NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
State University of New York at Buffalo

# EFFECTS OF PROTECTIVE CUSHION AND SOIL COMPLIANCY ON THE RESPONSE OF EQUIPMENT WITHIN A SEISMICALLY EXCITED BUILDING

by

J. A. HoLung
Center for Applied Stochastics Research
Florida Atlantic University
Boca Raton, Florida 33431-0991

Technical Report NCEER-89-0001

February 16, 1989

This research was conducted at Florida Atlantic University and was partially supported by the National Science Foundation under Grant No. ECE 86-07591.

# EFFECTS OF PROTECTIVE CUSHION AND SOIL COMPLIANCY ON THE RESPONSE OF EQUIPMENT WITHIN A SEISMICALLY EXCITED BUILDING

by

J.A. HoLung[1]

February 16, 1989

Technical Report NCEER-89-0001

NCEER Contract Number 88-2007

NSF Master Contract Number ECE 86-07591

1 Research Assistant, Center for Applied Stochastics Research, Florida Atlantic University

NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
State University of New York at Buffalo
Red Jacket Quadrangle, Buffalo, NY 14261

## PREFACE

The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion and dissemination of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

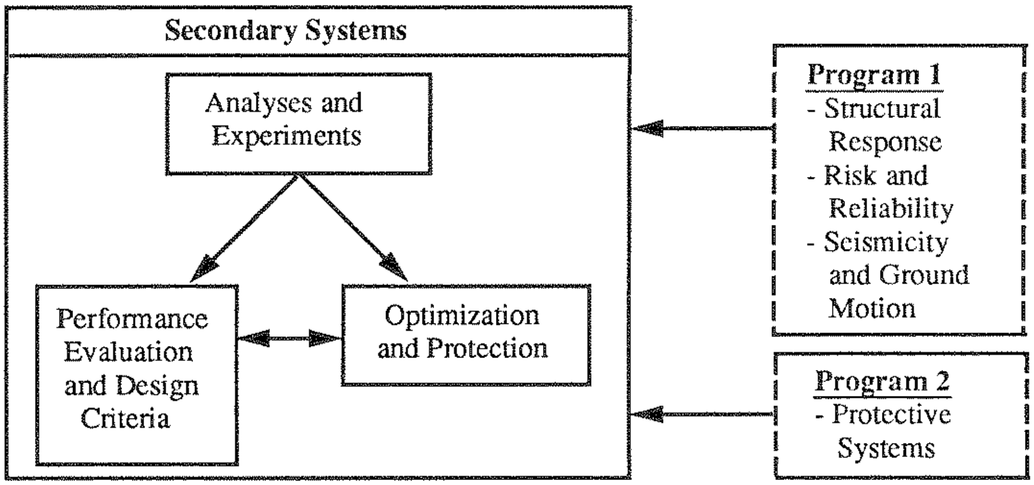
- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to the second program area and, more specifically, to secondary systems.

In earthquake engineering research, an area of increasing concern is the performance of secondary systems which are anchored or attached to primary structural systems. Many secondary systems perform vital functions whose failure during an earthquake could be just as catastrophic as that of the primary structure itself. The research goals in this area are to:

1. Develop greater understanding of the dynamic behavior of secondary systems in a seismic environment while realistically accounting for inherent dynamic complexities that exist in the underlying primary-secondary structural systems. These complexities include the problem of tuning, complex attachment configuration, nonproportional damping, parametric uncertainties, large number of degrees of freedom, and nonlinearities in the primary structure.
2. Develop practical criteria and procedures for the analysis and design of secondary systems.
3. Investigate methods of mitigation of potential seismic damage to secondary systems through optimization or protection. The most direct route is to consider enhancing their performance through optimization in their dynamic characteristics, in their placement within a primary structure or in innovative design of their supports. From the point of view of protection, base isolation of the primary structure or the application of other passive or active protection devices can also be fruitful.

Current research in secondary systems involves activities in all three of these areas. Their interaction and interrelationships with other NCEER programs are illustrated in the accompanying figure.




*This report is the second of a series of two reports dealing with dynamics of secondary systems under earthquake excitations. In the first (NCEER-87-0013), two approaches, a component mode synthesis technique and a modified cascade approach, were developed for determining the response of secondary systems to ground motion excitations. In this report, these two approaches are extended to the study of effects of soil compliancy and linear viscoelastic cushioning devices on the secondary system behavior. An important conclusion to be drawn is that the cushioning effect can be significant. It can reduce the secondary system response by an order of magnitude, even when a detuned condition is shifted to a tuned condition.*

## ABSTRACT

Protective cushion is shown to be effective for reducing seismically induced response of a piece of equipment, such as a computer, installed in a building. The cushioning device is assumed to be linearly viscoelastic in the analysis and the compliancy of the soil under the building is taken into consideration.

The input earthquake excitation is modelled as a nonstationary random process.

Since the building is generally a dynamic system of many degrees of freedom, approximate procedures are used in which only a limited number of important modes is included in calculating the equipment response. Guidelines for choosing such important modes are given, and the approximate results are compared with mathematically exact results in numerical examples.



**KEY WORDS:** Secondary system; primary-secondary system; earthquakes; seismic analysis; nonstationary random process.

**ACKNOWLEDGEMENT**

Financial support from the IBM Corp., for J. A. HoLung while pursuing his Ph.D. degree in Mechanical Engineering at the Florida Atlantic University is gratefully acknowledged.

## TABLE OF CONTENTS

## LIST OF ILLUSTRATIONS

## LIST OF ILLUSTRATIONS (CONT'D)

## LIST OF TABLES

## SECTION 1
## INTRODUCTION

A secondary system is a lighter appendage to a more massive primary system. It may be life-sustaining equipment in a hospital, process equipment, or computers in a nuclear facility. In such cases, performance is critical even under severe loadings such as earthquake induced loads. The response of the secondary system must be determined in order to assess its reliability.

The response of secondary systems has been studied by many researchers [e.g., 1, 4, 10, 11, 12, 21, 22, 23, 24], and continues to be an important area for study because of its importance.

Several complicating factors revealed from earlier works make direct analysis of the combined p-s system unattractive when one attempts to do conventional analysis of practical size problems. Several approaches have evolved to account for a number of these factors, while avoiding conventional analysis procedures. The secondary system is usually considered to be light compared to its supporting primary structure, so that perturbation techniques can be employed. However, Suarez and Singh's approach [23], permits exact treatment of heavy secondary systems as well.

These approaches are suited when the primary system is known, and while the properties of the secondary system are fixed or can be varied in parametric studies. For example, this approach may be used for nuclear facilities or other important primary structures where a model and data from prior analyses may be available. In studies requiring both the primary and the secondary system parameters to vary, these approaches are less attractive. A manufacturer of floor-mounted equipment, for example, would find the present approaches cumbersome. The primary system may be any building; the variety of secondary equipment may be large. A procedure which would easily permit parametric studies where both the primary and secondary system parameters can be varied is needed.

With this in mind, two accurate methods were developed for computing the frequency response of a single degree-of-freedom secondary equipment supported on an arbitrary floor in a multi-story building which is subjected to input horizontal ground acceleration [7]. Parametric studies carried out in that study show that the methods give accurate results for light as well as relatively heavy secondary equipment.

The first method, based on well-known component mode synthesis techniques, is accurate and efficient when the natural frequency of the secondary equipment is low. This study showed that when the equipment is tuned to a low primary mode of an example building (an idealized N-story building with identically constructed story units), sufficiently accurate results were obtained by including the tuned primary mode, one immediately higher than the tuned primary mode, and all the lower primary modes. The method is not limited to the idealized periodic building, which was only chosen for convenience: exact closed-form solutions were derived, and used to evaluate the accuracy of the approximate results. The method is applicable to a less idealized primary building, which need not be classically damped.

When the secondary system is tuned to a high primary mode, the second approach, referred to as a modified cascade approach, is more efficient. In this approach, the response near the natural frequencies of the tuned primary mode and a few of the lowest primary modes are calculated on the basis of only these primary modes, whereas the response for the remaining frequency region is calculated using a traditional cascade procedure.

The two methods are extended to study the effects of soil compliancy and linear viscoelastic cushioning devices that are sometimes employed between the equipment and the supporting floor of the building. The nonstationary nature of the input earthquake excitation is also included in the numerical computations.

# SECTION 2
# ANALYSIS

In order to be concise, both soil compliancy and protective cushion will be included in our derivation. Their individual effects will be evaluated separately in our numerical calculations.

As shown in figure 2-1, a protective cushion is inserted between a secondary equipment and the supporting primary structure. This, in turn, is supported by a flexible soil mass.

The interaction among the three subsystems (soil-structure-equipment) will be represented in the frequency domain. The soil-structure interaction is assumed to be describable in terms of frequency-dependent impedance functions. The protective cushion is assumed to be linear viscoelastic and the well known complex modulus representation [5, 19] will be used to describe its behavior.

The combined system is linear. The deformation of the combined system is characterized not only by horizontal translations of individual floors, but also the translation and rotation of the footing, due to soil compliancy. For simplicity, the footing is assumed to be rigid, although its flexibility can be accounted for with much more involved computations [9]. It is further assumed that: (1) the mass of each story unit is concentrated at the floor level, (2) linear elasticity is provided by massless columns or shear walls between neighboring floors, (3) linear viscous damping is generated by the relative motion between neighboring floors, and for the secondary system, (4) the equipment is a simple linear oscillator with mass $M_e$, stiffness $K_e$, and viscous damping coefficient $C_e$.

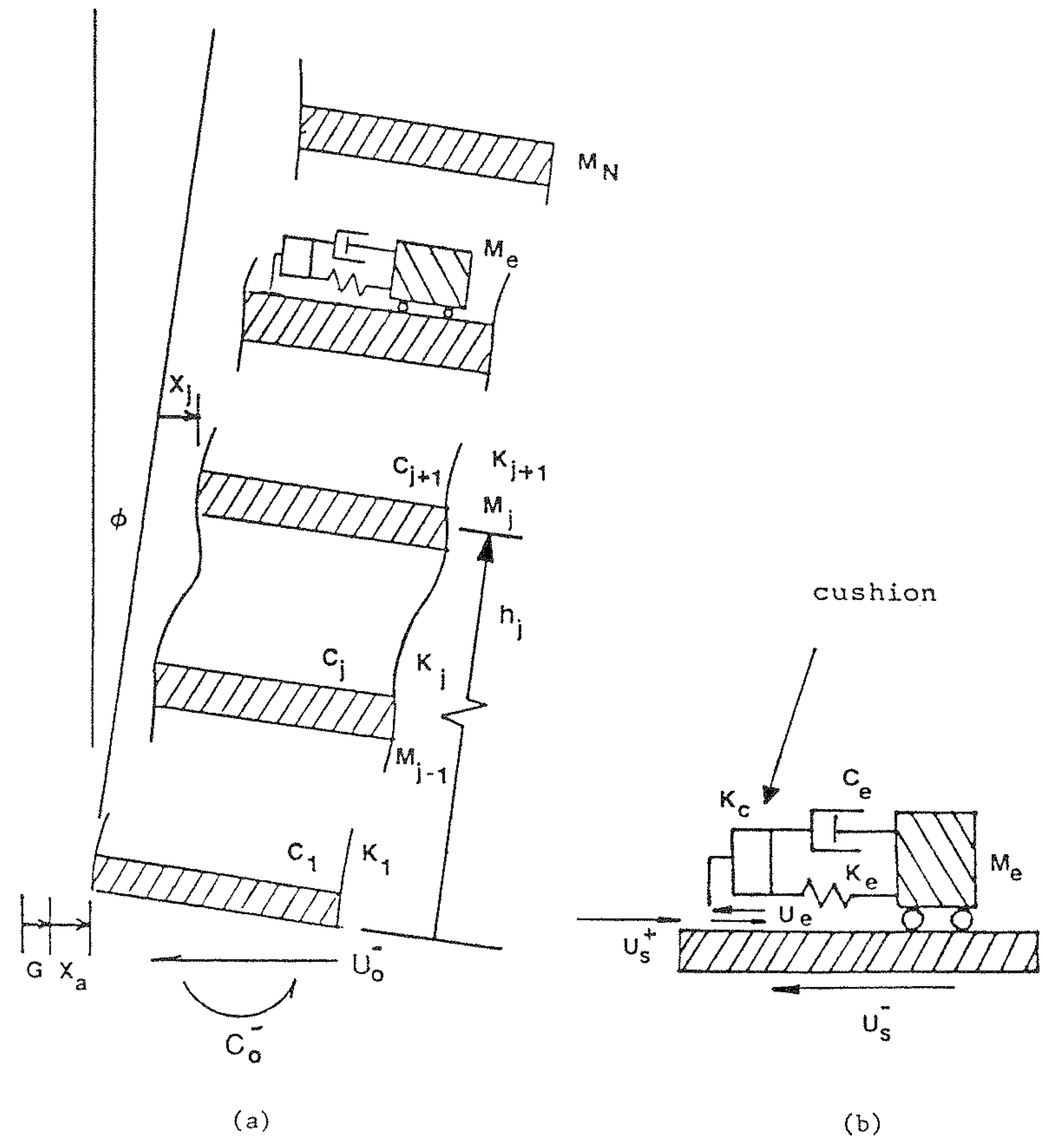



FIGURE 2-1. STRUCTURAL MODEL: (a) N+3 DEGREE-OF-FREEDOM COMBINED P-S SYSTEM, (b) FORCES ON THE S-TH FLOOR.

Denoting the reactionary force and couple from the soil by $U_0^-$ and $C_0^-$, respectively, and the interaction force between the primary and secondary system by $U_e$, as shown in figure 2-1, the following system of equations for the combined system can be obtained by inspection:

$$M_j \ddot{X}_j^t - C_{j+1} \dot{X}_{j+1} + ( C_j + C_{j+1} ) \dot{X}_j - C_j \dot{X}_{j-1} - K_{j+1} X_{j+1}$$
$$+ ( K_j + K_{j+1}) X_j - K_j X_{j-1} - U_e \delta_{js} = 0 \qquad (2.1)$$
$$j=1,\ldots,N$$

$$M_e \ddot{Y}_e^t + U_e = 0 \qquad (2.2)$$

$$\sum_{j=0}^{N} M_j \ddot{X}_j^t + M_e \ddot{Y}_e^t + U_0^- = 0 \qquad (2.3)$$

$$\sum_{j=0}^{N} M_j \ddot{X}_j^t h_j + M_e \ddot{Y}_e^t h_e + ( \sum_{j=0}^{N} I_j + I_e ) \ddot{\varphi} + C_0^- = 0 \qquad (2.4)$$

In the above equations, $X_j$= translation of the j-th floor relative to the footing (which may be referred to as the 0-th floor), $Y_e$= translation of the equipment relative to the footing, $I_j$= rotational inertia of the j-th floor about its own centroidal axis, $I_e$= rotational inertia of the equipment about its centroidal axis, $h_j$= height of the j-th floor above the footing, $\varphi$= rotation of the footing which is assumed to be rigid, $\delta_{js}$ is a Kronecker delta, and a superscript t denotes total translation. For small motion this can be approximated as

$$X_j^t = G + X_a + X_j + h_j \varphi \qquad (2.5)$$

where G is the free-field horizontal ground translation during an earthquake, and $X_a$ is an additional translation of the footing due to soil compliancy.

