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Screw-theoretic analysis models for felid jaw mechanisms

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Abstract

In this paper, we examine the development of quasi-static computational models for musculoskeletal analysis, leveraging screw-theoretic techniques traditionally employed for the analysis of articulated multibody systems (MBS). The case study of analysis of bite- and muscle-forces in the articulated jaws of members of the felid (cat) family is used to highlight the critical aspects. In particular, musculoskeletal systems with multiple muscles superimposed on an underlying articulated skeleton share many features with the subclass of cable actuated parallel MBS (including redundancy in actuation and unidirectional nature of actuation forces). The screw-theoretic formulation facilitates the development of a computational model for resolving such redundancy while retaining explicit geometric meaning in terms of lines-of-action, motions and forces. The low-computational-complexity of the ensuing quasi-static models makes them well-suited both for: (a) iterative/parametric studies of the roles of geometry (muscle locations) or physiology (muscle-parameters) on skeletal load-distributions, as well as (b) implementing online inverse-dynamics-based muscle-force planners for biomimetic physical prototypes. A MATLAB Graphical User Interface was also developed to aid lay users (non-computational scientists) in performing iterative parametric force optimization and muscle location studies.

Keywords: Screw-theory; Musculoskeletal systems; Optimization; Cable-driven parallel manipulators

1. Introduction

In recent times, the ubiquitous availability of inexpensive computation has fostered a trend towards greater use of computational analysis in many scientific arenas. Virtual prototyping (also known as simulation-based design) is an engineering methodology that takes advantage of iterative parametric computational studies for studying the functional performance of designed products (and any subsequent design refinements). However, this approach is critically dependent upon the availability of suitable parametric models, methods and tools that can adequately capture and effectively simulate the underlying physics of the problem. While this approach has made tremendous inroads in certain engineering related fields, the shortfalls in the availability of subject-specific computational methods limit its direct application in biological fields (such as musculoskeletal

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analysis). Therefore, the principal underlying goal of this effort is to develop low-computational-complexity models for musculoskeletal system analysis that are well suited for both: (a) preliminary iterative parametric studies: as well as (b) real-time online computations at high-sampling rates.

The case study of bite- and muscle-force estimation in the skull/mandible structure of members of the cat family (from extinct saber-tooth cats to modern day large cats) helps unify various aspects of this paper. The felid jaws of interest can be modeled within a multibody analysis setting as an articulated skeletal structure with superimposed musculature. Accurate information pertaining to skeletal geometry and the underlying articulated structures may be obtained from the various anatomical databases/fossil records. Anatomical studies of modern-day large cats [1] also enable us to approximately locate the origin and insertion points of various associated antagonistic jaw muscles.

The goal then is to create a mathematical model to estimate the muscle forces associated with an applied/desired bite force, and ultimately estimate the maximal bite force of the animal. To this end, we note the similarities between the musculoskeletal models and redundantly-actuated parallel multibody systems (MBS). This permits us to leverage the considerable literature on of parallel MBS [2,3] for resolution of this problem. In particular, such Musculoskeletal systems share a number of features with the subclass of cable-actuated articulated MBS [4,5]. These systems require careful handling principally due to the unidirectional nature of application of actuation forces through the attached cables. The analysis is undertaken using screw-theoretic methods – which retain explicit geometric meaning in terms of lines-of-action, velocities, forces, and moments while providing a simplified yet computationally efficient analysis framework. The ensuing low-order parametric computational model is well suited for an online implementation of inverse dynamics based actuated force determination. Such an analysis model sets the stage for iterative performance of a number of virtual simulation-based studies – pertaining to muscle-force optimization as well as muscle-location. A lay-user, without an extensive computational background (such as a biologist), now has a ready means to perform numerous and rapid parametric studies to improve their understanding and validate various hypotheses.

Additionally, we have developed a biomimetic prototype (Fig. 1b) with an articulated skeletal structure and with wire-tendon type DC servo actuators to emulate the action of muscles. This testbed is intended to be used for repeated experimentation with changes in muscle location, applied external forces, system configuration such as jaw gap angle and initial conditions. As shown in Fig. 1a, wire-tendon type DC servo actuators are used in a hardware-in-the-loop (HIL) setting to emulate the action of muscles. The redundancy in actuation arising because of multiple cable drive actuators must be resolved in order to actively drive the physical prototype. Our developed models and methods also find ready applicability here, as will be discussed later.

The paper is organized as follows: Section 2 provides a brief overview of literature related to musculoskeletal and redundant system modeling. Section 3 discusses mathematical background. Sections 4 and 5 present the modeling of the skull/mandible as an articulated MBS and development of the quasi-static screw-theoretic model. Sections 6 and 7 discuss the development of two variants of a secondary optimization strategy to ensure non-negativity of muscle forces. Section 8 presents the MATLAB GUI based implementation of the framework and discusses aspects of validation of results. Section 9 concludes the paper.

2. Literature

There have been several attempts at using computer-based tools for analyzing biomechanical systems [6–12]. The Fauna Group [8,9] considers detailed musculoskeletal models, replete with skin, joint motions and tissue deformation, purely from a viewpoint of realistic animation. While their algorithms are not based on any formal biological and mechanical principles, they emulate the appearance, movement and behavior of individual animals and groups very ‘‘realistically’’. Others, such as the Primate Evolution and Morphology Group of Liverpool [10,11], ‘‘retroengineer’’ gait and masticatory behavior of early hominids and other primates using EMG to determine muscle firing and gauge comparative energetic costs of different behaviors. Yet others, such as Thow-Hing and Fiume [12], consider very detailed muscle models including fiber orientation, but with limited consideration of their physical/mechanical impact on the system.

