A Frequency-Domain Entropy-Based Detector for Robust Spectrum Sensing in Cognitive Radio Networks

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***Abstract*—Sensitivity to noise uncertainty is a fundamental limitation of current spectrum sensing strategies in cognitive radio networks (CRN). Because of noise uncertainty, the performance of traditional detectors such as matched filters, energy detectors, and even cyclostationary detectors deteriorates rapidly at low Signal-to-Noise Ratios (SNR). To counteract noise uncertainty, a new entropy-based spectrum sensing scheme is introduced in this letter. The entropy of the sensed signal is estimated in the frequency domain with a probability space partitioned into fixed dimensions. It is proven that the proposed scheme is robust against noise uncertainty. Simulation results confirm the robustness of the proposed scheme and show 6dB and 5dB performance improvement compared with energy detectors and cyclostationary detectors, respectively. In addition, the sample size is significantly reduced compared to an energy detector.**

***Index Terms*—Cognitive radio, spectrum sensing, entropy, noise uncertainty.**

I. Introduction

Sensitivity to noise uncertainty is a fundamental limitation of current spectrum sensing strategies designed to detect the presence of primary users in cognitive radio networks (CRN). Because of noise uncertainty, the performance of traditional detectors such as matched filters, energy detectors, and even cyclostationary detectors deteriorates rapidly at low Signal-to-Noise Ratios (SNR). Without accurate estimation of background noise, there exists an absolute "SNR wall" below which a detector may fail to be robust, no matter how long the detector can observe the channel[1]. Noise uncertainty can be alleviated by on-line calibration, but it cannot be completely eliminated. The SNR wall problem might be overcome by macroscale features under the assumption of prior knowledge of channel characteristics and infinite sample size [2]. However, these assumptions usually do not hold in practice.

The idea to use entropy for spectrum sensing in CR is presented in [3], where the information entropy is estimated directly from the output sequence of a matched filter in the time domain. Entropy-based sensing is based on the fact that the entropy of a stochastic signal is maximized if the signal is Gaussian noise. If the received signal contains the primary user's modulated signal, the entropy is reduced. However, the adaptability of the scheme is limited by the assumptions of perfect knowledge of primary signal and synchronization, which usually do not hold in practice.

In this letter, we investigate entropy-based detection to counteract noise uncertainty in the frequency domain. Frequency transform is first applied to the received signal, and the spectrum magnitude is regarded as a random variable. The probability space is then partitioned into fixed dimensions and the Shannon entropy is finally calculated as the information measure of the received signal for test statistic. We demonstrate that the entropy is independent of noise power under a fixed-dimension probability space and that the proposed scheme is thus robust to noise uncertainty. Simulation results show that significant performance improvements can be obtained compared with energy detectors and cyclostationary detectors, with strong robustness against noise uncertainty.

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II. A Frequency-Domain Entropy-Based Detector

We consider a signal with frequency bandwidth B_w and central frequency f_c. The general discrete signal $x(n)$ at the cognitive receiver can be expressed as

$$x(n) = s(n) + w(n),\ n = 0, 1, \cdots, N-1, \tag{1}$$

where $s(n)$ is the primary signal of interest, $w(n)$ represents background noise, and N is the sample size. We assume that noise is Gaussian white noise (WGN) with variance σ_0^2 and $s(n)$ can be any stochastic signal accounting for channel characteristics like fading and multipath.

The spectrum sensing problem is formulated as binary hypotheses: H_0 denotes absence of primary signal, and H_1 denotes presence of the primary signal. The entropy quantifies the information contained in a signal. However, the entropy of the received signal in the time domain is related to the signal power and is sensitive to noise uncertainty. We thus consider entropy based detector in frequency domain.