Taking the Fourier transform of the above equations, and denoting the transform of a variable by an overbar, the following frequency domain equations are obtained:

$$- M_j\omega^2 \bar{X}_j^t + i\omega\left\{-C_{j+1} \bar{X}_{j+1} + (C_j+C_{j+1}) \bar{X}_j - C_j\bar{X}_{j-1}\right\} + \left\{-K_{j+1} \bar{X}_{j+1} + (K_j+K_{j+1}) \bar{X}_j - K_j \bar{X}_{j-1}\right\} - \bar{U}_e \delta_{js} = 0 \qquad (2.6)$$

$$j = 1,2,\ldots,N$$

$$- M_e\omega^2 \bar{Y}_e^t + \bar{U}_e = 0 \qquad (2.7)$$

$$- \sum_{j=0}^{N} M_j\omega^2 \bar{X}_j^t - M_e\omega^2 \bar{Y}_e^t + \bar{U}_0^- = 0 \qquad (2.8)$$

$$- \sum_{j=0}^{N} M_j\omega^2 \bar{X}_j^t h_j - M_e\omega^2 \bar{Y}_e^t h_e - \omega^2 \left(\sum_{j=0}^{N} I_j+I_e\right) \bar{\varphi} + \bar{C}_0^- = 0 \qquad (2.9)$$

$$\bar{X}_j^t = \bar{X}_j + \bar{X}_a + \bar{G} + h_j\bar{\varphi} \qquad (2.10)$$

When applying the preceding equations, note that $h_0 = K_{N+1} = C_{N+1} = 0$, in addition to the boundary condition $\bar{X}_0=0$. The interaction force $\bar{U}_e$ between the building and the secondary equipment, appearing in equations (2.6) and (2.7), can be expressed as

$$\bar{U}_e = K_{eq}(\omega) ( \bar{Y}_e - \bar{X}_s) \qquad (2.11)$$

where $K_{eq}(\omega)$ denotes an equivalent frequency dependent stiffness. Referring to figure 2-1, $K_{eq}(\omega)$ depends on the equipment parameters $K_e$ and $C_e$ as well as $K_c(\omega)$, the equivalent stiffness of the viscoelastic cushioning device. The following relationship follows from elementary considerations:

$$\frac{1}{K_{eq}(\omega)} = \frac{1}{K_c(\omega)} + \frac{1}{K_e + i\,\omega C_e} \tag{2.12}$$

$K_c(\omega)$, in turn, depends on the modulus of the viscoelastic material as well as the physical design of the cushioning device (e.g., size, shape, construction, etc.). Since our interest is not on any one device, the following approach has been adopted. First, a material was selected. The choice is Butyl B252, a rubber whose viscoelastic properties are defined in terms of its complex modulus $G_c(\omega)$, already described over a wide range of frequencies ( $0 < f < 10^4$ Hz) as follows [2]:

$$G_c(\omega) = G_0 + G_1\sqrt{\omega}\; e^{i\,\pi/4} \tag{2.13}$$

where $G_0 = 7.6 \times 10^5$ N/m², the static modulus, and $G_1 = 2.95 \times 10^5$ N/m². By factoring $G_0$ out of this expression, the stiffness of the cushioning device fashioned from this material is represented as

$$K_c(\omega) = a\, G_0 \left\{ 1 + \frac{G_1}{G_0}\sqrt{\omega}\; e^{i\,\pi/4} \right\} \tag{2.14}$$

where the parameter `a' can be adjusted to incorporate the physical design features mentioned above. In the numerical computations, the value for `a' will be varied to simulate different designs. Therefore, different cushions in the numerical computations can be simulated quite easily. The real and imaginary parts of the second term within the brackets represent the recoverable and dissipated portion of the energy, respectively, of the cushion material Butyl B252. These can be changed for another viscoelastic material once its complex modulus is known. Thus, the present formulation is quite convenient for optimization studies in which the cushion construction or material or both are varied.

The interaction force $U_0^-$ and the interaction couple $C_0^-$ between the footing and the soil are related to the additional translation $X_a$ and rotation $\varphi$ of the footing due to soil compliancy as follows:

$$\begin{bmatrix} \bar{U}_0^- \\ \bar{C}_0^- \end{bmatrix} = \begin{bmatrix} K_{HH} & K_{HM} \\ K_{MH} & K_{MM} \end{bmatrix} \begin{bmatrix} \bar{X}_a \\ \bar{\varphi} \end{bmatrix} \qquad (2.15)$$

The K matrix in equation (2.15) is an impedance matrix with frequency dependent elements. Earlier published results for the elements of this matrix (or the elements of its inverse i.e., the compliance matrix) were obtained under the assumptions that the soil was a linearly elastic half-space and the footing was circular and rigid [3, 6, 17, 27]. More recent results included viscoelastic soil [26], noncircular footing shapes [28], and footing flexibility [9]. The elements of the impedance matrix in equation (2.15) will be obtained by inverting a compliance matrix given in graphical form by Luco and Westmann, and will be defined more completely later.

Substituting equations (2.10), (2.11), and (2.15), into equations (2.6) through (2.9) results in N+3 equations for the N+3 unknowns $X_j$ (j=1,...,N), $Y_e$, $X_a$, and $\varphi$, for a given earthquake excitation G. The ideal case of a rigid soil is obtained if we set $X_a$ and $\varphi$ equal to zero. Then equations (2.6) and (2.7), after substituting equations (2.10) and (2.11), would result in N+1 equations for N+1 unknowns $X_j$ ($j=1,\ldots,N$), and $Y_e$. If, in addition, $K_c$ is taken to be infinitely large, the simpler equations for the case without cushion are recovered [7]. The differences in equipment response for these cases are illustrated in numerical computations.

For convenience, assume that each story of the primary building structure is again identically constructed. This simplifying assumption, while not required for application of the two approximate methods previously developed, nevertheless permit the exact frequency response of the equipment (as well as the frequency response of the building itself at various floors) to be obtained in closed form. The details of the exact solution are summarized in the Appendices. Therefore, the response obtained from the two approximate methods can be compared with the exact results.

When applying the two approximate methods, the following expression for the normal modes of the primary building with identically constructed story units is used [7]:

$$u_i(j) = (-1)^{i+1} \cos(N+\tfrac{1}{2}-j)\pi \frac{2i-1}{2N+1} \sin \pi \frac{2i-1}{2N+1} \qquad (2.16)$$

$$i,j=1,\ldots,N$$

where $u_i(j)$ is the j-th element of mode i. These expressions are not restricted however, to the idealized N-story building; they apply equally well to more general N-story buildings. For completeness, the following expressions for the natural frequencies and damping ratios of the idealized N-story building are reproduced:

$$\omega_i = 2\left(\frac{K}{M}\right)^{\frac{1}{2}} \sin \frac{\pi}{2} \quad \frac{2i-1}{2N+1} \; ; \quad i=1,\ldots,N \tag{2.17}$$

$$\zeta_i = \frac{C}{\sqrt{(MK)}} \sin \frac{\pi}{2} \quad \frac{2i-1}{2N+1} \; ; \quad i=1,\ldots,N \tag{2.18}$$

# SECTION 3
# EARTHQUAKE EXCITATION MODEL

In the following numerical computations, the input ground acceleration due to an earthquake will be modelled as an evolutionary process [20]. This is a nonstationary random process having a Stieltjes integral representation of

$$\ddot{G}(t) = \int_{-\infty}^{\infty} \Gamma(t, \omega) e^{i\omega t} \, d\tilde{P}(\omega) \tag{3.1}$$

where $\tilde{P}(\omega)$ is another random process with uncorrelated increments. Equation (3.1) is a generalization of the Fourier-Stieltjes representation of a stationary random process [30] for which the function $\Gamma(t, \omega)$ reduces to a constant. The uncorrelated-increment random process $\tilde{P}(\omega)$ has the property

$$E[\, d\tilde{P}(\omega_1) \, d\overset{*}{P}(\omega_2)] = \begin{cases} 0, & \omega_1 \neq \omega_2 \\ \Phi(\omega) \, d\omega, & \omega_1 = \omega_2 = \omega \end{cases} \tag{3.2}$$

where $E[\cdot]$ denotes an ensemble average, an asterisk represents the complex conjugate, and $\Phi(\omega)$ is the spectral density of some stationary random process. Given the impulse response function $h_x(t)$ of a general response variable $X(t)$ of a linear system, the autocorrelation function of $X(t)$ can be computed as

$$E[X(t_1)X(t_2)] = \int_{-\infty}^{\infty} M(t_1,\omega)\overset{*}{M}(t_2,\omega) \, e^{i\omega(t_1-t_2)} \, \Phi(\omega)d\omega \tag{3.3}$$

in which

$$M(t,\omega) = \int_0^t \Gamma(t-u, \omega)h_x(u) \, e^{-i\omega u} \, du \tag{3.4}$$

By setting $t_1=t_2$ in equation (3.3), the integrand yields the evolutionary spectral density of X(t), and the mean-square response can be computed upon integration.

In the subsequent numerical computations, $\Gamma$ will be taken to be a function of t alone. For this special case, the evolutionary process of equation (3.1) is called a uniformly modulated process and $\Gamma(t)$ is called the modulating or envelope function. The evolutionary spectrum of a uniformly modulated process does not change its frequency content with time. The envelope function in the present study will be taken as

$$\Gamma(t)= \begin{cases} 0, & t<0 \\ (t/t_1)^2, & 0\le t < t_1 \\ 1, & t_1\le t < t_2 \\ \exp[-c(t-t_2)] & t > t_2 \end{cases} \tag{3.5}$$

and the spectrum $\Phi(\omega)$ to be used in equation (3.3) will be taken to be the well known Kanai-Tajimi spectrum which has the form

$$\Phi(\omega) = \frac{1 + 4 \zeta_g \left(\frac{\omega}{\omega_g}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_g}\right)^2\right]^2 + 4 \zeta_g^2 \left(\frac{\omega}{\omega_g}\right)^2} S \tag{3.6}$$

in which $\omega_g$ is a characteristic frequency, $\zeta_g$ is a characteristic damping, and S is a spectral level. These parameters can be adjusted to represent ground resonance, attenuation, and intensity of seismic waves.

It is of interest to note that Lin and Yong [15] have proposed a general random pulse train model for earthquakes, which includes as a special case the evolutionary Kanai-Tajimi model, and admits a nonstationary spectral representation as given in equation (3.1).

# SECTION 4
# NUMERICAL RESULTS

Numerical calculations have been carried out for a 20 story building (N=20) with the following properties:

$h$=story height=3.6 m
$M$=floor mass=3.456 x $10^6$ kg
$M_0$=footing mass=0.5 x M
$K$=shear-wall stiffness=3.404 x $10^9$ N/m
$C$=damping coefficient=1.0 x $10^6$ N/m/s
$I$=rotational inertia of each floor=7 x $10^7$ kg-m²
$I_0$=rotational inertia of footing=2.5 x $10^7$ kg-m²

The damping ratio for the equipment model was assumed to be 0.03. Other equipment properties were selected as follows:

$h_e$=height=$h_s$=4h (equipment located on the 4-th floor)
$M_e$=mass=0.01 M (mass ratio $M_e/M$ =0.01)
$I_e$=0 kg-m²

The equipment stiffness and the cushion parameter `a' were varied to simulate "tuned" and "detuned" conditions. Of course, these descriptions are strictly meaningful for buildings located on rigid soils and without any cushion since the frequency variation of soil and cushion parameters results in a frequency variation in the parameters for the combined system also. However, for convenience, these conditions are referred in the following manner. They aptly describe the essential behavior, and permit easy discussion when comparing the results for different cases.

The soil under the footing was assumed to be a linear elastic half space, characterized by:

$G_s$=shear modulus=$1.62 \times 10^8$ N/m²
$\rho$=mass density=1800 kg/m$^3$
$\sigma$=Poisson's ratio=1/3

The footing was assumed to have an equivalent radius r=7.5 m, and the elements of the impedance matrix were obtained from inversion of a compliance matrix obtained numerically by Luco and Westmann. Moreover, the off-diagonal terms in the compliance matrix were neglected following a suggestion of Veletsos and Wei [27]. The two non-negligible elements of the impedance matrix were fitted in the following forms according to Wu [29]:

$$K_{HH}(\omega)= \frac{p + i\omega}{b_1 \, p} \tag{4.1}$$

$$K_{MM}(\omega)= \frac{c_1 - \omega^2+i\omega c_2}{q \, (c_1+i\omega c_2)} \tag{4.2}$$

where the coefficients are expressed in terms of the soil properties as follows:

$$p= \frac{1.5}{r} \left(\frac{\rho}{G_s}\right)^{-\frac{1}{2}} \tag{4.3a}$$

$$b_1= \frac{2-\sigma}{8 \, G_s r} \tag{4.3b}$$

$$c_1= \frac{3.265 \, G_s}{r^2 \rho} \tag{4.3c}$$

$$c_2= (2.393 \, c_1)^{\frac{1}{2}} \tag{4.3d}$$

Parameters selected for the envelope function and the Kanai-Tajimi spectrum were $t_1$=3 s, $t_2$=13 s, c=0.26, $\omega_g$=18.85 rad/s, $\zeta_g$=0.65, and S=$4.65 \times 10^{-4}$ m²/s$^3$.

## 4.1 Equipment Tuned to Ninth Primary Mode

### 4.1.1 Rigid Soil Case

It will be instructive to study the equipment response under different assumptions. As a basis for comparison, first consider an equipment tuned to the ninth primary mode of the building, without a protective cushion for the equipment and neglecting the soil compliancy. The modulus of the frequency response function for the interactive force $U_e$, obtained from the modified cascade approach using primary modes 1,2, and the tuned 9-th primary mode, was compared with the exact results in the previous work [7] - it is reproduced here as figure 4-1 for convenience. The corresponding nonstationary root-mean-square response, computed from equations (3.3) and (3.4) is shown in figure 4-2. It can be seen that the approximate solution agrees very well with the exact results.

### 4.1.2 Viscoelastic Cushion Effect

Next, the effect of a cushion placed between the equipment and the supporting floor of the building is explored. To do this, the cushion parameter $a=66.0 \times 10^{-3}$ m is chosen. According to equations (2.12) and (2.14), this choice corresponds approximately to an equivalent equipment frequency of 1.2 rad./sec., if one neglects the frequency dependence in these equations. The value 1.2 rad./sec. is one half the first natural frequency of the building which is located on a rigid soil (refer to equations (2.17), (2.18) and table 4-1). Therefore, it is anticipated that with this choice, the response of the secondary system will exhibit characteristics similar to those of a detuned case. In figure 4-3 the approximate frequency response solution is compared with the exact result. Note that a peak appears in the figure at approximately 1.2 radian/second which is the equivalent natural frequency of the secondary system. The corresponding nonstationary equipment response is shown in figure

4-4. Comparing figure 4-4 with figure 4-2, it can be seen that introducing the cushion results in a decrease in equipment response by an order of magnitude. The usefulness of such devices is readily apparent. It can also be seen that the modified cascade solution is accurate as well.

### 4.1.3 Compliant Soil Case

When soil compliancy is taken into account, the approximate frequency response solution is compared with the exact solution in figure 4-5. It is clear that the approximate solution is not as accurate as in the previous cases. This is partially explained by comparing the exact solutions for figures 4-1 and 4-5. This comparison shows that the frequency dependent soil parameters causes a change in both the magnitude and shape of the response curve, although the two curves share some common features. This is consistent with the conclusion reached by Lin and Wu [16] for an example multi-story building that soil compliancy does modify the frequency response function of the building at each floor. Compared with the rigid soil case, more primary modes may have to be included in the approximations when soil compliancy is considered. For the present case, the approximate solution for the nonstationary equipment response and the exact solution are compared in figure 4-6. It can be seen that soil compliancy in this case reduces the peak response of the equipment by approximately 40 percent when compared with figure 4-2.

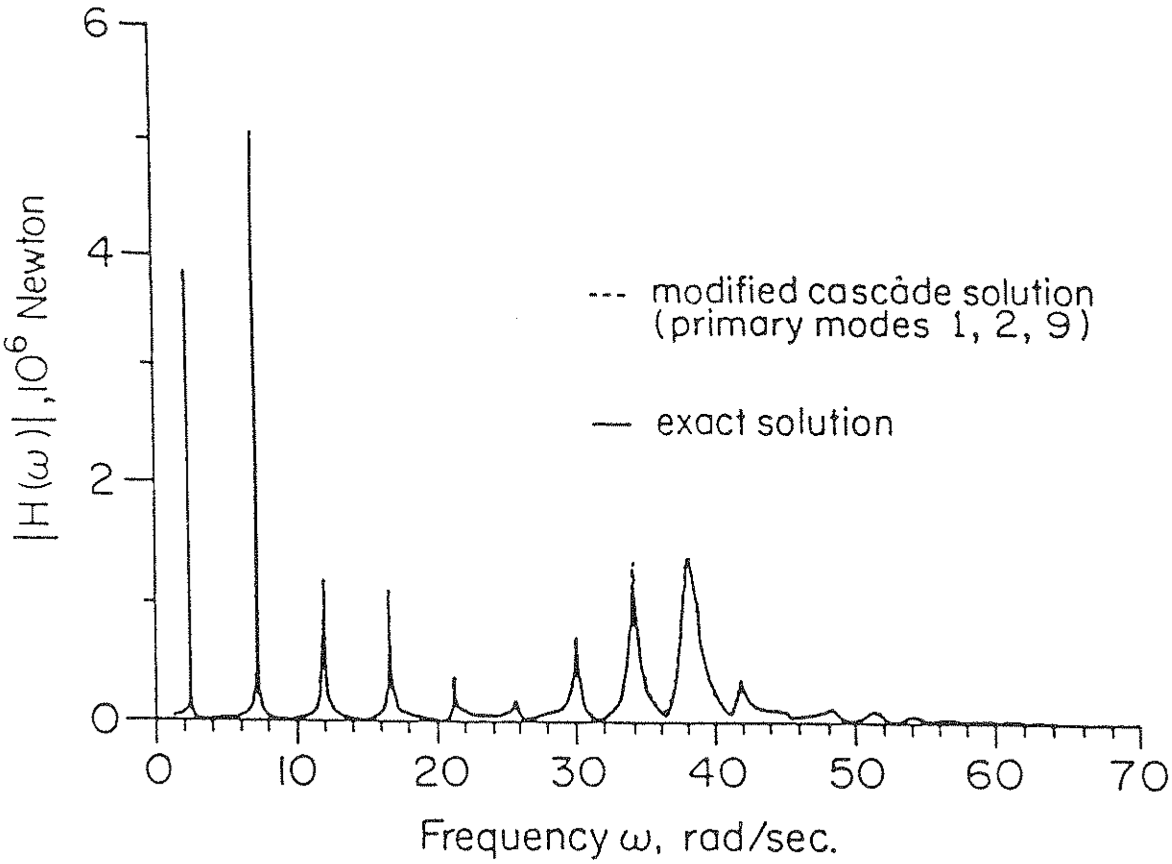



FIGURE 4-1. MODULUS OF FREQUENCY RESPONSE FUNCTION FOR INTERACTIVE FORCE ACTING AT THE EQUIPMENT-BUILDING INTERFACE: MODIFIED CASCADE VS. EXACT SOLUTION. EQUIPMENT ON 4-TH FLOOR TUNED TO NINTH MODE OF BUILDING. (RIGID SOIL).

