NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
State University of New York at Buffalo

NONNORMAL ACCELERATIONS DUE TO YIELDING IN A PRIMARY STRUCTURE

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NOTICE

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PREFACE

The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. The emphasis is on structures and lifelines that are found in zones of moderate to high seismicity throughout the United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

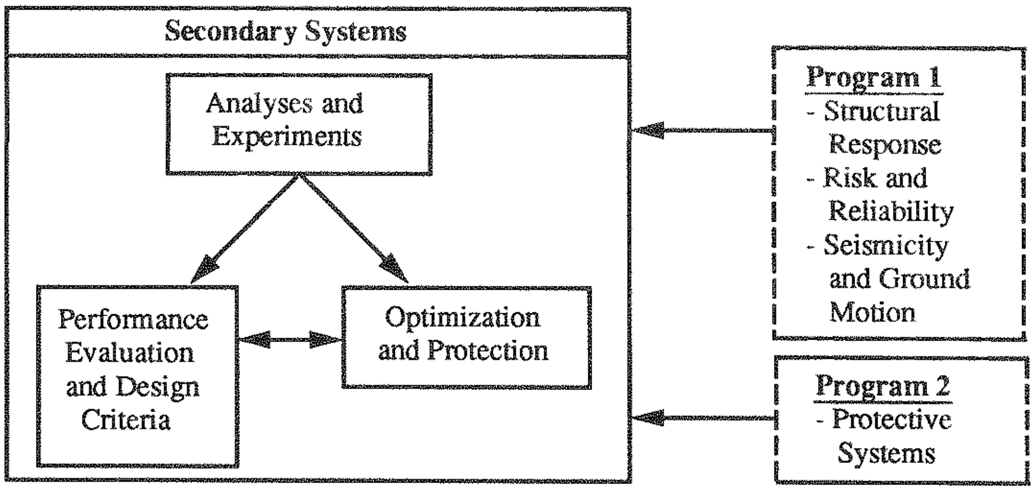
- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

This technical report pertains to the second program area and, more specifically, to secondary systems.

In earthquake engineering research, an area of increasing concern is the performance of secondary systems which are anchored or attached to primary structural systems. Many secondary systems perform vital functions whose failure during an earthquake could be just as catastrophic as that of the primary structure itself. The research goals in this area are to:

1. Develop greater understanding of the dynamic behavior of secondary systems in a seismic environment while realistically accounting for inherent dynamic complexities that exist in the underlying primary-secondary structural systems. These complexities include the problem of tuning, complex attachment configuration, nonproportional damping, parametric uncertainties, large number of degrees of freedom, and nonlinearities in the primary structure.
2. Develop practical criteria and procedures for the analysis and design of secondary systems.
3. Investigate methods of mitigation of potential seismic damage to secondary systems through optimization or protection. The most direct route is to consider enhancing their performance through optimization in their dynamic characteristics, in their placement within a primary structure or in innovative design of their supports. From the point of view of protection, base isolation of the primary structure or the application of other passive or active protection devices can also be fruitful.

Current research in secondary systems involves activities in all three of these areas. Their interaction and interrelationships with other NCEER programs are illustrated in the accompanying figure.



Response of a yielding primary structure under normal white noise excitations is studied in this report. The focus is on the nonnormality of the absolute acceleration. While the report does not address its effect on the response characteristics of secondary systems, results suggest that secondary system response can be decidedly nonnormal as well and this nonnormality should be investigated.

ABSTRACT

Response nonnormality is investigated for a yielding primary structure subjected to a normally distributed ground acceleration. This is a preliminary step in the nonlinear investigation of primary-secondary systems, since this primary acceleration is the base excitation of a light secondary system. The nonlinearity considered is bilinear hysteretic yielding in the primary structure. Results are presented from simulation, and from analysis of a simplified nonhysteretic substitute structure.

This study of the nonnormality of acceleration is an extension of previous investigations which focused on displacement nonnormality. The nonlinear but nonhysteretic substitute structural model used here is a substantially improved version of an earlier model. The substitute structure is one for which the stationary Fokker-Planck equation can be solved to obtain values for probability distribution and moments of response. The coefficient of excess is used as a measure of the nonnormality.

Analytical and numerical results for the substitute structure are compared with those from simulation of the bilinear hysteretic system. Such comparisons are made for both the relative displacement and the absolute acceleration of the primary structure, and include both the mean squared levels and the coefficients of excess of those responses. The agreement between the two sets of results for acceleration is shown to be quite good.

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INTRODUCTION

If the excitation of a linear system has a normal or Gaussian probability distribution then the linear system response is also normal. The theory of the stationary response of linear systems to random excitation is quite well developed and is available in common reference books [7,13,17,18]. Unfortunately, structural systems under dynamic loading often exhibit nonlinear behavior, and the response of a nonlinear system under normal excitation is not normal. Some recent studies of fatigue damage accumulation [10,14,22] and of first-excursion [8] probabilities have shown that these two quantities can be significantly affected by nonnormality of the random process studied. Such nonnormality is particularly likely to occur in a situation involving significant nonlinearity, like the yielding effect in a hysteretic system.

The dynamics of linear primary-secondary systems have been quite extensively investigated for both deterministic and stochastic excitations [1,9,12,19]. One aspect of the dynamic response which cannot be found from such linear studies, though, is the nonnormality which may result from nonlinearities in the system. As for simpler systems, this nonnormality could have a very important effect on the probability of damage or failure. One very simple nonlinear primary-secondary situation is when a very light secondary system is attached to a yielding single-degree-of-freedom (SDF) primary structure. In this situation the absolute acceleration of the response of the primary structure becomes a nonnormal base excitation of the secondary system. The current study is primarily concerned with quantification of the nonnormality of the absolute acceleration for such a yielding primary structure.

A simple and natural way to include nonnormality in the analysis of a random variable is through consideration of moments higher than the second. In particular, the fourth moments are important for characterizing nonnormality (especially if the random variable is symmetric about its mean value so that the third moment gives no new information). In this study, the kurtosis or the coefficient of excess (i.e. kurtosis minus 3) will serve as the index to represent the degree of nonnormality of a random process.

The objective of this study is to investigate the nonnormality of the response behavior of a SDF bilinear hysteretic structure under normal white excitation. Particular attention will be paid to the nonnormality of the absolute acceleration. Both approximate analytical methods and computer simulation results will be presented. The approximate technique uses a nonlinear nonhysteretic substitute structure. Numerical results from computer simulation and from the substitute structure will be presented, and appropriate conclusions will be drawn regarding the extent of nonnormality and the usefulness of the substitute structure model.

SECTION II

HYSTERETIC SYSTEM

A mechanical system which exhibits a bilinear hysteretic restoring force is shown in figure 2-1. The equation of motion can be written as :

$$m\ddot{x} + c\dot{x} + (k_1 + k_2)\phi(x) = \overline{n}(t)$$

or

$$\ddot{x} + 2\beta_0\omega_0\dot{x} + \omega_0^2\phi(x) = \frac{\overline{n}(t)}{m} = n(t) \tag{2.1}$$

where:

m is the mass

$\omega_0 = \sqrt{\frac{k_1+k_2}{m}}$, small amplitude undamped natural circular frequency

$\beta_0 = \frac{c}{2\omega_0 m}$, small amplitude fraction of critical damping, and

$\phi(x)$ is the bilinear hysteretic restoring force as shown in figure 2-2. Note that $\phi(x)$ is chosen to have a unit slope for small amplitudes and a second slope $\alpha = \frac{k_2}{(k_1+k_2)}$. Also, $\phi(x)$ depends on previous value of $x(t)$ but with the limitation that if $x(t)$ is periodic, then $\phi(x)$ is also periodic.

