Capacity Design of Vertical Boundary Elements in Steel Plate Shear Walls

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Design requirements now appear in the 2005 AISC *Seismic Provisions for Structural Steel Buildings* (AISC, 2005b), referred to herein as *The Provisions*, for steel plate shear walls (SPSWs) that are designed such that their web plates buckle in shear and develop diagonal tension field action when resisting lateral loads. Energy dissipation and ductility during seismic events is principally achieved through yielding of the web plates along the diagonal tension field. Consistent with capacity design principles, *The Provisions* require that the vertical and horizontal boundary elements (VBEs and HBEs) of SPSWs, as shown in Figure 1, be designed to remain essentially elastic with the exception of plastic hinging at the ends of horizontal boundary elements. The commentary of *The Provisions* provides some guidance on how to achieve this requirement. However, the methods described in the commentary, as shown in this paper, do not necessarily lead to VBEs that meet the requirement of essentially elastic behavior under the forces generated by fully yielded web plates.

This paper reviews the current approaches provided in *The Provisions* commentary for determination of capacity design loads for the VBEs of SPSWs and also describes how the capacity design objective may be achieved using nonlinear static analysis. Then, a new procedure is proposed that uses a fundamental plastic collapse mechanism and linear beam analysis to approximate the design actions for VBEs of SPSWs for given web plates and horizontal boundary member sizes. The proposed procedure does not involve nonlinear analysis, making it practical for use in design. VBE design loads are estimated using both the current and proposed procedures for two example SPSW configurations. The resulting design loads are then compared with the VBE design loads as determined by nonlinear pushover analysis.

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CURRENT VBE DESIGN PROCEDURES

Three methods for ensuring capacity design of VBEs and HBEs for SPSWs are described in the commentary of *The Provisions*, two of the methods are linear and one is nonlinear. These are: the combined linear elastic computer programs and capacity design concept (LE+CD); the

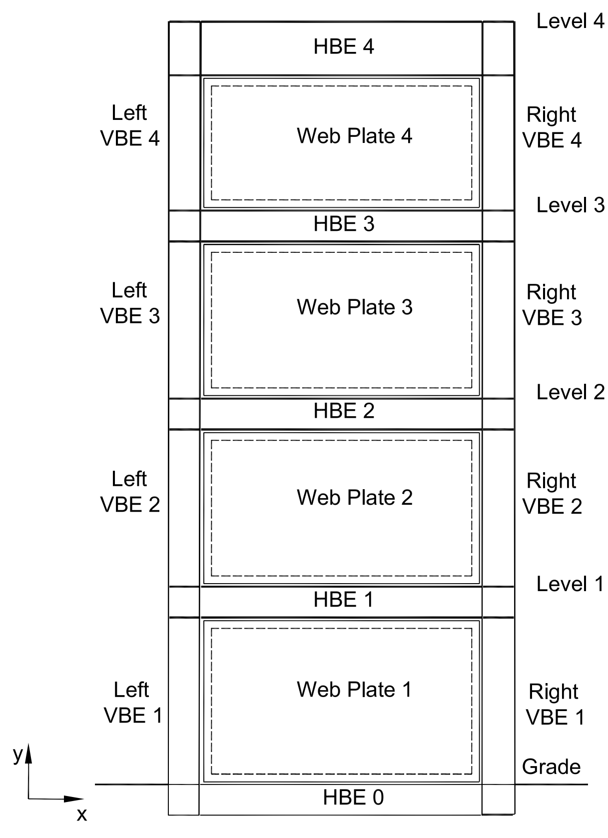


Fig. 1. General SPSW Configuration.

indirect capacity design approach (ICD); and nonlinear static analysis (pushover analysis). Each is reviewed briefly below and the steps in the two linear procedures that result in significant inaccuracies are identified. Note that nonlinear static analysis of SPSWs with web plates modeled as a series of strips oriented at α from the vertical, the angle of inclination of the tension field as calculated per *The Provisions*, has been shown to adequately represent the behavior of SPSW (Driver, Kulak, Kennedy and Elwi, 1998; Berman and Bruneau, 2004; and Berman and Bruneau, 2005; among others). That strip model is also used in elastic analyses of SPSW in the two linear procedures described below.

Combined Linear Elastic Computer Program and Capacity Design Concept (LE+CD)

The combined linear elastic computer program and capacity design concept as described in Commentary Section C17.4a of *The Provisions* consists of four steps (note that R_y is the ratio of mean to nominal yield stress of the web plate, A_s is the area of a strip in the strip model representation of a SPSW, and F_{yp} is the web plate yield stress):

1. Lateral forces: Use combined model, boundary elements and web elements, to come up with the demands in the web and boundary elements for the code required base shear. The web elements shall not be considered as vertical-load carrying elements.

2. Gravity load (dead load and live load): Apply gravity loads to a model with only gravity frames. The web elements shall not be considered as vertical-load carrying elements.

3. Without any overstrength factors, design the boundary elements using demands based on combination of the forces from the above steps 1 and 2.

4. Boundary element capacity design check: Check the boundary element for the maximum capacity of the web elements in combination with the maximum possible axial load due to overturning moment. Use the axial force obtained from step 1 above and multiply by the overstrength factor, Ω_o. Apply load from the web elements ($R_yF_{yp}A_s$) in the direction of α. For this capacity design check use a material strength reduction factor of 1.0. For the determination of the required strength of boundary elements and their connection to the web, neither the resistance factor (LRFD), nor the safety factor (ASD), are applied to the strength of the web.

While this procedure may, in some cases, result in proper capacity design of boundary elements, there is an inconsistency with respect to equilibrium. In step 4 the procedure requires that forces equal to the expected yield strength of the web panel be applied to the surrounding framing members in the direction of α. However, without simultaneous application of the lateral loads that caused the web plates to yield, the frame member moment diagrams resulting from the application of just the web plate forces will be incorrect because equilibrium with the applied loads is not satisfied and the deformed shape of the system is incorrect for those loads. To illustrate this, consider Figure 2, which shows a SPSW with the web plate represented by strip elements subjected to lateral loads (Figure 2a), and the model implied by step 4 of the LE+CD procedure where the strips have been replaced with plate yield forces, but no corresponding lateral loads have been applied. It is clear that the moment diagrams for the columns from these two cases will have different shapes regardless of the dimensions, geometry, or configuration of the SPSW. Unfortunately, calculation of those lateral loads and displacements is complex and since the procedure is meant to provide a simple linear method for capacity design of framing elements it is not useful to recommend the addition of such calculations. However, the concept that the forces from the yielding web plates may be applied to an otherwise elastic model in lieu of using full nonlinear analysis is promising and is the motivation for the proposed capacity design procedure below.

Indirect Capacity Design Approach (ICD)

The indirect capacity design approach presented in Commentary Section C17.4a of *The Provisions* is from the Canadian Standards Association steel design standard CAN/CSA-S16-01 (CSA, 2001) and proposes that the loads in the

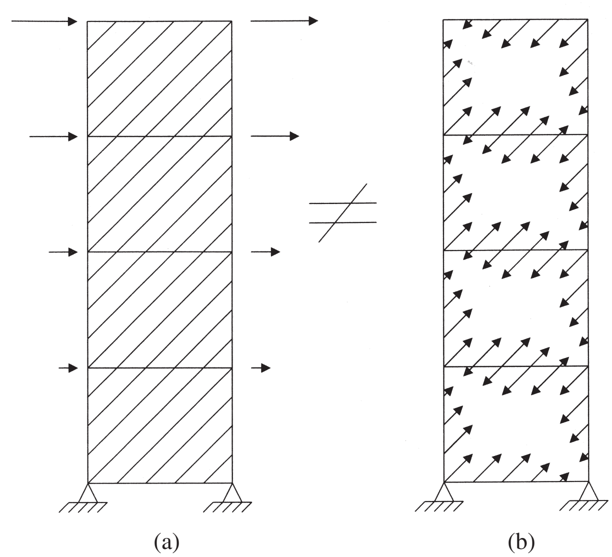


Fig. 2. Conceptual difference between: (a) SPSW subject to lateral load and (b) SPSW boundary frame subject to only infill plate yield forces.