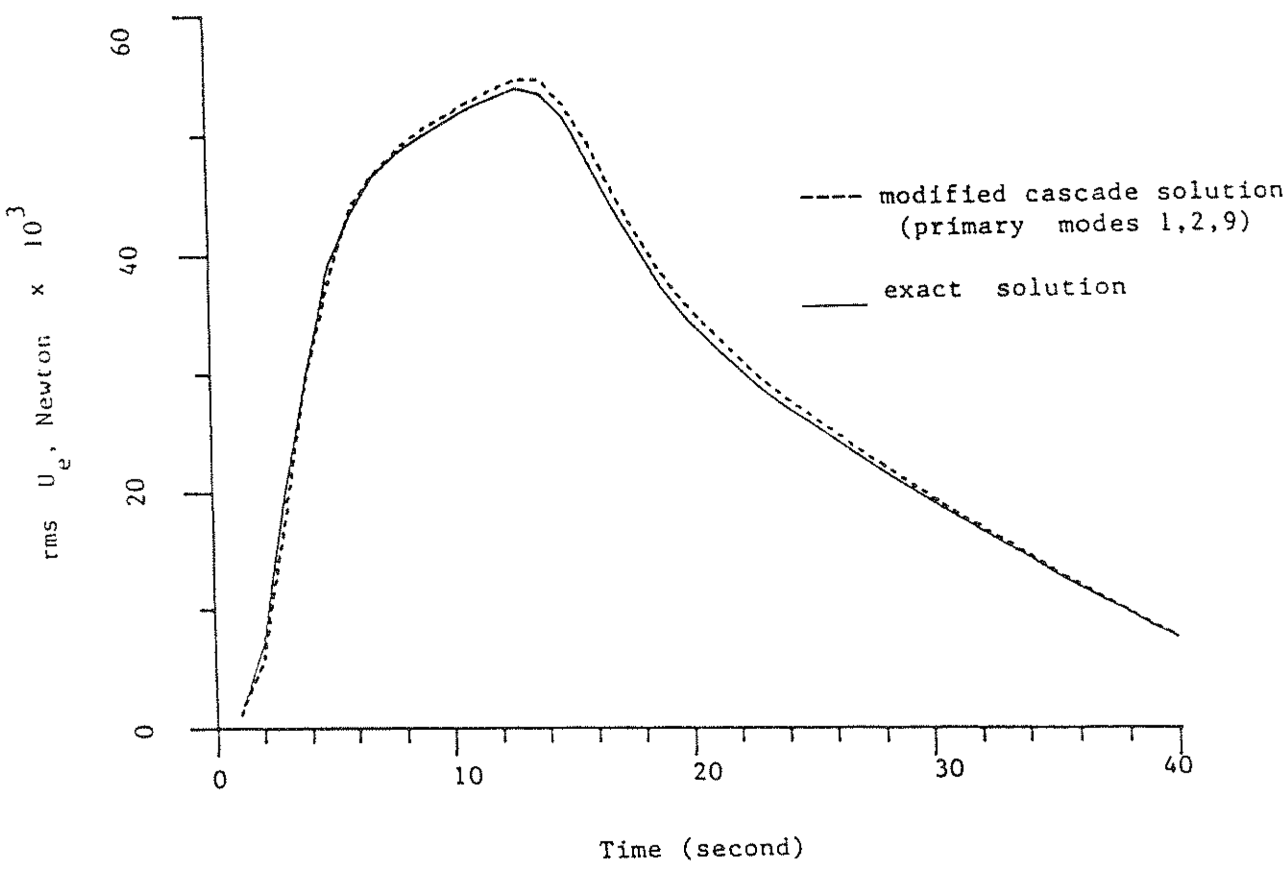



FIGURE 4-2. ROOT-MEAN-SQUARE SHEAR FORCE ACTING AT THE EQUIPMENT-BUILDING INTERFACE: MODIFIED CASCADE VS. EXACT SOLUTION. EQUIPMENT ON 4-TH FLOOR TUNED TO NINTH MODE OF BUILDING. ( RIGID SOIL).

**TABLE 4-1. NATURAL FREQUENCIES AND DAMPING RATIOS OF AN EXAMPLE 20-STORY PERIODIC BUILDING**

| Mode | Frequency (rad/s.) | Damping ratio (for c=1 x $10^6$ N/m/s) |
|---|---|---|
| 1 | 2.40418 | 0.00035 |
| 2 | 7.19844 | 0.00106 |
| 3 | 11.95046 | 0.00176 |
| 4 | 16.63235 | 0.00244 |
| 5 | 21.21662 | 0.00312 |
| 6 | 25.67640 | 0.00377 |
| 7 | 29.98550 | 0.00440 |
| 8 | 34.11862 | 0.00501 |
| 9 | 38.05153 | 0.00559 |
| 10 | 41.76113 | 0.00613 |
| 11 | 45.22567 | 0.00664 |
| 12 | 48.42479 | 0.00713 |
| 13 | 51.33975 | 0.00754 |
| 14 | 53.95342 | 0.00793 |
| 15 | 56.25048 | 0.00826 |
| 16 | 58.21743 | 0.00855 |
| 17 | 59.84274 | 0.00879 |
| 18 | 61.11687 | 0.00898 |
| 19 | 62.03234 | 0.00911 |
| 20 | 62.58378 | 0.00919 |

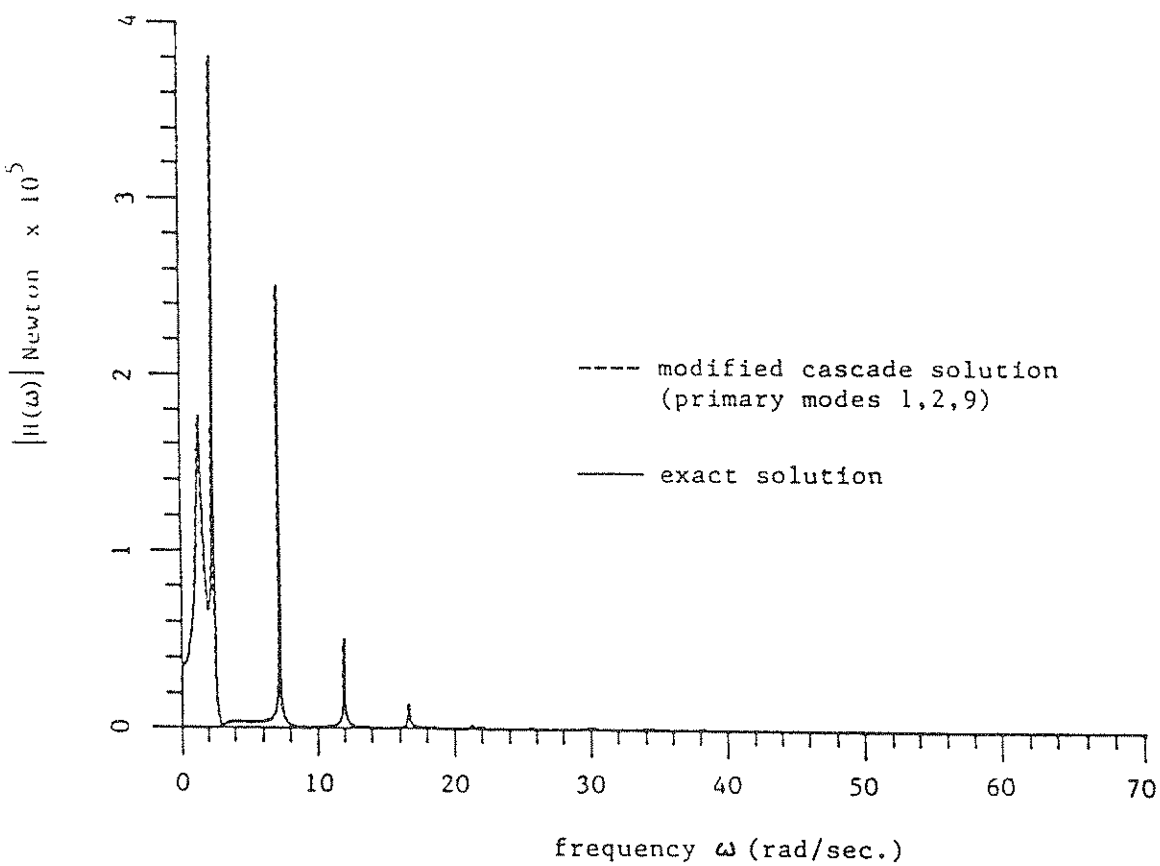



FIGURE 4-3. MODULUS OF FREQUENCY RESPONSE FUNCTION FOR INTERACTIVE FORCE ACTING AT THE EQUIPMENT-BUILDING INTERFACE: MODIFIED CASCADE VS. EXACT SOLUTION. EQUIPMENT ON 4-TH FLOOR TUNED TO NINTH MODE OF BUILDING. (RIGID SOIL AND VISCOELASTIC CUSHION WITH PARAMETER $a=66.0 \times 10^{-3}$ m).

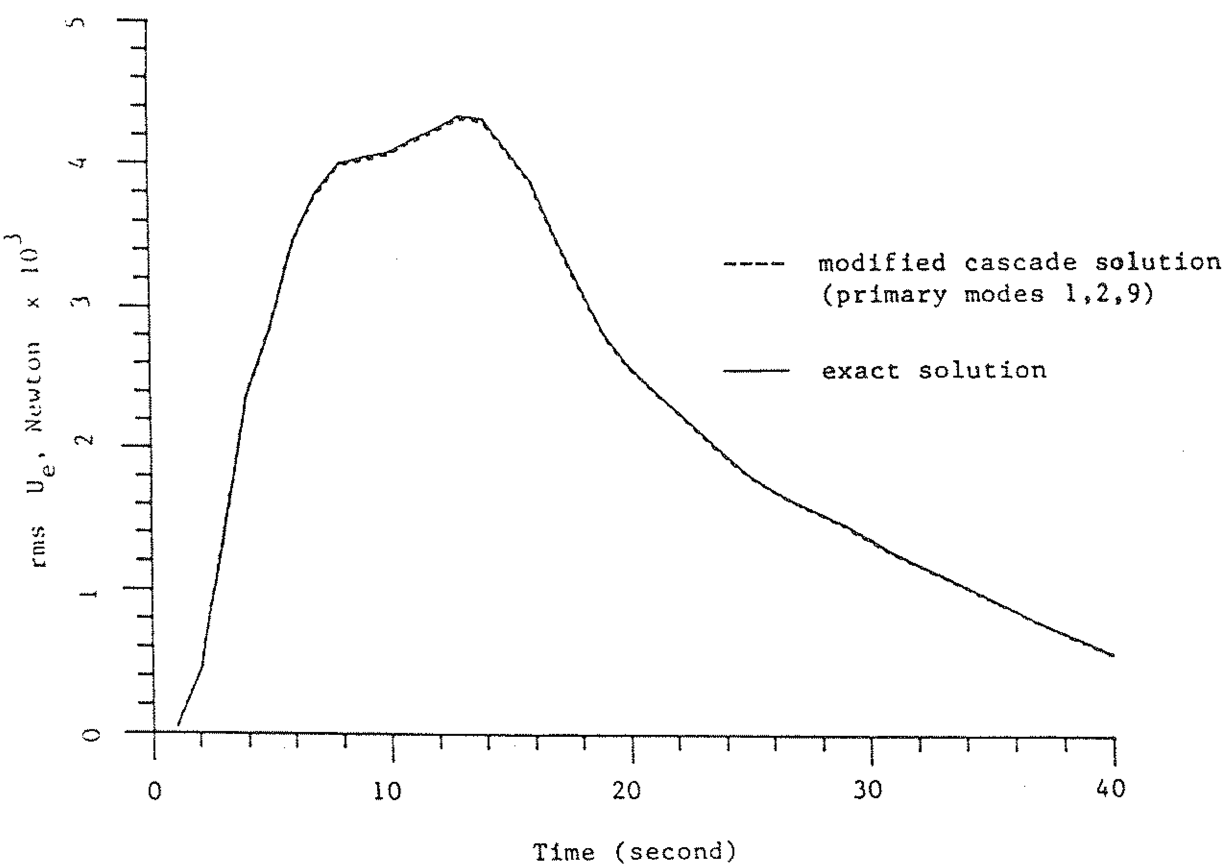



FIGURE 4-4. ROOT-MEAN-SQUARE SHEAR FORCE ACTING AT THE EQUIPMENT-BUILDING INTERFACE: MODIFIED CASCADE VS. EXACT SOLUTION. EQUIPMENT ON 4-TH FLOOR TUNED TO NINTH MODE OF BUILDING. (RIGID SOIL AND VISCOELASTIC CUSHION WITH PARAMETER $a=66.0 \times 10^{-3}$ m).

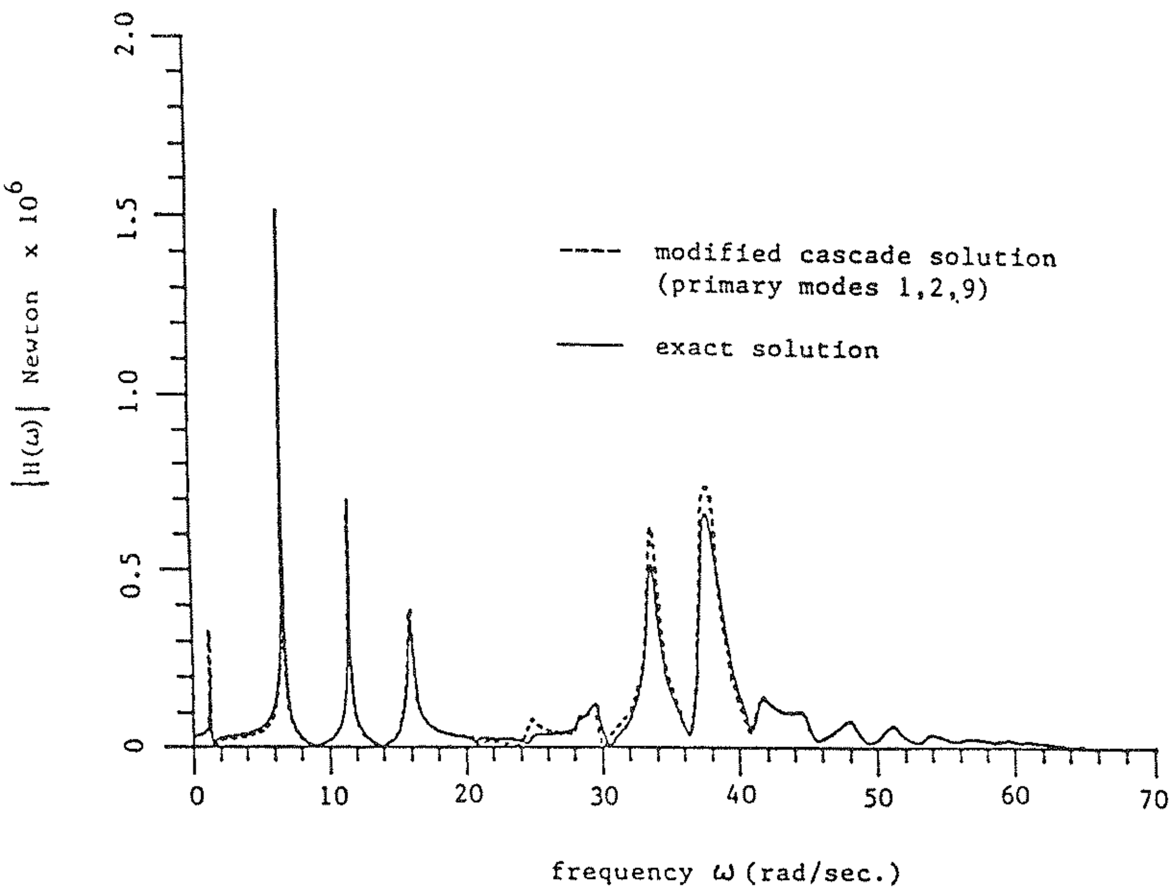



FIGURE 4-5. MODULUS OF FREQUENCY RESPONSE FUNCTION FOR INTERACTIVE FORCE ACTING AT THE EQUIPMENT-BUILDING INTERFACE: MODIFIED CASCADE VS. EXACT SOLUTION. EQUIPMENT ON 4-TH FLOOR "TUNED" TO NINTH MODE OF BUILDING. (COMPLIANT SOIL).

