Dynamic Control of Wheeled Mobile Robots (WMR)

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Abstract

- The control of nonholonomic systems has been the subject of considerable research effort over the last few years. There are a large number of mechanical systems such as robot manipulators, mobile robots, wheeled vehicles, and space and underwater robots that possesses such non-integrable constraints.
- The majority of the research efforts on the control of such system concentrated on the kinematic models that are controlled directly by velocity inputs. Less attention paid to forces/torques control, where the dynamic of the system is influential.
- Inclusion of dynamics models in the control problem in nonholonomic constrained systems is an important issues for further research.
- Our goal is to develop and implement effective dynamic control schemes to our test bed of the form of differentially-driven wheeled mobile robots (DD-WMR), and for our future work on the cooperative payload transport and manipulation by collectives of such DD-WMR.

Motivation

Why do we need dynamic control of WMR?

From the viewpoint of applications, there exists certain dynamic parameters that can affect the dynamic performance of the WMR, for instance:

-WMR is usually required to carry massive payload vehicles in real world application.

-WMR is usually required to navigate on different frictional ground surface with different curvature. It may need to run at different slops, e.g. up/down hill.

-WMR is sometime required to operate at high speeds

Consideration of robot dynamics is necessary for the above realistic control design.

Robotically Instrumented Segway Scooter — Autonomous Hummer — NASA Cooperative Rovers

Equation of motion

$$\dot{q} = S(q)u \ : \ u = (\dot{\theta}_r, \dot{\theta}_l)^T$$
$$M(q)\ddot{q} + V(q,\dot{q}) = B(q)\tau - A^T(q)\lambda$$
$$\ddot{q} = \dot{S}(q)u + S(q)\dot{u} \implies$$
$$M(q)S(q)\dot{u} + \left(V(q,\dot{q}) + M(q)\dot{S}(q)u\right) = B(q)\tau - A^T(q)\lambda$$
$$x = \begin{bmatrix} q^T & \upsilon^T \end{bmatrix}^T = [x_c, y_c, \phi, \theta_r, \theta_l, \dot{\theta}_r \quad \dot{\theta}_l]^T$$

State space representation of the equation of motion subject to nonholonomic constraint

$$\dot{x} = \begin{bmatrix} S\upsilon \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u$$

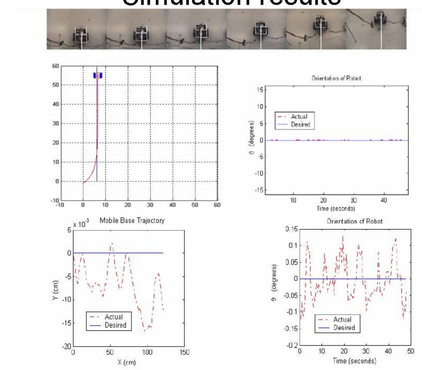
Nonlinear feedback

$$\tau = S^T MS(u - f_2)$$
$$f_2 = (S^T MS)^{-1}(-S^T MS\upsilon - S^T V).$$

$$M(q) = \begin{bmatrix} m & 0 & 2m_w d\sin\phi & 0 & 0 \\ 0 & m & -2m_w d\cos\phi & 0 & 0 \\ 2m_w d\sin\phi & -2m_w d\cos\phi & I & 0 & 0 \\ 0 & 0 & 0 & I_w & 0 \\ 0 & 0 & 0 & 0 & I_w \end{bmatrix} \quad V(q,\dot{q}) = \begin{bmatrix} 2m_w d\dot{\phi}^2\cos\phi \\ 2m_w d\dot{\phi}^2\sin\phi \\ 0 \\ 0 \\ 0 \end{bmatrix} ; \quad B(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} ; \quad \tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}$$

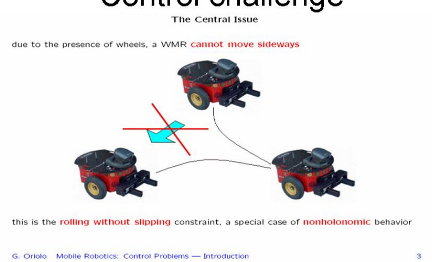
$$I = I_c + 2m_w(d^2 + b^2) + 2I_m \ ; \quad m = m_c + 2m_w$$