For the present investigation, the excitation $n(t)$ represents ground acceleration. It is taken to be a stationary, white, random process with a normal probability distribution, and a uniform power spectral density equal to D/π (per radian) for all frequency. That is, the auto-correlation function is given by

$$E[n(t_1)n(t_2)] = 2D\delta(t_1 - t_2) \tag{2.2}$$

in which $\delta(t)$ is the Dirac delta function.

The system described by eq. 2.1 has probably been more widely studied than any other class of nonlinear hysteretic oscillator[3,10,15]. Two particular values of the slope ratio were chosen to illustrate important situations. These are $\alpha=1/2$, a moderately nonlinear system, and $\alpha=1/21$, a nearly elastoplastic system.

No exact solution for the statistics of the response of such a hysteretic system to random excitation have yet been obtained by an analytical technique. Thus, a computer simulation program has been used to obtain empirical data, and a substitute structure concept is used for analysis.

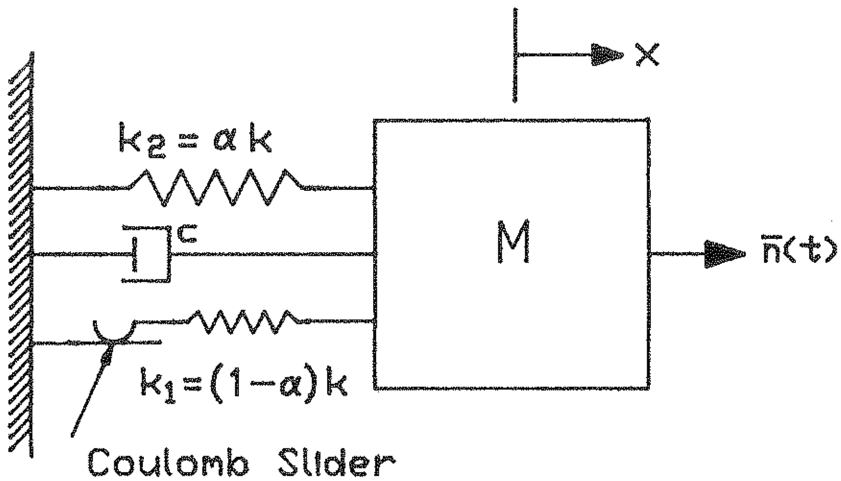


Figure 2-1 A SDF bilinear hysteretic system

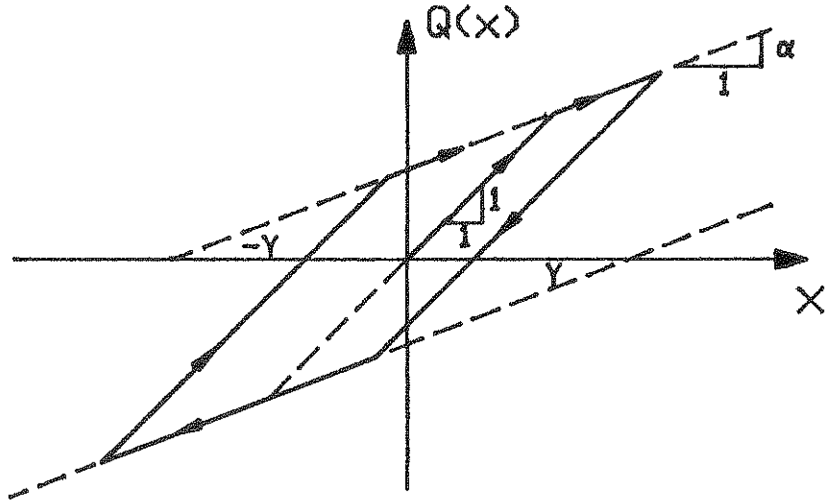


Figure 2-2 The restoring force of a SDF bilinear hysteretic system

SECTION III

EQUIVALENT NONHYSTERETIC SYSTEM

A general class of nonlinear oscillators for which the exact analytical probability density function of stationary response has been obtained is described by

$$\ddot{x} + f(H)\dot{x} + g(x) = n(t) \tag{3.1}$$

where g(x) is an odd nonhysteretic function and g(x)>0 for x>0, $H=G(x)+\dot{x}^2/2$ (energy in the system),

$$G(x) = \int_0^x g(y)dy$$

$f(H)$ is a positive function, and $n(t)$ is a Gaussian, white process with mean zero.

The method used in this study involves choosing $f(H)$ and $g(x)$ in eq.(3.1) in such a way as to give an approximate equivalence between the systems described by eq. 3.1 and eq. 2.1. Approximate response statistics for the hysteretic system are then predicted from the analytical or numerical solution for eq. 3.1.

The actual choice of a nonhysteretic system to approximate a given hysteretic system could be based on any of several methods of comparison of the two systems. Two rather mathematical approaches have been applied to specific problems by Caughey and Ma [4] and by Cai and Lin [13]. In the former work, the choice is based on a minimization of the mean squared difference between the two equations of motion, and in the latter the comparison is in terms of the stationary probability distribution solution of the appropriate Fokker-Planck equations. The method of comparison chosen here is less mathematical and somewhat intuitive. It is a modification of the procedure used in [15]. The nonlinear stiffness is taken to be the same as in that earlier work, but the damping function is modified based on more recent results regarding the energy dissipation of hysteretic structures. Specifically, two different nonlinear oscillators will be considered to be approximately equivalent herein if the following two statements hold true :

1. The two oscillators have the same functional relationship between resonant frequency and amplitude of vibration, and

2. The two oscillators have equal energy dissipation per unit time.

To make equivalence statement 1 be precise, the resonant frequency w_r for eq. 2.1 will be taken as

$$w_r^2 = \frac{1}{\pi A}\int_0^{2\pi} cosu\ \phi(Acosu)\ du \tag{3.2}$$

where A is amplitude of vibration. For eq. 3.1, one merely replaces ϕ by g in eq. 3.2. eq. 3.2 can be shown to be closely related to other common definitions of resonance. For example, eq. 3.2 approximates the frequency of free vibration of an undamped nonlinear system, and the frequency of maximum response of a hysteretic system with harmonic excitation [15]. Equivalence statement 1 requires that $g(x)$ in eq. 3.1 be chosen such that eq. 3.2 gives the same w_r as for eq. 2.1 with some particular functional ϕ. For the ϕ of the bilinear hysteretic system it has been shown [15] that an appropriate $g(x)$ function is

$$\begin{aligned} \frac{g(x)}{w_0^2} &= x \qquad for\ |x| \leq Y \\ &= (1-\alpha)Y^{\frac{3}{2}}x^{-1}|x|^{\frac{1}{2}} + \alpha x \qquad for\ |x| > Y \end{aligned} \tag{3.3}$$