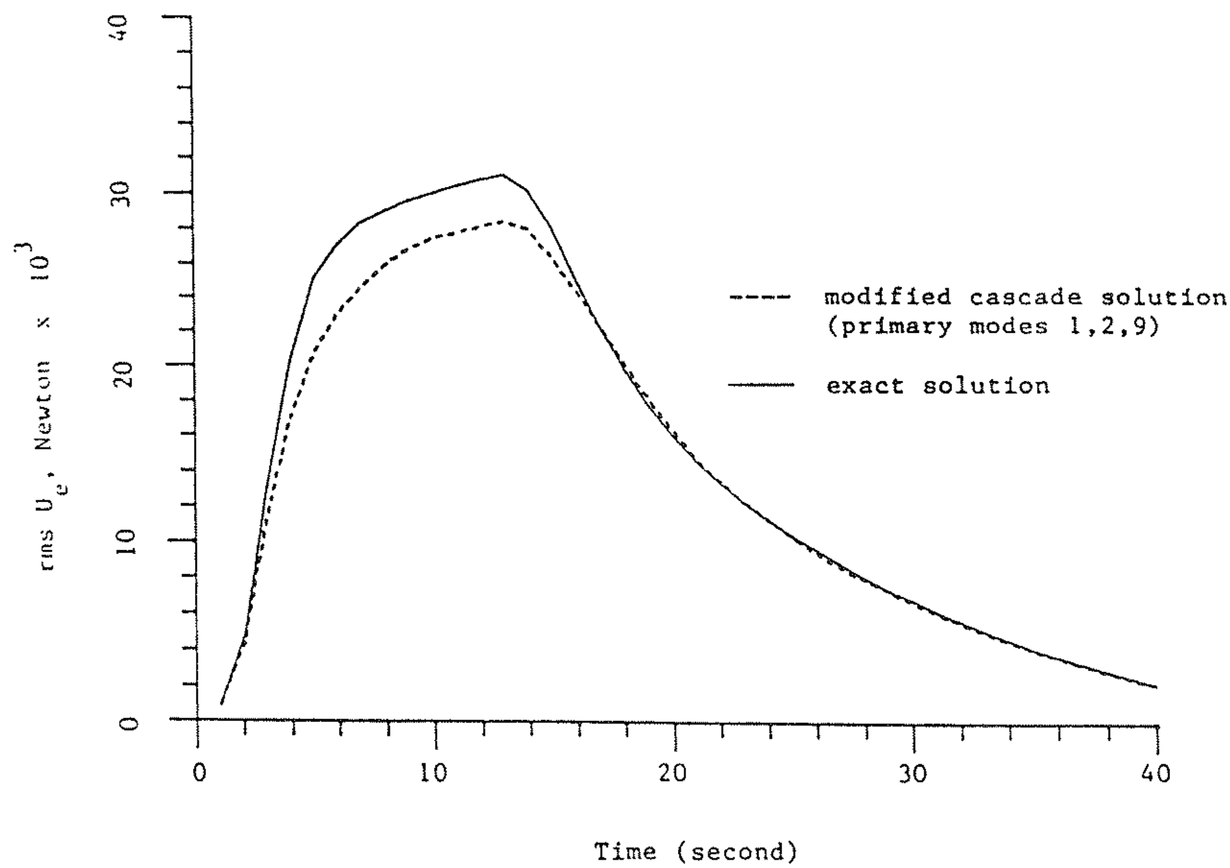



FIGURE 4-6. ROOT-MEAN-SQUARE SHEAR FORCE ACTING AT THE EQUIPMENT-BUILDING INTERFACE: MODIFIED CASCADE VS. EXACT SOLUTION. EQUIPMENT ON 4-TH FLOOR "TUNED" TO NINTH MODE OF BUILDING. (COMPLIANT SOIL).

## 4.2 Equipment Tuned to First Primary Mode

### 4.2.1 Viscoelastic Cushion Effect

Next, the effect of a cushioning device between the equipment and the supporting floor is considered when the equipment is tuned originally to the first primary mode of the building. The same cushion parameter ($a=66.0 \times 10^{-3}$ m) was used in the computation. When the equipment frequency is low, the component-mode approach alone is adequate. The exact and component-mode results are compared in figure 4-7. The results show an even greater reduction in equipment response than that obtained in figure 4-4. The component-mode approach is again very accurate, with only the tuned primary mode included in the analysis.

### 4.2.2 Compliant Soil Case

Figure 4-8 shows the corresponding results when soil compliancy is also taken into account. Two approximate results are shown in the figure, obtained when only the first mode is included in the analysis, and when two additional modes are also included. It can be seen that the three-mode approximation is still inaccurate, and indicates that for this case even more modes should be included for the required accuracy.

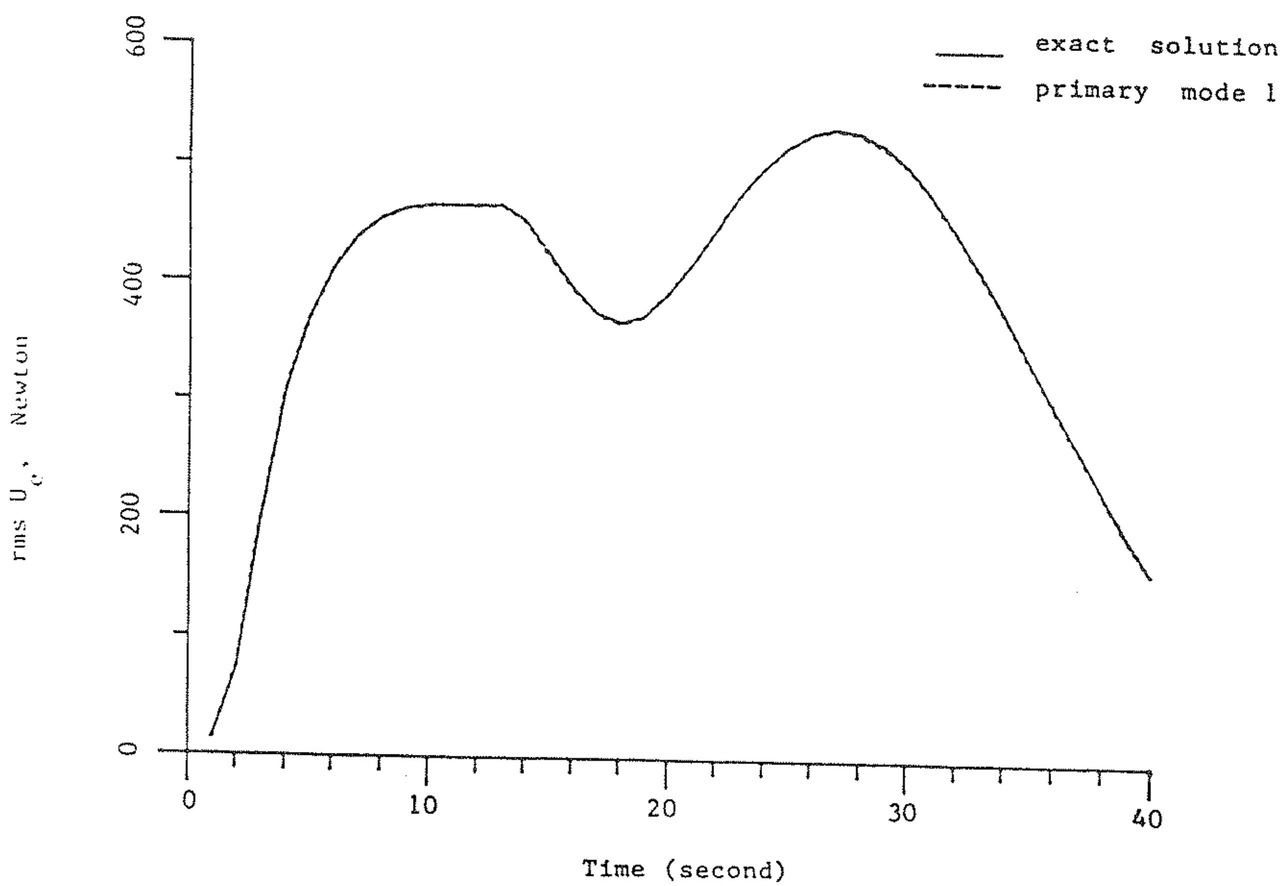



FIGURE 4-7. ROOT-MEAN-SQUARE SHEAR FORCE ACTING AT THE EQUIPMENT-BUILDING INTERFACE: COMPONENT-MODE APPROXIMATION VS. EXACT SOLUTION. EQUIPMENT ON 4-TH FLOOR TUNED TO FIRST MODE OF BUILDING. (RIGID SOIL AND VISCOELASTIC CUSHION WITH PARAMETER $a=66.0 \times 10^{-3}$ m).

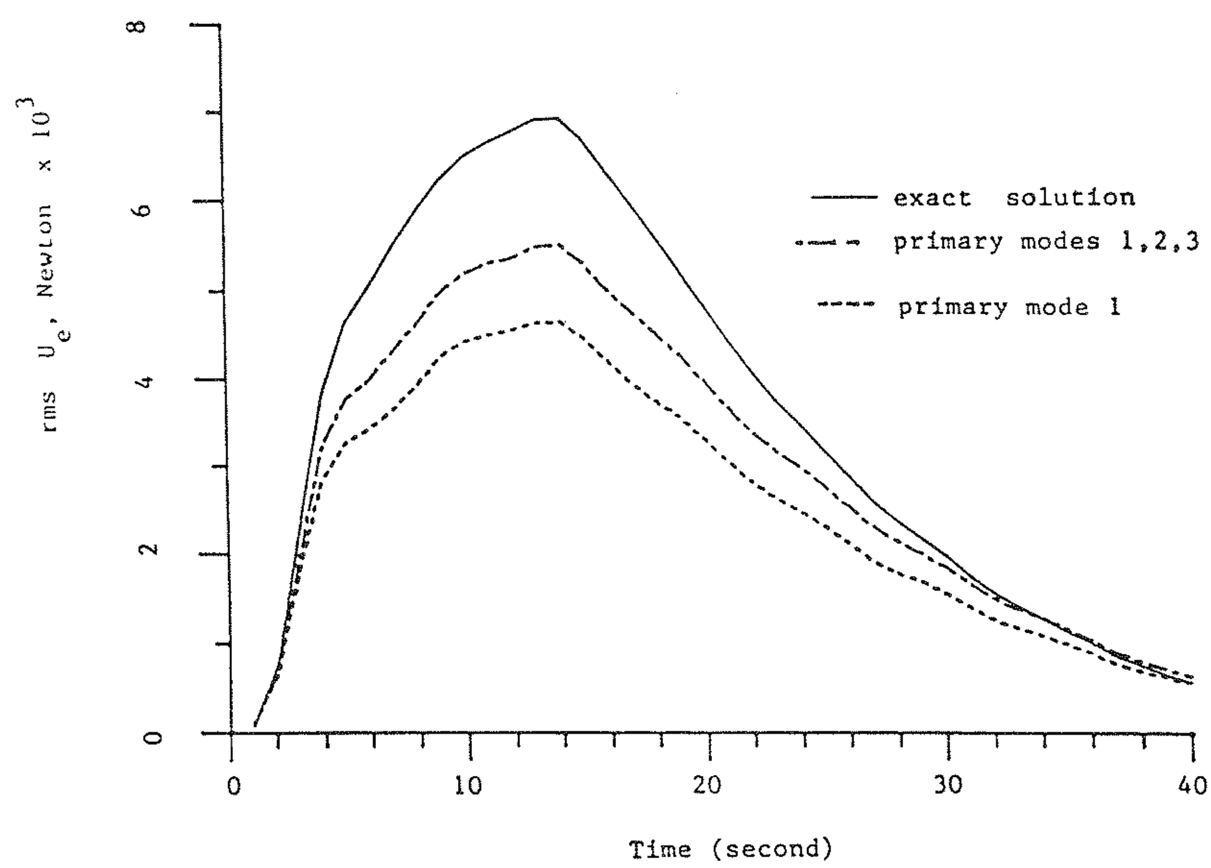



FIGURE 4-8. ROOT-MEAN-SQUARE SHEAR FORCE ACTING AT THE EQUIPMENT-BUILDING INTERFACE: COMPONENT-MODE APPROXIMATION VS. EXACT SOLUTION. EQUIPMENT ON 4-TH FLOOR "TUNED" TO FIRST MODE OF BUILDING. (COMPLIANT SOIL).

## 4.3 From Detuned Condition to Tuned Condition

Finally, a case where the introduction of a cushion shifts the equipment from a detuned condition to a tuned condition is investigated. The detuned condition is chosen to correspond to an equipment frequency mid-way between the third and fourth natural frequency of the building (i.e., 14.29 rad./sec. - see table 4-1), and the tuned condition corresponding to a tuning with the first building frequency (2.40 rad./sec.).

### 4.3.1 Detuned Case

For the detuned case, the frequency response and nonstationary response of the equipment are shown in figures 4-9 and 4-10, respectively. Again, it is found that the component-mode approximation, which employs the first four primary modes, compares very well with the exact results. A comparison of figures 4-10 and 4-2 indicate that the response of a detuned equipment can be greater than that of a tuned equipment, provided the frequency of the detuned equipment is low in comparison to the tuned frequency. This behavior was also noted in other numerical examples [8].

### 4.3.2 Tuned Case

For the tuned case, the frequency response and nonstationary response are shown in figures 4-11 and 4-12, respectively. Comparing figures 4-12 and 4-10, it can be seen that the cushion is still effective in reducing the equipment response, even though the equipment is tuned to the first building frequency. As expected, the reduction is not as great as the previous results in figure 4-4.

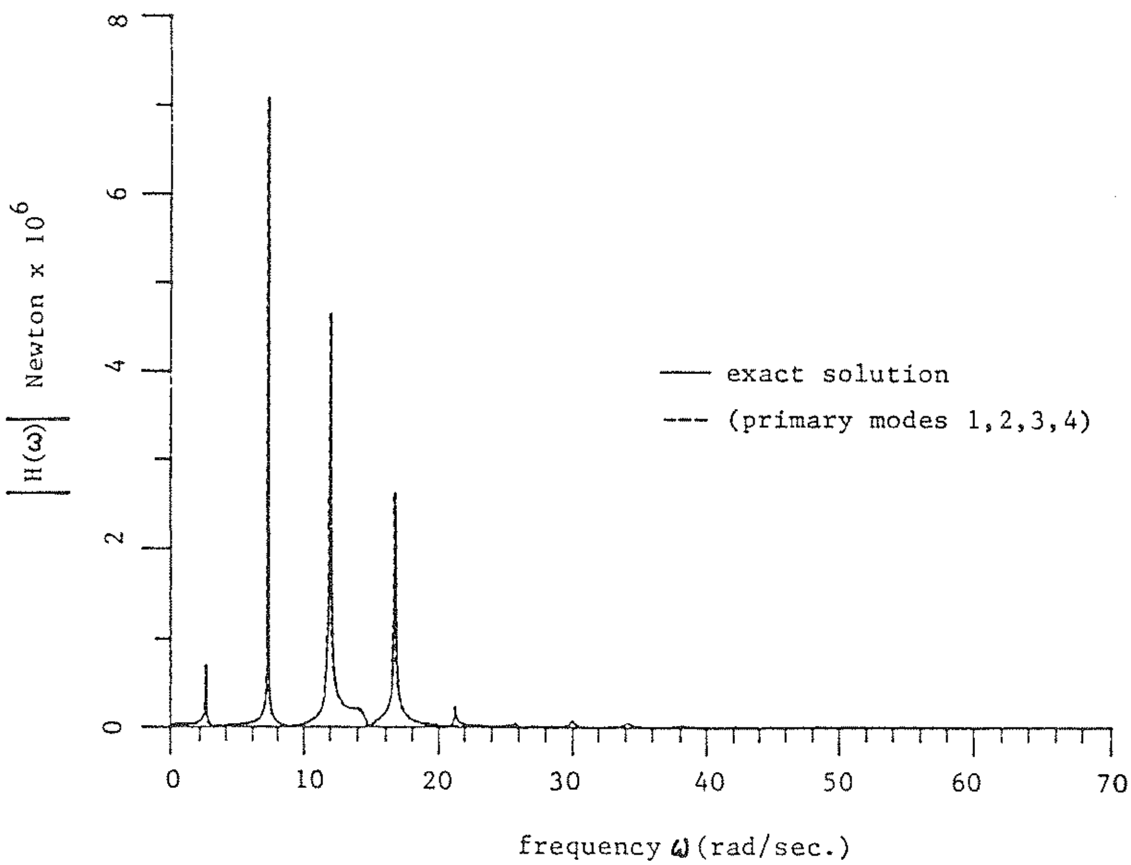



FIGURE 4-9. MODULUS OF FREQUENCY RESPONSE FUNCTION FOR INTERACTIVE FORCE ACTING AT THE EQUIPMENT-BUILDING INTERFACE: COMPONENT-MODE APPROXIMATION VS. EXACT SOLUTION. EQUIPMENT ON 4-TH FLOOR "DETUNED" MID-WAY BETWEEN 3RD AND 4TH MODES OF BUILDING. (RIGID SOIL).