VBEs of a SPSW may be found from gravity loads combined with seismic loads amplified by:

$$B = V_e / V_u \qquad (1)$$

where

V_e = expected shear strength, at the base of the wall, determined for the web thickness supplied
= $0.5R_y F_{yp} t_w L \sin 2\alpha$, where L is the width of the wall

V_u = factored lateral seismic force at the base of the wall

In determining the loads in VBEs, the amplification factor, B, need not be taken larger than the seismic force reduction factor, R.

The VBE design axial forces shall be determined from overturning moments defined as follows:

1. The moment at the base of the wall is BM_u, where M_u is the factored seismic overturning moment at the base of the wall corresponding to the force V_u;

2. The moment, BM_u, extends for a height H but not less than two stories from the base, and;

3. The moment decreases linearly above a height H to B times the factored seismic overturning moment at one story below the top of the wall, but need not exceed R times the factored seismic overturning moment at any story under consideration corresponding to the force V_u.

The local bending moments in the VBE due to tension field action in the web shall be multiplied by the amplification factor, B.

This procedure relies on elastic analysis of a strip model (or equivalent) for the design seismic loads, followed by amplification of the resulting VBE moments by the factor B. Therefore, it produces moment diagrams and SPSW deformations that are similar in shape to what one would obtain from a pushover analysis. Similarly, the determination of VBE axial forces from overturning calculations based on the design lateral loads amplified by B results in axial force diagrams that are of the proper shape. However, the amplification factor used in both instances is found only for the first story and does not include the possibly significant strength of the surrounding frame. This leads to estimates of VBE demands that are less than those required to develop full web yielding on all stories prior to development of hinges in VBEs. For example, it is conceivable that the ratio of web thickness provided to web thickness needed for the design seismic loads is larger on the upper stories than on the lower stories. In these situations, the indirect capacity design approach would significantly underestimate the VBE design loads for the upper stories and capacity design would not be achieved. Additionally, frame members for SPSW may be large and the portion of the base shear carried by the surrounding moment frame could be substantial. For example, a recent SPSW implementation detailed in Monnier and Harasimowicz (2007) had VBEs at the bottom stories that were W14×730, the second largest W14 shape available. Neglecting the strength of the surrounding moment resisting frame in the ICD procedure when that frame has such substantial sections will certainly result in an underestimation of the expected SPSW shear strength, V_e, and VBE design loads that are underestimated for true capacity design.

Nonlinear Static (Push-Over) Analysis

Nonlinear static analysis of VBE strip models has been shown to give reasonable results for HBE and VBE moments and axial forces. Capacity design may be achieved by accounting for the actual thickness of the web plates and the ratio of mean to nominal web plate yield stress. However, as a design tool, nonlinear static analysis is time consuming as several iterations may be necessary to ensure capacity design of VBEs. Additional complexity results from having to properly account for possible formation of flexural-axial plastic hinges in the HBEs. Despite these issues, nonlinear static analysis of SPSW strip models generally gives accurate results for VBE demands and will be used to compare the adequacy of the proposed procedure below for two example SPSWs.

PROPOSED VBE DESIGN PROCEDURE

Objective

As discussed above there are two rather simple linear procedures for capacity design of SPSW VBEs given in the commentary of *The Provisions*. However, those methods, for different reasons, do not necessarily achieve the goal of VBE capacity design. Furthermore, the nonlinear static analysis procedure, which results in a more accurate estimation of the capacity design demands for VBEs, is tedious for broad use in design. Therefore, a need exists to develop a reasonably accurate and relatively efficient method for estimating the demands in VBEs when full yielding occurs in web plates for SPSW. This method should preferably involve only linear computer analyses without development of the complete strip model, and should account for the strength of surrounding framing (in other words, include the strength demands associated with hinging at the HBE ends).

The procedure proposed below to estimate VBE design loads to ensure capacity design of SPSWs combines a linear elastic beam model and plastic analysis. A model of the VBE on elastic supports is used to determine the axial forces in the HBEs and a plastic collapse mechanism is assumed to estimate the lateral seismic loads that cause full web plate yielding and plastic hinging of HBEs at their ends. A simple VBE free body diagram is then used to determine the design VBE axial forces and moments. For use in design, iteration may be necessary as certain parameters are assumed at the beginning of the process that may need revision as the design progresses.

Plastic Collapse Mechanism

Plastic collapse mechanisms for SPSWs subject to lateral loads have been proposed by Berman and Bruneau (2003) and have been shown to agree well with experimental results for ultimate capacity of single and multistory SPSWs. They examined two types of plastic mechanisms for multistory SPSW, namely, a uniform collapse mechanism and a soft-story collapse mechanism which are shown schematically in Figures 3a and 3b respectively. For the purpose of capacity design of VBEs, it is conservative to use the uniform plastic collapse mechanism as it will result in larger base shear forces and larger VBE demands. Furthermore, if a soft-story mechanism is found to be likely, it is recommended that the SPSW be redesigned to develop more uniform yielding of the web plates over the height. This can be achieved, even for web plates of equal thickness over the height, by adjusting the sizes and moments of inertia of the surrounding HBEs and VBEs. Therefore, the uniform collapse mechanism shown in Figure 3a will be used in the proposed procedure for determination of capacity design loads for SPSW VBEs.

Proposed VBE Design Procedure

Free Body Diagrams of VBEs

Assuming that the web plates and HBEs of a SPSW have been designed according to *The Provisions* to resist the factored loads (or, for the case of HBE design the maximum of the factored loads or web plate yielding), the required capacity of VBEs may be found from VBE free body diagrams such as those shown in Figure 4 for a generic four-story SPSW. Those free body diagrams include distributed loads representing the web plate yielding at story i, ω_{xci} and ω_{yci}; moments from plastic hinging of HBEs, M_{prli} and M_{prri}; axial forces from HBEs, P_{bli} and P_{bri}; applied lateral seismic loads, found from consideration of the plastic collapse mechanism, F_i; and base reactions for those lateral seismic loads, R_{yL}, R_{xL}, R_{yR}, and R_{xR}. The following describes how the components of the VBE free body diagrams are determined. Note that for the purpose of this discussion lateral forces are assumed to be acting from left to right on the SPSW of Figure 4.