Equivalence statement 2 involves the rate of energy dissipation. For convenience, this will be taken as energy per unit mass. The energy dissipation of eq. 3.1 will depend on f, which is given to be a function of the energy H. If one postulates that H varies slowly as compared to displacement and velocity variations, then the energy for a cycle of amplitude A can be approximated as

$$H = G(A) \tag{3.4}$$

Hence, the damping function f is only a function of the amplitude of vibration. Neglecting higher-harmonic contributions, the energy dissipated per cycle for eq. 3.1 is

$$ED = \pi f w_r A^2 \tag{3.5}$$

The energy dissipated per unit time is then found by dividing by the period of the cycle $(2\pi/w_r)$ and averaging over all possible amplitudes:

$$E(EDT) = \frac{1}{2}E[f[G(A)]w_r^2(A)A^2] \tag{3.6}$$

The energy dissipated per unit time for the hysteretic system of eq. 2.1 is slightly more complicated. The viscous damping term can be handled as above, with $f[G(A)]$ in eq. 3.6 replaced by $2\beta_0 w_0$. The hysteretic energy dissipation, though, will be treated somewhat differently. Previous studies of the power balance in hysteretic oscillators [16,20] have shown that this rate of energy dissipation can be approximated by using the area of the hysteresis loop (EDH), which is the energy dissipated per hysteretic cycle, and a period based on the tangent stiffness of ϕ. For the bilinear hysteretic system this tangent stifness is k for $A \leq Y$ and $k_2 = \alpha k$ for $A > Y$. Combining these results gives the energy dissipation per unit time in eq. 2.1 as

$$E(EDT) = \beta_0 w_0 E[w_r^2(A)A^2] + \frac{\sqrt{\alpha} w_0}{2\pi} E(EDH) \tag{3.7}$$

Comparing eq. 3.6 and eq. 3.7 shows that equal energy dissipation per unit time is achieved if f is defined by

$$f = 2\beta_0 w_0 + \frac{\sqrt{\alpha}}{\pi} \frac{w_0}{w_r^2} \frac{(EDH)}{A^2} \tag{3.8}$$

where

$$(EDH) = 4(1-\alpha) w_0^2 Y(A - Y) \tag{3.9}$$

for the bilinear hysteretic restoring force.

SECTION IV

RESPONSE OF THE NONHYSTERETIC SYSTEM

Caughey solved the Fokker-Planck equation for the system of eq. 3.1 to obtain the joint, stationary, probability density function as [5]

$$p_{x\dot{x}}(q,\dot{q}) = Cexp\{-F[H(q,\dot{q})/D\} \tag{4.1}$$

where

$$F(H) = \int_0^H f(h)dh$$

Note that $p_{x\dot{x}}$ depends on the energy H associated with particular combinations of x and $\dot{x}$, rather than depending on x and $\dot{x}$ independently.

For the present purpose, it is more convenient to write the results in a slightly different form. Let a variable $A \geq 0$ be defined by $H = G(A)$, then one can compute the probability density function of this variable from eq. 4.1 as

$$p_A(a) = CT(a)g(a)exp\{-F[G(a)]/D\} \tag{4.2}$$

where

$$T(A) = 4\int_0^A [2G(A) - 2G(x)]^{-\frac{1}{2}} dx \tag{4.3}$$

One can show that $T(A)$ is exactly the period of free vibration of amplitude A of an undamped system, and it will be approximated by

$$T(A) \simeq \frac{2\pi}{w_r} \tag{4.4}$$

where w_r is given by eq. 3.2, which can be written for the bilinear hysteretic system as

$$\begin{aligned} \frac{w_r^2 - \alpha w_0^2}{w_0^2(1-\alpha)} &= 1 \qquad for\ A \leq Y \\ &= \frac{1}{\pi} cos^{-1}(1 - \frac{2Y}{A}) - \frac{2}{\pi}(1 - \frac{2Y}{A})[\frac{Y}{A}(1 - \frac{Y}{A})]^{\frac{1}{2}} \qquad for\ A \geq Y \end{aligned} \tag{4.5}$$

The $g(A)$ in eq. 4.2 will be evaluated from eq. 3.3, for the corresponding nonhysteretic system, and $G(A)$ is obtained by integration as

$$\begin{aligned} G(A) &= \frac{w_0^2 A^2}{2} \qquad for\ A \leq Y \\ &= \frac{(1-\alpha) w_0^2 Y^2}{2}[4(\frac{A}{Y})^{1/2} - 3] + \frac{\alpha w_0^2 A^2}{2} \qquad for\ A > Y \end{aligned} \tag{4.6}$$

Using eq. 3.9, the EDH for a bilinear hysteretic system, in eq. 3.8 to obtain the damping function f in eq. 3.1 yields

$$\begin{aligned} f[G(A)] &= 2\beta_0 w_0 \qquad for\ A \leq Y \\ &= 2\beta_0 w_0 + \frac{4}{\pi}\sqrt{\alpha}(1-\alpha) w_0 (\frac{w_0}{w_r})^2 \frac{Y}{A}(1 - \frac{Y}{A}) \qquad for\ A > Y \end{aligned} \tag{4.7}$$

Since both f and $H = G(A)$ can now be easily evaluated as functions of the amplitude A, it is straightforward to use numerical integration to evaluate F (for $H > w_0^2 Y^2/2$) for use in eq. 4.2. This gives all the terms in eq. 4.2 except C, which can be obtained from the necessary normalization that the integral of $p_A(a)$ from zero to infinity must be unity.

Next consider the evaluation of the desired response moments. Let the response be written as

$$x(t) = A(t)cos[w_r t + \theta(t)] \tag{4.8}$$

where w_r is a function of A, and assume that the random phase angle θ is uniformly distributed from zero to 2π and is statistically independent of A. Then, the mean-squared value of x can be written as

$$E(x^2) = \sigma_x^2 = \frac{E(A^2)}{2} = \frac{1}{2}\int_0^\infty a^2 p_A(a)da \tag{4.9}$$

the fourth moment of x is

$$E(x^4) = \frac{3}{8}E(A^4) = \frac{3}{8}\int_0^\infty a^4 p_A(a)da \tag{4.10}$$

and these two integrals can be easily evaluated by numerical integration.

Let $\ddot{z}$ denote the absolute acceleration $(\ddot{x} - n)$ of eq. 3.1. Then

$$\ddot{z} = -[f(H)\dot{x} + g(x)] \tag{4.11}$$

Unfortunately, some of the terms in $E(\ddot{z}^2)$ and $E(\ddot{z}^4)$ are not quite so easily converted into integrals over A as were the expectations for displacements. Looking at the conditional expectations given A, the moments of $\ddot{z}$ can be shown to be

$$E(\ddot{z}^2|A) = f^2[G(A)]\frac{w_r^2 A^2}{2} + E[g^2(x)|A] \tag{4.12}$$

$$E(\ddot{z}^4|A) = f^4[G(A)]\frac{3w_r^4 A^4}{8} + 6f^2[G(A)]E[\dot{x}^2 g^2(x)|A] + E[g^4(x)|A] \tag{4.13}$$

After some simplification the conditional expectations involving g (from eq. 3.3) can be shown to be

$$E[g^2(x)|A]/w_0^4 = \frac{A^2}{2} - \frac{(1-\alpha^2)}{\pi}A^2(\theta^* + \frac{Y}{A}sin\theta^*) + \frac{4\alpha(1-\alpha)}{\pi}Y^{3/2}A^{1/2}\int_0^{\theta^*}(cos\theta)^{1/2}d\theta + \frac{(1-\alpha)^2}{\pi}\frac{Y^3}{A}log(\frac{1+sin\theta^*}{1-sin\theta^*}) \tag{4.14}$$

$$E[\dot{x}^2 g^2(x)|A]/(w_0^4 w_r^2) = \frac{A^4}{8} - \frac{(1-\alpha^2)}{4\pi}A^4[\theta^* + \frac{Y}{A}(1-2\frac{Y^2}{A^2})sin\theta^*] + \frac{8\alpha(1-\alpha)}{5\pi}Y^{3/2}A^{5/2}[\int_0^{\theta^*}(cos\theta)^{1/2}d\theta - (\frac{Y}{A})^{3/2}sin\theta^*] + \frac{(1-\alpha)^2}{\pi}Y^3A[\log(\frac{1+sin\theta^*}{1-sin\theta^*}) - 2sin\theta^*] \tag{4.15}$$

$$E[g^4(x)|A]/w_0^8 = \frac{3A^4}{8} - \frac{(1-\alpha^4)}{2\pi}A^4[\frac{3}{2}(\theta^* + \frac{Y}{A}sin\theta^*) + (\frac{Y}{A})^3 sin\theta^*] + \frac{8\alpha^3(1-\alpha)}{5\pi}Y^{3/2}A^{5/2}[3\int_0^{\theta^*}(cos\theta)^{1/2}d\theta + 2(\frac{Y}{A})^{3/2}sin\theta^*] + \frac{12\alpha^2(1-\alpha)^2}{\pi}Y^3 A sin\theta^* + \frac{2(1-\alpha)^4}{\pi}\frac{Y^5}{A}sin\theta^* + \frac{8\alpha(1-\alpha)^3}{\pi}\frac{Y^{9/2}}{A^{1/2}}[3\int_0^{\theta^*}(cos\theta)^{3/2}d\theta - 2(\frac{Y}{A})^{1/2}sin\theta^*] \tag{4.16}$$

in which $cos\theta^* = Y/A$.