Applying discrete Fourier transform (DFT) to (1), we obtain

$$\overline{X}(k) = \overline{S}(k) + \overline{W}(k),\ k = 0, 1, \cdots, K-1 \tag{2}$$

where $K = N$ is the DFT size, $\overline{X}$, $\overline{S}$, and $\overline{W}$ denote the complex spectrum of the received signal, primary signal, and noise respectively. The random variable Y for which we want to estimate the probability density function (PDF) represents the spectrum magnitude of the measured signal. Hence, a possible detection strategy consists of testing information entropy based on a frequency model as

$$H_{L0}(Y)\ vs.\ H_{L1}(Y) \tag{3}$$

where $H_{Li}(Y)$ denotes the entropy with number of states L in hypothesis H_i.

For simplicity, we consider the histogram method to estimate the probability of each state. The number of states of the random variable is equal to the bin number L (dimension of the probability space). Let k_i denote the total number of occurrences in the i^{th} bin, we then have $\sum_{i=1}^{L} k_i = N$. The probability p_i is defined as the frequency of occurrences in the i^{th} bin, i.e., $p_i = k_i/N$. In the probability space, we have $\sum_{k=1}^{L} p_i = 1$. The bin width $\Delta = Y_m/L$, where Y_m denotes the maximum spectrum amplitude of the signal.

A. Spectrum statistics of the received signal

In hypothesis H_0, the received signal $x(n) = w(n)$ consists of noise (i.e., independently distributed Gaussian random variables). Hence, $\forall k \in \{0, 1, \dots, N-1\}$, both the real part W_r and the imaginary part W_i of the spectrum follow a Gaussian distribution because they are linear combinations of Gaussian random variables. Consequently, the spectrum $\overline{W}$ is a complex Gaussian variable with mean and variance expressed by

$$E\{\overline{W}(k)\} = \frac{1}{N}\sum_{n=0}^{N-1} \exp(-j\frac{2\pi}{N}kn)E[w(n)] = 0, \quad (4)$$

$$D\{\overline{W}(k)\} = \frac{1}{N^2}\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} E[w^2(n)] = \frac{\sigma_0^2}{N}. \quad (5)$$

Hence, $Y = \sqrt{W_r^2 + W_i^2}$ follows a Rayleigh distribution with parameter $\sigma_1 = \sigma_0/\sqrt{2N}$, and differential entropy H_d as

$$H_d(Y) = 1 + \ln\frac{\sigma_1}{\sqrt{2}} + \gamma/2, \quad (6)$$

where γ is the Euler-Mascheroni constant.

In hypothesis H_1, the received signal consists of both primary signal and noise. For a primary signal carrying certain information, the real part $S_r(k)$ and imaginary part $S_i(k)$ of the spectrum are not always zero. Hence, for $X_r(k) \neq 0$, the real part $X_r(k)$ and imaginary part $X_i(k)$ follow a Gaussian distribution such that $X_r(k) \sim N(S_r(k), \sigma_0^2/2N), X_i(k) \sim N(S_i(k), \sigma_0^2/2N)$. Hence, the spectrum magnitude of the received signal in H_1 generally follows a Rice distribution without analytical expression of differential entropy.

B. Robustness to noise uncertainty

Proposition 1. *With probability space partitioned into fixed dimensions, the discrete entropy of the spectrum of white Gaussian noise can be approximated by a constant.*

Proof: Let L denote the total number of bins, Y_m denote the maximum spectrum amplitude and Δ represent the bin width with $\Delta = Y_m/L$. Since the spectrum amplitude is a random variable, its maximum value is also a random variable, with its mean value $\mathbb{E}(Y_m) = \sigma_1$, where $\mathbb{E}(y)$ denotes the expectation of random variable y. Under a given bin number L, the bin width Δ is also a random variable with $\mathbb{E}(\Delta) = \sigma_1/L$.