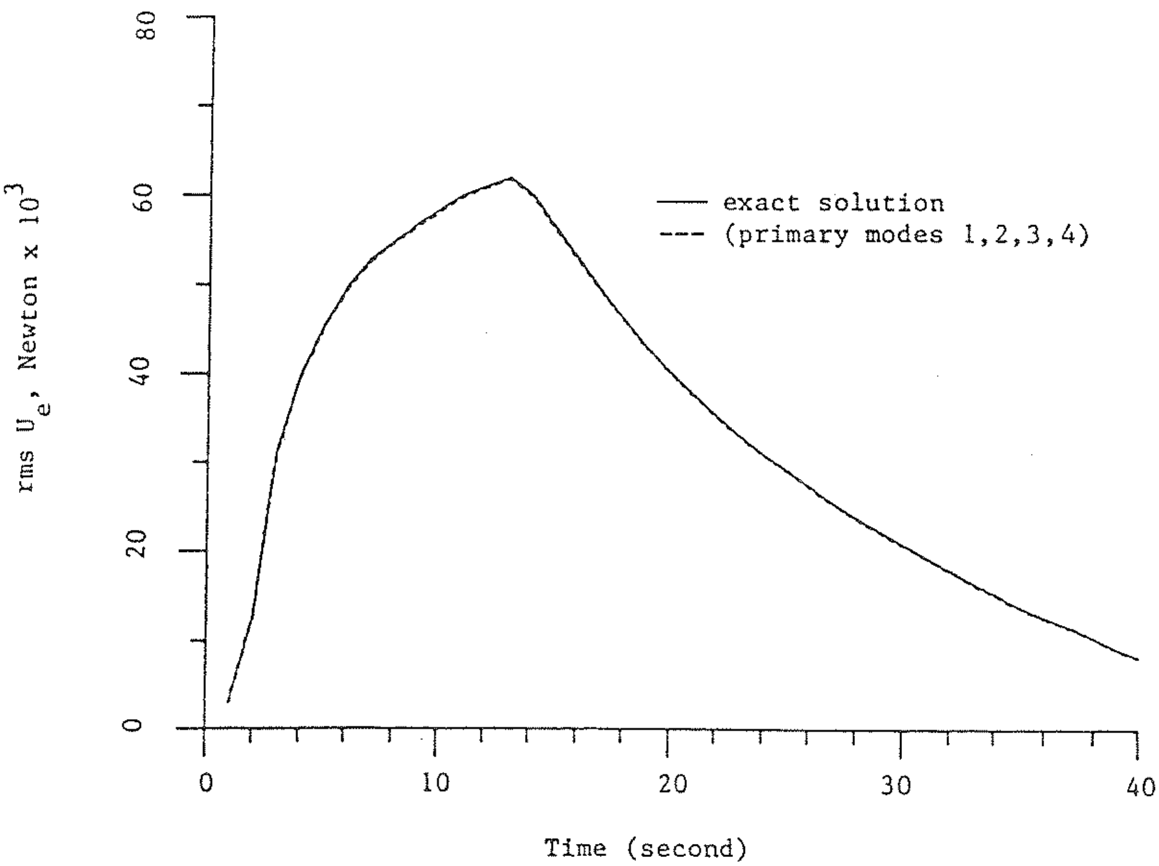



FIGURE 4-10. ROOT-MEAN-SQUARE SHEAR FORCE ACTING AT THE EQUIPMENT-BUILDING INTERFACE: COMPONENT-MODE APPROXIMATION VS. EXACT SOLUTION. EQUIPMENT ON 4-TH FLOOR "DETUNED" MID-WAY BETWEEN 3RD AND 4TH MODES OF BUILDING. (RIGID SOIL).

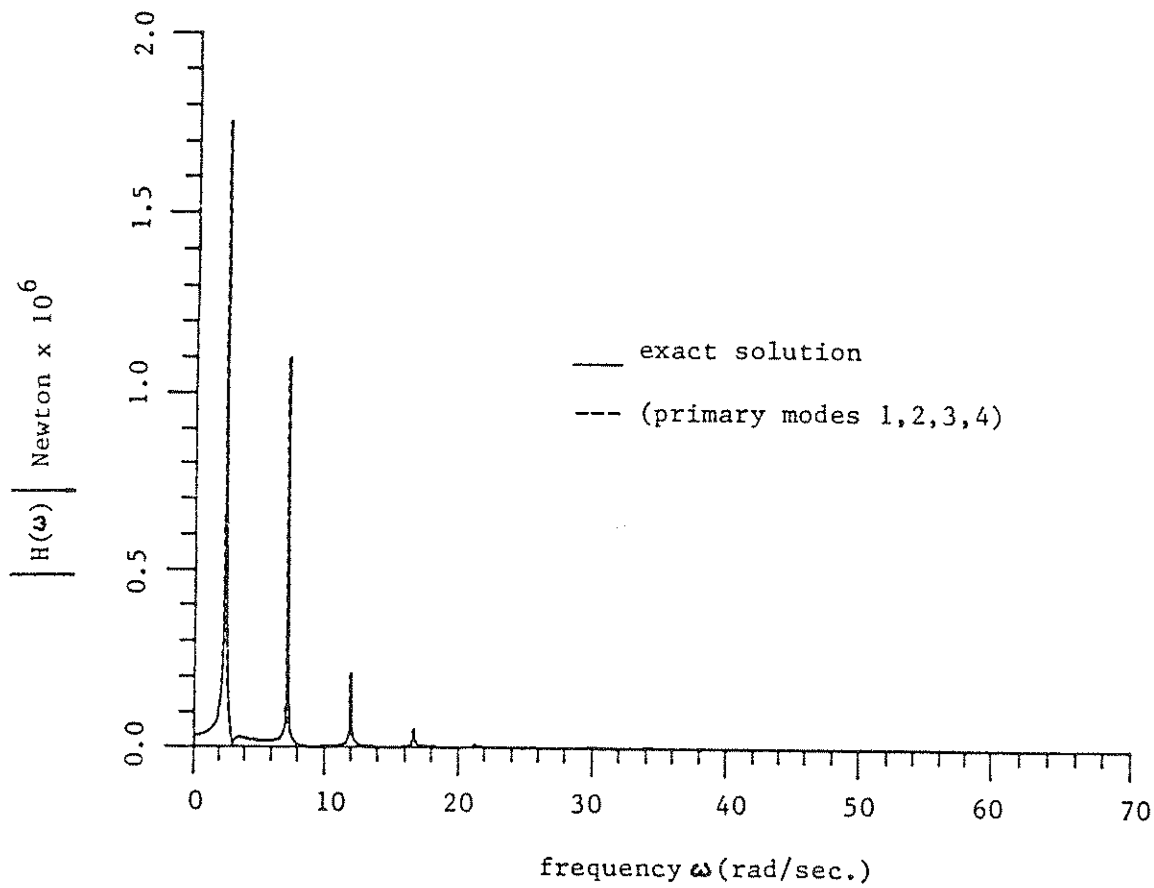



FIGURE 4-11. MODULUS OF FREQUENCY RESPONSE FUNCTION FOR INTERACTIVE FORCE ACTING AT THE EQUIPMENT-BUILDING INTERFACE: COMPONENT-MODE APPROXIMATION VS. EXACT SOLUTION. EQUIPMENT ON 4-TH FLOOR "DETUNED" MID-WAY BETWEEN 3RD AND 4TH MODES OF BUILDING. (RIGID SOIL AND VISCOELASTIC CUSHION WITH PARAMETER $a=270 \times 10^{-3}$ m, TO EFFECT A FIRST MODE TUNING).

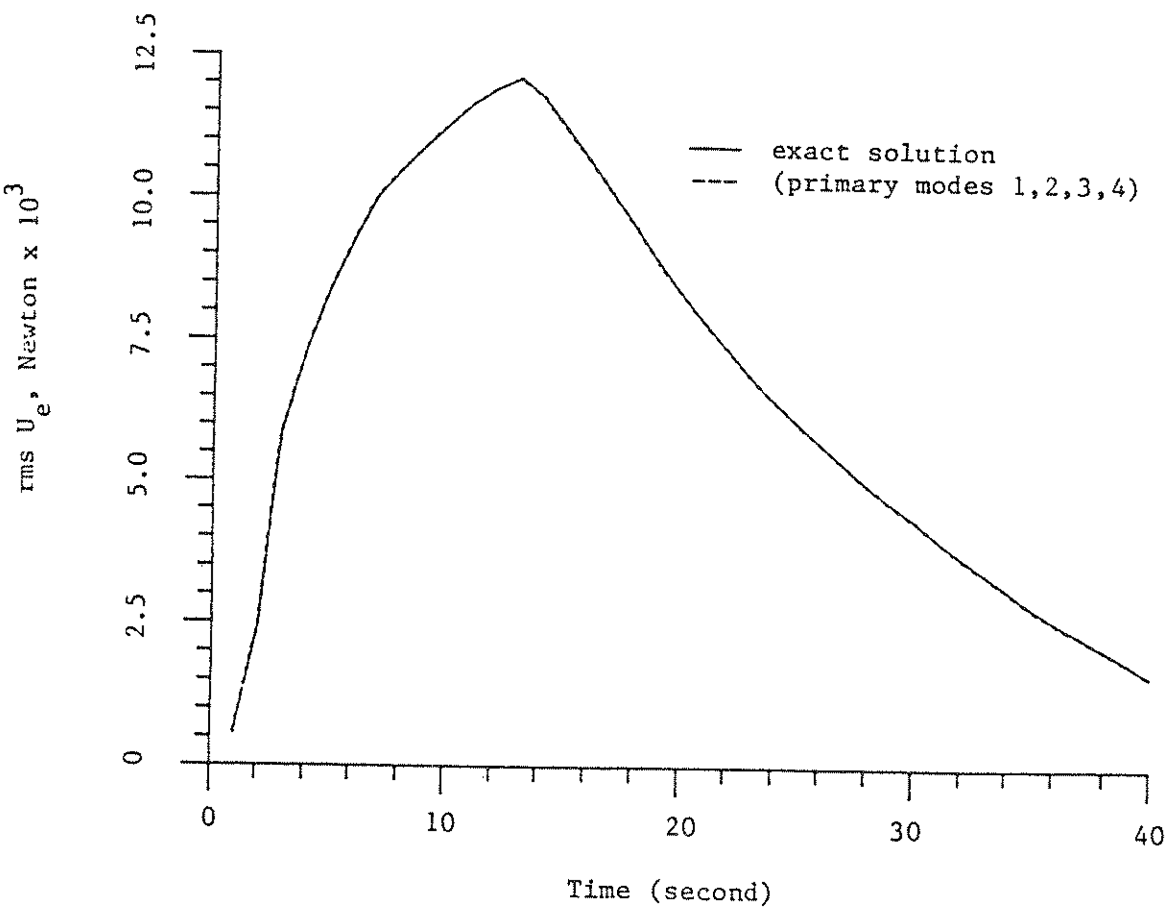



FIGURE 4-12. ROOT-MEAN-SQUARE SHEAR FORCE ACTING AT THE EQUIPMENT-BUILDING INTERFACE: COMPONENT-MODE APPROXIMATION VS. EXACT SOLUTION. EQUIPMENT ON 4-TH FLOOR "DETUNED" MID-WAY BETWEEN 3RD AND 4TH MODES OF BUILDING. (RIGID SOIL AND VISCOELASTIC CUSHION WITH PARAMETER a=270 x $10^{-3}$ m, TO EFFECT A FIRST MODE TUNING).

# SECTION 5
# CONCLUDING REMARKS

The effects of cushioning devices and soil compliancy on the response of secondary equipment has been illustrated. Further, it has been shown that while both factors are significant, the effect of cushioning devices is much greater. In particular, a cushioning device can reduce the equipment response by an order of magnitude, even when it shifts an equipment from a detuned condition to a tuned condition.

When comparing two pieces of equipments, one with a low frequency not tuned to any of the primary-system frequencies and another with a high frequency but tuned to a high primary mode, the response of the former can be higher than that of the latter. Thus, the response of a detuned equipment can be larger than that of a tuned equipment.

A comparison of the accuracy of the two approximate methods previously developed was performed, which showed that a greater number of modes in the approximate procedures is required to accurately evaluate the soil compliancy effects.

# SECTION 6
# REFERENCES


1. Asfura, A., and Der Kiureghian, A., "Floor Response Spectrum Method For Seismic Analysis of Multiply Supported Secondary Systems," Earthquake Engineering and Structural Dynamics, Vol. 14, 1986, pp. 245-265.

2. Bagley, R. L., and Torvik, P. J., "Fractional Calculus in the Transient Analysis of Viscoelastically Damped Structures" AIAA Journal, Vol. 23, No. 6, June 1985, pp. 918-925.

3. Bielak, J., "Earthquake Response of Building-Foundation Systems," Ph.D. Thesis, California Institute of Technology, Pasadena, California, 1971.

4. Burdisso, R. A., and Singh, M. P., "Seismic Response Analysis of Multiply Connected Secondary Systems," Report No. VPI-E-86-10, Virginia Polytechnic Institute and State University, Blacksburgh,VA 24061, April 1986.

5. Flugge, W., Viscoelasticity, Blaisdell Publishing Company, 1976.

6. Gladwell, G. M. L., "Forced Tangential and Rotatory Vibrations of a Rigid Circular Disc on a Semi-infinite Solid," International Journal of Engineering Science, No. 6, 1968, pp. 591-607.

7. HoLung, J. A., Cai, J., and Lin, Y. K. "Frequency Response of Secondary Systems Under Seismic Excitation," Technical Report NCEER-87-0013, National Center For Earthquake Engineering Research, State University of New York at Buffalo, NY 14261, July, 1987.

8. HoLung, J. A. "Response of Secondary Systems to Seismic Excitation," Dissertation submitted to the Department of Mechanical Engineering at Florida Atlantic University, Boca Raton, Florida, April, 1988, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

9. Iguchi, M., and Luco, J. E. "Dynamic Response of Flexible Rectangular Foundations on an Elastic Half-Space," Earthquake Engineering Structural Dynamics, Vol. 9, 1981, pp. 239-249.

10. Igusa, T., and Der Kiureghian, A., "Dynamic Response of Multiply Supported Secondary Systems," Journal of Engineering Mechanics, ASCE, Vol. 111, No. 1, January, 1985, pp. 20-41.

11. Igusa, T., and Der Kiureghian, A., "Generation of Floor Response Spectra Including Oscillator-Structure Interaction," Earthquake Engineering and Structural Dynamics, Vol. 13, 1985, pp. 661-676.

12. Igusa, T., Der Kiureghian, A., and Sackman, J. L., "Modal Decomposition Method for Stationary Response of Non-Classically Damped Systems," Earthquake Engineering and Structural Dynamics, Vol. 12, 1984, pp. 121-136.

13. Lin, Y. K., Probabilistic Theory of Structural Dynamics, McGraw-Hill, New York, 1967, Reprinted by Kreiger Publishing Co., Malibar, Fl, 1976.

14. Lin, Y. K., Random Vibration of Periodic and Almost Periodic Structures," Mechanics Today, ed. S. Nemat-nasser, Vol. 3, Pergamon, 1976, pp. 93-124.

15. Lin, Y. K., and Yong, Y., "Evolutionary Kanai-Tajimi Earthquake Models," Journal of Engineering Mechanics, ASCE, Vol. 113, No. 8, August, 1987, pp. 1119-1137.