The two integrals of non-integer powers of $cos\theta$ in eqs. 4.14 to 4.16 have been approximated analytically (see Appendix A) as

$$\int_0^{\theta^*}(cos\theta)^{1/2}d\theta \simeq \theta^*[1 - \frac{1}{3}(\frac{\theta^*}{2})^2 - \frac{1}{11}(0.5784\ \theta^*)^{10}] \tag{4.17}$$

$$\int_0^{\theta^*} (cos\theta)^{3/2} d\theta \simeq \theta^*\{1 - (\frac{\theta^*}{2})^2 + \frac{9}{20}(\frac{\theta^*}{2})^4 - [\frac{2}{11} - \frac{3}{13}(\frac{\theta^*}{2})^2](0.4912\ \theta^*)^{10} + \frac{1}{21}(0.4912\ \theta^*)^{20}\} \tag{4.18}$$

With appropriate substitutions it is now possible to obtain the desired moments of x and $\ddot{z}$ in one pass of numerical integration incrementing A from Y to some very large values. Specifically, integration of eq. 4.7 gives F in eq. 4.2, integration of $(g/w_r)\exp(-F/D)$ gives $(2\pi C)^{-1}$ for the same equation, integration of eqs. 4.9 and 4.10 gives the moments of x, and integration of eq. 4.2 times 4.12 and 4.13, respectively, gives the moments of $\ddot{z}$.

If the excitation is either very small or very large, then one can use simpler approximations for $f(H)$ and $g(x)$ than those proposed above and thereby avoid numerical integration. That is, one can obtain analytic expressions for mean-squared response as well as fourth moment response. Appendix B summarizes the results of two such approximations.

In the computer simulation, the "white noise" excitation was obtained from a pulse method. The acceleration at the base of a structure was taken to be a sequence of uniformly spaced Dirac delta functions, with each acceleration pulse giving an instantaneous change in the relative velocity $\dot{x}$ [2]. The pulse magnitude, in this study, is a standard normal random number, obtained from subroutine RNNOA in the IMSL-Library [21], scaled by a constant R, which is given by

$$R = \sqrt{2D\Delta t} \tag{4.19}$$

where

D/π is the power spectral density of the white noise excitation.

Δt is the time interval between two adjacent pulses.

The interval Δt was chosen to give $w_0 \Delta t$ =0.1 radian, giving approximately 63 pulses per cycle of the unyielded system. Each sample of simulated response was long enough to contain approximately 2000 cycles of response of the unyielded system ($w_0 t$=4000 π). The first 100 cycles of each sample were omitted from calculations, though, on the basis of possible nonstationarity. Statistical accuracy was improved by using an ensemble of 10 such samples for each process investigated. The reproducibility of the results was verified by comparing numbers obtained from different ensembles and from ensembles of different lengths. The results for the displacement x were also found to agree very well with analog computer results [15].

SECTION V

NUMERICAL RESULTS

Recall that *the coefficient of excess* (coe) is a measure of how much the distribution of a random variable departs from a normal distribution, and it can be written as :

$$r_x = \frac{E(x^4)}{E(x^2)^2} - 3 \tag{5.1}$$

$$r_{\ddot{z}} = \frac{E(\ddot{z}^4)}{E(\ddot{z}^2)^2} - 3 \tag{5.2}$$

This measure is zero for a normal distribution, negative for an amplitude-limited type of distribution, and positive for a distribution with greater than normal probability of large amplitudes. It will be used here to present the fourth moment results.

It is also convenient to characterize the excitation level by a measure with dimension length, so that the ratios of yield levels to excitation level and root-mean-square response level to excitation level can be plotted as dimensionless quantities. Such a length measure of the excitation level is

$$N = \frac{(2Dw_0/\pi)^{\frac{1}{2}}}{w_0^2} \tag{5.3}$$

The numerical results of root-mean-square response from eqs. 4.9 and 4.12 are plotted in figures 5-1 to 5-4, and of coefficient of excess from eqs. 5.1 and 5.2 are plotted in figures 5-5 to 5-8, for the cases of $\alpha = 1/2$ and $1/21$, of damping $\beta = 1\%$ and 5%. The simulation results for the bilinear hysteretic system are also included in figures 5-1 to 5-8, as are the analytical approximations from Appendix B for very large and very small Y/N values.

5.1 RMS VALUES:

Figures 5-1 to 5-4 show that for both the moderately nonlinear system ($\alpha = 1/2$) and the nearly elastoplastic system ($\alpha = 1/21$), the response levels of rms displacement and absolute acceleration obtained from numerical integration for the nonhysteretic system agree quite well with the simulation results for the corresponding hysteretic system.

The rms displacement levels have been investigated in various previous studies. The values obtained here for the substitute nonhysteretic structure are similar to those previously obtained by a power balance method [16], and are substantially better than those previously obtained by Lutes [15] for a nonlinear nonhysteretic model. The largest error is for the nearly elasto-plastic structure with intermediate values of the yield level and 5% viscous damping. In ref. 16, it was concluded that similar errors were primarily due to errors in the calculation of the average frequency of the system. If the same conclusion applies in the present study, then modification of eq. 3.2 or 3.3 could be expected to improve these rms displacement predictions. The rms levels of absolute acceleration predicted for the nonhysteretic system are seen to be in very good agreement with those for the bilinear hysteretic system. The errors are sometimes significantly smaller for acceleration than for displacement. The authors are not aware of prior studies of the acceleration levels.

Recall that the "small Y/N" and "large Y/N" results in the figures have been obtained from purely analytical solution of simplified equations (as detailed in Appendix B). These results are always good for the appropriate Y/N ranges. For intermediate Y/N values no such simplified approximation has been obtained, but the figures show that the results for large and small Y/N do also give considerable information regarding the intermediate range.

One may also note that because the stiffness term dominates the acceleration response when Y/N is either very large or very small, $\sigma_{\ddot{z}}/w_0^2 N$ tends to $\alpha\sigma_x/N$ for small values of Y/N, and to σ_x/N for large values of Y/N. Thus, contrary to displacement response, generally $\alpha = 1/21$ has lower mean-squared acceleration response than does $\alpha = 1/2$.