Simulation results



Wheeled Mobile Robots (WMR)

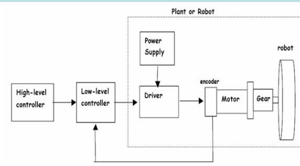
Control challenge



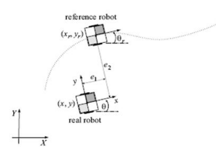
- The feedback control problem of *nonholonomic system* is much more nontrivial, because:

— a WMR is **underactuated**: less control inputs than generalized coordinates

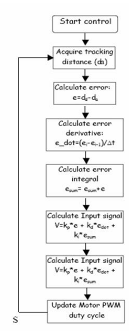
— a WMR is **not smoothly stabilizable** at a point



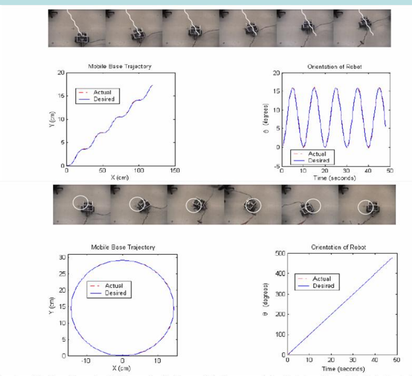
Control elements of a WMR system



Robot following error transformation



Control flow chart



Courtesy M. Abou-Samah, "A Kinematically Compatible Framework for Collaboration of Multiple Nonholonomic Wheeled Mobile Robots", M. Eng. Thesis, Nov 2001.

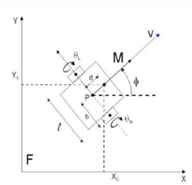
Why WMR?

WMR is popular because:

- •wheels are simpler to control
- •pose fewer stability problems
- •use less energy
- •possesses reasonable speed
- •are reasonably maneuverable

Lagrangian formulation for DD-WMR with nonholonomic constraints

Dynamic Equations

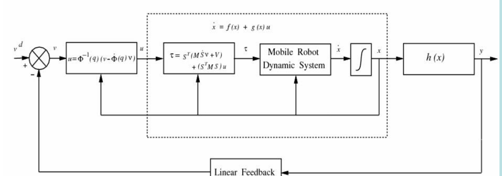


$$q = (x_c, y_c, \phi, \theta_r, \theta_l)^T$$
$$T = \frac{1}{2}m_c v_c^2 + \frac{1}{2}I_c\dot{\phi}^2 + T_m$$
$$A(q) = \begin{bmatrix} -\sin\phi & \cos\phi & -d & 0 & 0 \\ -\cos\phi & -\sin\phi & -b & r & 0 \\ -\cos\phi & -\sin\phi & b & 0 & r \end{bmatrix}$$
$$S(q) : A(q)S(q) = 0$$

Introduce nullspace of the constraint Jacobian matrix (these are all the *feasible motion directions* of the WMR, i.e. the control input must lie in the span of this space.)

$$S(q) = \begin{bmatrix} c(b\cos\phi - d\sin\phi) & c(b\cos\phi + d\sin\phi) \\ c(b\sin\phi + d\cos\phi) & c(b\sin\phi - d\cos\phi) \\ c & -c \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

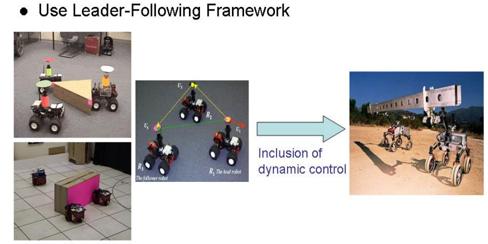
Schematic of the control algorithms



Two type of control algorithms: (a) trajectory tracking; (b) path following. While they differ in the selection of output equations, the basic scheme is the same as shown above. The nonlinear feedback cancels the nonlinearity in the dynamics so that the state equation is simplified as represented by the dotted block above. The overall system is decoupled into two linear subsystem.

Future Avenue: Payload Transport and Manipulation by WMR collectives

- Goal: Enabling multiple WMRs to cooperatively carry, push, or manipulate common objects with developed dynamic control scheme
  - Use Leader-Following Framework



Inclusion of dynamic control