16. Lin, Y. K., and Wu, W. F., "A Closed Form Earthquake Response Analysis of Multistory Building on Compliant Soil," Journal of Structural Mechanics, Vol. 12, No. 1, 1984, pp. 87-110.

17. Luco, J. E., and Westmann, R. A., "Dynamic Response of Circular Footings," Journal of Engineering Mechanics Division, ASCE, Vol. 97, No. EM5, October, 1971, pp. 1381-1395.

18. Sackman, J. L., and Kelly, J. M., "Seismic Analysis of Internal Equipment and Components in Structures," Engineering Structures, Vol. 1, No. 4, July 1979, pp. 179-190.

19. Nashif, A. D., Jones, D. I. G., and Henderson, J. P., "Vibration Damping," John Wiley & Sons, 1985.

20. Priestley, M. B., "Evolutionary Spectra and Nonstationary Processes," Journal of the Royal Statistical Society, B27, pp 204-228, 1965.

21. Singh, A. K., and Ang, A. H-S., "A Stochastic Model for Predicting Seismic Response of Light Secondary Systems," 5th World Conference on Earthquake Engineering, 1973.

22. Suarez, L. E., and Singh, M. P., "Seismic Response of SDF Equipment-Structure System," Journal of Engineering Mechanics, Vol. 113, No. 1, January, 1987.

23. Suarez, L. E., and Singh, M. P., "A Modal Coupling Procedure for Structural Dynamics," Dynamics of Structures, Proceedings of the Sixth Structures Congress of ASCE, August, 1987, pp. 1-14.

24. Suarez, L. E., and Singh, M. P., "Perturbed Complex Eigenproperties of Classically Damped Primary Structure And Equipment Systems," Journal of Sound and Vibration, Vol. 116, No. 2, 1987, pp. 199-219.

25. Veletsos, A. S., and Meek, J. W., "Dynamic Behavior of Building-Foundation Systems," Earthquake Engineering Structural Dynamics, Vol. 3, 1974, pp. 121-138.

26. Veletsos, A. S., and Verbic, B., "Vibration of Viscoelastic Foundations," Earthquake Engineering Structural Dynamics, Vol. 2, 1973, pp. 87-102.

27. Veletsos, A. S., and Wei, Y. T., "Lateral and Rocking Vibration of Footings," Journal of Soil Mechanics and Foundation Division, ASCE, Vol. 97, 1971, pp. 1227-1248.

28. Wong, H. L., and Luco, J. E., "Dynamic Response of Rigid Foundations of Arbitrary Shape," Earthquake Engineering Structural Dynamics, Vol. 4, 1976, pp. 579-587.

29. Wu, W. F., "Some Closure Problems in Stochastic Dynamics of Solids," Ph.D. Thesis, University of Illinois at Urbana-Champaign, 1985, reproduced in Report CAS-85-4, at the Center For Applied Stochastics Research, September 1985.

30. Yaglom, A. M., An Introduction to the Theory of Stationary Random Functions (English Translation by R. A. Silverman), Prentice-Hall, Englewood Cliffs, New Jersey, 1962.

## SECTION 7

## APPENDICES

### APPENDIX A: EXACT RESULTS FOR BUILDINGS WITH IDENTICAL STORY UNITS, CONSIDERING SOIL COMPLIANCY AND PROTECTIVE CUSHION FOR THE EQUIPMENT

Referring to figure 2-1, the motion of the j-th story unit of the building model, is governed by:

$$M_j(\ddot{X}_j+\ddot{G}+\ddot{X}_a+h_j\ddot{\varphi}) = U_j^+ - U_j^- + U_e\,\delta_{js} \tag{A-1}$$

$$U_j^- = U_{j-1}^+ = K_j\,(X_j - X_{j-1}) + C_j\,(\dot{X}_j-\dot{X}_{j-1}) \tag{A-2}$$

$$j = 1,2,\ldots,N$$

where $U^+$ = shear force from above, and $U^-$ = shear force from below. Taking the Fourier transform of the above equations and rearranging terms, the following frequency domain equations is obtained in matrix form:

$$\begin{bmatrix}\bar{X}_j\\ \bar{U}_j^+\end{bmatrix} = \begin{bmatrix}1 & \dfrac{1}{K_j + i\,\omega\,C_j}\\ -M_j\omega^2 & 1-\dfrac{M_j\,\omega^2}{K_j + i\,\omega\,C_j}\end{bmatrix}\begin{bmatrix}\bar{X}_{j-1}\\ \bar{U}_{j-1}^+\end{bmatrix} - M_j\omega^2\begin{bmatrix}0\\ \bar{G}+\bar{X}_a\end{bmatrix}$$

$$-M_j\omega^2\begin{bmatrix}0\\ h_j\,\bar{\varphi}\end{bmatrix} - \delta_{js}\begin{bmatrix}0\\ \bar{U}_e\end{bmatrix} \quad ; \qquad j = 1,2,\ldots,N \tag{A-3}$$

More concisely, equation (A-3) may be written as

$$Z_j = T_j \, Z_{j-1} - M_j \omega^2 \, G_o - M_j \omega^2 \, H_j - \delta_{js} F; \quad j = 1,2,...N \tag{A-4}$$

where $T_j$ is known as a transfer matrix. It represents the mechanism that transfers the state at station j-1 to station j, in the absence of G(t), $U_e$, $X_a$, and $\varphi$. The boundary conditions are $\bar{X}_0 = 0$ at the footing, and $\bar{U}_N = 0$ at the top floor, j=N. Similarly, for the equipment we have

$$Z_e = T_e \, Z_{e-1} - M_e \, \omega^2 \, G_0 - M_e \, \omega^2 \, H_e \tag{A-5}$$

where $Z_e = [\ \bar{Y}_e,\ 0\ ]^T$, and

$$T_e = \begin{bmatrix} 1 & \dfrac{1}{K_{eq}(\omega)} \\ -M_e \omega^2 & 1 - \dfrac{M_e \, \omega^2}{K_{eq}(\omega)} \end{bmatrix} \tag{A-6}$$

The shear force in the equipment obtained from the second row of equation (A-5), can be simplified to the form

$$\bar{U}_e = - \left(\frac{\tau_{21}}{\tau_{22}}\right)_e \left( \bar{X}_s + \bar{G} + \bar{X}_a + h_e \, \bar{\varphi} \right) \tag{A-7}$$

where $\tau_{ij}$ denotes the (i,j) element of a transfer matrix, and in the following discussion $\tau_{ij}(n)$ represents the (i,j) element of $T^n$.

**BUILDING WITH IDENTICAL STORY UNITS**

The analysis is simplified considerably when all the story units are identical. For this case, $h_j$ = jh. The subscript for the transfer matrices for different stories of the primary system can be discarded, since they are identical. The following simplified expressions can be obtained:

$$Z_j = T^j Z_o - M\omega^2 \sum_{r=1}^{j} T^{j-r} G_o - M\omega^2 \sum_{r=1}^{j} r\, T^{j-r} H \qquad j < s \qquad \text{(A-8)}$$

$$Z_s = T^s Z_o - M\omega^2 \sum_{r=1}^{s} T^{s-r} G_o - M\omega^2 \sum_{r=1}^{s} r\, T^{s-r} H - F \qquad j = s \qquad \text{(A-9)}$$

$$Z_j = T^{j-s} Z_s - M\omega^2 \sum_{r=1}^{j-s} T^{j-s-r} G_o - M\omega^2 \sum_{r=1}^{j-s} T^{j-s-r} (s+r) H \qquad j > s \qquad \text{(A-10)}$$

$$Z_N = T^{N-s} Z_s - M\omega^2 \sum_{r=1}^{N-s} T^{N-s-r} G_o - M\omega^2 \sum_{r=1}^{N-s} T^{N-s-r} (s+r) H \qquad j = N \qquad \text{(A-11)}$$

From the first row of equation (A-9):

$$\bar{X}_s = \tau_{12}(s)\, \bar{U}_o^{+} - M\omega^2 \sum_{r=1}^{s} \tau_{12}(s-r)\, (\bar{G} + \bar{X}_a)$$

$$- M\omega^2 \sum_{r=1}^{s} r\, \tau_{12}(s-r)\, ( h\, \bar{\varphi} ) \qquad \text{(A-12)}$$

By substituting equation (A-9) into equation (A-11), and making use of equations (A-7) and (A-12), we obtain from the second row:

$$\bar{U}_{\circ}^{+} = \left(\frac{1}{Q}\right)\left[M\,\omega^2\,(\bar{G} + \bar{X}_a)\,P_1 + M\,\omega^2\,(\bar{h}\,\varphi)_2\,P_3 + P_e\,(\bar{h}\,\varphi)\right]$$

(A-13a)

where

$$P_1 = \sum_{r=1}^{N} \tau_{22}\,(N-r)\left\{1 - \frac{M_e\,\omega^2}{K_{eq}(\omega)}\right\}$$

$$- \tau_{22}\,(N-s) \sum_{r=1}^{s} \tau_{12}\,(s-r)\; M_e\,\omega^2 + \tau_{22}\,(N-s)\left(\frac{M_e}{M}\right) \qquad \text{(A-13b)}$$

$$P_2 = \sum_{r=1}^{N} r\,\tau_{22}\,(N-r)\left\{1 - \frac{M_e\,\omega^2}{K_{eq}(\omega)}\right\}$$

$$- M_e\,\omega^2\,\tau_{22}\,(N-s) \sum_{r=1}^{s} r\,\tau_{12}\,(s-r) \qquad \text{(A-13c)}$$

$$P_3 = M_e \omega^2 \tau_{22}(N-s) \tag{A-13d}$$

and

$$Q = \tau_{22}(N) \left\{ 1 - \frac{M_e \omega^2}{K_{eq}(\omega)} \right\} - M_e \omega^2 \tau_{12}(s) \tau_{22}(N-s) \tag{A-13e}$$

Using equations (B-2) through (B-7) in Appendix B, it can be shown that equation (A-13) can be written in closed form, resulting in

$$\bar{U}_o^+ = \frac{2(K+i\omega C)\sin\alpha \sin\tfrac{1}{2}\alpha}{\cos(N+\tfrac{1}{2})\alpha} \left(\frac{1}{J_2}\right) \left\{ \frac{J_1 \sin N\alpha}{\sin\alpha} (\bar{G} + \bar{X}_a) + J_3 (h\bar{\varphi}) + \left(\frac{M_e}{M}\right) \frac{\cos(N-s+\tfrac{1}{2})\alpha}{\cos\tfrac{1}{2}\alpha} (h_e \bar{\varphi}) \right\} \tag{A-14}$$

where

$$J_1 = 1 - \frac{M_e \omega^2}{K_{eq}(\omega)} - \left(\frac{2M_e}{M}\right) \frac{\sin(s\alpha/2)\sin\tfrac{1}{2}(s-1)\alpha \sin\alpha \cos(N-s+\tfrac{1}{2})\alpha}{\cos^2\alpha/2 \sin(N\alpha)} + \left(\frac{M_e}{M}\right) \frac{\cos(N-s+\tfrac{1}{2})\alpha \sin\alpha}{\cos(\alpha/2)\sin(N\alpha)} \tag{A-15}$$

$$J_2 = 1 - \frac{M_e \omega^2}{K_{eq}(\omega)} - \left(\frac{4\, M_e}{M}\right) \frac{\sin^2(\alpha/2)\ \sin\ (s\alpha)\ \cos(N-s+\frac{1}{2})\alpha}{\cos\ (N+\frac{1}{2})\alpha\ \sin\ \alpha} \tag{A-16}$$

$$J_3 = \frac{\sin\ \frac{1}{2}(N+1)\alpha\ \sin\ \frac{1}{2}\ N\alpha}{\sin\ \frac{1}{2}\ \alpha\ \sin\ \alpha} \left\{1 - \frac{M_e\ \omega^2}{K_{eq}(\omega)}\right\} - \left(\frac{M_e}{M}\right)\left(\frac{\cos\ (N-s+\frac{1}{2})\alpha}{\cos\ \frac{1}{2}\ \alpha}\right)\left(\frac{s\ \sin\ \alpha\ -\ \sin\ s\alpha}{\sin\ \alpha}\right) \tag{A-17}$$

Similarly, with the knowledge of $\overline{U}_o^+$, the response of the building and the equipment can also be obtained in closed form by making use of the formulas in Appendix B. The resulting expressions for the building are:

$$\bar{X}_j = (\bar{G} + \bar{X}_a)\left\{\beta_1(j) - 1 - \frac{\sin(j-s)\alpha\ \beta_1(s)\ U(j-s)}{(K + i\omega C)\sin\alpha}\ \frac{M_e\ \omega^2}{1 - \dfrac{M_e\ \omega^2}{K_{eq}(\omega)}}\right\}$$

$$+ (h\bar{\varphi})\left\{\beta_2(j) + \beta_3(j) - \frac{\sin(j-s)\alpha\ [\beta_2(s)+\beta_3(s)+(h_e/h)]U(j-s)}{(K + i\omega C)\sin\alpha}\ \frac{M_e\ \omega^2}{1 - \dfrac{M_e\ \omega^2}{K_{eq}(\omega)}}\right\}$$

$$j = 1,\ldots,N \qquad \text{(A-18)}$$

$$\bar{U}_j^{+} = (\bar{G} + \bar{X}_a)\left\{2(K + i\omega C)\mu(j) - \frac{\cos(j-s+\frac{1}{2})\alpha\ \beta_1(s)\ U(j-s)}{\cos\frac{1}{2}\alpha}\ \frac{M_e\ \omega^2}{1 - \dfrac{M_e\ \omega^2}{K_{eq}(\omega)}}\right\}$$

$$+ (h\bar{\varphi})\left\{2(K + i\omega C)\left[\frac{2\cos(j+\frac{1}{2})\alpha\ \sin^2\frac{1}{2}\alpha}{\cos(N+\frac{1}{2})\alpha}\ \frac{J_3}{J_2} - \frac{\sin\frac{1}{2}(j+1)\alpha\ \sin\frac{1}{2}(j\alpha)}{\cos\frac{1}{2}\alpha} + \frac{\beta_3(j)\ \sin\frac{1}{2}\alpha\ \cos(j+\frac{1}{2})\alpha}{\sin j\alpha}\right]\right.$$

$$\left. - \frac{\cos(j-s+\frac{1}{2})\alpha[\beta_2(s)+\beta_3(s)+(h_e/h)]U(j-s)}{\cos\frac{1}{2}\alpha}\ \frac{M_e\ \omega^2}{1 - \dfrac{M_e\ \omega^2}{K_{eq}(\omega)}}\right\}$$

$$j = 1,\ldots,N \qquad \text{(A-19)}$$

where

$$\beta_1(j) = \frac{\sin(j\alpha)\ \sin(N\alpha)\ (J_1/J_2)}{\cos(N+\frac{1}{2})\alpha\ \ \cos\alpha/2} + \frac{\cos(j-\frac{1}{2})\alpha}{\cos\alpha/2} \qquad \text{(A-20)}$$

$$\beta_2(j) = \frac{\sin(j\alpha)\ \sin\alpha\ \ (J_3/J_2)}{\cos(N+\frac{1}{2})\alpha\ \ \cos\alpha/2} + \frac{\sin j\alpha}{\sin\alpha} - j \qquad \text{(A-21)}$$

$$\beta_3(j) = \left(\frac{M_e}{M}\right)\left(\frac{h_e}{h}\right)\left(\frac{1}{J_2}\right)\ \frac{\sin j\alpha\ \sin\alpha\ \cos(N-s+\frac{1}{2})\alpha}{\cos(N+\frac{1}{2})\alpha\ \ \cos^2\alpha/2} \qquad \text{(A-22)}$$

$$\mu(j) = \frac{2\cos(j+\frac{1}{2})\alpha\ \sin^2(\alpha/2)\ \sin(N\alpha)\ (J_1/J_2)}{\cos(N+\frac{1}{2})\alpha\ \sin\alpha} - \frac{\sin(\alpha/2)\ \sin(j\alpha)}{\cos(\alpha/2)} \qquad \text{(A-23)}$$

and

$$U(j-s) = 1 \quad \text{if } j \geq s; \qquad = 0 \quad \text{if } j < s \qquad \text{(A-24)}$$