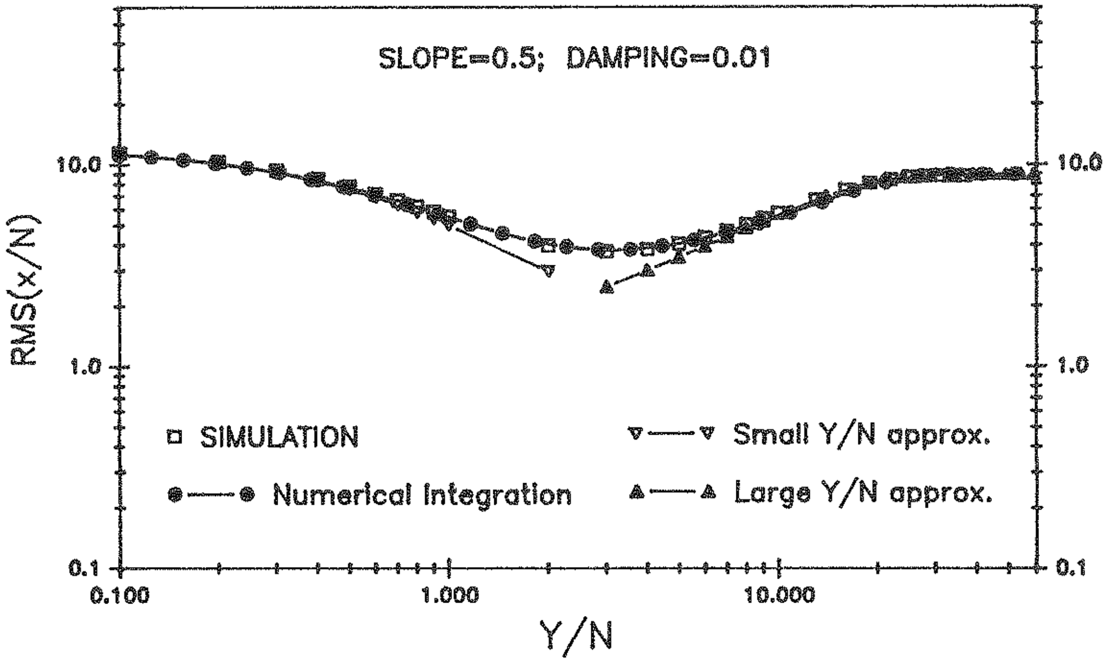
5.2 COE VALUES:

There are essentially no prior data with which to compare the *coe* values obtained here. Analytical approximation of the fourth moment of x for the bilinear hysteretic system with $\alpha = 1/2$ was included in ref. 16, but the corresponding simulation values were not obtained. The $coe(\ddot{z})$ is the primary focus of this study, and it had apparently not been previously studied by either analytical or empirical methods.

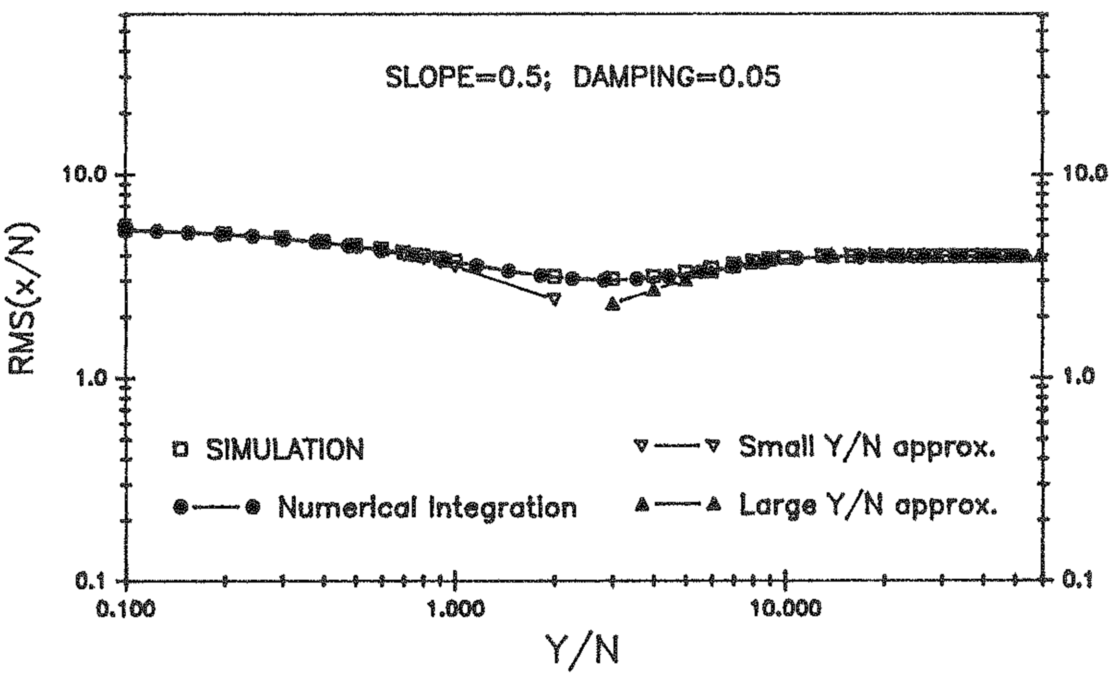
From figures 5-5 and 5-6, one can see that the coe of the displacement response of the nonhysteretic system agrees well with simulation results for $\alpha = 1/2$ but not for $\alpha = 1/21$. The excessively large values of $coe(x)$ predicted when α is small may reflect an inadequate choice of $g(x)$ according to eq 3.3.

Figures 5-7 and 5-8 show that the *coe* of the absolute acceleration response of the nonhysteretic system agrees quite well with the simulation results for the corresponding hysteretic system for both $\alpha = 1/2$ and $\alpha = 1/21$. The accuracy of this approximation is somewhat surprising in light of the large errors in $coe(\ddot{x})$ for $\alpha = 1/21$ and intermediate Y/N values. Apparently $\ddot{z}$ is much more affected by $\dot{x}$ than by x in this situation for a nearly elastoplastic structure. Figures 5-7 and 5-8 show that $\ddot{z}$ can be quite substantially nonnormal, and that this is often in the direction of a negative *coe* value. The fact that $\ddot{z}$ is nonnormal implies that the response of a secondary system attached to this primary system will also be nonnormal. If the mass of the secondary system is much smaller than that of the primary system, then $\ddot{z}$ can be considered as the secondary excitation. If the mass ratio is not very small, then interaction between the two systems may change the $coe(\ddot{z})$ value.

The fact that the $coe(\ddot{z})$ is often negative is good news in that it implies that very large values of $\ddot{z}$ are less likely than for a normal process with the same *rms* value. This should somewhat reduce the probability of damage in the secondary system. The opposite and undesirable situation of $coe(\ddot{z}) > 0$ does occur in a some circumstances though. Specifically, moderately small Y/N values lead to $coe(\ddot{z}) > 0$ unless the system is nearly elastoplastic and β_0 is not very small.

