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Decentralized Kinematic Control of Payload Transport by a System of Mobile Manipulators

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***Abstract*—In this paper, we examine creation of a decentralized kinematic control scheme for a composite system of two (or more) wheeled mobile manipulators that can team up to cooperatively transport a common payload. Each mobile manipulator module consists of a differentially-driven wheeled mobile robot (WMR) with a mounted planar two-degree-of-freedom (d.o.f) manipulator. A composite multi-degree-of-freedom system is formed when a payload is placed at the end-effectors of multiple such modules with significant advantages. However, the nonholonomic/holonomic constraints and active/ passive components within the composite vehicle need careful treatment for realizing the payload transport task. Hence, we first verify that arbitrary desired end-effector motions can be *accommodated,* within the feasible motion distributions of the articulations and the wheeled base. Then, we develop motion-plans by which this desired end-effector motion can be *actively realized,* using only the limited active motion-distribution of the differentially-driven wheels. Finally, we deploy this in the form of a two-level hierarchical control framework, with an upper-level planning of the steerable active vector-fields and a lower-level posture stabilization control of the individual WMRs. Preliminary experimental results from the decentralized-control implementation for a two-module composite vehicle are also presented.**

Keywords*—*Mobile Manipulator, Nonholonomic Constraints, Twist Analysis, Decentralized Control.

I. INTRODUCTION

In this paper, we present a decentralized control scheme for a cooperative composite system of multiple mobile manipulator modules. Autonomous multi-agent systems, such as these, are intended to be capable of performing tasks cheaper and better than would be possible by a single agent. The potential application arenas include material-handling [1], environmental cleanup [2], planetary-exploration [3], among others**.**

Our overall goal is to design and develop a flexible, scalable and modular framework for multiple individually autonomous mobile manipulators that can team up to cooperatively transport a large common payload. In our system, each basic module, seen in Figure 1(a), consists of a passive, planar, two-degree-of-freedom (d.o.f.) manipulator mounted on a differentially-driven wheeled mobile robot (WMR). The planar arms of adjacent modules supporting the common object create an effective articulated compliant linkage between the multiple wheeled bases, as depicted in Figure 1(b).

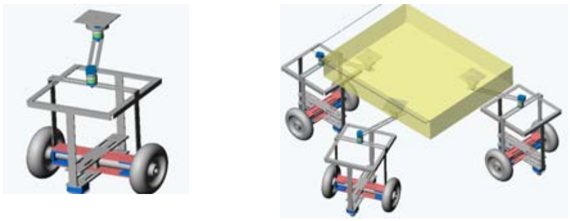
(a) (b)

Figure 1. (a) Representative mobile manipulator module and (b) representative composite system formation

Our selection of the topology of the individual mobile manipulator modules is guided by the requirement for modularity (in terms of easy attachment/detachment of multiple such modules to a common payload while maintaining at least three-d.o.f. within each sub-chain). Careful selection of the parameters of the of configuration of the intermediate compliant linkage is also critical, and these aspects are examined elsewhere [4],[5]. The resulting composite vehicle possesses: (i) ability to accommodate changes in the relative configuration (by virtue of the compliant linkage); (ii) a mechanism for detecting such changes (using sensed articulations); and (iii) means to compensate for such disturbances (using the redundant actuation of the bases), while performing the payload transport task.

A. Research Issues

The overall motion-planning and control of such systems creates a number of challenges. First, we must explicitly take into account on both the nonholonomic constraints due to the wheel assemblies, and the holonomic constraints due to the closed kinematic-loops formed by the manipulators. Second, while combining the d.o.f. of the mobile base and manipulator provides increased mobility and workspace, such system features considerable redundancy which needs to be suitably resolved. Finally, we note that some of the articulations are passive while the composite system is actuated solely by the differentially-driven wheels of the individual WMRs. Therefore, any formulation needs to take into account and coordinate mixtures of such active and passive components in the system.