Similarly, the expressions for the equipment are

$$\bar{Y}_e = \left\{ \frac{\beta_1(s)}{1 - \frac{M_e\,\omega^2}{K_{eq}(\omega)}} - 1 \right\} (\bar{G} + \bar{X}_a)$$

$$+ \frac{h\,\bar{\varphi}}{1 - \frac{M_e\,\omega^2}{K_{eq}(\omega)}} \left\{ \beta_2(s) + \beta_3(s) + \frac{(h_e/h) M_e\,\omega^2}{K_{eq}(\omega)} \right\} \qquad \text{(A-25)}$$

$$\bar{U}_e = \left\{ \frac{M_e\,\omega^2}{1 - \frac{M_e\,\omega^2}{K_{eq}(\omega)}} \right\} \left\{ \beta 1(s)\ (\bar{G} + \bar{X}_a) \right.$$

$$\left. + (\beta_2(s) + \beta_3(s) + (h_e/h))\ (h\,\bar{\varphi}) \right\} \qquad \text{(A-26)}$$

By setting $\bar{X}_a$ and $\bar{\varphi} = 0$ in equations (A-18), (A-19), (A-25), and (A-26), and taking $K_c$ infinitely large, the simpler expressions obtained by Holung, Cai, and Lin [7] for a rigid soil are recovered. The translation of the footing is governed by

$$M_0\ (\ddot{G} + \ddot{X}_a) = U_0^{+} - U_0^{-} \qquad \text{(A-27)}$$

where $M_0$ = mass of the footing and $U_0^-$ = reactive horizontal shear force from the soil. The rotation of the building and equipment as a whole is governed by

$$\sum_{j=1}^{N} M\, j\, h \left( \ddot{G} + \ddot{X}_a + \ddot{X}_j + j\, h\, \ddot{\varphi} \right)$$

$$+ M_e\, h_e \left( \ddot{G} + \ddot{X}_a + \ddot{Y}_e + h_e\, \ddot{\varphi} \right) + \left( I_t + I_e \right) \ddot{\varphi} + C_0^{-} = 0 \tag{A-28}$$

in which $I_t = \Sigma\, I_j$ ( $j=0,\ldots N$). The frequency domain versions of equations (A-27) and (A-28) are:

$$- M_0\, \omega^2 \left( \bar{G} + \bar{X}_a \right) = \bar{U}_0^{+} - \bar{U}_0^{-} \tag{A-29}$$

$$-M\, h\, \omega^2 \sum_{j=1}^{N} j\, \bar{X}_j - \omega^2 \left( \bar{G} + \bar{X}_a \right) \left\{ M\, h\, \frac{N\,(N+1)}{2} + m_e\, h_e \right\} - \omega^2\, \bar{\varphi} \left\{ M\, h^2\, \frac{N\,(N+1)\,(2N+1)}{6} + M_e\, h_e^2 + I_t + I_e \right\} - M_e\, h_e\, \omega^2\, \bar{Y}_e + \bar{C}_0^{-} = 0 \tag{A-30}$$

Using equations (A-18), (A-20) through (A-22), and (B-8) through (B-11) in Appendix B, the summation term in equation (A-30) can be carried out, resulting in

$$\sum_{j=1}^{N} j\,\bar{X}_j = (\bar{G}+\bar{X}_a)\left\{\frac{\Omega_1 \sin N\alpha}{\cos(N+\frac{1}{2})\alpha \cos\frac{1}{2}\alpha}\;\frac{J_1}{J_2} + \frac{\Omega_3}{\cos\frac{1}{2}\alpha}\right.$$

$$\left. - \frac{N(N+1)}{2} - \frac{\Omega_2\,\beta_1(s)}{(K+i\omega C)\sin\alpha}\;\frac{M_e\,\omega^2}{1-\dfrac{M_e\,\omega^2}{K_{eq}(\omega)}}\right\}$$

$$+ (h\,\bar{\varphi})\left\{\frac{\Omega_1 \sin\alpha}{\cos(N+\frac{1}{2})\alpha\cos\frac{1}{2}\alpha}\;\frac{J_3}{J_2} - \frac{N(N+1)(2N+1)}{6}\right.$$

$$+ \frac{M_e}{M}\;\frac{h_e}{h}\;\frac{\Omega_1}{J_2}\;\frac{\sin\alpha\cos(N-s+\frac{1}{2})\alpha}{\cos(N+\frac{1}{2})\alpha\cos^2\frac{1}{2}\alpha} + \frac{\Omega_1}{\sin\alpha}$$

$$\left. - \frac{\Omega_2\,[\beta_2(s)+\beta_3(s)+(h_e/h)]}{(K+i\omega C)\sin\alpha}\;\frac{M_e\,\omega^2}{1-\dfrac{M_e\,\omega^2}{K_{eq}(\omega)}}\right\} \qquad \text{(A-31)}$$

By combining equations (A-14), (A-25), (A-29), (A-30), (A-31), and (2-15), two equations involving only two unknowns can be obtained. Following Lin and Wu [16], $(G + X_a)$ and $(h\,\varphi)$ are chosen as unknowns, and the equations are cast in matrix form as follows:

$$\begin{bmatrix} A_{11}-B_{11} & A_{12}-B_{12} \\ A_{21}-B_{21} & A_{22}-B_{22} \end{bmatrix} \begin{bmatrix} \bar{G}+\bar{X}_a \\ h\,\bar{\varphi} \end{bmatrix} = -\begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix} \bar{G} \qquad \text{(A-32)}$$

where

$$A_{11} = \frac{2\sin N\alpha \sin\frac{1}{2}\alpha}{\cos(N+\frac{1}{2})\alpha}\left(\frac{J_1}{J_2}\right) + \frac{M_0\,\omega^2}{K+i\omega C} \qquad \text{(A-33)}$$

$$A_{12} = \frac{2 \sin \frac{1}{2} \alpha}{J_2 \cos (N+\frac{1}{2}) \alpha} \left\{ J_3 \sin \alpha \right.$$

$$\left. + \left(\frac{2 M_e}{M}\right)\left(\frac{h_e}{h}\right) \sin \tfrac{1}{2} \alpha \cos (N-s+\tfrac{1}{2})\alpha \right\} \tag{A-34}$$

$$A_{21} = \left(\frac{J_1}{J_2}\right) \frac{\Omega_1 \sin N\alpha}{\cos (N+\frac{1}{2})\alpha \cos \frac{1}{2} \alpha} + \frac{\Omega_3}{\cos \frac{1}{2} \alpha}$$

$$- \left\{ \frac{M_e \ \omega^2}{1 - \dfrac{M_e \ \omega^2}{K_{eq}(\omega)}} \right\} \frac{\Omega_2 \ \beta_1(s)}{(K + i\omega C) \sin \alpha}$$

$$+ \left(\frac{M_e}{M}\right)\left(\frac{h_e}{h}\right) + \left\{ \frac{\beta_1(s)}{1 - \dfrac{M_e \ \omega^2}{K_{eq}(\omega)}} - 1 \right\} \left(\frac{M_e}{M}\right)\left(\frac{h_e}{h}\right) \tag{A-35}$$

$$A_{22} = \left(\frac{\sin\alpha}{\cos\,(N+\tfrac{1}{2})\alpha\;\cos\tfrac{1}{2}\alpha}\right)\left(\frac{J_3\;\Omega_1}{J_2}\right) + \frac{\Omega_1}{\sin\alpha}$$

$$+\left(\frac{M_e}{M}\right)\left(\frac{h_e}{h}\right)\left(\frac{\Omega_1}{J_2}\right)\frac{\sin\alpha\;\cos\,(N-s+\tfrac{1}{2})\alpha}{\cos\,(N+\tfrac{1}{2})\alpha\;\cos^2\alpha/2}$$

$$-\left\{\frac{M_e\,\omega^2}{1-\dfrac{M_e\,\omega^2}{K_{eq}(\omega)}}\right\}\frac{\Omega_2\,[\beta_2(s)+\beta_3(s)+(h_e/h)]}{(K+i\omega C)\sin\alpha}$$

$$+\left(\frac{M_e}{M}\right)\left(\frac{h_e}{h}\right)\left\{\frac{1}{1-\dfrac{M_e\,\omega^2}{K_{eq}(\omega)}}\right\}\left\{\beta_2(s)+\beta_3(s)+\frac{M_e\,\omega^2\,(h_e/h)}{K_{eq}(\omega)}\right\}$$

$$+\left(\frac{I_t+I_e}{M\,h^2}\right)+\left(\frac{M_e}{M}\right)\left(\frac{h_e}{h}\right)^2 \qquad \text{(A-36)}$$

$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{K_{HH}}{K+i\omega C} & \dfrac{K_{HM}}{h\,(K+i\omega C)} \\ \dfrac{K_{MH}}{M\,h\,\omega^2} & \dfrac{K_{MM}}{M\,h^2\,\omega^2} \end{bmatrix} \qquad \text{(A-37)}$$

The solution of equation (A-32) is given by

$$(\,\bar{G} + X_a\,) = \Delta^{-1}\;\bar{G}\;\left\{B_{11}\,(\,B_{22}-A_{22}) + B_{21}(\,A_{12}-B_{12})\right\}$$

(A-38)

$$h\,\bar{\varphi} = \Delta^{-1}\,\bar{G}\left\{B_{11}\,(A_{21} - B_{21}) + B_{21}(B_{11} - A_{11})\right\} \qquad \text{(A-39)}$$

where $\Delta$= determinant of [A]-[B]. Substituting equations (A-38) and (A-39) into equations (A-18) and (A-19) the following expressions for the building are obtained:

$$\bar{X}_j = \Delta^{-1}\,\bar{G}\Bigg\{\beta_1(j) - 1$$

$$- \frac{\beta_1(s)\ \sin\ (j-s)\alpha\ U(j-s)}{(K + i\omega C)\ \sin\ \alpha}\ \frac{M_e\,\omega^2}{1 - \dfrac{M_e\,\omega^2}{K_{eq}(\omega)}}\Bigg]\Bigg[B_{11}(B_{22}$$

$$- A_{22}) + B_{21}\,(A_{12} - B_{12})\Bigg] + \Bigg[\beta_2(j) + \beta_3(j)$$

$$- \frac{\sin(j-s)\alpha[\beta_2(s)+\beta_3(s)+(h_e/h)]U(j-s)}{(K + i\omega C)\ \sin\ \alpha}\ \frac{M_e\,\omega^2}{1- \dfrac{M_e\,\omega^2}{K_{eq}(\omega)}}\Bigg]\Bigg[B_{11}(A_{21}$$

$$- B_{21}) + B_{21}\,(B_{11} - A_{11})\Bigg]\Bigg\} \qquad \text{(A-40)}$$

$$\bar{U}_j^{+} = \Delta^{-1}\,\bar{G}\left\{\left[2\,(K + i\,\omega C)\,\mu(j)\right.\right.$$

$$\left. - \frac{M_e\,\omega^2}{1 - \frac{M_e\,\omega^2}{K_{eq}(\omega)}}\;\frac{\cos\,(j-s+\frac{1}{2})\alpha\;U(j-s)\;\beta_1(s)}{\cos\,\frac{1}{2}\,\alpha}\right]\left[B_{11}(\,B_{22}\right.$$

$$\left. - A_{22}\,) + B_{21}(\,A_{12} - B_{12})\right]$$

$$+ \left[\,2\,(\,K + i\,\omega\,C\,)\left(\frac{J_3}{J_2}\;\frac{\cos\,(j+\frac{1}{2})\alpha\,\sin^2\,\frac{1}{2}\,\alpha}{\cos\,(N+\frac{1}{2})\alpha}\right.\right.$$

$$\left. - \frac{\sin\,\frac{1}{2}\,(j+1)\alpha\;\sin\,\frac{1}{2}(j\alpha)}{\cos\,\frac{1}{2}\,\alpha} + \frac{\beta_3(j)\;\sin\,\frac{1}{2}\,\alpha\;\cos\,(j+\frac{1}{2})\alpha}{\sin\,j\alpha}\right)$$

$$\left. - \frac{M_e\,\omega^2}{1 - \frac{M_e\,\omega^2}{K_{eq}(\omega)}}\;\frac{[\beta_2(s)+\beta_3(s)+(h_e/h)]U(j-s)\;\cos\,(j-s+\frac{1}{2})\alpha}{\cos\,\frac{1}{2}\,\alpha}\right] \times$$

$$\left.\left[B_{11}(\,A_{21} - B_{21}) + B_{21}(\,B_{11} - A_{11})\right]\right\} \qquad \text{(A-41)}$$

Similarly, by substituting equations (A-38) and (A-39) into equations (A-25) and (A-26) we obtain the following expressions for the equipment:

$$\bar{Y}_e = \Delta^{-1}\bar{G}\left\{\left[\frac{\beta_1(s)}{1 - \frac{M_e\,\omega^2}{K_{eq}(\omega)}} - 1\right]\left[B_{11}(B_{22} - A_{22}) + B_{21}(A_{12} - B_{12})\right] + \frac{1}{1 - \frac{M_e\,\omega^2}{K_{eq}(\omega)}}\left[\beta_2(s) + \beta_3(s) + \frac{M_e\,\omega^2}{K_{eq}(\omega)}\,\frac{h}{h_e}\right] x \left[B_{11}(A_{21} - B_{21}) + B_{21}(B_{11} - A_{11})\right]\right\} \quad \text{(A-42)}$$

$$\bar{U}_e = \Delta^{-1}\bar{G}\left(\frac{M_e\,\omega^2}{1 - \frac{M_e\,\omega^2}{K_{eq}(\omega)}}\right)\left\{\beta_1(s)\left[B_{11}(B_{22} - A_{22}) + B_{21}(A_{12} - B_{12})\right] + \left[\beta_2(s) + \beta_3(s) + (h_e/h)\right] * \left[B_{11}(A_{21} - B_{21}) + B_{21}(B_{11} - A_{11})\right]\right\} \quad \text{(A-43)}$$

By setting $M_e$ and $I_e = 0$ in equations (A-40) and (A-41) it can be shown that the results reduce to the simpler expressions obtained by Lin and Wu [16].

## APPENDIX B: USEFUL CLOSED FORM SOLUTIONS

Equation (A-13) can be simplified using two interesting properties common to all transfer matrices; namely, the determinant of a transfer matrix is equal to one, and its eigenvalues are reciprocal pairs. Denoting the two eigenvalues of the present 2x2 transfer matrix by exp $(\pm i\alpha)$, it can be shown that

$$\cos \alpha = 1 - \frac{M\omega^2}{2(K + i\omega C)} \qquad \text{(B-1)}$$

To simplify some of the expressions encountered in the present study, it is sometimes convenient to rewrite equation (B-1) in the following form:

$$M\omega^2 = 4\,(K + i\omega C)\sin^2 \tfrac{1}{2}\alpha \qquad \text{(B-1b)}$$

Closed form expressions for several functions of the transfer matrix defined by equation (A-3) have already been derived [16]. A summary of these expressions, as well as other required formulas that are needed in the present study are:

$$\tau_{12}(j) = \frac{\sin j\alpha}{(K + i\omega C)\sin \alpha} \qquad \text{(B-2)}$$

$$\tau_{22}(j) = \frac{\cos (j + \frac{1}{2})\alpha}{\cos \frac{1}{2}\alpha} \qquad \text{(B-3)}$$

$$\sum_{r=1}^{j} \tau_{22}(j-r) = \frac{\sin j\alpha}{\sin \alpha} \qquad \text{(B-4)}$$

$$\sum_{r=1}^{j} \tau_{12}(j-r) = \frac{\sin \frac{1}{2}(j-1)\alpha \sin \frac{1}{2} j\alpha}{(K + i\omega C) \sin \frac{1}{2} \alpha \sin \alpha} \qquad \text{(B-5)}$$

$$\sum_{r=1}^{j} r \, \tau_{22}(j-r) = \frac{\sin \frac{1}{2}(j+1)\alpha \sin \frac{1}{2} j\alpha}{\sin \frac{1}{2} \alpha \sin \alpha} \qquad \text{(B-6)}$$

$$\sum_{r=1}^{j} r \, \tau_{12}(j-r) = \frac{j \sin \alpha - \sin j\alpha}{4 (K + i\omega C) \sin \alpha \sin^2 \frac{1}{2} \alpha} \qquad \text{(B-7)}$$

$$\Omega_1 = \sum_{j=1}^{N} j \sin j\alpha = \frac{(N+1) \sin N\alpha - N \sin (N+1)\alpha}{4 \sin^2 \frac{1}{2} \alpha} \qquad \text{(B-8)}$$

$$\sum_{j=1}^{N} j \cos (N-j+\tfrac{1}{2})\alpha = \frac{\sin \frac{1}{2} (N+1)\alpha \ \sin \frac{1}{2} N\alpha}{2 \sin^2 \frac{1}{2} \alpha} \qquad \text{(B-9)}$$

$$\Omega_2 = \sum_{j=1}^{N} j \sin (j-s)\alpha \, U(j-s)$$

$$= \frac{(N-s+1) \sin (N-s)\alpha - (N-s) \sin (N-s+1)\alpha}{4 \sin^2 \frac{1}{2} \alpha}$$

$$+ \frac{s}{4 \sin^2 \frac{1}{2} \alpha} \left( \sin \alpha - \sin (N-s+1)\alpha + \sin (N-s)\alpha \right) \qquad \text{(B-10)}$$

$$\Omega_3 = \sum_{j=1}^{N} j \cos (j-\tfrac{1}{2})\alpha$$

$$= \frac{(N+1) \cos (N-\frac{1}{2})\alpha - N \cos (N+\frac{1}{2})\alpha - \cos \frac{1}{2} \alpha}{4 \sin^2 \frac{1}{2} \alpha} \qquad \text{(B-11)}$$

# NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
## LIST OF PUBLISHED TECHNICAL REPORTS

The National Center for Earthquake Engineering Research (NCEER) publishes technical reports on a variety of subjects related to earthquake engineering written by authors funded through NCEER. These reports are available from both NCEER's Publications Department and the National Technical Information Service (NTIS). Requests for reports should be directed to the Publications Department, National Center for Earthquake Engineering Research, State University of New York at Buffalo, Red Jacket Quadrangle, Buffalo, New York 14261. Reports can also be requested through NTIS, 5285 Port Royal Road, Springfield, Virginia 22161. NTIS accession numbers are shown in parenthesis, if available.