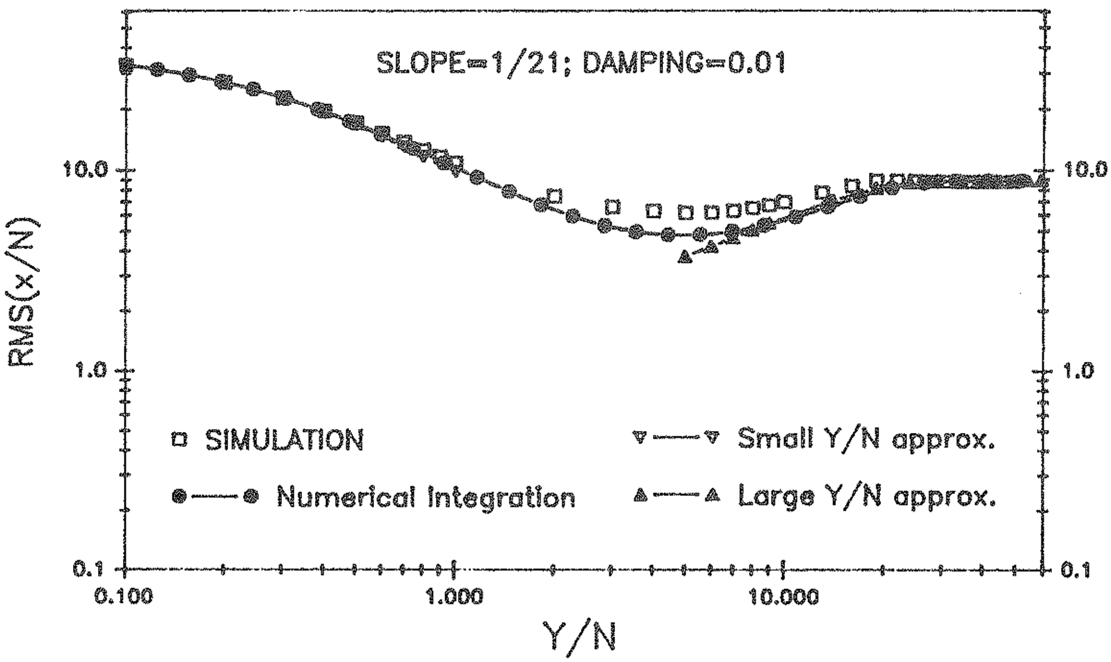


(a) damping = 1 %

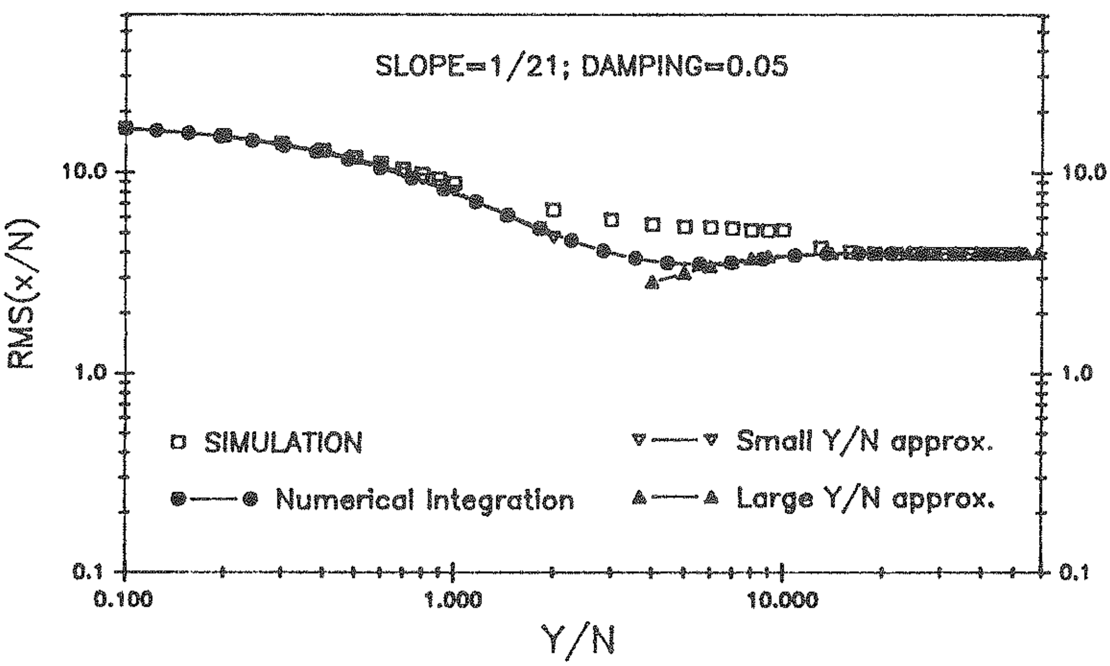


(b) damping = 5 %

Figure 5–1 RMS of displacement for slope=0.5



(a) damping = 1 %