Forces from Plate Yielding

The distributed loads to be applied to the VBEs (ω_{yci} and ω_{xci}) and HBEs (ω_{ybi} and ω_{xbi}) from plate yielding on each story i may be determined as:

$$\omega_{yci} = \tfrac{1}{2}F_{yp}t_{wi}\sin 2\alpha \qquad \omega_{xci} = F_{yp}t_{wi}\left(\sin\alpha\right)^2 \tag{2}$$

$$\omega_{ybi} = F_{yp}t_{wi}\left(\cos\alpha\right)^2 \qquad \omega_{xbi} = \tfrac{1}{2}F_{yp}t_{wi}\sin 2\alpha \tag{3}$$

These are found from resolving the plate yielding force, occurring at an angle α from the vertical, into horizontal and vertical components acting along the VBEs and HBEs as demonstrated for a VBE in Figure 5. In that figure, ds is an incremental plate width perpendicular to the tension field,

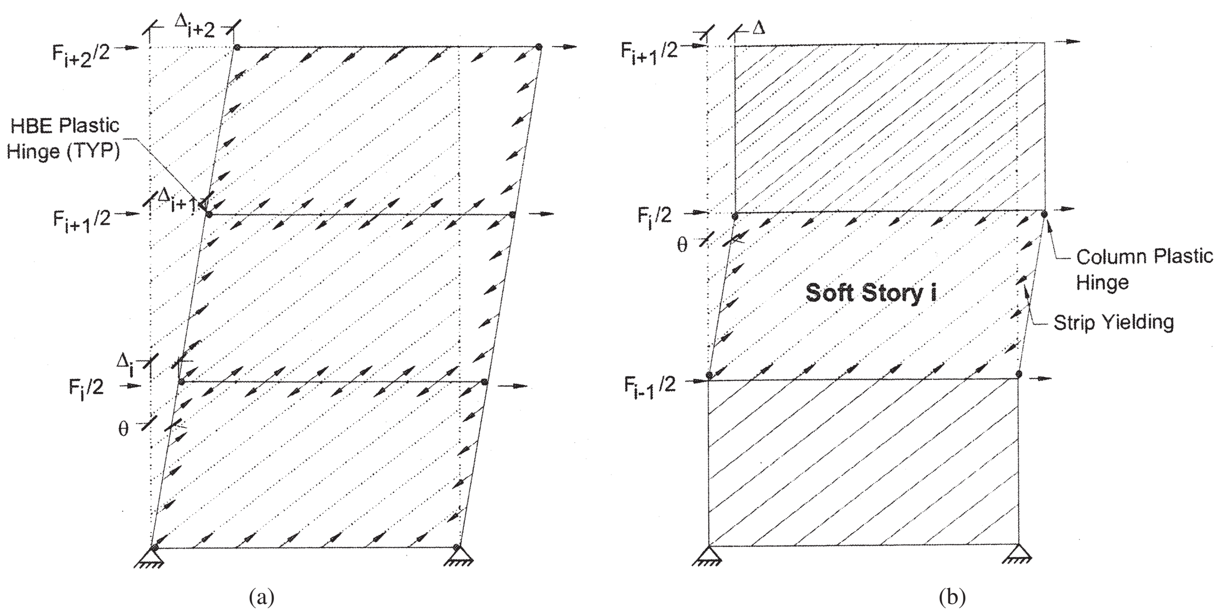


Fig. 3. SPSW Collapse Mechanisms: (a) Uniform Yielding Mechanism and (b) Soft-Story Mechanism.

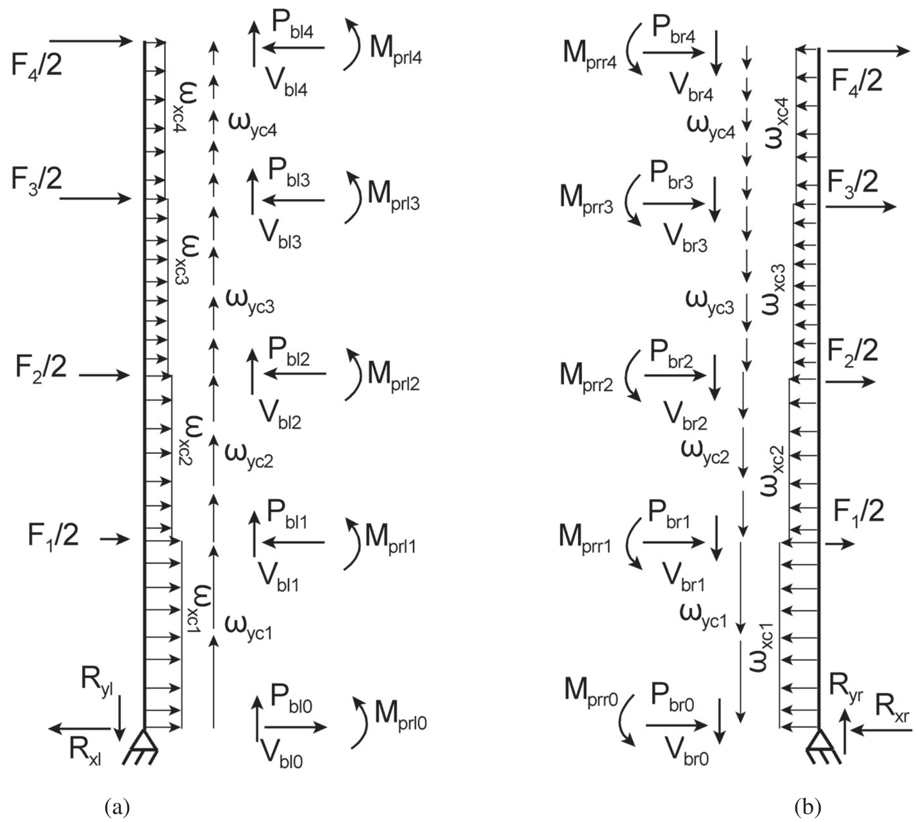


Fig. 4. VBE free body diagrams.

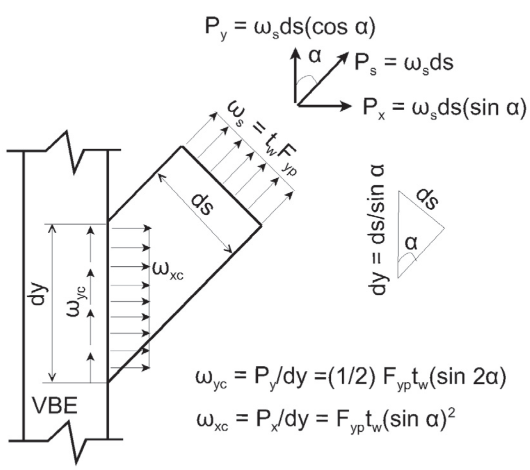


Fig. 5. Resolution of plate forces applied to a VBE.

dy is an incremental VBE height corresponding to ds, and P_s, P_x and P_y, are forces from plate yielding summed over ds in the tension field, horizontal and vertical directions, respectively. Note that the inclination angle, as determined per *The Provisions*, depends on the VBE cross-sectional area and moment of inertia and will have to be assumed at the beginning of a design procedure and then revised once VBEs have been selected. An initial assumption of 45º is suggested.

HBE Axial Forces

As part of estimating the axial load in the HBEs, an elastic model of the VBE is developed as shown in Figure 6. The model consists of a continuous beam element representing the VBE which is pin-supported at the base and supported by elastic springs at the intermediate and top HBE locations. HBE spring stiffnesses at each story i, k_{bi}, can be taken as the axial stiffness of the HBEs considering one-half the bay width (or HBE length for considerably deep VBEs), in other words:

$$k_{bi} = \frac{A_{bi}E}{L/2} \tag{4}$$

where

A_{bi}	=	HBE cross-sectional area
L	=	bay width
E	=	modulus of elasticity

This VBE model is then loaded with the horizontal component of the forces from the web plates yielding over each story, namely, ω_{xci}. An initial VBE size will have to be assumed for use in this model and some iteration may be required once that VBE size is revised. Additionally, it is reasonable to neglect the rotational restraint provided by the HBEs. This assumption has been observed to have a negligible impact on the resulting spring forces, P_{si}. Note that it is also reasonable, although less accurate, to estimate the HBE axial forces from the horizontal component of web plate yielding on the VBEs, P_{si}, considering VBE lengths tributary to each HBE, in other words:

$$P_{si} = \omega_{xci}\frac{h_i}{2} + \omega_{xci+1}\frac{h_{i+1}}{2} \tag{5}$$

Regardless of the method used, the spring forces are used below to determine the HBE axial forces. Note that these spring forces correspond to compression forces in the HBE, and can be of significant magnitude. Physically, one can envision the SPSW VBEs as being pulled toward each other by the uniformly distributed forces applied by the yielding webs, and the HBEs acting as regularly spaced "shoring" to keep the VBEs apart.