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B. Background

Interest in the use of teams of wheeled mobile robots for collective foraging, map-building and reconnaissance applications has grown tremendously in recent years. In many cases, the formation paradigm has emerged as a convenient mechanism for complexity-reduction, abstraction and coordination between multiple agents with approaches including leader-following [6], virtual structures [7] and mixture of these approaches [8],[9]. However, in most of the above cases, there is no physical interaction between the agents within the formation.

Our situation is one where the agents physically interact with each other, where far less literature exists (but with considerable variety in their proposed approaches). Many approaches emphasize cooperative physical manipulation by teams of relatively simple pushing mobile robots [10]-[12]. Khatib et al. [13] used a decentralized control structure for cooperative tasks with mobile manipulation systems, but with holonomic bases and fully actuated manipulators. Others have considered development of optimal motion-planning/control schemes for nonholonomic cooperating mobile manipulators grasping and transporting objects [14], including the effects of flexibility [15] but from a centralized perspective.

Furthermore, in almost all cases, the focus is on a fully actuated manipulator, without any passive or semi-passive joints, which is a dominant feature in our system. Relatively few papers discuss design/control modifications intended to aid the decentralization of cooperation task. These include approaches for use of impedance-based controllers [16], selective locking/unlocking of joints [17] and/or special mechanical designs of the couplings between the multiple manipulators [18].

C. Our Approach

In this paper, we focus on realizing an arbitrary set of desired motions at the end-effector of the individual mobile manipulator modules, which would be critical for the overall payload transport task.

We will first verify that arbitrary desired end-effector motions can be *accommodated* within the feasible motion vector-field distribution of the articulations and the wheeled base. In a second stage, we partition this feasible motion distribution into actively-realizable and passively-accommodating distributions, both of which are *configuration dependent* and *steerable*. We then develop motion-plans to *actively realize* this desired end-effector motion using the restricted active motion-distribution of the differentially-driven wheels. Finally, we develop a control strategy using the actuation solely from the mobile bases (and employ the articulations primarily to absorb environmental disturbances and controller errors). We adopt a two level hierarchical control implementation which combines an upper-level design of the steerable active vector-fields with the lower-level posture stabilization control of the individual bases.

The remainder of the paper is organized as follows: in Section II, we develop the kinematic model of an individual mobile manipulator by using twist approach and determine the condition such that any arbitrary end-effector twist can be generated by the mobile platform. Section III describes the decentralized kinematic control between two mobile manipulators; followed by some preliminary experimental results in Section IV. Section V concludes the paper.

II. TWIST-BASED MODELING OF MOBILE MANIPULATOR

A. *Mathematical Preliminaries* [19]

In the two-dimensional Euclidean task space, the relative configuration of a moving frame $\{E\}$ relative to a fixed frame $\{F\}$ can be defined by the homogeneous transformation:

$$ {}^F A_E = \begin{bmatrix} {}^F R_E & {}^F \underset{\sim}{d} \\ 0 \quad 0 & 1 \end{bmatrix} \tag{1} $$

where ${}^F R_E$ is a 2×2 rotation matrix, and ${}^F \underset{\sim}{d}$ is a 2×1 displacement vector from the origin of frame $\{F\}$ to the origin of frame $\{E\}$. The twist corresponding to the motion of moving frame $\{E\}$ with respect to its immediately preceding frame $\{F\}$, and expressed in the moving frame $\{E\}$ is:

$$ {}^E\left[{}^F T_E\right] = {}^F A_E^{-1} \cdot \frac{d}{dt}\left[{}^F A_E\right] = \begin{bmatrix} {}^E\left[{}^F \Omega_E\right] & {}^E\left[{}^F \underset{\sim}{v}_E\right] \\ 0 \quad 0 & 0 \end{bmatrix} \tag{2} $$

where ${}^E\left[{}^F \Omega_E\right] = \begin{bmatrix} 0 & -\omega_{Ez} \\ \omega_{Ez} & 0 \end{bmatrix}$ and ${}^E\left[{}^F \underset{\sim}{v}_E\right]^T = \begin{bmatrix} v_{Ex} & v_{Ey} \end{bmatrix}$. The matrix can also be rewritten as a 3×1 twist vector as:

$$ {}^E\left[{}^F \underset{\sim}{t}_E\right]^T = \begin{bmatrix} \omega_{Ez} & \underset{\sim}{v}_E^T \end{bmatrix} \tag{3} $$