NCEER-87-0001 "First-Year Program in Research, Education and Technology Transfer," 3/5/87, (PB88-134275/AS).

NCEER-87-0002 "Experimental Evaluation of Instantaneous Optimal Algorithms for Structural Control," by R.C. Lin, T.T. Soong and A.M. Reinhorn, 4/20/87, (PB88-134341/AS).

NCEER-87-0003 "Experimentation Using the Earthquake Simulation Facilities at University at Buffalo," by A.M. Reinhorn and R.L. Ketter, to be published.

NCEER-87-0004 "The System Characteristics and Performance of a Shaking Table," by J.S. Hwang, K.C. Chang and G.C. Lee, 6/1/87, (PB88-134259/AS).

NCEER-87-0005 "A Finite Element Formulation for Nonlinear Viscoplastic Material Using a Q Model," by O. Gyebi and G. Dasgupta, 11/2/87, (PB88-213764/AS).

NCEER-87-0006 "Symbolic Manipulation Program (SMP) - Algebraic Codes for Two and Three Dimensional Finite Element Formulations," by X. Lee and G. Dasgupta, 11/9/87, (PB88-219522/AS).

NCEER-87-0007 "Instantaneous Optimal Control Laws for Tall Buildings Under Seismic Excitations," by J.N. Yang, A. Akbarpour and P. Ghaemmaghami, 6/10/87, (PB88-134333/AS).

NCEER-87-0008 "IDARC: Inelastic Damage Analysis of Reinforced Concrete Frame - Shear-Wall Structures," by Y.J. Park, A.M. Reinhorn and S.K. Kunnath, 7/20/87, (PB88-134325/AS).

NCEER-87-0009 "Liquefaction Potential for New York State: A Preliminary Report on Sites in Manhattan and Buffalo," by M. Budhu, V. Vijayakumar, R.F. Giese and L. Baumgras, 8/31/87, (PB88-163704/AS). This report is available only through NTIS (see address given above).

NCEER-87-0010 "Vertical and Torsional Vibration of Foundations in Inhomogeneous Media," by A.S. Veletsos and K.W. Dotson, 6/1/87, (PB88-134291/AS).

NCEER-87-0011 "Seismic Probabilistic Risk Assessment and Seismic Margins Studies for Nuclear Power Plants," by Howard H.M. Hwang, 6/15/87, (PB88-134267/AS). This report is available only through NTIS (see address given above).

NCEER-87-0012 "Parametric Studies of Frequency Response of Secondary Systems Under Ground-Acceleration Excitations," by Y. Yong and Y.K. Lin, 6/10/87, (PB88-134309/AS).

NCEER-87-0013 "Frequency Response of Secondary Systems Under Seismic Excitation," by J.A. HoLung, J. Cai and Y.K. Lin, 7/31/87, (PB88-134317/AS).

NCEER-87-0014 "Modelling Earthquake Ground Motions in Seismically Active Regions Using Parametric Time Series Methods," by G.W. Ellis and A.S. Cakmak, 8/25/87, (PB88-134283/AS).

NCEER-87-0015 "Detection and Assessment of Seismic Structural Damage," by E. DiPasquale and A.S. Cakmak, 8/25/87, (PB88-163712/AS).

NCEER-87-0016 "Pipeline Experiment at Parkfield, California," by J. Isenberg and E. Richardson, 9/15/87, (PB88-163720/AS).

NCEER-87-0017 "Digital Simulation of Seismic Ground Motion," by M. Shinozuka, G. Deodatis and T. Harada, 8/31/87, (PB88-155197/AS). This report is available only through NTIS (see address given above).

NCEER-87-0018 "Practical Considerations for Structural Control: System Uncertainty, System Time Delay and Truncation of Small Control Forces," J.N. Yang and A. Akbarpour, 8/10/87, (PB88-163738/AS).

NCEER-87-0019 "Modal Analysis of Nonclassically Damped Structural Systems Using Canonical Transformation," by J.N. Yang, S. Sarkani and F.X. Long, 9/27/87, (PB88-187851/AS).

NCEER-87-0020 "A Nonstationary Solution in Random Vibration Theory," by J.R. Red-Horse and P.D. Spanos, 11/3/87, (PB88-163746/AS).

NCEER-87-0021 "Horizontal Impedances for Radially Inhomogeneous Viscoelastic Soil Layers," by A.S. Veletsos and K.W. Dotson, 10/15/87, (PB88-150859/AS).

NCEER-87-0022 "Seismic Damage Assessment of Reinforced Concrete Members," by Y.S. Chung, C. Meyer and M. Shinozuka, 10/9/87, (PB88-150867/AS). This report is available only through NTIS (see address given above).

NCEER-87-0023 "Active Structural Control in Civil Engineering," by T.T. Soong, 11/11/87, (PB88-187778/AS).

NCEER-87-0024 Vertical and Torsional Impedances for Radially Inhomogeneous Viscoelastic Soil Layers," by K.W. Dotson and A.S. Veletsos, 12/87, (PB88-187786/AS).

NCEER-87-0025 "Proceedings from the Symposium on Seismic Hazards, Ground Motions, Soil-Liquefaction and Engineering Practice in Eastern North America," October 20-22, 1987, edited by K.H. Jacob, 12/87, (PB88-188115/AS).

NCEER-87-0026 "Report on the Whittier-Narrows, California, Earthquake of October 1, 1987," by J. Pantelic and A. Reinhorn, 11/87, (PB88-187752/AS). This report is available only through NTIS (see address given above).

NCEER-87-0027 "Design of a Modular Program for Transient Nonlinear Analysis of Large 3-D Building Structures," by S. Srivastav and J.F. Abel, 12/30/87, (PB88-187950/AS).

NCEER-87-0028 "Second-Year Program in Research, Education and Technology Transfer," 3/8/88, (PB88-219480/AS).

NCEER-88-0001 "Workshop on Seismic Computer Analysis and Design of Buildings With Interactive Graphics," by W. McGuire, J.F. Abel and C.H. Conley, 1/18/88, (PB88-187760/AS).

NCEER-88-0002 "Optimal Control of Nonlinear Flexible Structures," by J.N. Yang, F.X. Long and D. Wong, 1/22/88, (PB88-213772/AS).

NCEER-88-0003 "Substructuring Techniques in the Time Domain for Primary-Secondary Structural Systems," by G.D. Manolis and G. Juhn, 2/10/88, (PB88-213780/AS).

NCEER-88-0004 "Iterative Seismic Analysis of Primary-Secondary Systems," by A. Singhal, L.D. Lutes and P.D. Spanos, 2/23/88, (PB88-213798/AS).

NCEER-88-0005 "Stochastic Finite Element Expansion for Random Media," by P.D. Spanos and R. Ghanem, 3/14/88, (PB88-213806/AS).

NCEER-88-0006 "Combining Structural Optimization and Structural Control," by F.Y. Cheng and C.P. Pantelides, 1/10/88, (PB88-213814/AS).

NCEER-88-0007 "Seismic Performance Assessment of Code-Designed Structures," by H.H-M. Hwang, J-W. Jaw and H-J. Shau, 3/20/88, (PB88-219423/AS).

NCEER-88-0008 "Reliability Analysis of Code-Designed Structures Under Natural Hazards," by H.H-M. Hwang, H. Ushiba and M. Shinozuka, 2/29/88, (PB88-229471/AS).

NCEER-88-0009 "Seismic Fragility Analysis of Shear Wall Structures," by J-W Jaw and H.H-M. Hwang, 4/30/88, (PB89-102867/AS).

NCEER-88-0010 "Base Isolation of a Multi-Story Building Under a Harmonic Ground Motion - A Comparison of Performances of Various Systems," by F-G Fan, G. Ahmadi and I.G. Tadjbakhsh, 5/18/88, (PB89-122238/AS).

NCEER-88-0011 "Seismic Floor Response Spectra for a Combined System by Green's Functions," by F.M. Lavelle, L.A. Bergman and P.D. Spanos, 5/1/88, (PB89-102875/AS).

NCEER-88-0012 "A New Solution Technique for Randomly Excited Hysteretic Structures," by G.Q. Cai and Y.K. Lin, 5/16/88, (PB89-102883/AS).

NCEER-88-0013 "A Study of Radiation Damping and Soil-Structure Interaction Effects in the Centrifuge," by K. Weissman, supervised by J.H. Prevost, 5/24/88, (PB89-144703/AS).

NCEER-88-0014 "Parameter Identification and Implementation of a Kinematic Plasticity Model for Frictional Soils," by J.H. Prevost and D.V. Griffiths, to be published.

NCEER-88-0015 "Two- and Three- Dimensional Dynamic Finite Element Analyses of the Long Valley Dam," by D.V. Griffiths and J.H. Prevost, 6/17/88, (PB89-144711/AS).

NCEER-88-0016 "Damage Assessment of Reinforced Concrete Structures in Eastern United States," by A.M. Reinhorn, M.J. Seidel, S.K. Kunnath and Y.J. Park, 6/15/88, (PB89-122220/AS).

NCEER-88-0017 "Dynamic Compliance of Vertically Loaded Strip Foundations in Multilayered Viscoelastic Soils," by S. Ahmad and A.S.M. Israil, 6/17/88, (PB89-102891/AS).

NCEER-88-0018 "An Experimental Study of Seismic Structural Response With Added Viscoelastic Dampers," by R.C. Lin, Z. Liang, T.T. Soong and R.H. Zhang, 6/30/88, (PB89-122212/AS).

NCEER-88-0019 "Experimental Investigation of Primary - Secondary System Interaction," by G.D. Manolis, G. Juhn and A.M. Reinhorn, 5/27/88, (PB89-122204/AS).

NCEER-88-0020 "A Response Spectrum Approach For Analysis of Nonclassically Damped Structures," by J.N. Yang, S. Sarkani and F.X. Long, 4/22/88, (PB89-102909/AS).

NCEER-88-0021 "Seismic Interaction of Structures and Soils: Stochastic Approach," by A.S. Veletsos and A.M. Prasad, 7/21/88, (PB89-122196/AS).

NCEER-88-0022 "Identification of the Serviceability Limit State and Detection of Seismic Structural Damage," by E. DiPasquale and A.S. Cakmak, 6/15/88, (PB89-122188/AS).

NCEER-88-0023 "Multi-Hazard Risk Analysis: Case of a Simple Offshore Structure," by B.K. Bhartia and E.H. Vanmarcke, 7/21/88, (PB89-145213/AS).

NCEER-88-0024 "Automated Seismic Design of Reinforced Concrete Buildings," by Y.S. Chung, C. Meyer and M. Shinozuka, 7/5/88, (PB89-122170/AS).

NCEER-88-0025 "Experimental Study of Active Control of MDOF Structures Under Seismic Excitations," by L.L. Chung, R.C. Lin, T.T. Soong and A.M. Reinhorn, 7/10/88, (PB89-122600/AS).

NCEER-88-0026 "Earthquake Simulation Tests of a Low-Rise Metal Structure," by J.S. Hwang, K.C. Chang, G.C. Lee and R.L. Ketter, 8/1/88, (PB89-102917/AS).

NCEER-88-0027 "Systems Study of Urban Response and Reconstruction Due to Catastrophic Earthquakes," by F. Kozin and H.K. Zhou, 9/22/88, to be published.

NCEER-88-0028 "Seismic Fragility Analysis of Plane Frame Structures," by H.H-M. Hwang and Y.K. Low, 7/31/88, (PB89-131445/AS).

NCEER-88-0029 "Response Analysis of Stochastic Structures," by A. Kardara, C. Bucher and M. Shinozuka, 9/22/88.

NCEER-88-0030 "Nonnormal Accelerations Due to Yielding in a Primary Structure," by D.C.K. Chen and L.D. Lutes, 9/19/88, (PB89-131437/AS).

NCEER-88-0031 "Design Approaches for Soil-Structure Interaction," by A.S. Veletsos, A.M. Prasad and Y. Tang, 12/30/88.

NCEER-88-0032 "A Re-evaluation of Design Spectra for Seismic Damage Control," by C.J. Turkstra and A.G. Tallin, 11/7/88, (PB89-145221/AS).

NCEER-88-0033 "The Behavior and Design of Noncontact Lap Splices Subjected to Repeated Inelastic Tensile Loading," by V.E. Sagan, P. Gergely and R.N. White, 12/8/88.

NCEER-88-0034 "Seismic Response of Pile Foundations," by S.M. Mamoon, P.K. Banerjee and S. Ahmad, 11/1/88, (PB89-145239/AS).

NCEER-88-0035 "Modeling of R/C Building Structures With Flexible Floor Diaphragms (IDARC2)," by A.M. Reinhorn, S.K. Kunnath and N. Panahshahi, 9/7/88.

NCEER-88-0036 "Solution of the Dam-Reservoir Interaction Problem Using a Combination of FEM, BEM with Particular Integrals, Modal Analysis, and Substructuring," by C-S. Tsai, G.C. Lee and R.L. Ketter, 12/31/88.

NCEER-88-0037 "Optimal Placement of Actuators for Structural Control," by F.Y. Cheng and C.P. Pantelides, 8/15/88.

NCEER-88-0038 "Teflon Bearings in Aseismic Base Isolation: Experimental Studies and Mathematical Modeling," by A. Mokha, M.C. Constantinou and A.M. Reinhorn, 12/5/88.

NCEER-88-0039 "Seismic Behavior of Flat Slab High-Rise Buildings in the New York City Area," by P. Weidlinger and M. Ettouney, 10/15/88, to be published.

NCEER-88-0040 "Evaluation of the Earthquake Resistance of Existing Buildings in New York City," by P. Weidlinger and M. Ettouney, 10/15/88, to be published.

NCEER-88-0041 "Small-Scale Modeling Techniques for Reinforced Concrete Structures Subjected to Seismic Loads," by W. Kim, A. El-Attar and R.N. White, 11/22/88.

NCEER-88-0042 "Modeling Strong Ground Motion from Multiple Event Earthquakes," by G.W. Ellis and A.S. Cakmak, 10/15/88.

NCEER-88-0043 "Nonstationary Models of Seismic Ground Acceleration," by M. Grigoriu, S.E. Ruiz and E. Rosenblueth, 7/15/88.

NCEER-88-0044 "SARCF User's Guide: Seismic Analysis of Reinforced Concrete Frames," by Y.S. Chung, C. Meyer and M. Shinozuka, 11/9/88.

NCEER-88-0045 "First Expert Panel Meeting on Disaster Research and Planning," edited by J. Pantelic and J. Stoyle, 9/15/88.

NCEER-88-0046 "Preliminary Studies of the Effect of Degrading Infill Walls on the Nonlinear Seismic Response of Steel Frames," by C.Z. Chrysostomou, P. Gergely and J.F. Abel, 12/19/88.

NCEER-88-0047 "Reinforced Concrete Frame Component Testing Facility - Design, Construction, Instrumentation and Operation," by S.P. Pessiki, C. Conley, T. Bond, P. Gergely and R.N. White, 12/16/88.

NCEER-89-0001 "Effects of Protective Cushion and Soil Compliancy on the Response of Equipment Within a Seismically Excited Building," by J.A. HoLung, 2/16/89.

NCEER-89-0001 "Effects of Protective Cushion and Soil Compliancy on the Response of Equipment Within a Seismically Excited Building," by J.A. HoLung, 2/16/89.

NCEER-89-0002 "Statistical Evaluation of Response Modification Factors for Reinforced Concrete Structures," by H.H-M. Hwang and J-W. Jaw, 2/17/89.

National Center for Earthquake Engineering Research
State University of New York at Buffalo