The axial force component in the intermediate and top HBEs resulting from the horizontal component of the plate yield forces on the HBEs, ω_{xbi}, is assumed to be distributed as shown in Figure 7. Note that for the bottom HBE, this distribution is the reverse of that in the top beam. These axial force components are then combined with the spring forces from the linear VBE model, resulting in the following equations for the axial force at the left and right sides of the intermediate and top HBEs (P_{bli} and P_{bri} respectively):

$$P_{bli} = -\left(\omega_{xbi} - \omega_{xbi+1}\right)\frac{L}{2} + P_{si} \tag{6}$$

$$P_{bri} = \left(\omega_{xbi} - \omega_{xbi+1}\right)\frac{L}{2} + P_{si} \tag{7}$$

where the spring forces should be negative indicating that they are adding to the compression in HBEs. As mentioned above, the axial forces from ω_{xbi} and ω_{xbi+1} in the bottom HBE may be taken as the mirror image of those shown in Figure 7, where ω_{xbi} is zero in that case as there is no web below the bottom HBE. Furthermore, there are no spring forces to consider at the bottom HBE location as the horizontal component

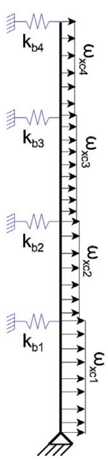


Fig. 6. Elastic VBE model with HBE springs.

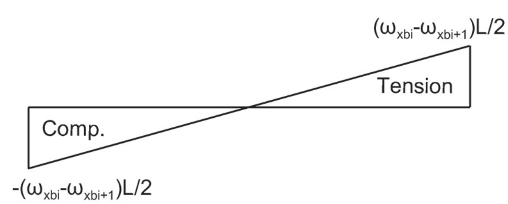


Fig. 7. Assumed HBE axial force distribution due to horizontal component of plate yield forces on the HBE.

of force from web plate yielding on the lower portion of the bottom VBE is added to the base reaction determined as part of the plastic collapse mechanism analysis, as described below. Therefore, the bottom HBE axial forces on the right and left hand sides, P_{br0} and P_{bl0} are:

$$P_{br0} = -\omega_{xb1}\frac{L}{2} \quad \text{and} \quad P_{bl0} = \omega_{xb1}\frac{L}{2} \tag{8}$$

HBE Reduced Plastic Moments and Corresponding Shear Forces

Once the HBE axial forces have been estimated it is possible to determine the plastic moment that will develop at the HBE ends for the assumed collapse mechanism, reduced for the presence of axial load. Note that it is conservative to assume that this reduction is negligible; however, since substantial axial loads may develop in the HBEs, resulting in significantly reduced plastic moment capacities, it can be advantageous to account for the reduced plastic moments at the left and right HBE ends, M_{prl} and M_{prr}, respectively.

The intermediate and top HBEs will have free body diagrams similar to that shown in Figure 8, except that there will be no plate forces acting above the top HBE. For the bottom HBE, the axial forces at the HBE ends will be in the opposite direction to those shown in Figure 8 and there will be no plate forces acting below the HBE. The reduced plastic moment capacity at the HBE ends, given here for the left end, can be approximated by (Bruneau, Whittaker and Uang, 1998):

$$\begin{aligned} &1.18\left(1-\frac{|P_{bli}|}{F_{yb}A_{bi}}\right)Z_{xbi}F_{yb} \quad \text{if} \quad 1.18\left(1-\frac{|P_{bli}|}{F_{yb}A_{bi}}\right) \le 1.0 \\ &Z_{xbi}F_{yb} \quad \text{if} \quad 1.18\left(1-\frac{|P_{bli}|}{F_{yb}A_{bi}}\right) > 1.0 \end{aligned} \tag{9}$$

where

F_{yb} = HBE yield strength
A_{bi} = HBE cross-sectional area for story i
Z_{xbi} = HBE plastic modulus for story i.

Using the reduced plastic moment capacities and the HBE free body diagram shown in Figure 8, the shear forces at the left and right ends of all HBEs, V_{bl} and V_{br} can be found from:

$$V_{bri} = \frac{M_{prri} + M_{prli}}{L} + \left(\omega_{ybi} - \omega_{ybi+1}\right)\frac{L}{2} \tag{10}$$

$$V_{bli} = V_{bri} - \left(\omega_{ybi} - \omega_{ybi+1}\right)L \tag{11}$$

Applied Lateral Loads

The final forces necessary to complete the free body diagram of the VBE are the applied lateral loads corresponding to the assumed collapse mechanism for the SPSW (Figure 3a). Following the derivation in Berman and Bruneau (2003) the governing equation for that collapse mechanism is:

$$\begin{aligned} \sum_{i=1}^{n_s} F_i H_i &= \sum_{i=0}^{n_s} M_{prli} + \sum_{i=0}^{n_s} M_{prri} \\ &+ \sum_{i=1}^{n_s} \frac{1}{2}\left(t_{wi} - t_{wi+1}\right) F_{yp} L H_i \sin\left(2\alpha_i\right) \end{aligned} \tag{12}$$

where

F_i = applied lateral load at each story to cause the mechanism
H_i = height from the base to each story

and other terms are as previously defined. Note that the indices for the HBE plastic moment summations begin at zero so that the bottom HBE (denoted HBE_0) is included.

To employ Equation 12 in calculating the applied lateral loads that cause this mechanism to form, it is necessary to assume some distribution of those loads over the height of the structure; in other words, a relationship between F_1, F_2, etc. For this purpose, a pattern equal to that of the design lateral seismic loads from the appropriate building code may be used. This is an approximation that is simple and that has been observed to provide reasonable results for SPSW. It would also be appropriate to use the deformation pattern of the first mode of vibration of the structure for this purpose (obtained from a modal analysis), but this more sophisticated approach is unnecessary given that the code specified distribution of lateral seismic forces vertically on a lateral force resisting system is meant to simulate first mode characteristics. Once a load pattern is assumed and a relationship between the applied collapse loads at each story is determined, Equation 12 may be used to solve for those collapse loads.

The base shear force, V, for the collapse loading is found by summing the applied lateral loads. Horizontal reactions at the column bases, R_{xL} and R_{xR}, are then determined by dividing the collapse base shear by 2 and adding the pin-support reaction from the VBE model, R_{bs}, to the reaction under the left VBE and subtracting it off the reaction under the right

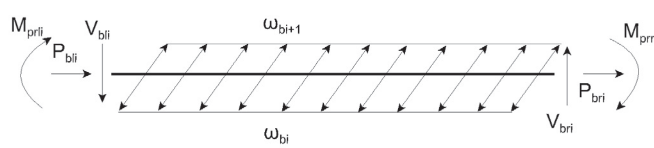


Fig. 8. HBE free body diagram.