Note that $\underset{\sim}{v}_E = \left(v_{Ex}, v_{Ey}\right)$ and ω_{Ez} are the instantaneous linear and angular velocities in frame $\{E\}$ with respect to frame $\{F\}$, and expressed in the frame $\{E\}$. The resulting motion description expression known in kinematics as body-fixed twists are particularly useful in the study of locomotion system since it is invariant to the changes with respect to the inertial-fixed frame $\{F\}$. The twist matrix in (2) can then be transformed to any arbitrary frame $\{N\}$ by a similarity transformation of:

$$ {}^N\left[{}^F T_E\right] = \left[{}^N A_E\right]{}^E\left[{}^F T_E\right]\left[{}^N A_E\right]^{-1} \tag{4} $$

The total twist of an object may be considered as a linear combination of various twist contributions by each d.o.f. in a system expressed in a common frame. For instance, for an N d.o.f. system:

$$ \begin{aligned} {}^E\left[{}^F T_E\right] = {} & \underbrace{\left[{}^E A_1\right]{}^1\left[{}^1 T_E\right]\left[{}^E A_1\right]^{-1}}_{T_1} \\ & + \underbrace{\left[{}^E A_2\right]{}^2\left[{}^2 T_E\right]\left[{}^E A_2\right]^{-1}}_{T_2} + \cdots + \underbrace{{}^E\left[{}^N T_E\right]}_{T_N} \end{aligned} \tag{5} $$

These twist matrices can be rewritten as linear combinations of twist vectors parameterized by the corresponding manipulation variable rates of each d.o.f. $\underset{\sim}{\eta}^{T} = \begin{bmatrix} v_1 & v_2 & \cdots & v_N \end{bmatrix}$ (For instance, $\dot{\theta}$ for revolute joint and $\dot{d}$ for prismatic joint). A geometrically assembled Jacobian matrix J may now be written as:

$$ {}^{E}\left[{}^{F}t_{E}\right] = J\underset{\sim}{\eta} \tag{6}$$

where $J = \begin{bmatrix} \underset{\sim}{t_1} & \underset{\sim}{t_2} & \cdots & \underset{\sim}{t_N} \end{bmatrix}$.

B. Kinematic Modeling of Single Mobile Manipulator

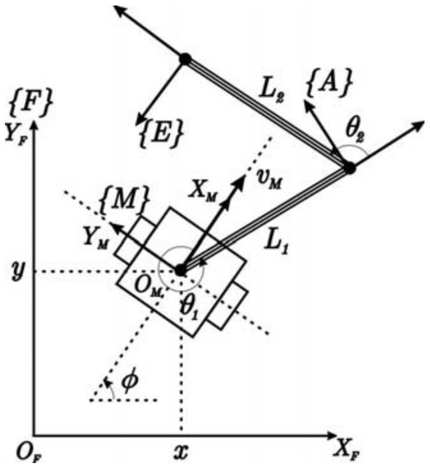


Figure 2. Schematic diagram of a two-link wheeled mobile manipulator

Figure 2 depicts a differentially-driven wheeled mobile robot (WMR) with a passive two-link manipulator mounted at the midpoint of the axle between the two driving wheels. The frame $\{M\}$ is rigidly attached to center of the WMR with the X_M-axis oriented in the direction of the forward travel of the mobile robot, and Y_M-axis oriented at direction perpendicular to X_M-axis, i.e. the direction where the WMR cannot move. Frame $\{E\}$ is attached to the end-effector of the 2 d.o.f. planar manipulator. The configuration of the manipulator with the two passive revolute joints can be parameterized by the two relative angles θ_1 and θ_2, with the link lengths L_1 and L_2. The frame $\{I\}$ is rigidly attached at the distal end of the first link in the Denavit-Hartenberg convention.