In the past few decades, there have also been many efforts at creating physics-based analysis models for musculoskeletal systems within the framework of MBS [6,7]. These efforts, reported primarily in the biomechanics literature, leverage the structure provided by the underlying articulated rigid body model to progres-

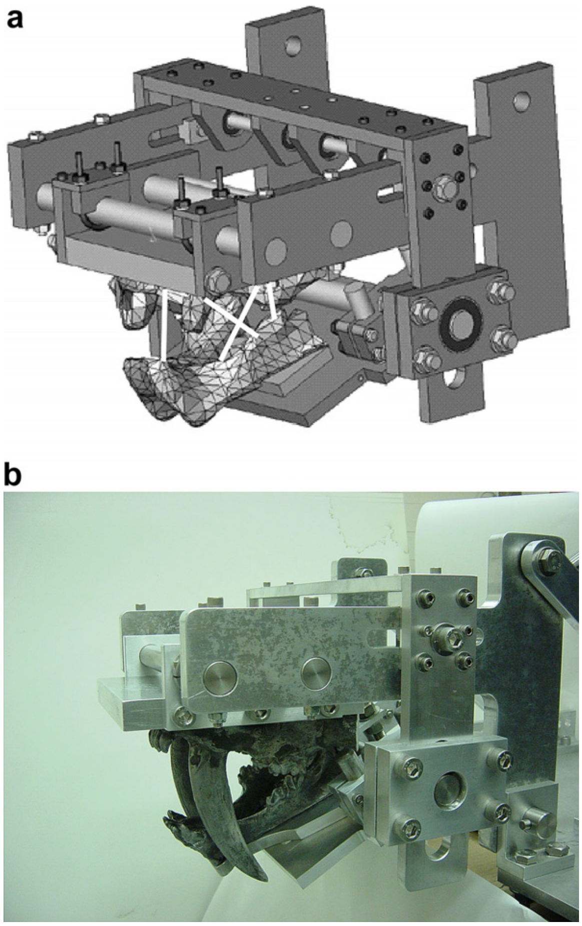

Fig. 1. Prototype of hardware-in-the-loop (HIL) testbed design: (a) virtual prototype and (b) physical prototype.

sively develop the constrained multibody dynamic models. Commercial-off-the-shelf (COTS) MBS packages face numerous challenges when used for musculoskeletal system analysis. The total number of muscles included in the model and the muscle modeling fidelity is dependent on the desired complexity of the model. Multiple muscle systems (MMS) [13], while better represents the actual animal, are redundant by nature. Since the system model has more actuators than degrees of freedom (DOF), they result in statically and dynamically indeterminate problems. Solution methodologies to these indeterminate problems have been examined by many authors [14–17] but tend to be computationally expensive (and not suitable for iterative use of the type seen in parametric analyses). Many of these approaches have also been implemented in the form of computational toolkits (and are available as COTS software or freeware) [6,7,15]. However, the generality desired in the multibody dynamics-based software implementation can potentially mask the underlying geometry inherent in the articulated structure. Hence, in our own work, we examine the applicability and utility of screw-theoretic modeling methods developed traditionally in the context of parallel MBS [2,3] to such problems. This enables us to develop the requisite equations for quasi-static musculoskeletal analysis while retaining the explicit geometric meaning in terms of lines-of-action, velocities, forces, and moments.

Over the past two decades numerous physical chewing/biting simulators have also been developed – see [18,19] for an overview. Most of the developed simulators have taken the form of a robotic/mechatronic machine that can appropriately produce the motions and forces of human mastication. For example, several studies have investigated the role of chewing behaviors on the force-cycling on dental materials/implants by such physical testing. In order to reproduce the complex chewing motions in three-dimensions, the masticators typically include complex articulations, spatial links and multiple closed kinematic loops. Using a biomimetic argument, all dominant masticatory muscles are individually implemented using artificial-muscle actuators or wire tendon-type DC-servo actuators, resulting in a prototype with considerable actuation redundancy. The determination of the sets of actuator forces required to create the desired chewing motions/forces – the solution to the inverse dynamics problem – is critical. Significant challenges arise due to the presence of both the multiple closed spatial kinematic loops and redundancy within a spatial mechanism. Daumas et al. [19] use SimMechanics, a COTS MBS simulation package, to perform this inverse dynamics analysis of a 3D human skull/jaw. However, their prototype uses at most six actuators, thereby avoiding the need to resolve redundancy within their system. However, as noted earlier, the characteristic feature of almost every musculoskeletal system is the redundancy of actuation, due to presence of multiple sets of antagonistic muscle pairs. The effective determination of actuator forces in the presence of such redundancy is an underdetermined problem, but needs to be resolved in order to use in the physical prototype.

A least-squares solution is always possible if the sole goal is to overcome the indeterminacy. However, it is also often useful to use the redundancy of actuation to optimize additional criteria and achieve secondary goals (such as redistribution of forces, improved stiffness, or to maintain the unidirectional force constraints). Such approaches have been pursued significantly in context of multi-arm cooperating systems [20], multi-legged locomotion systems [21,22] and multi-fingered hands [23] – and has direct applicability in the context of musculoskeletal systems. Hence, in our work, we focus on developing a redundancy resolution technique with a particular focus on theory of selectively non-reciprocal screws (SNRS) to aid the solution process.