VBE. Vertical base reactions can be estimated from overturning calculations using the collapse loads as:

$$R_{yl} = \frac{\sum_{i=1}^{n_s} F_i H_i}{L} \qquad \text{and} \qquad R_{yr} = -R_{yl} \tag{13}$$

Determination of VBE Design Loads

The moment, axial, and shear force diagrams for the VBEs are established once all the components of the VBE free body diagrams are estimated. The diagrams give minimum design actions for those VBEs such that they can resist full web plate yielding and HBE hinging.

Additional Considerations

Though not explicitly considered in the above formulations, use of the ratio of the expected yield stress to the specified minimum yield stress, R_y, may be incorporated into the procedure when determining the distributed loads from plate yielding and when determining HBE plastic moment capacity. Additionally, when deep VBEs are used, the length between VBE flanges, L_{cf}, may be substituted for the column centerline bay width, L, when applying the plate yielding loads to the HBEs. Furthermore, using the schematic structure shown in Figure 3 for which structural members have no width, the HBE plastic hinges are assumed to form at the VBE centerlines, which is not the actual case. HBE hinges will typically form $d_b/2$ from the column face, where d_b is the HBE depth. This can be accounted for by either including in the VBE free body diagrams the distance from the column centerlines to the HBE hinge locations or by calculating the projected column centerline moment as is done for moment frames. This calculation is not included here for simplicity and because the increase in moment applied to the VBE is generally small relative to the magnitude of the moments generated by web plate yielding and HBE hinging. Gravity loads are another consideration that has not been included; however, they can easily be added to the vertical components of the web plate yield forces that are applied to the HBEs in Figure 8. They will then be accounted for in the resulting HBE shear forces and VBE axial forces. Finally, this procedure will provide reasonable VBE design forces for SPSWs that can be expected to yield over their entire height—typically shorter SPSWs. This procedure will likely be overly conservative for tall SPSWs where nonlinear time history analysis indicates that simultaneous yielding of the web plates over the entire SPSW height is unlikely. In those situations it may be acceptable to reduce the VBE axial forces obtained from this proposed procedure (following a procedure similar to that proposed by Redwood and Channagiri, 1991) to account for some web plates remaining partially elastic while others yield. However, at each story the VBEs should be designed to resist the moments generated by yielding of the web plates at that level and the corresponding frame moments.

EXAMPLE ESTIMATION OF VBE DESIGN LOADS

Two examples of the proposed procedure for estimating VBE design loads for their capacity design are described below. Since the primary inelastic elements in SPSWs are the web plates and the web plate strength is a function of thickness, the two examples explore the cases of variable and constant web plate thickness over the height of a four-story SPSW. In the SPSW with variable web plate thickness, a different thickness is used at each story and not limited to those available for common plate stock. For the case of constant web plate thickness over the height of the SPSW, the web plates were designed for the required story shear at the first story and that thickness is used over all four stories. Furthermore, that thickness was constrained to be available from common plate stock.

Structure Description and SPSW Design

The MCEER (Multidisciplinary Center for Earthquake Engineering Research) demonstration hospital was used as the prototype structure (Yang and Whittaker, 2002) for which the SPSWs were designed. For simplicity, four SPSWs were assumed to carry equal portions of the seismic load resulting from the active seismic weight of the structure of 9,800 kips (2,613 kips, 2,542 kips, 2,542 kips, and 2,103 kips at the 1st story, 2nd story, 3rd story and roof HBEs, respectively). The geometries of the two SPSWs are shown in Figure 9 and the structure is assumed to be located on Class D soil. Design seismic loads were calculated using FEMA 450 (FEMA, 2003) and the associated spectral acceleration maps. Design short and 1-second spectral ordinates, S_{DS} and S_{D1}, were calculated to be 1.17g and 0.44g respectively. The period of the structure was estimated using the FEMA procedures as 0.38 sec and using a response modification coefficient, R, of 7 and importance factor, I, of 1.5, the base shear for the structure was found to be 2,450 kips, or 612.5 kips for each SPSW. Distribution of the base shear up the height of the structure resulted in lateral loads of 215 kips, 195 kips, 132 kips and 71 kips, at each story from the roof down to the level of HBE_1 for each SPSW.

Figure 9 shows the web plate thicknesses selected for the two SPSWs designed for the above loading in accordance with *The Provisions*, and it also shows the selected HBE and VBE sizes. As mentioned above, cases of variable and constant web plate thickness, denoted SPSW-V and SPSW-C, respectively, have been considered. SPSW-V uses plate thicknesses that may not be available but correspond to the minimum required for the design story shear forces and SPSW-C uses the assumed minimum available plate thickness of 0.1875 in. Note that ASTM A36 steel has been assumed for the web plates and ASTM A992 for the HBEs and VBEs. VBE selection was done after a first iteration of the proposed procedure for evaluation of design loads with an assumed tension field inclination angle of 45º. Described below are the calculations for a second iteration of the proposed procedure. Additionally, for simplicity and to provide a direct comparison between design loads resulting from the current and proposed procedures, the effects of gravity loads and vertical ground motion were neglected (in other words, only VBE design loads resulting from the horizontal seismic loading will be calculated).

Calculation of VBE Design Loads

The proposed procedure for evaluating VBE design loads was employed for the SPSWs shown in Figure 9. Calculation results for the tension field inclination angle and distributed loads from web plate yielding given by Equations 2 and 3, are shown for both SPSW-C and SPSW-V in Table 1. Note that the inclination angle, as given by *The Provisions*, depends mostly on the aspect ratio, h/L, of the bays and therefore, does not show substantial difference between SPSW-C and SPSW-V.

Table 2 shows the HBE spring stiffnesses, k_b, used in the linear VBE models for each wall and the resulting spring loads, P_s, where the entry for the bottom HBE is the horizontal reaction at the pin base of the VBE. The HBE axial forces, P_{bl} and P_{br}, reduced plastic moments, M_{prl} and M_{prr}, and shear forces, V_{bl} and V_{br}, at each HBE end are then calculated per Equations 6 through 11. Results for all HBEs in both example walls are given in Table 2. For comparison purposes, the HBE spring forces were also estimated by considering the horizontal component of web plate yielding forces applied to the VBE tributary to each HBE, as given by Equation 5. The results given in Table 2 show some deviation relative to the spring forces from the VBE model, although for preliminary design of VBEs the difference may be insignificant.

Next, the applied seismic lateral loads at each story were found for the assumed collapse mechanism, similar to that shown in Figure 3a, using Equation 12. Note that it was assumed that those applied loads to cause collapse were in the