The kinematic model of the mobile manipulator is developed by composition of two parts: (a) mobile platform and (b) manipulator part. For the *mobile platform*, the velocity kinematic expressed in frame $\{M\}$ that includes the effect of nonholonomic constraints is:

$$ {}^{M}\left[{}^{F}T_{M}\right] = {}^{M}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\phi} + {}^{M}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v_M \tag{7}$$

where $\dot{\phi}$ and v_M are the angular and linear forward velocities of the mobile platform. The twists can be expressed in frame $\{E\}$ by a similarity transformation. For the *manipulator part*, the total twist can be expressed as a linear combination of the twist matrices of each d.o.f in the form of:

$$ {}^{E}\left[{}^{M}T_{E}\right] = {}^{E}\left[{}^{M}T_{I}\right]\dot{\theta}_1 + {}^{E}\left[{}^{I}T_{E}\right]\dot{\theta}_2 \tag{8}$$

Extracting the twist vectors of each part, we can assemble the Jacobian matrix of the entire system into:

$$ {}^{E}\left[{}^{F}\underset{\sim}{t_E}\right] = {}^{E}\begin{bmatrix} \omega_{Ez} \\ \underset{\sim}{v_E} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{t_1} & \underset{\sim}{t_2} & \underset{\sim}{t_3} & \underset{\sim}{t_4} \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ v_M \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{9}$$

where

$$ \underset{\sim}{t_1} = \begin{bmatrix} 1 \\ L_1 S_2 \\ L_1 C_2 + L_2 \end{bmatrix}, \ \underset{\sim}{t_2} = \begin{bmatrix} 0 \\ C_{12} \\ -S_{12} \end{bmatrix}, \ \underset{\sim}{t_3} = \begin{bmatrix} 1 \\ L_1 S_2 \\ L_1 C_2 + L_2 \end{bmatrix}, \ \underset{\sim}{t_4} = \begin{bmatrix} 1 \\ 0 \\ L_2 \end{bmatrix} $$

and $S_{ab\cdots} = \sin\left(\theta_a + \theta_b + \cdots\right)$ and $C_{ab\cdots} = \cos\left(\theta_a + \theta_b + \cdots\right)$.

Depending on the context, the columns of this Jacobian can be interpreted as the vector-fields spanning the distribution of feasible twists. For the remainder of the paper, we will denote the end-effector twist ${}^{E}\underset{\sim}{t}$ instead of ${}^{E}\left[{}^{F}\underset{\sim}{t_E}\right]$ for notation simplicity.

C. Twist Distribution-Based Analysis

Given an arbitrary end-effector twist ${}^{E}\underset{\sim}{t}$, we would often like to see if such twists lie within various distributions $[J]$ (or in some case sub-distributions such as $[J_a]$). Conceptually, one could perform this check by augment the twist ${}^{E}\underset{\sim}{t}$ to the columns in $[J]$ to form a matrix:

$$ G = \begin{bmatrix} {}^{E}\underset{\sim}{t} & [J] \end{bmatrix} \tag{10}$$

If $rank(G) = rank(J)$, then ${}^{E}\underset{\sim}{t}$ lies in the span of the distribution $[J]$. While this offers a method to verify if a specific twist ${}^{E}\underset{\sim}{t}$ falls into the span of $[J]$, it offers no guidance in determining to the conditions under which condition ${}^{E}\underset{\sim}{t}$ would lie within the span of $[J]$. Hence we prefer to use the alternate constructive method.

The *alternate method* is by determining a set of wrenches that span the constraint co-distribution that annihilates the vector-fields in $[J]$. If a candidate end-effector twist ${}^E\underset{\sim}{t}$ is in fact spanned by the vector fields of $[J]$, then this reciprocal wrenches would completely annihilate this end-effector twist [19]. Performing this test with the full twist distribution $[J]$ results in a null set for the reciprocal wrenches. Thus, this permits us to unambiguously determine that all possible end-effector twists can either be created or accommodated by the distribution $[J]$.