3. Mathematical background

The various motions of the system as well as the external bite force may be expressed in terms of screw coordinates. We briefly summarize the critical aspects here and refer the reader to several of the excellent books for more details [3,24,25]. Given a unit vector pointing along the direction of the screw axis, $\hat{\mathbf{u}}$, the location of a point on this axis $\vec{\mathbf{r}}$, and the pitch λ, defined as the ratio of translation to rotation, we can define a *unit screw* as shown in Fig. 2 in axis coordinates, $\widehat{\$} = [S_1\ S_2\ S_3\ S_4\ S_5\ S_6]^{\mathrm{T}}$ as

$$\widehat{\$} = \begin{bmatrix} \hat{\mathbf{u}} \\ \hat{\mathbf{u}}_0 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{u}} \\ \vec{\mathbf{r}} \times \hat{\mathbf{u}} + \lambda \hat{\mathbf{u}} \end{bmatrix} \tag{1}$$

where $\hat{\mathbf{u}}_0 = \vec{\mathbf{r}} \times \hat{\mathbf{u}} + \lambda \hat{\mathbf{u}}$ is the moment of the screw axis about the origin of a reference frame. The reciprocity relation between two unit screws, $\hat{\$}_1 = \begin{bmatrix} S_1^1 & S_2^1 & S_3^1 & S_4^1 & S_5^1 & S_6^1 \end{bmatrix}^{\mathrm{T}}$ and $\hat{\$}_2 = \begin{bmatrix} S_1^1 & S_2^2 & S_3^2 & S_4^2 & S_5^2 & S_6^2 \end{bmatrix}^{\mathrm{T}}$ is given as:

$$\begin{aligned} \hat{\$}_1 \otimes \hat{\$}_2 &= \hat{\$}_1 \cdot \hat{\$}_2^r \\ &= S_1^1 \cdot S_4^2 + S_2^1 \cdot S_5^2 + S_3^1 \cdot S_6^2 + S_4^1 \cdot S_1^2 + S_5^1 \cdot S_2^2 + S_6^1 \cdot S_3^2 \\ &= 0 \end{aligned} \tag{2}$$

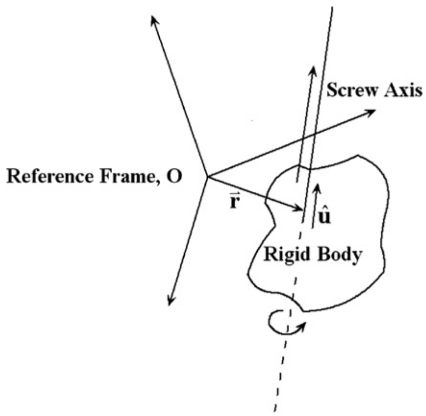

Fig. 2. Schematic of a general unit screw.

where $\hat{\$}^r_2 = \begin{bmatrix} S^2_4 & S^2_5 & S^2_6 & S^1_1 & S^2_2 & S^2_3 \end{bmatrix}^T$ represents a general unit screw in ray coordinates. Such screw systems have been used to define both finite and infinitesimal motions (twists) and forces (wrenches). When dealing with velocities of rigid bodies in space, a twist vector can be defined using the underlying screw basis as

$$\widehat{\$}_t = \begin{bmatrix} w_{0,x} \\ w_{0,y} \\ w_{0,z} \\ v_{0,x} \\ v_{0,y} \\ v_{0,z} \end{bmatrix} = \begin{bmatrix} \mathbf{w}_0 \\ \mathbf{v}_0 \end{bmatrix} = w_0 \begin{bmatrix} \hat{\mathbf{u}} \\ \vec{\mathbf{r}} \times \hat{\mathbf{u}} + \lambda\hat{\mathbf{u}} \end{bmatrix} \tag{3}$$

where $\mathbf{w}_0$ is the angular velocity of the body and $\mathbf{v}_0$ is the linear velocity of a point on the body that is instantaneously coincident with the origin of a reference frame in which the screws are expressed [3]. Similarly forces can be represented using the underlying screw vector and screw coordinates as a wrench:

$$\widehat{\$}_w = \begin{bmatrix} F_{0,x} \\ F_{0,y} \\ F_{0,z} \\ M_{0,x} \\ M_{0,y} \\ M_{0,z} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_0 \\ \mathbf{M}_0 \end{bmatrix} = f_0 \begin{bmatrix} \hat{\mathbf{u}} \\ \vec{\mathbf{r}} \times \hat{\mathbf{u}} + \lambda\hat{\mathbf{u}} \end{bmatrix} \tag{4}$$

where $\mathbf{F}_0$ is the force applied at a point on the body in terms of the reference frame in which the screws are expressed and $\mathbf{M}_0$ is the moment created by $\mathbf{F}_0$ at the origin of the same reference frame. It is important to note that multiple twists (or wrenches) can be combined and represented as a single equivalent twist (or wrench) acting on a body. When these spatial screws are restricted to operate in the plane, the corresponding expressions for the planar twist and planar wrench can be written (assuming without lost of generality that the Z-axis points out of the plane) as

$$\widehat{\$}_t = \begin{bmatrix} w_{0,z} \\ v_{0,x} \\ v_{0,y} \end{bmatrix}, \quad \widehat{\$}_w = \begin{bmatrix} F_{0,x} \\ F_{0,y} \\ M_{0,z} \end{bmatrix} \tag{5}$$

4. RPR serial chain modeling

For simplicity and without lost of generality, we present a planar screw-theoretic model of the skull/mandible musculoskeletal system. However, this developed frame work can be extended to the 3D case with minimal effort. The skull (upper jaw) and mandible (lower jaw) are considered to be rigid bodies with the mandible assumed to be grounded in space. In the felid family, the motion of the jaws can be very closely approximated as a pure rotation [1]. Thus we assume the skull to be attached to the mandible via a revolute joint (with axis normal to the display plane). The three main coordinate systems used in the analysis are shown in Fig. 3. The inertial (fixed) frame, $O_0X_0Y_0$, is fixed in space and is the principal calculation frame of the model. An Upper Jaw Frame, $O_UX_UY_U$, is attached to the skull (upper jaw) and is related to the inertial frame through the jaw gape angle, θ and an Inertial End Effector Frame, $O_EX_EY_E$, is created with the application point of the external/desired or bite force.