By the mean value theorem, when the bin width Δ is adequately small, there exists a value Y_i within each bin Δ, such that the probability p_i can be expressed as

$$p_i = \int_{i\Delta}^{(i+1)\Delta} f(Y)dY = f(Y_i)\Delta. \quad (7)$$

Then, the entropy of the quantized version can be written as

$$H_L(Y) = -\sum_{i=1}^{L} p_i \log p_i = -\sum_{i=1}^{L}(f(Y_i)\Delta)\log(f(Y_i)\Delta) \quad (8)$$

$$= -\sum_{i=1}^{L} f(Y_i)\Delta \log f(Y_i) - \mathbb{E}(\log\Delta) \quad (9)$$

$$= -\sum_{i=1}^{L} f(Y_i)\Delta \log f(Y_i) - \log(\mathbb{E}(\Delta)). \quad (10)$$

If the density $f(Y)$ of the random variable Y is Riemann integrable, the first term in (10) can be approximated [4] by the differential entropy in (6). With natural logarithm, we obtain

$$H_L(Y) \approxeq H_d(Y) - \ln(\mathbb{E}(\Delta)) \quad (11)$$

$$= 1 + \ln\frac{\sigma_1}{\sqrt{2}} + \gamma/2 - \ln\frac{\sigma_1}{L} \quad (12)$$

$$= \ln\frac{L}{\sqrt{2}} + \frac{\gamma}{2} + 1 \quad (13)$$

It is seen that the noise power term σ_1^2 is canceled out and discrete entropy is approximated by a constant for a given bin number L, which implies that the false alarm ratio is almost fixed for a given threshold. In this sense, the proposed detector is intrinsically robust to noise uncertainty.

In implementation, Y_m can be known accurately to the detector by obtaining the maximum spectrum amplitude of the sampled signal, and the noise uncertainty is thus transferred to fluctuations of the bin width Δ by fixing the bin number L. Therefore, the bin number L is a design parameter determined by nominal noise, while the entropy does not depend on the noise term as shown in (13).

For a given bin number L, the entropy is estimated by (8) for both hypotheses and the test statistic is obtained as

$$T(Y) = -\sum_{i=1}^{L}\frac{k_i}{N}\log\frac{k_i}{N} \begin{cases} \leq \lambda : & \text{decide } H_1 \\ > \lambda : & \text{decide } H_0 \end{cases} \quad (14)$$

where λ is the detection threshold determined by a target false alarm ratio P_f.

Because of quantization errors and of the ideality of the Gaussian noise model, the noise term cannot be perfectly canceled out in practice. Assuming that the estimated noise entropy follows a Gaussian distribution with theoretical value H_L in (13) as the mean value and variance σ_e^2, the threshold is determined by

$$\lambda = H_L + Q^{-1}(1 - P_f)\sigma_e, \quad (15)$$

where $Q(u) = \frac{1}{\sqrt{2\pi}}\int_u^{\infty} exp(-\tau^2/2)d\tau$.

The test statistic $T(Y)$ is generally related to the bin number L and the sample size N. Since the detection is essentially based on the relative entropy $H(Y|H_1) - H(Y|H_0)$ and the influence of absolute errors of entropy estimation can be eliminated. Different bin numbers reveal different features of the data and $L > 10$ is required in the detection for a better description of the received signal. Once the bin number is fixed, the entropy in H_0 is a constant independent of sample size and the false alarm ratio keeps invariant; however, the sample size affects detection probability in H_1.Under a given

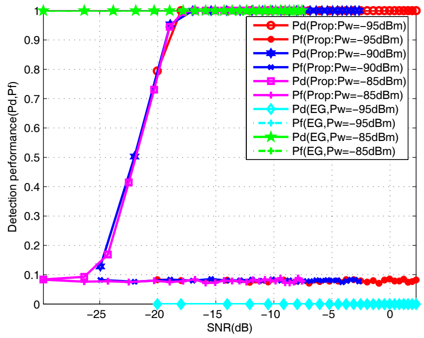

Fig. 1. Detection performance vs. SNR with ± 5dB noise uncertainty ("Prop" denotes the proposed detector, "EG" denotes energy detector).

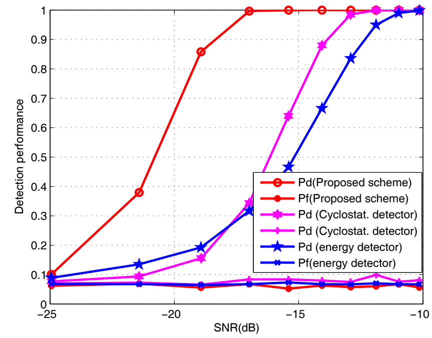

Fig. 2. Detection performance vs. SNR of the proposed scheme, cyclostationary detection and energy detection(N=5000,without noise uncertainty).

