRB axial force. Longitudinal drift decreases as d/a increases and also decreases as s/a decreases. Global longitudinal ductility ($\mu_{GL}$) decreases as s/a increases, but increases as the local (BRB) ductility ($\mu$) increases. Expectedly, hysteretic energy dissipation per volume increases as local ductility increases.

6. Numerical examples

Two systems, namely S1 and S2, representing the bridge end diaphragms (for Retrofit Scheme-1 and 2, respectively) are selected to numerically show the impact of several structural parameters on the inelastic behavior of the systems considered. Special emphasis is placed on hysteretic energy dissipation. The numerical results of this example are useful to assess the effects of various BRB configurations on the seismic behavior of bridge end diaphragms. Fig. 14a and b show the selected systems. The geometrical dimensions of these systems are arbitrarily chosen for simplicity and not intended to correspond to a specific bridge. For this purpose, a cube with a side length of 914.4 mm (36") is selected for the analyses. Both unidirectional and bidirectional loadings are considered to show the effect of bidirectional earthquake effects.

Please cite this article in press as: Celik OC, Bruneau M. Seismic behavior of bidirectional-resistant ductile end diaphragms with buckling restrained braces in straight steel bridges. Engineering Structures (2008), doi:10.1016/j.engstruct.2008.08.013
All supports are taken as simple supports. In the analyses, the BRBs are assumed to be pinned at their ends, with no strain hardening and equal tension and compression capacities (i.e., $T_Y = C_Y$).

The BRBs are also assumed to have a target displacement ductility of $\mu = 4$, a yield point of $F_y = 345$ MPa (50 ksi), and a modulus of elasticity of $E = 200,000$ MPa (29,000 ksi). Other system properties are summarized in Table 1. Static unidirectional (in X or Y directions) and bidirectional (in X and Y directions, labeled X + Y in the table) pushover analyses are conducted using SAP2000. Note that X and Y indicate the transverse and longitudinal directions, respectively. Using SAP2000 results and the formulas developed in the previous sections, the system parameters and responses of each system are summarized in Table 1.

To compare the effectiveness of each system, similar systems are defined as having either braces with the same cross sectional area (SA), braces with the same base shear strength (SBS) in the governing direction, and braces with the same initial stiffness (SIS). For each case, results are presented for the base shear at yield ($V_B$) in the governing direction (X or Y as depicted in the table). The corresponding yield displacement ($\Delta_Y$), maximum displacement ($\Delta_{\text{max}}$), hysteretic energy dissipated ($E_H$) at an assumed brace (or member) displacement ductility of $\mu = 4$, volumetric energy dissipation ($E_H/\text{Vol}$), i.e., the energy dissipated per BRB material used, and the effectiveness ratio (with respect to an arbitrarily chosen reference system with similar properties) of each system in terms of hysteretic energy dissipation are provided. Note that $E_H$ is calculated using the area under 1/4 of a complete hysteretic loop as illustrated in Table 1.

The following observations are made:

- Under unidirectional loading and for the same cross sectional area, S1 has greater initial stiffness and base shear capacity, but lower total and volumetric hysteretic energy dissipations and yield and maximum displacement capacities than S2. 63% more bracing material is used in S1 (i.e., shorter braces in S1, but more of them). For the same base shear, S1 has greater initial stiffness but lower required cross sectional area, yield and maximum displacements, and total and volumetric hysteretic energy dissipation. In this case, 33% more bracing material is used in S1. For the same initial lateral stiffness in the direction of loading, S2 has greater base shear capacity, yield and maximum displacements, and total and volumetric hysteretic energy dissipations. S2 also has greater required cross sectional area. In this case, 12% more bracing material is used in S2. In all cases, the volumetric energy dissipation is always twice for S2 than S1. However, note that under unidirectional loading, when the volumetric ratios of hysteretic energy dissipation in S2 are greater than for S1, the braces in the orthogonal direction to the loading direction are inactive (unloaded) and do not dissipate any energy. It could be argued that a more fair comparison for the unidirectional loading case would discount these inactive BRBs. In such a case, S1 and S2 would share identical values of $E_H/\text{Vol}$.