Muscle forces are considered to act along the line-of-action joining the origin and insertion points as shown in Fig. 4. In the simplest form the effective actuation due to a muscle can be treated as arising due to a single lumped muscle. However, greater fidelity may be obtained by modeling the multiple (near collinear) fascicles that form the muscle bundle. One of the underlying goals of our longer-term research efforts is to examine the role of fidelity of modeling of the muscles – lumped versus distributed models – on the accuracy of the results.

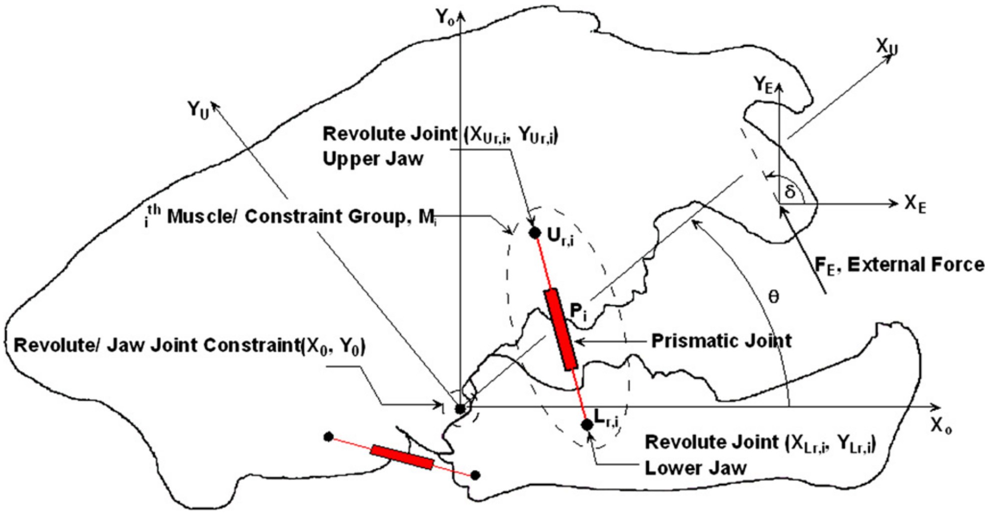

Fig. 3. Schematic illustrating model nomenclature.

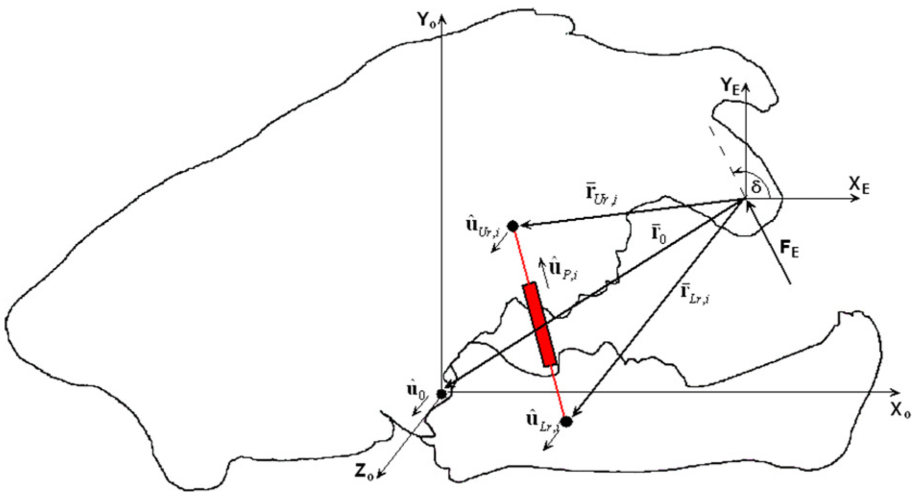

Fig. 4. Illustration depicting the systems various unit direction and distance vectors.

Hence, although we principally present results using a single lumped muscle for each major muscle, the framework can be used directly with the distributed muscle models.

Each muscle is modeled as an RPR serial chain – consisting of a revolute joint on the upper jaw ($U_{r,i}$), a prismatic joint (P_i), and a revolute joint on the lower jaw ($L_{r,i}$). A total of n_m such muscles are assumed to couple the upper and lower jaws. The combined end-effector twist due to the linear combination of joint twists can be written as

$$\$_{T,E} = [\mathbf{J}_i]\dot{\Theta}_i = \begin{bmatrix} | & | & | \\ \widehat{\$}_{Ur,i} & \widehat{\$}_{P,i} & \widehat{\$}_{Lr,i} \\ | & | & | \end{bmatrix} \begin{bmatrix} \dot{\theta}_{Ur,i} \\ \dot{d}_{P,i} \\ \dot{\theta}_{Lr,i} \end{bmatrix} \tag{6}$$

where $\mathbf{J}_i$ is a 3×3 Jacobian matrix whose column vectors represent the unit screws associated with each joint and $\dot{\Theta}_i$ is the column vector of the joint velocities.

5. Reciprocal wrench formulation

A selectively non-reciprocal screw (SNRS), $W_{k,i}$, in the given RPR serial chain is reciprocal to all other screws except the given screw [2,26], and can be defined as

$$W_{k,i} \otimes \$_{j,i} \begin{cases} = 0, & k \neq j \\ \neq 0, & k = j \end{cases} \quad \forall k,j \in \{\, U_r \quad P \quad L_r \,\} \tag{7}$$

For example, $W_{P,i}$ is the selectively non-reciprocal screw to the unit screw corresponding to the P joint that satisfies:

$$\begin{aligned} W_{P,i} \otimes \$_{Ur,i} &= 0 \\ W_{P,i} \otimes \$_{Lr,i} &= 0 \\ W_{P,i} \otimes \$_{P,i} &\neq 0 \end{aligned} \tag{8}$$