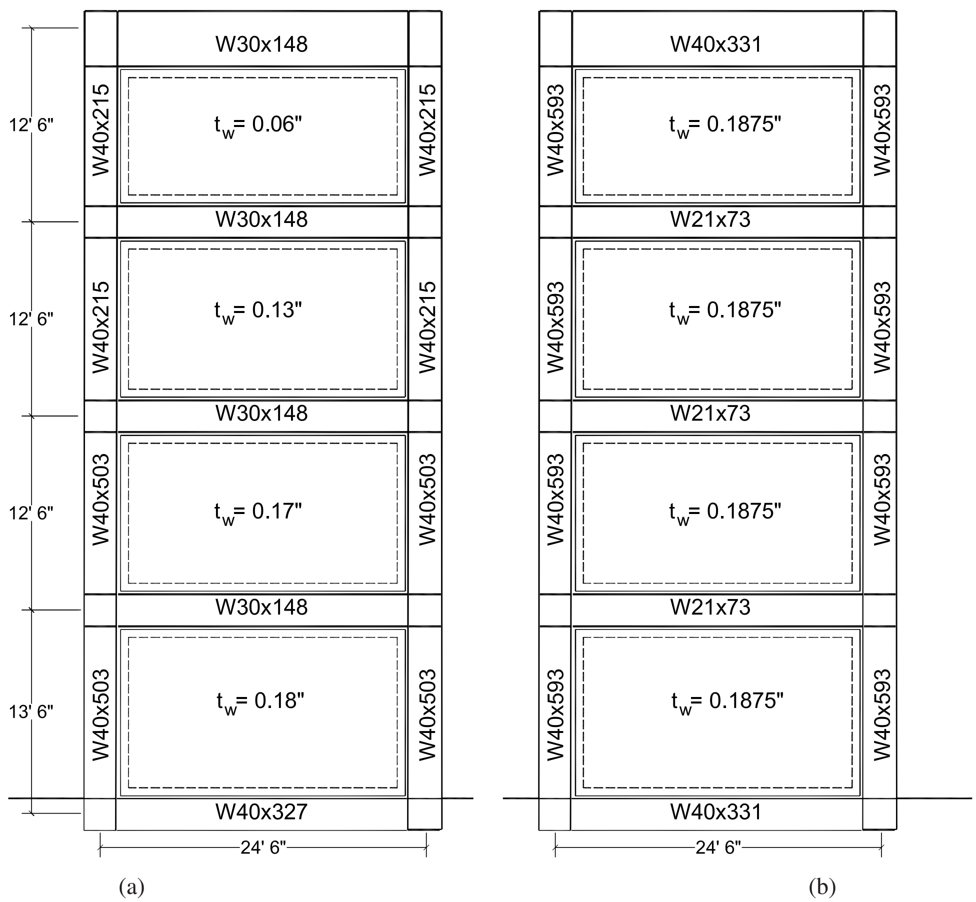


Fig. 9. Example SPSWs: (a) SPSW-V and (b) SPSW-C.

Table 1. Distributed Loads from Yielding Web Plates						
Wall	Story	α (deg)	ω_{yc} (kip/in.)	ω_{xc} (kip/in.)	ω_{yb} (kip/in.)	ω_{xb} (kip/in.)
SPSW-C	1	46	3.37	3.49	3.26	3.37
	2	48	3.35	3.75	3.00	3.35
	3	48	3.35	3.75	3.00	3.35
	4	48	3.35	3.75	3.00	3.35
SPSW-V	1	46	3.24	3.36	3.13	3.24
	2	48	3.06	3.42	2.74	3.06
	3	48	2.23	2.49	1.99	2.23
	4	48	1.13	1.26	1.01	1.13

Table 2. Linear VBE Model Parameters, Results and Corresponding HBE End Actions										
Wall	HBE	k_b (kip/in.)	P_s [(b)] (kips)	P_s [(c)] (kips)	P_{bl} (kips)	P_{br} (kips)	M_{prl} (kip-in.)	M_{prr} (kip-in.)	V_{bl} (kips)	V_{br} (kips)
SPSW-C	0	(a)	-277	-283	496	-496	71500	71500	965	8
	1	4242	-540	-564	-543	-537	4811	4862	-5	71
	2	4242	-622	-563	-622	-622	4095	4095	28	28
	3	4242	-537	-563	-537	-537	4864	4864	33	33
	4	19235	-277	-281	-770	216	68030	71500	915	34
SPSW-V	0	(a)	-250	-272	477	-477	70500	70500	940	19
	1	8582	-542	-529	-569	-515	20860	21560	86	202
	2	8582	-463	-443	-585	-341	20650	23820	42	261
	3	8582	-295	-281	-456	-134	22320	25000	17	305
	4	8582	-71	-95	-237	95	25000	25000	22	319

[(a)] Not applicable, in the linear VBE models there are pin supports at the bottom HBE locations.
[(b)] Values at the bottom HBE locations are the horizontal reactions at the pin supports.
[(c)] Spring forces approximated by Equation 5.

same pattern of distribution as the design lateral loads given above. Resulting lateral loads, F, are given in Table 3 and Table 4 gives the corresponding base shear, V, and base reactions, R_{xl}, R_{yl}, R_{xr}, and R_{yr}, for each of the example walls.

Axial, moment and shear force diagrams for the VBEs of the two SPSWs are shown for the left VBE of SPSW-C in Figures 10a, 10b and 10c, respectively. The resulting forces at the bases of the columns, where they are a maximum, are given in Table 5 for both SPSWs. Assuming lateral bracing of the columns at each story, the moment-axial capacity interaction equation values, given by Equation H1-1 of the AISC *Specification for Structural Steel Buildings* (AISC, 2005a), were 0.96 and 1.0 for the VBE sizes for SPSW-C and SPSW-V respectively, considering the VBE sections shown in Figure 9. Note that SPSW strength and VBE demand are proportional to the bay width, L, and in the case considered here the bay width is large. Lower SPSW overstrength and VBE demands may be achieved by reducing the bay width, however, the aspect ratio of the bay, L/h, must be greater than 0.8 and less than or equal to 2.5, as specified in *The Provisions* Section 17.2b.

Comparison with Current Procedures

To judge the adequacy of both the current and proposed approximate procedures for determining VBE design loads, nonlinear static analysis of strip models of SPSW-C and SPSW-V are used. The strip models had tension only strip

Table 3. Seismic Loading for Assumed Collapse Mechanism

Wall	Story	*F* (kips)
SPSW-C	1	236
	2	440
	3	649
	4	716
SPSW-V	1	197
	2	367
	3	541
	4	597

Table 4. Base Shear and Reactions for Assumed Collapse Mechanism

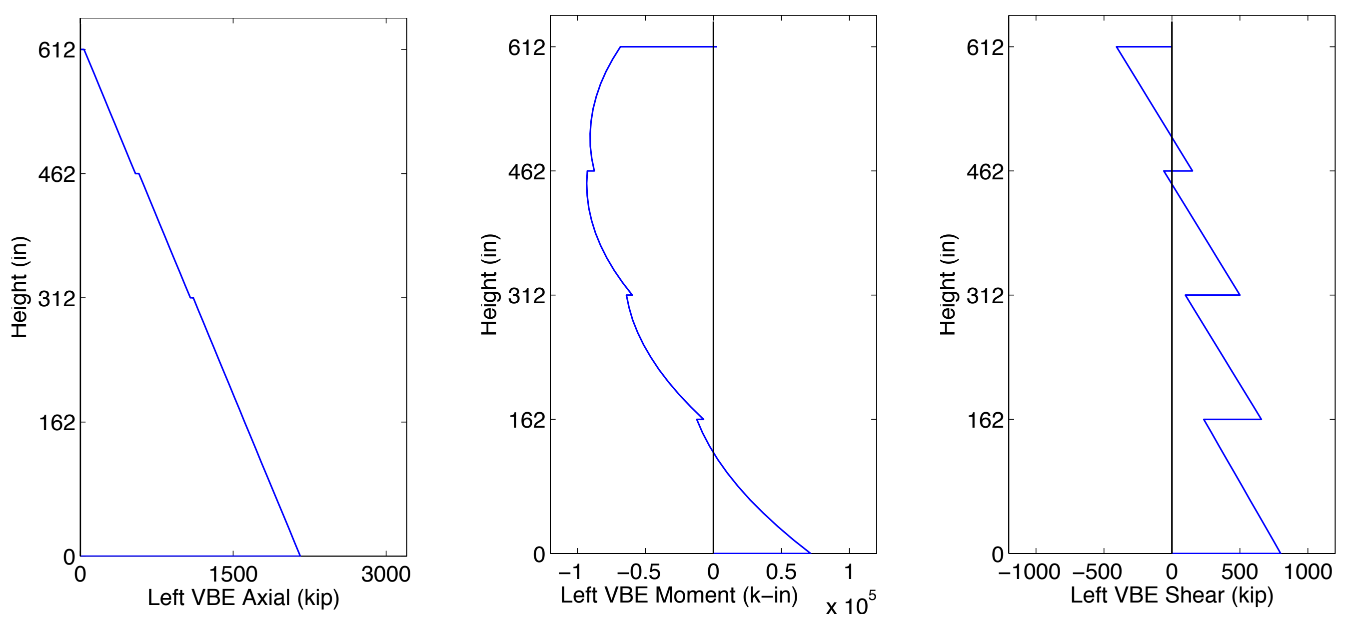
Wall	*V* (kips)	R_{xl} [(a)] (kips)	R_{yl} [(a)] (kips)	R_{xr} [(a)] (kips)	R_{yr} [(a)] (kips)
SPSW-C	2041	-1298	-3107	-744	3100
SPSW-V	1702	-1101	-2591	-601	2600