In our system, the joints of the manipulator are passive and create a set of vector-fields forming the *passive distribution.* The entire system is actuated by the mobile platforms which create the vector-fields forming the *active distribution.* Therefore, we wish to project an arbitrary end-effector twist first into active distribution $\left[J_a\right]$ in order to determine active actuator rates $\underset{\sim}{u}_a$ (and in the absence of a solution into the passive distribution $\left[J_p\right]$ to determine $\underset{\sim}{u}_p$). The end-effector twist can be decomposed into:

$$ {}^E\underset{\sim}{t} = \left[J_a\right]\underset{\sim}{u}_a + \left[J_p\right]\underset{\sim}{u}_p \tag{11}$$

where $\underset{\sim}{u}_a = \left[\dot{\phi} \quad v_M\right]^T$, $\underset{\sim}{u}_p = \left[\dot{\theta}_1 \quad \dot{\theta}_2\right]^T$

$$\left[J_a\right] = \begin{bmatrix} 1 & 0 \\ L_1S_2 & C_{12} \\ L_1C_2 + L_2 & -S_{12} \end{bmatrix}, \left[J_p\right] = \begin{bmatrix} 1 & 1 \\ L_1S_2 & 0 \\ L_1C_2 + L_2 & L_2 \end{bmatrix}$$

The reciprocal wrench to all the twists in the active distribution $\left[J_a\right]$ can be determined as:

$$\begin{bmatrix} -L_1C_1 - L_2C_{12} \\ S_{12} \\ C_{12} \end{bmatrix}$$

and it is guaranteed to be reciprocal to any twist that lies in the span of the distribution $\left[J_a\right]$. Hence, the reciprocity requirement yields the condition for an arbitrary end-effector twist ${}^E\underset{\sim}{t} = \left[\omega_{Ez} \quad v_{Ex} \quad v_{Ey}\right]^T$ to lie in this distribution as:

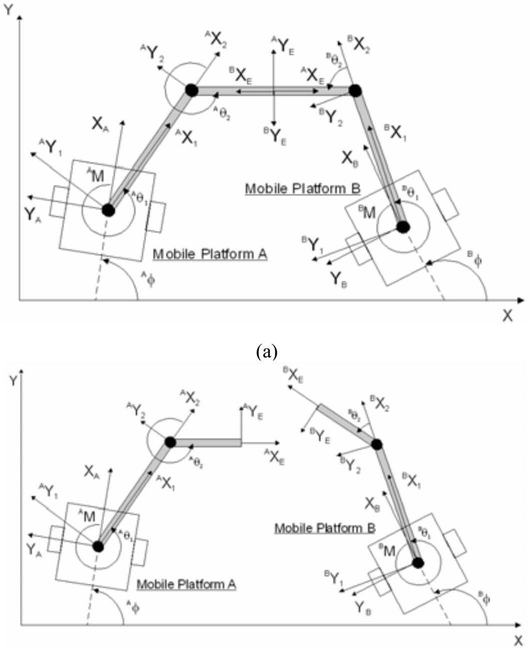
$$\left[-L_1C_1 - L_2C_{12}\right]\omega_{Ez} + \left[S_{12}\right]v_{Ex} + \left[C_{12}\right]v_{Ey} = 0 \tag{12}$$

We note that a twist ${}^E\underset{\sim}{t}$ applied to the end-effector would naturally create a helicoidal velocity vector-field (that corresponds to rigid motions of all attached points). Expressing this twist in the mobile robot frame $\{M\}$ by a similarity transformation, we obtain:

$$ {}^M\left[{}^E\underset{\sim}{t}\right] = \begin{bmatrix} \omega_{Ez} \\ \left[L_1S_1 + L_2S_{12}\right]\omega_{Ez} + C_{12}v_{Ex} - S_{12}v_{Ey} \\ \left[-L_1C_1 - L_2C_{12}\right]\omega_{Ez} + S_{12}v_{Ex} + C_{12}v_{Ey} \end{bmatrix} \tag{13}$$

In light of (13), we see that the condition in (12) is nothing more than the requirement that the Y-component of the helicoidal velocity vector-field in the mobile robot frame $\{M\}$ to be identically zero. This can be easily achieved by aligning the forward direction of travel (the X_M-axis of the robot) with the helicoidal velocity field. Thus this simple strategy guarantees that any arbitrary end-effector twist ${}^E\underset{\sim}{t}$ lies within the span of the active distributions $\left[J_a\right]$ alone and will be employed for control of our systems in subsequent sections.