The goal now is to find the feasible wrench space for the entire constrained mechanical system. To this end, we need to first determine the wrenches exertable by each individual chain. If the chain has lesser DOF (m) than the dimension of the task space (n), then a non-null SNRS system of dimension $(n - m)$ is guaranteed to exist. The revolute joint between the upper and lower jaw constitutes one such low-DOF serial chain whose SNRS system is now of dimension 2 and denoted as W_0. Thus the span of W_0 represents the passively equilibrated reaction forces that may be sustained even when the revolute joint is left inactive. If the serial chain has as many or greater DOF compared to the dimension of the task space, then only the SNRS corresponding to the active joint in each chain is capable of exerting/resisting forces. Since each muscle is modeled as a 3-DOF RPR chain with an active prismatic joint, the corresponding SNRS system of each serial chain is one dimensional and denoted as $W_{P,i}$. Combining the span of various SNRS systems into a single matrix allows us to write the system force and moment equilibrium equation as

$$\begin{bmatrix} | & | & | & | & | & | \\ W_{0x} & W_{0y} & W_{p,1} & W_{p,2} & \dots & W_{p,n_m} \\ | & | & | & | & | & | \end{bmatrix} \begin{bmatrix} f_{0x} \\ f_{0y} \\ f_1 \\ \vdots \\ f_{n_m} \end{bmatrix} = \$_w \tag{9}$$

$$[\mathbf{W}]_{m\times(n_m+2)}\{\mathbf{f}\}_{(n_m+2)\times 1} = \$_w$$

where $\$_w = [F_x \; F_y \; M_z]^{\mathrm{T}}$ is the external wrench created by the application of the external bite force (F_{E}), and $\mathbf{f}$ represents the wrench intensities to the corresponding selectively non-reciprocal wrenches, which in this case correspond to the magnitudes of the muscle forces, $f_1, \dots, f_{n_m}$, and the reaction forces at the jaw joint, f_{0x}, f_{0y}. Traditionally, musculoskeletal systems feature surplus actuation (in the form of antagonistic muscle-pairs). This manifests itself here in the form of a rectangular $\mathbf{W}$ matrix with more columns then rows. Thus, a pseudo-inverse based solution to this linear system can be found by

$$\mathbf{f} = \mathbf{W}^{\#}\$_w + [\mathbf{I} - \mathbf{W}^{\#}\mathbf{W}]\,\vec{\mathbf{z}} = \mathbf{f}_P + \mathbf{f}_H \tag{10}$$

where $\mathbf{W}^{\#}$ is the pseudo-inverse of the $\mathbf{W}$. Since the system under consideration is almost always redundantly actuated, i.e. $m <$ n, the $\mathbf{W}^{\#}$ can be computed as

$$\mathbf{W}^{\#} = \mathbf{W}^{\mathrm{T}}(\mathbf{W}\mathbf{W}^{\mathrm{T}})^{-1} \tag{11}$$

The first term of Eq. (10) corresponds to the particular solution ($\mathbf{f}_P$) and the second term corresponds to the homogeneous solution ($\mathbf{f}_H$). Due to the redundancy in the system, solving the system equilibrium equation in the inverse dynamics setting yields an indeterminate solution with an infinite set of possible solutions since $\mathbf{f}_H$ in Eq. (10) can take on any value. The least squares minimum-norm solution to the system of equations corresponds to the case where $\mathbf{f} = \mathbf{f}_P$ and $\mathbf{f}_H = \mathbf{0}$. However, this solution may possess non-positive values for the wrench intensity vector – the implication in this setting is that the muscles are required to both push and pull which is not possible. As shown in Kumar and Waldron [21], these terms can be interpreted as the equilibrating force field and interaction force field respectively. The equilibrating force field gives the least squares solution to the problem, and one can now add multiples of the interaction force field without changing the output. This becomes important because we require the wrench intensities corresponding to the muscle forces $f_1, \ldots, f_{n_m}$ to be positive, which can now be ensured using the interaction force field.

6. Muscle force optimization

We first examine the development of a constrained secondary optimization to ensure non-negativity of the wrench intensities. The pseudo-inverse solution to the system equilibrium equation can be re-written as

$$\mathbf{f} = \underbrace{\mathbf{W}^{\#}\$_W}_{\mathbf{f}_P} + \underbrace{\mathbf{H}\vec{\mathbf{z}}}_{\mathbf{f}_H} \tag{12}$$

$$\mathbf{H}_{(n_m+2)\times(n_m+2)} = [\mathbf{I} - \mathbf{W}^{\#}\mathbf{W}] \tag{13}$$

where $\mathbf{f}_P$ and $\mathbf{H}\,\vec{\mathbf{z}}$ represent the particular and homogenous components of the solution respectively. The homogenous component, $\mathbf{H}\,\vec{\mathbf{z}}$, is typically found to be rank deficient resulting in non-independent components within the nullspace solution. The length of the vector $\vec{\mathbf{z}}$, and thus the number design variables, can be reduced by finding the full rank nullspace of the system. The full rank nullspace component of the system can be found by performing a singular value decomposition of the $\mathbf{H}$ matrix.

$$[\mathbf{U}\ \Sigma\ \mathbf{V}^{\mathrm{T}}] = \mathrm{svd}(\mathbf{H}) \tag{14}$$

The matrices $\mathbf{U}$ and $\mathbf{V}$ now represent the orthonormalized eigenvectors of $\mathbf{H}\mathbf{H}^{\mathrm{T}}$ and $\mathbf{H}^{\mathrm{T}}\mathbf{H}$ respectively, and the matrix Σ contains the singular values of $\mathbf{H}$ along its diagonal in descending order [27]. We define ρ as the number of columns of Σ containing non-zero singular values. The full rank nullspace of $\mathbf{H}$, $\mathbf{S}'$, can now be defined as the first ρ columns of $\mathbf{U}$.