[(a)] Positive *x* and *y* directions are right and up, respectively.

Table 5. Actions at Column Bases from Proposed Procedure

Wall	Column Side	Axial (kips)	Moment (kip-in.)	Shear (kips)
SPSW-C	Left	2142	71500	802
	Right	-3115	71500	248
SPSW-V	Left	1651	70500	624
	Right	-2610	70500	164

[(a)] Positive *x* and *y* directions are right and up, respectively.



(a) (b) (c)

Fig. 10. Force diagrams for the VBE of SPSW-C: (a) axial, (b) moment, and (c) shear.

elements with elastic-perfectly-plastic behavior. Flexural-axial hinges were also added at all possible HBE and VBE locations. The HBE and VBE hinges were defined to have moment-axial interaction per FEMA 356 (FEMA, 2000) and a low strain hardening level (in other words, 0.5%) introduced to ensure numerical stability but close to zero because none of the design procedures that will be compared consider strain hardening. Resulting pushover curves for the two SPSWs are shown in Figure 11. The analyses showed that all web elements yielded and many HBEs formed hinges prior to yielding of any VBEs. Note that linear analyses of the strip models for the design seismic loads were also performed for use in the LE+CD and ICD approaches as described below.

VBE design loads were calculated again using the LE+CD procedure as per the commentary of *The Provisions*. The distributed loads from web plate yielding given in Table 1 were applied to linear models of the surrounding moment resisting frames for each SPSW and resulting VBE moments were recorded. VBE axial loads were then found by multiplying the axial loads from the linear strip model analysis factored by the 2.5 overstrength factor.

The indirect capacity design approach (ICD) was also used to estimate the VBE design loads for each SPSW. Moments from the linear strip models were factored by B, which was 2.10 and 2.03 for SPSW-C and SPSW-V, respectively. Axial loads were found from B times the overturning moment as described in the commentary of *The Provisions*.

Figures 12 through 15 compare axial loads and moments from the three procedures for approximating VBE design loads (in other words, LE+CD, ICD, and the proposed procedure) with those from pushover analysis for both SPSW-C and SPSW-V. As shown, the proposed procedure agrees well with pushover results in terms of both VBE axial force and moment. Furthermore, the proposed procedure is able to capture the important aspects of SPSW behavior that effect the VBE force diagrams, such as moment-axial interaction in HBEs, and proper distribution of HBE axial load to the right and left VBEs.

The LE+CD procedure agrees well with pushover analysis for VBE axial force as shown in Figures 12 and 14. However, because the procedure neglects the application of the lateral loads to cause web plate yielding when evaluating the VBE moments, the moment diagrams in Figures 13 and 15 are not in agreement. Neglecting the applied loads to cause infill yielding (recall Figure 2), results in moment diagrams that do not include the significant contributions of frame action under those loads, which in these cases are actually large enough to not only change the magnitude but also the sign of the VBE moments. Although it appears the VBE moment diagrams from the LE+CD for the VBEs from SPSW-V may simply be reversed, that is not the case, and those for SPSW-C would not agree even if they were converted into their mirror image.

Finally, the ICD approach reasonably estimates the VBE axial forces; however, because the overstrength is very large for these SPSWs, it is not able to adequately estimate the VBE moments. As shown in Figure 13 and 15, the ICD results in VBE moment diagrams that have similar shape to those from pushover analyses but significantly underestimates the values. Therefore, the proposed procedure is the only one of the three methods available for estimating design loads for VBEs that ensures web plate yielding is able to fully develop prior to hinging in VBEs.

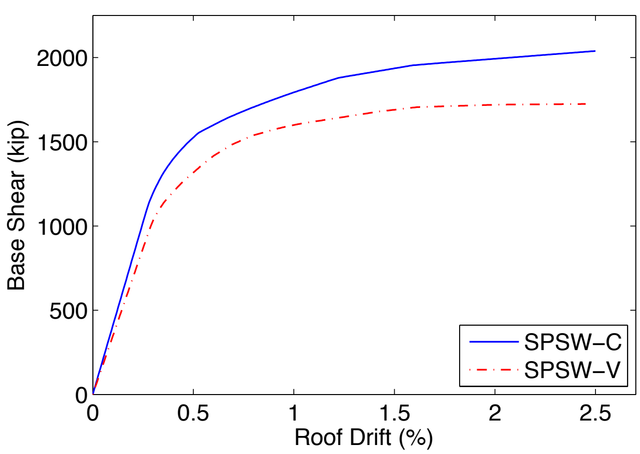


Fig. 11. Pushover Curves for Example SPSW.

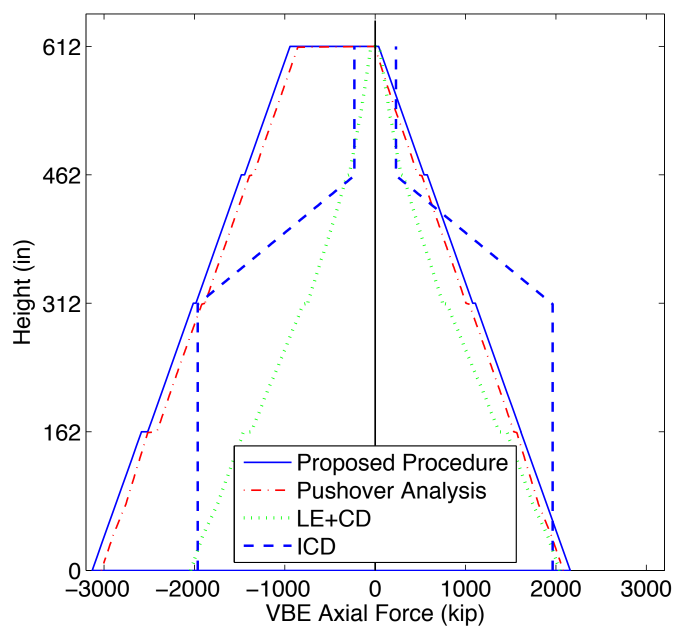


Fig. 12. Comparison of VBE Axial Forces from Various Methods for SPSW-C.