III. DECENTRALIZED KINEMATIC COLLABORATION OF A SYSTEM OF TWO MOBILE MANIPULATORS



(a)

(b)

Figure 3. Overall system considered as: (a) composite system; or (b) independent mobile manipulators.

In this section, we consider the decentralized kinematic collaboration between *two* mobile manipulators carrying a common payload. The end-effectors of the physical manipulators are assumed to be rigidly attached to the payload and that we are given apriori, the locations of the attachments of these fixture sites, with respect to the object reference frame.

We then implement decentralized cooperation by considering a frame attached to a point of interest on the common object as the common end-effector frame of both flanking mobile manipulator systems (see Figure 3). The actuated bases of each mobile manipulator are controlled using the "Virtual Robot Algorithm," which combines the continuous time-varying nonlinear feedback control law for posture tracking [20] and point stabilization [21]. We refer the reader to [22] for greater discussion on the development and the stability analysis of the controller.

IV. PREMILINARY EXPERIMENTS

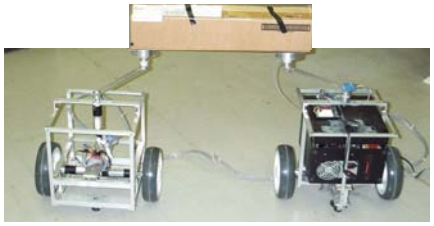


Figure 4. Experimental hardware testbed used for testing of the decentralized kinematic control scheme.

We tested this decentralized online planning scheme using the hardware test bed illustrated in Figure 4. Under ideal situations and perfect tracking by the two mobile bases, the entire system behaves like one large rigid body following a prescribed trajectory. However, in reality, small disturbances and deviations from perfect trajectory tracking by the two mobile bases are inevitable. The unactuated d.o.f.'s in the articulated manipulator arm accommodate these relative positioning errors of the mobile bases. Internal measurement of configuration of the linkage using the encoders permit detection and correction of the relative positioning between the mobile bases, without the need for external sensing. Key aspects of the design, analysis, development of the experimental test bed, and hardware-in-the-loop testing are discussed in [5].

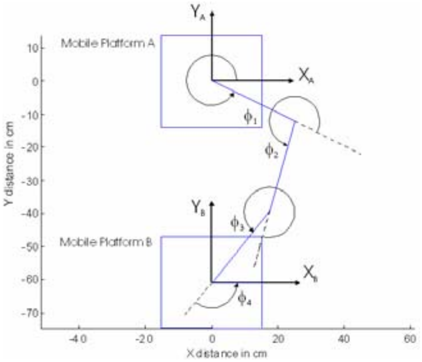
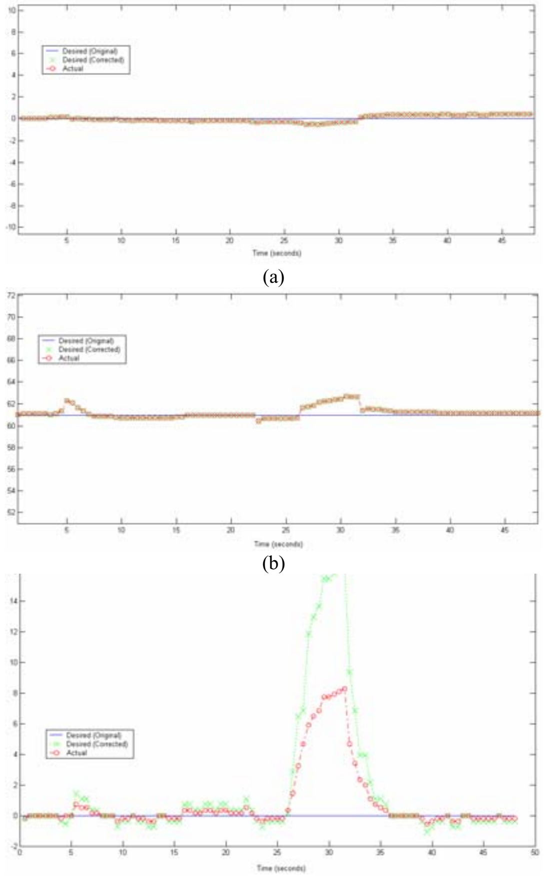