$$\mathbf{S}' = \begin{bmatrix} | & | & & | \\ \mathbf{U}_1 & \mathbf{U}_2 & \cdots & \mathbf{U}_\rho \\ | & | & & | \end{bmatrix} \tag{15}$$

The pseudo-inverse solution can be rewritten as

$$\mathbf{f} = \mathbf{f}_P + \mathbf{S}'\,\vec{\boldsymbol{v}} \tag{16}$$

In our analysis we only wish to optimize the muscle forces, thus we rewrite pseudo-inverse solution to separate the components of the solution pertaining to the reaction forces at the jaw joint and the forces generated by the muscles.

$$\begin{bmatrix} \mathbf{f}_{0(2\times1)} \\ \mathbf{f}_{m(n_m\times1)} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{P0(2\times1)} \\ \mathbf{f}_{Pm(n_m\times1)} \end{bmatrix} + \begin{bmatrix} [\mathbf{S}'_0]_{(2\times\rho)} \\ [\mathbf{S}'_{\mathbf{m}}]_{(n_m\times\rho)} \end{bmatrix} \vec{\boldsymbol{v}}_{(\rho\times1)} \tag{17}$$

where $\mathbf{f}_{P0}$ and $\mathbf{f}_{Pm}$ represent the components of the particular or least squares solution pertaining to the reaction and muscle forces respectively. From this we can now define the general form of the optimization for the minimization of the forces generated by the muscles.

$$\begin{aligned} &\min && \left(f_1^2 + f_2^2 + \cdots + f_{n_m}^2\right) \\ &\text{subject to:} && -[\mathbf{S}'_{\mathbf{m}}]\,\vec{\boldsymbol{v}} \leqslant \mathbf{f}_{Pm} \end{aligned} \tag{18}$$

where $\mathbf{f}_m = \mathbf{f}_{Pm} + [\mathbf{S}'_m]\,\vec{\boldsymbol{v}}$.

7. Muscle activity optimization

Since the peak muscle force capacities of different muscles can vary significantly, often a good case can be made for development of a normalized muscle force measure (as is done frequently in the musculoskeletal literature). The "activity" of a muscle is defined as the ratio of the current muscle force to the maximal muscle force that the muscle can apply. Thus, by definition, all muscle activities are normalized to lie within the range of [0, 1].

$$\bar{f}_i = \frac{f_i}{f_{\max,i}}, \quad f_i = f_{\max,i}\bar{f} \tag{19}$$

All the muscle forces within our musculoskeletal system can be expressed in terms of their normalized activities as

$$\mathbf{f} = \begin{bmatrix} f_{\max,Rx} & 0 & 0 & \cdots & 0 \\ 0 & f_{\max,Ry} & 0 & \cdots & 0 \\ 0 & 0 & f_{\max,1} & \cdots & \vdots \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \cdots & f_{\max,n_m} \end{bmatrix} \begin{bmatrix} \bar{f}_{0x} \\ \bar{f}_{0y} \\ \bar{f}_1 \\ \vdots \\ \bar{f}_{n_m} \end{bmatrix}$$
$$\mathbf{f} = [\Omega]\bar{\mathbf{f}} \tag{20}$$

Substituting Eq. (20) into the system equilibrium Eq. (9), the following relationship is obtained.

$$\$_w = [\mathbf{W}][\Omega]\bar{\mathbf{f}} = [\overline{\mathbf{W}}]\bar{\mathbf{f}} \tag{21}$$

The structure of the muscle activity-based equilibrium equations (in Eq. (21)) is very similar to the muscle-force based equilibrium equation (developed in Eq. (9)). Hence, the solution steps outlined in Eqs. (12)–(18) can now be applied to the Eq. (21) in order to facilitate the determination of the unknown muscle-activities.

8. Implementation

Our overall implementation is implemented within MATLAB which gives access to powerful optimization solver routines as well as allows creation of GUIs. Optimization is implemented using the function *fmincon* which uses a sequential quadratic programming (SQP) solution methodology [28]. In this method, the function solves a quadratic programming (QP) sub-problem at each iteration. An interactive GUI interface of the system was developed in MATLAB Graphical User Interface (GUI) [29] – see Fig. 5.

The GUI serves as a user-friendly interface to the low-order computational simulation analysis tool. Within the GUI, the user specifies the magnitude and location of the applied desired bite force and the location or location range of multiple separate muscles (with four such muscles shown in the example in Fig. 6). The muscle-force optimization methods shown in Sections 6 and 7 are used to compute the non-negative magnitudes of muscle-forces needed to produce the desired applied bite force. The GUI also helps the user with visualizing these results and for performing various parametric studies (muscle-location etc.).

Finally, the results obtained were verified using virtual simulation studies using Simulink and Visual Nastran (VN). Fig. 7 provides a high-level overview of the steps to be taken. The user supplies information on the desired bite force, the prescribed jaw motion and upper limits on the actuator forces. The inverse dynamics scheme with redundancy resolution is used for muscle force/activity optimization to determine the muscle forces (and any reaction forces) for a desired bite force. The calculated muscle forces are then fed into a VN model to actuate the various prismatic actuators. The VN simulation (operating in the co-simulation mode) performs the forward dynamics then returns the resultant bite force generated by the model along with other pertinent information (reaction forces and motion trajectories). Finally, we study the error between the desired bite force and computed bite force.