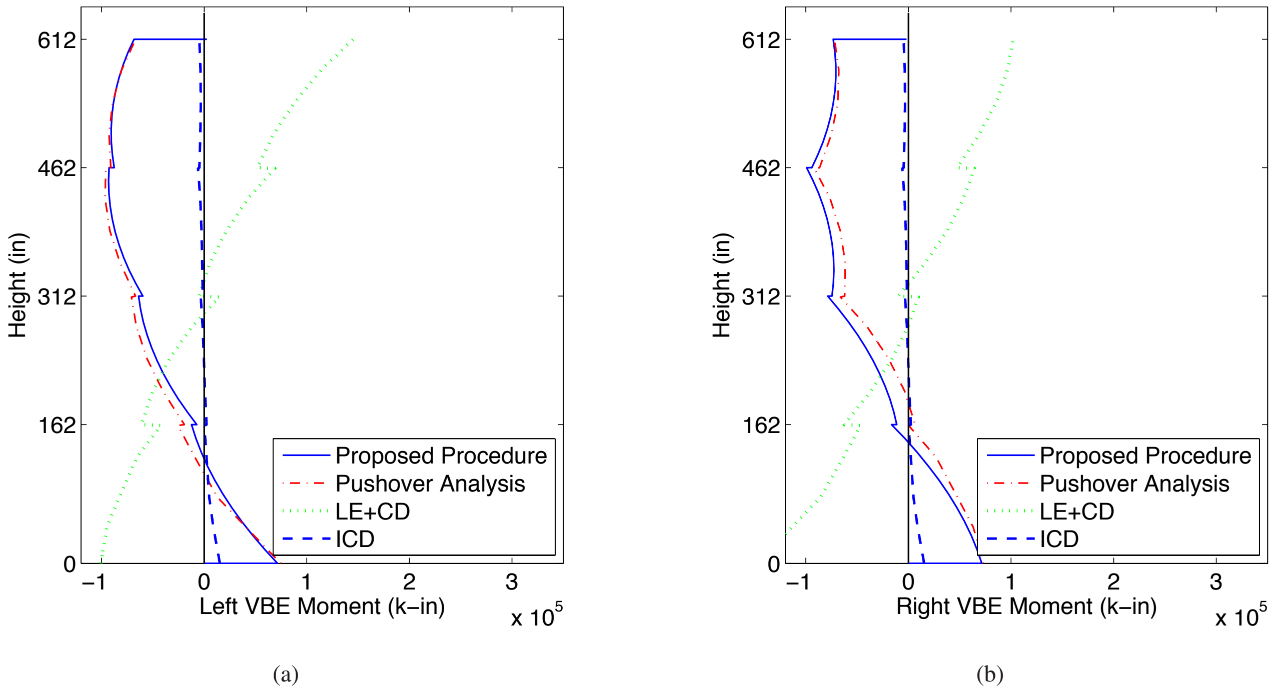


Fig. 13. Comparison of VBE moments from various methods for SPSW-C.

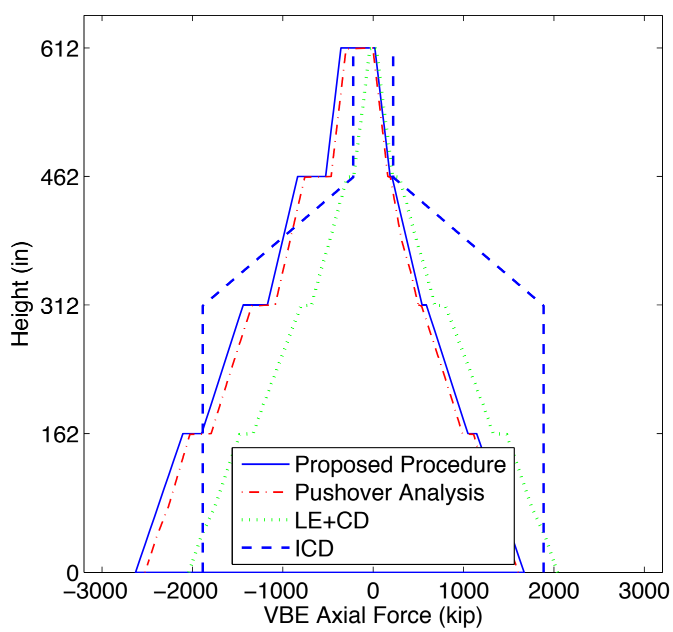


Fig. 14. Comparison of VBE axial forces from various methods for SPSW-V.

CONCLUSIONS

A procedure for estimating the design loads for VBEs of SPSWs has been proposed. The procedure does not involve nonlinear analysis and is based on an assumed plastic collapse mechanism and linear model of one of the vertical boundary elements. Moment and axial force diagrams from the proposed procedure were shown to agree well with results from pushover analyses of two example steel plate shear walls. Furthermore, deficiencies in the two approximate methods for capacity design of VBEs in the commentary of the *The Provisions* were identified and they were found to result in moment diagrams that differed significantly from those observed in pushover analyses.

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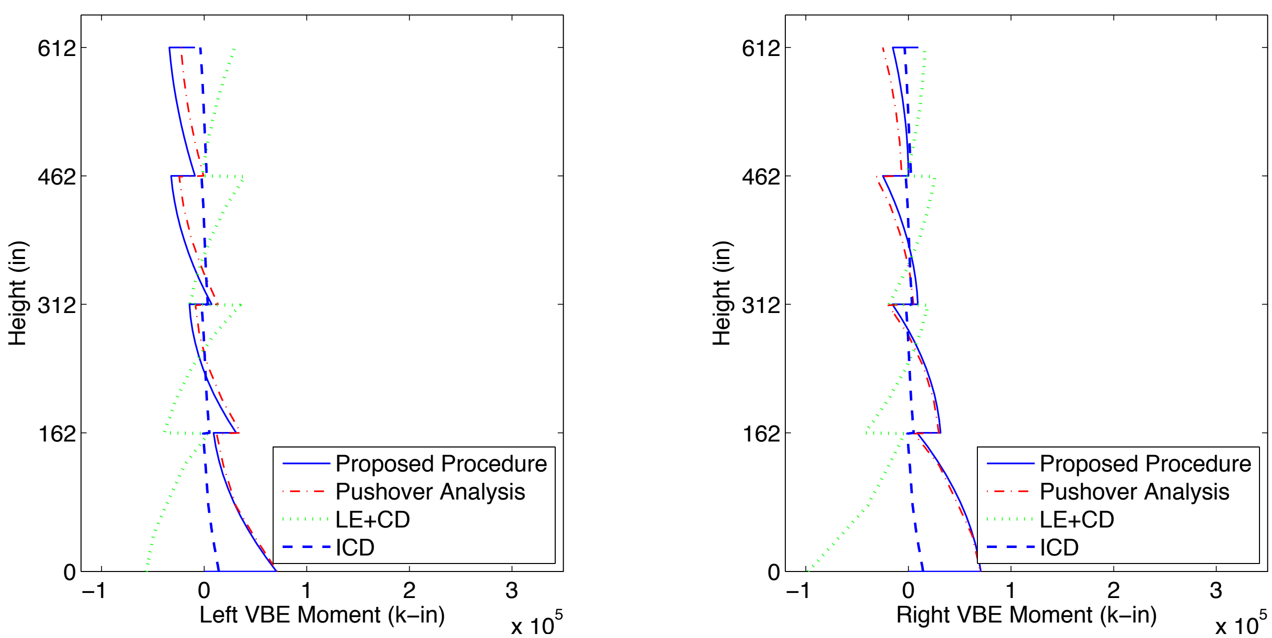

(a) (b)

Fig. 15. Comparison of VBE Moments from Various Methods for SPSW-V.

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