- For bidirectional and orthogonal loading with the same intensity of loading in each direction (in X and Y directions, the loading ratio is $P_x/P_y = 1.00$), for the case in which all BRBs have the same cross sectional area, S1 has greater initial stiffness, base shear capacity, and total and volumetric hysteretic energy dissipations than S2. Note that 63% more bracing material is used.

---

**Table 1**

<table>
<thead>
<tr>
<th>System</th>
<th>Info</th>
<th>$V_B$ (kN)</th>
<th>$K_e^x$ (kN/mm)</th>
<th>$\Delta_Y$ (mm)</th>
<th>$\Delta_{\text{max}}$ (mm)</th>
<th>$\mu_c$</th>
<th>$A$ (mm$^2$)</th>
<th>$L_y$ (mm)</th>
<th>Load</th>
<th>$E_H$ (kN mm)</th>
<th>$\text{Vol.}^1$ (mm$^3$)</th>
<th>$E_H/\text{Vol.}^1$</th>
<th>Eff. ratio$^{\text{a}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 SA</td>
<td>629.20</td>
<td>199.7</td>
<td>3.15</td>
<td>12.60</td>
<td>4.00</td>
<td>645.16</td>
<td>1293,11</td>
<td>$X^a$</td>
<td>5945.94</td>
<td>6674.103</td>
<td>0.89</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>S2 SA</td>
<td>513.90</td>
<td>108.7</td>
<td>4.73</td>
<td>18.92</td>
<td>4.00</td>
<td>645.16</td>
<td>1583,69</td>
<td>$X^a$</td>
<td>7292.24</td>
<td>4086.934</td>
<td>1.78</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>S1 SBS</td>
<td>177.93</td>
<td>56.5</td>
<td>3.15</td>
<td>12.60</td>
<td>4.00</td>
<td>182.45</td>
<td>1293,11</td>
<td>$X$</td>
<td>1681.44</td>
<td>1887.423</td>
<td>0.89</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>S2 SBS</td>
<td>177.93</td>
<td>37.6</td>
<td>4.73</td>
<td>18.92</td>
<td>4.00</td>
<td>223.35</td>
<td>1583,69</td>
<td>$X$</td>
<td>2524.83</td>
<td>1414.869</td>
<td>1.78</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>S1 S2 SBS</td>
<td>177.93</td>
<td>56.5</td>
<td>3.15</td>
<td>12.60</td>
<td>4.00</td>
<td>182.45</td>
<td>1293,11</td>
<td>$X + Y$</td>
<td>3362.88</td>
<td>1887.423</td>
<td>1.78</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

$^a$ Same cross sectional area.
$^b$ Same base shear in X direction.
$^c$ Same initial stiffness in X direction.
$^d$ Base shear strength in X direction.
$^e$ Initial stiffness in X direction.
$^f$ Yield displacement in X direction.
$^g$ Maximum displacement in X direction.
$^h$ Global ductility demand.
$^i$ Cross sectional area of each BRB.
$^j$ Brace length.
$^k$ Total hysteretic energy dissipation in X and Y directions.
$^l$ Total volume of braces used.
$^m$ Ratio of hysteretic energy dissipation per volume to max. One
$^n$ Loading in X direction.
$^o$ Same loading in both orthogonal directions.