Many simulations were carried out for varying combinations of jaw prescribed motion trajectories and applied external loads, one of which is shown in Fig. 8. This figure shows plots of force magnitude vs. time and force magnitude error vs. time as jaw gape angle decreases from 30° to 10° at a constant speed over 0.5 s while the applied external force is decreased from 1000 N to 500 N. The plots show that while there is

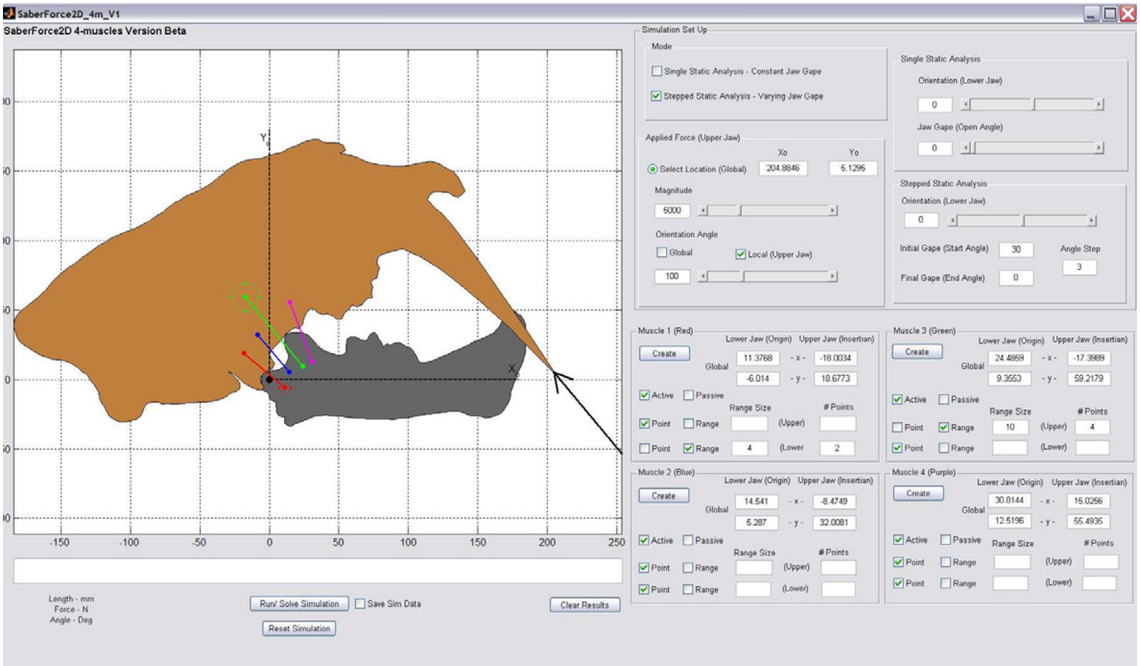

Fig. 5. MATLAB GUI developed for bite/muscle force analysis of the felid jaw.

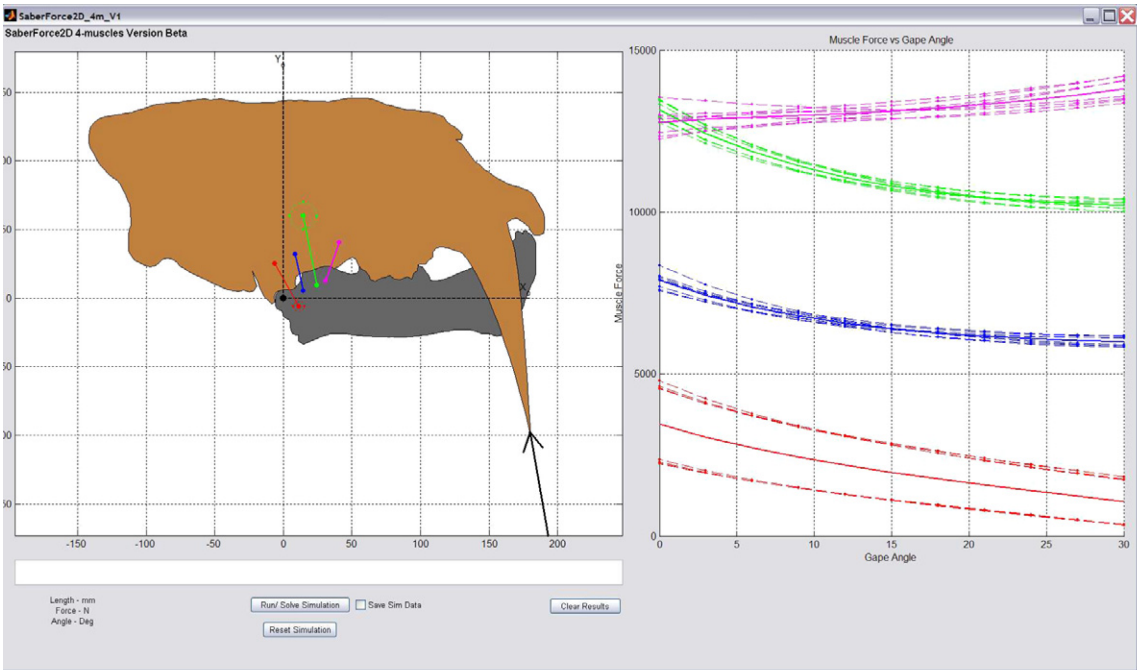

Fig. 6. Simulation results from MATLAB GUI – stepped static analysis.

an initial spike in the force magnitude error plot at the start of the simulation, the average error for this significant load and motion trajectory is about 5%. Note that the spikes are suspected to have arisen due to the stabilization implementation in VN and is being investigated. Nonetheless, the results allow us to develop