(b) damping = 5 %

Figure 5–2 RMS of displacement for slope=1/21

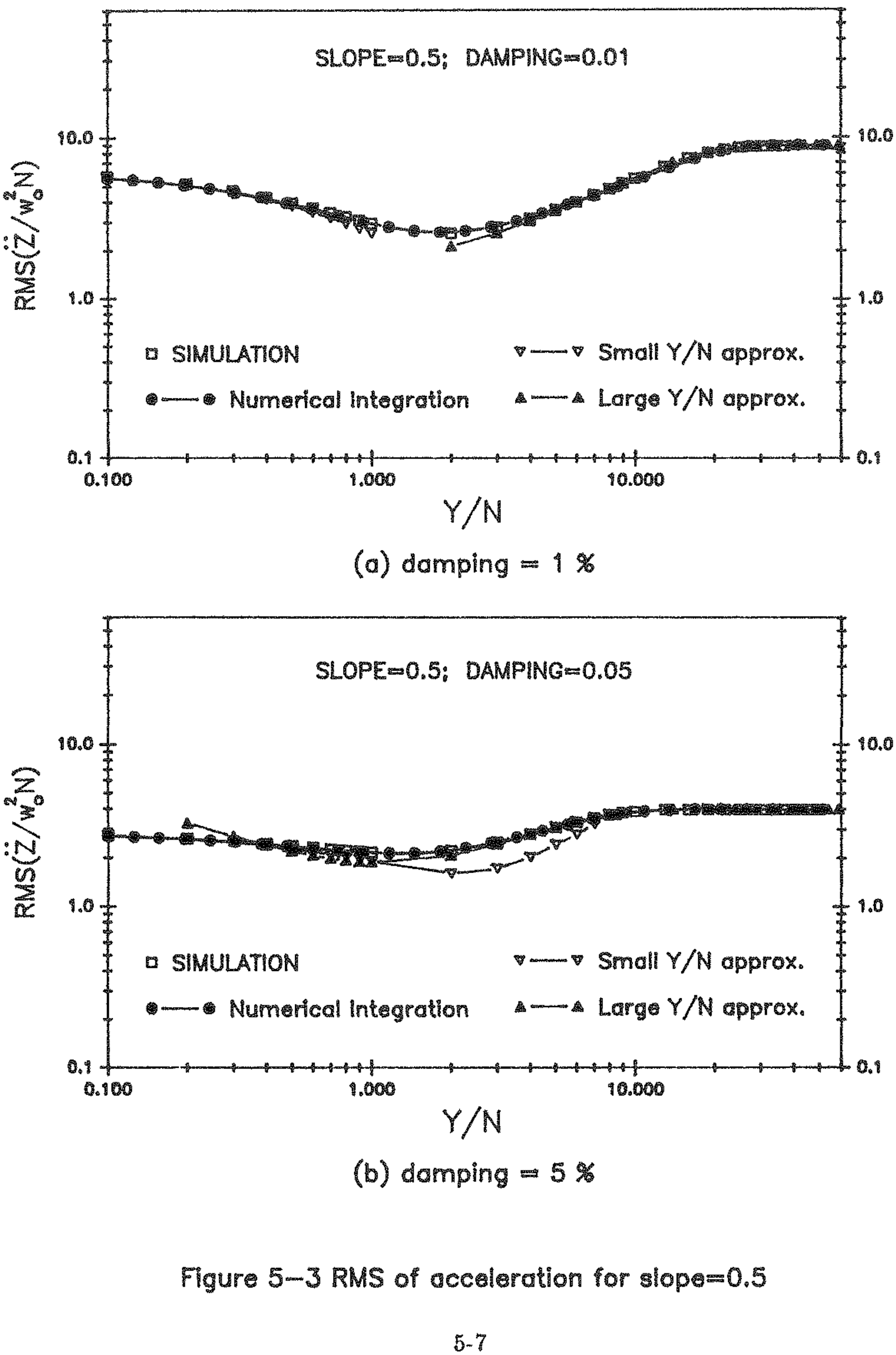
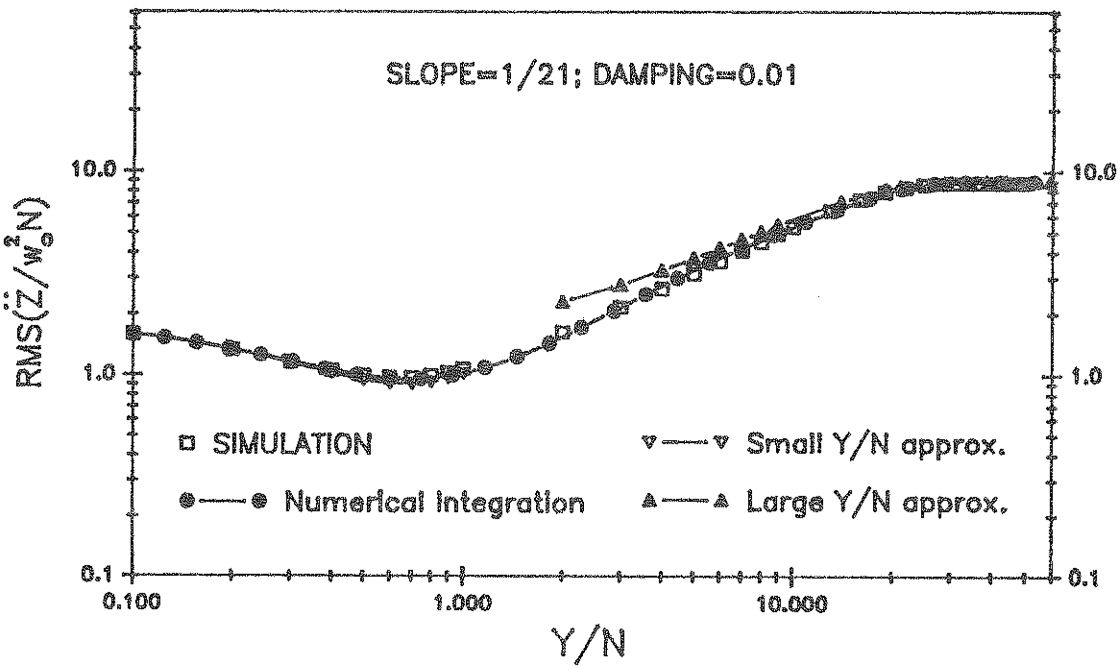
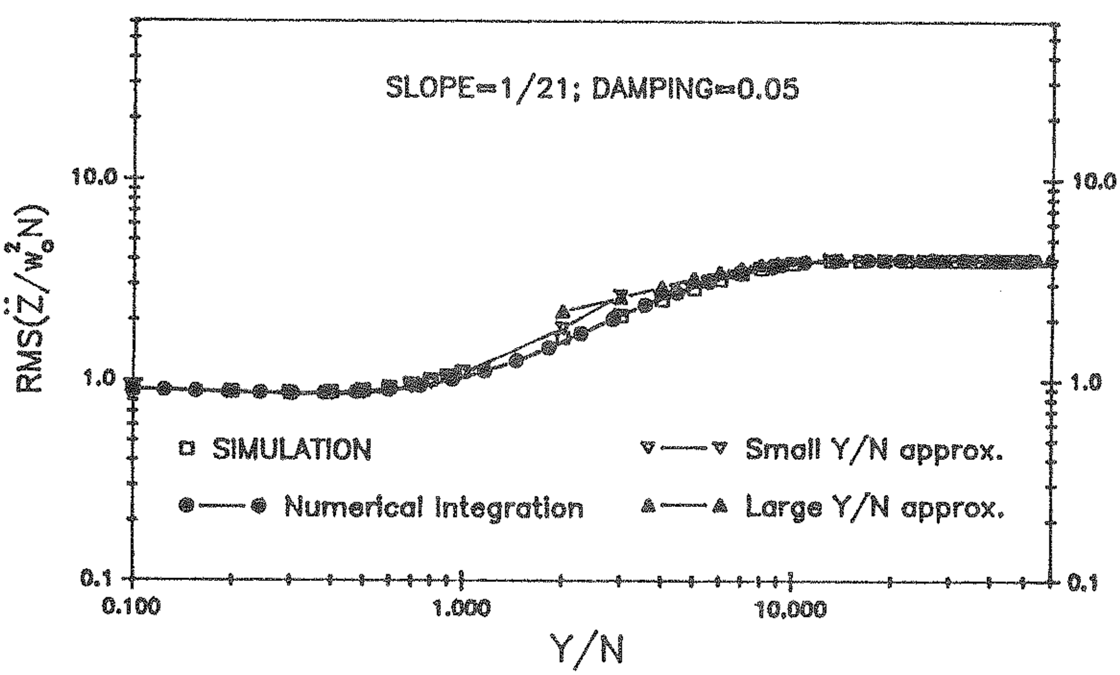