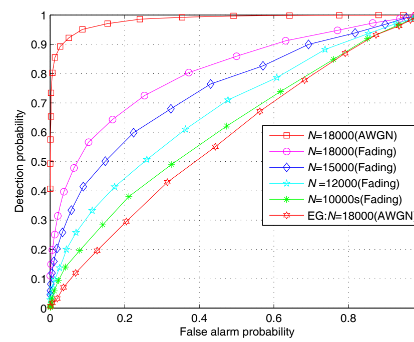

Fig. 3. ROC of the proposed scheme and energy detector (denoted as "EG in blue line") with different sample size (SNR=-25 dB, without noise uncertainty).

sensing time period, a possible way to improve detection performance is to increase the sampling frequency.

III. Performance Evaluation

The performance of the proposed scheme is evaluated through the probabilities of detection and false alarm, denoted by P_d and P_f respectively. Monte Carlo experiments are carried out with each result averaged over 10000 runs, nominal noise power $P_w = -90$ dBm, $L = 15$, $P_f = 0.08$ and $\sigma_e^2/H_L = 10^{-3}$. The theoretical entropy of noise is $H_L = 2.198$ by (13) and test threshold $\lambda = 2.149$ by (15).

The detection performance curves with and without noise uncertainty are plotted in Fig. 1, where the primary signal is a quadrature phase shift keying (QPSK) modulated signal with symbol ratio $R_s = 10\,\text{kbit/s}$, carrier frequency $f_c = 40\,\text{KHz}$, sampling frequency $f_s = 100\,\text{KHz}$, sampling time $t = 5\,\text{ms}$.

It is seen that the false alarm ratio with and without noise uncertainty is a constant (P_f=0.08). Moreover, for a fixed SNR, the detection probability curves do not shift. The results verify Proposition 1 that the proposed scheme is robust to noise uncertainty. While the performance of energy detector is deeply degraded under noise uncertainty in low SNR.

Figure 2 shows the performance of the proposed detector, cyclostationary detector and energy detector, where the primary signal is a Rayleigh fading double sideband (DSB) signal with main parameters the same as in Fig. 1 and $B_w = 12\,\text{KHz}$. For cyclostationary detection, the length of the time window for frequency estimation is 256, and hamming window is used for smoothing with decimation factor equal to 1.

It is seen that the proposed scheme is superior to cyclostationary detectors and energy detectors in terms of the required SNR. In particular, when the detection probability $P_d = 0.9$ under $P_f = 0.08$, the SNR of the proposed scheme is -18.8 dB, as compared to -13.8 dB and -12.4 dB for cyclostationary detector and energy detector, with more than 5dB and 6dB gain achieved respectively by the proposed detector.

Figure 3 shows receiver operation characteristics (ROC) curves of the proposed detector for different sample sizes at $\text{SNR} = -25\text{dB}$, in comparison to energy detection in AWGN.

It is observed that the detection probability generally increases as the sample size increases. Moreover, the proposed scheme with $N = 10000$ outperforms energy detector with $N = 18000$, which indicates that the sample size is reduced to about 65% by the proposed detector compared to the energy detector.

IV. Conclusions

A frequency-domain entropy-based spectrum sensing scheme for CRN was proposed and shown to improve the detection performance with respect to energy detector and cyclostationary detectors. The entropy of the measured signal is estimated in the frequency domain with the probability space partitioned into fixed dimensions. We analytically proved that the proposed scheme is robust against noise uncertainty. Through Monte Carlo experiments, we demonstrated that the proposed detector greatly outperforms energy detectors and cyclostationary detectors, with 6dB and 5dB performance improvements respectively. The sample size is significantly reduced by the proposed scheme compared to energy detector under the same detection performance.

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