Figure 5. Initial configuration of the two-module composite system.

The desired task is prescribed as motion of the frame attached to the midpoint of the common object along a straight line trajectory (forward velocity of 2.54 cm/s and zero angular velocity). Figure 5 depicts the nominal desired relative configuration of the overall system with frames $\{M\}$ of MPA and MPB initially aligned in the same direction but offset by a distance of 62 cm in the Y direction. With this information, the algorithm developed in the previous section can be used to prescribe the desired trajectories for both mobile bases (MPA and MPB). Further, in order to experimentally validate the efficiency of the method, we introduce a significant disturbance to the relative configuration of the system by causing the left wheels of MPA to run over a small bump.



(a)

(b)

(c)

Figure 6. Articulation-based estimation of location of frames $\{M\}$ of MP A and MP B, used for the system (a) X distance (in cm) vs. time; (b) Y distance (in cm) vs. time; and (c) Relative Orientation (in deg.) vs. time.

The two platforms, MPA and MPB, obtain an estimate of their current position relative to the payload frame using the sensed articulations. This is used in the online planning scheme to create a 'Corrected Desired' trajectory for use in the control and the results of the implementation are shown in Figure 6. In each of the figures (a-c), the 'Desired (Original)' (— line) is the nominal desired trajectory that was computed offline; the 'Desired (Corrected)' (–x– line) is the desired trajectory resulting from the online sensor-based computation that deviates from the nominal desired trajectory in response to the changed relative configuration; and 'Actual' (–o– line) is the actual trajectory followed by the system as determined by post-processing the measurements of the instrumented articulations.

The data obtained from the articulations accurately captures the effective separation of the frames of reference of MPA and MPB from the payload frame. The use of such articulation-based estimation of relative configuration/separation allows the system to not only detect disturbances to the relative configuration but also to successfully correct these and restore the original configuration of the articulation, as seen in Figure

6. While the relative system configuration is maintained, errors relative to a global reference frame cannot be detected if both mobile bases undergo identical simultaneous disturbances. Detection of such absolute errors would require an external reference and is not considered here.

V. CONCLUSION

In this paper, we examined the design, development, and successful implementation of a decentralized control scheme for a system of two collaborating wheeled mobile manipulators transporting a common object while maintaining a desired relative configuration. The proposed method creates *feasible desired trajectories* for the wheeled mobile platforms, while maintaining a desired manipulator configuration, for arbitrary end-effector trajectories. Further, the developed algorithm is well-suited for implementation as an online planning algorithm. This served as the basis of the decentralized control scheme for controlling the collaborating system of two mobile manipulators. The framework developed here lends itself well to implementations on larger systems with further addition of mobile manipulator modules. Experimental evaluation verified the ability of the combined system to accommodate, detect, and correct disturbances to the relative configuration that arise due to its interaction with the world.

Finally, we also note that each pair of differentially-driven wheels may be modeled by a single "virtual" unicycle attached at the center of the axle. In this context, our composite system may be viewed as a design enhancement of a tricycle cobot [23] – one in which the steerable wheels are no longer fixed rigidly but can move (within a limited range) relative to a formation frame. Thus the methods developed here, for steering the active and passive distribuitions, can also be directly applied for generation of kinematically-feasible motion plans for the tricycle cobot.

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