---

Please cite this article in press as: Celik OC, Bruneau M. Seismic behavior of bidirectional-resistant ductile end diaphragms with buckling restrained braces in straight steel bridges. Engineering Structures (2008), doi:10.1016/j.engstruct.2008.08.013
in S1. For the same base shear capacity, S1 has greater yield and maximum displacements, and total and volumetric energies, but lower initial stiffness and required cross sectional area. In this case, 50% more bracing material is used in S2. However, note that in both cases, when brace ductility reaches a value of $\mu = 4$ (the premise of how the data in this table were derived), S2 consistently dissipated less total hysteretic energy (as a system) than S1, and less volumetric hysteretic energy (exactly half). This is logical, since S2 and S1 have 4 and 8 braces, respectively. Overall, for a given required design base shear in each direction, S1 achieves the same displacement demand performance as S2 (i.e., $\mu = 4$) with a lesser volume of material (while providing braces in an X configuration in both cases). The orthogonal brace configuration therefore seems to be more effective. On the other hand, S2 has the advantage over S1 (again for the case of same design base shear) to result in a more flexible ductile diaphragm, which can be advantageous when trying to implement the system in bridges with relatively flexible substructures in which the diaphragms need to reach a larger lateral displacement for a given ductility (see Alfawakhiri and Bruneau [19,20]). However, note that highly flexible (in transverse or longitudinal directions) BRB end diaphragm systems must also be checked to prevent the occurrence of excessive drift and deformations in other parts of the bridge superstructure (for example, in deck truss bridges). In stiff substructures, it is implicit that both retrofit schemes can be used. Finally, the smaller braces in case S1 will develop smaller yield forces than those in S2, resulting in simple connections to the superstructure and substructure (although using a larger number of BRBs is always possible to minimize this problem).

In addition to these evaluations, other observations regarding the overall behavior of the systems are as follows:

- **Under the selected boundary conditions for the idealized systems considered herein, in all cases, BRBs of the same geometric properties (i.e., same cross sectional area and geometric configuration) yield at the same displacement level since their axial forces are equal. In other words, base shear vs. lateral displacement curves are typically bilinear. When the yield level is reached, a group of similar braces yields simultaneously, maximum system strength is reached, and the structure displaces up to the maximum target displacement at $\Delta_{\text{y}}$, which depends on a predetermined displacement ductility for the braces. Simultaneous yielding of a group of BRBs is especially important to ensure a stable seismic behavior, enhance hysteretic energy dissipation capability, and minimize the potential differences of local displacement demands in the braces. Note that no effort has been made to calculate actual ductility demands for the BRBs; instead a maximum ductility of $\mu = 4.4$ is assumed in this example.**

- **For some systems considered, some braces may not yield and remain elastic (or may unload depending on the loading ratio in the orthogonal directions).**

### 7. Conclusions

Two ductile end diaphragms configurations incorporating BRBs have been developed and analytically investigated for the seismic retrofit of steel bridge superstructures (labeled Retrofit Scheme-1 and Retrofit Scheme-2). Both bidirectional earthquake effects and generic bridge geometrical properties were considered in the analysis. Some shortcomings of the known ductile end diaphragm concepts have been resolved using the selected bidirectional bracing configurations (Retrofit Scheme-1 and Retrofit Scheme-2), which were able to resist both transverse and longitudinal seismic effects. The proposed retrofit schemes are promising and viable compared to the alternatives commonly used in bridge seismic retrofit (or design) applications.

Analytical investigations suggest that an appropriate value for $d/s$ for optimal seismic response could be between 0.5 and 1.0. When the ratio of $d/a$ is greater than 0.5, the variation in longitudinal drift is relatively insignificant, suggesting that appropriate $d/a$ ratios could also be selected between 0.5 and 1.0. For constant values of $s/a$, the local (BRB) ductility ($\mu$) increases with

For non-skewed bridges and under unidirectional loading, for a given required design base shear, Retrofit Scheme-1 achieves the same displacement demand as Retrofit Scheme-2 with a lesser volume of material. The orthogonal brace configuration therefore seems to be more effective. On the other hand, Retrofit Scheme-2 results in a more flexible ductile diaphragm. Smaller braces used in Retrofit Scheme-1 will develop smaller yield forces than those in Retrofit Scheme-2, resulting in simpler connections to the superstructure and substructure.

### Acknowledgments

This research was conducted at the State University of New York (SUNY) at Buffalo and was supported by the Federal Highway Administration (FHWA) under contract number DTFH61-98-C-00094 to the Multidisciplinary Center for Earthquake Engineering Research (MCEER). However, any opinions, findings, conclusions, and recommendations presented in this paper are those of the authors and do not necessarily reflect the views of the sponsors.

### References