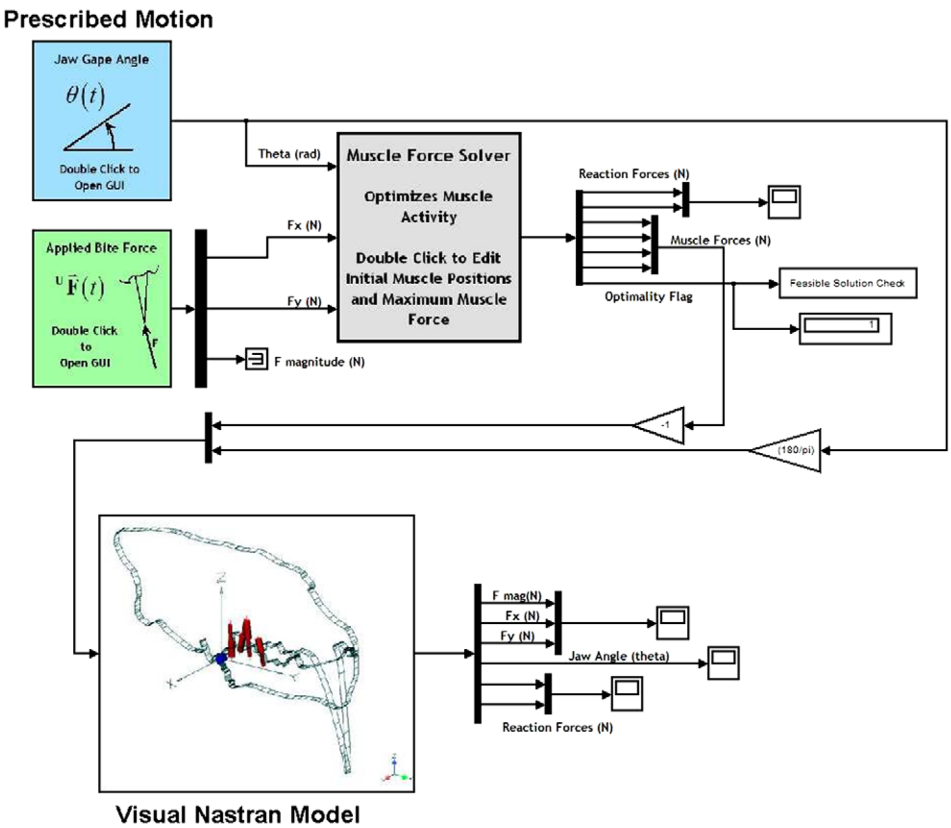

Fig. 7. Simulink/Visual Nastran model developed to facilitate virtual simulation-based parametric design analysis and validation.

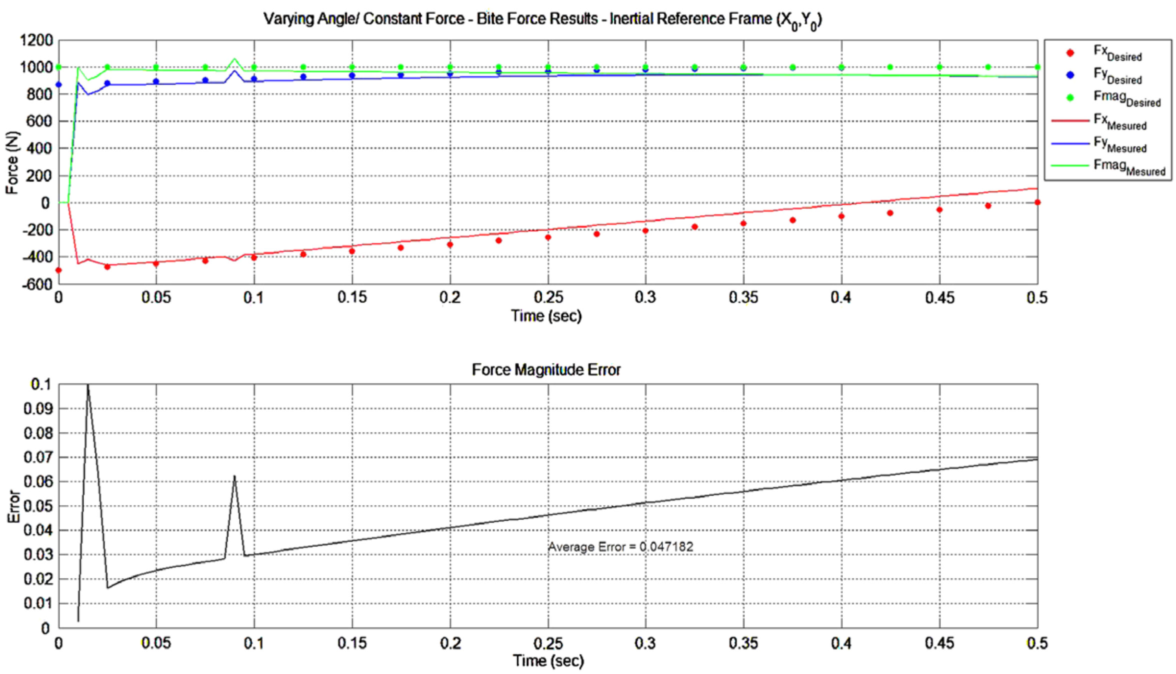

Fig. 8. Validation of the muscle force optimization results.

greater confidence in the accuracy of the developed low-order computational model and set the stage for its use in the real-time setting on the HIL prototype. Further details of GUI creation, interpretation of results of numerous case studies and validation efforts are presented in [30].

9. Conclusion

Our focus was on development of computational tools that can rapidly and accurately analyze redundantly actuated multiple-muscle musculoskeletal systems so that they may be employed in an iterative simulation-based design process or used for real-time model based control. The mathematical system model was developed using screw-theoretic modeling methods typically seen in the context of the analysis of parallel MBS. The developed model provided a convenient computational basis for implementation of rapid redundancy resolution and muscle optimization schemes. The development of such tools was presented in the context of a specific case scenario – the musculoskeletal analysis of the jaw closure of the members of the felid (cat) family. A GUI was developed to allow a user to rapidly perform parametric analysis of the muscle forces necessary to produce a desired bite force. Finally, a Simulink/Visual Nastran framework was employed to validate the solutions from the low-order computational model which is well-suited for online actuator force determination at high sampling rates (for use with the real-time HIL testbed under development).

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