Figure 5—3 RMS of acceleration for slope=0.5

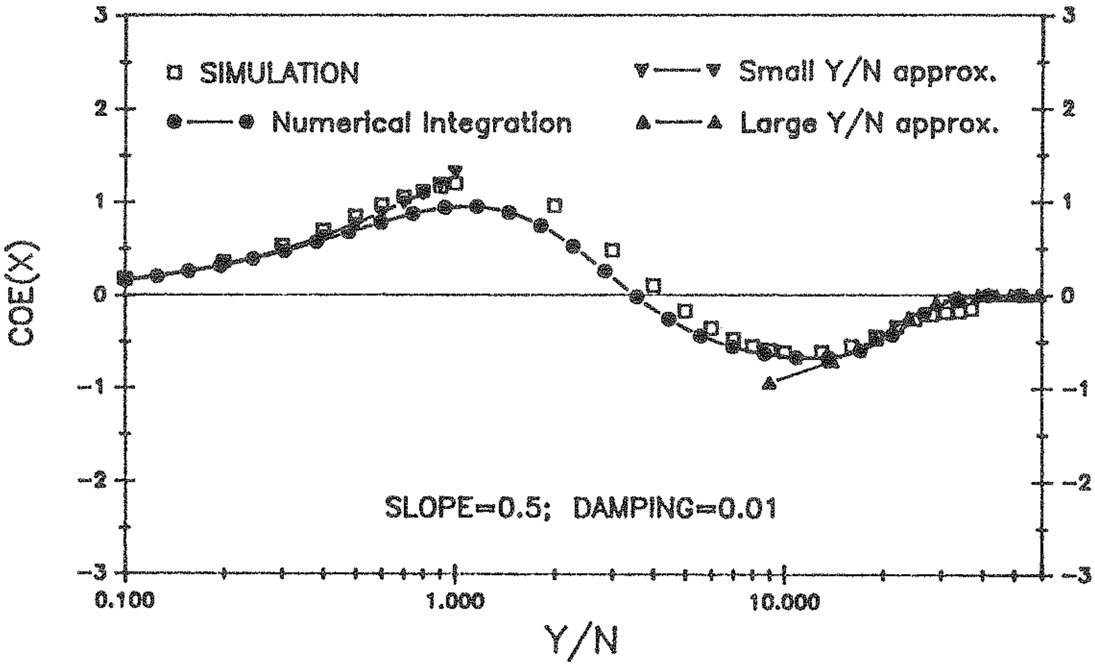


(a) damping = 1 %

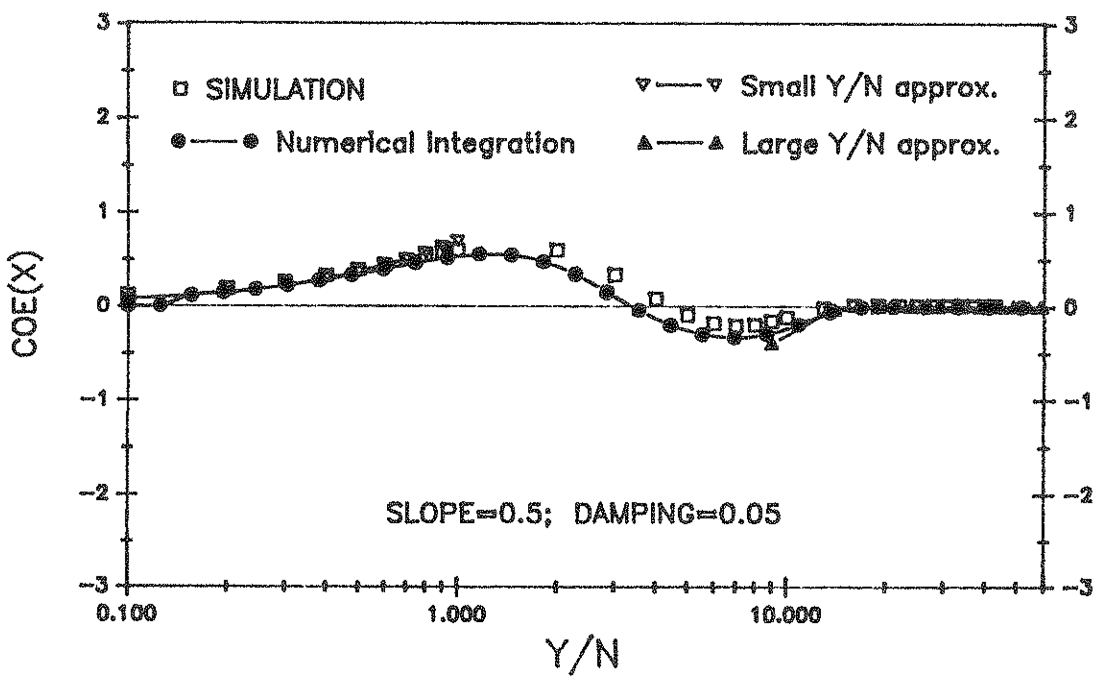


(b) damping = 5 %

Figure 5-4 RMS of acceleration for slope=1/21

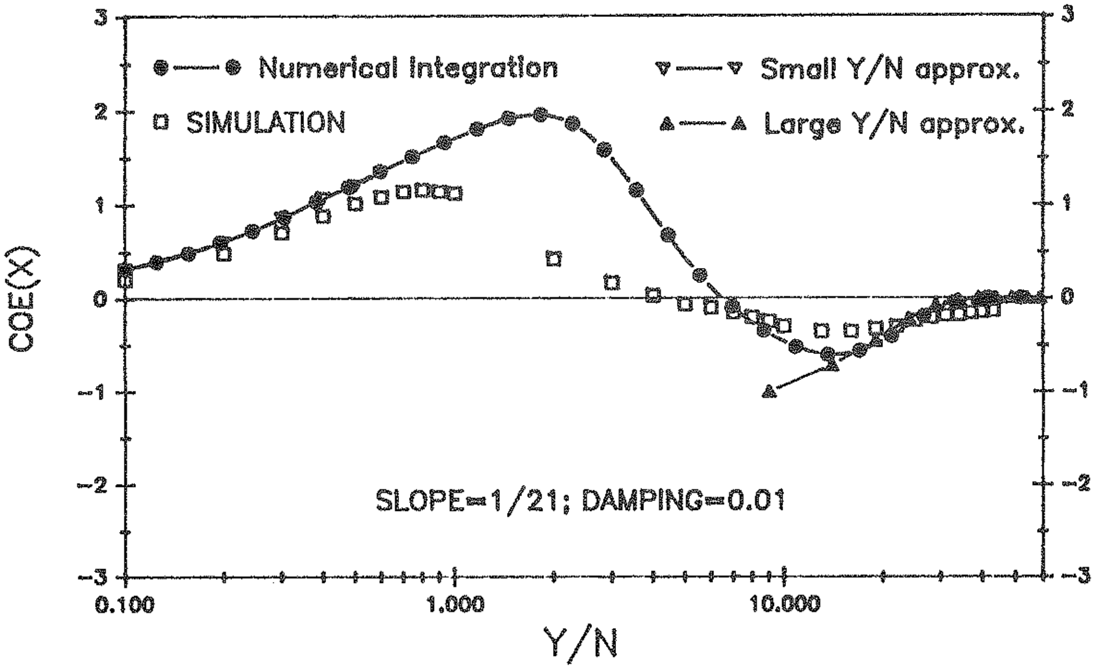


(a) damping = 1 %

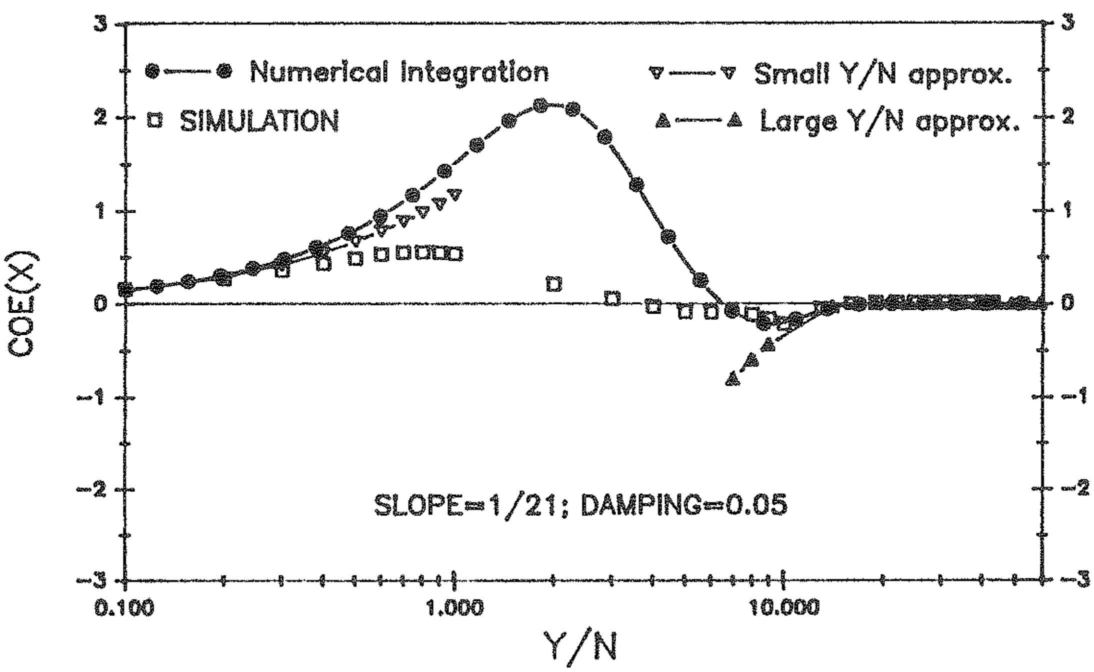


(b) damping = 5 %

Figure 5–5 COE of displacement for slope=0.5

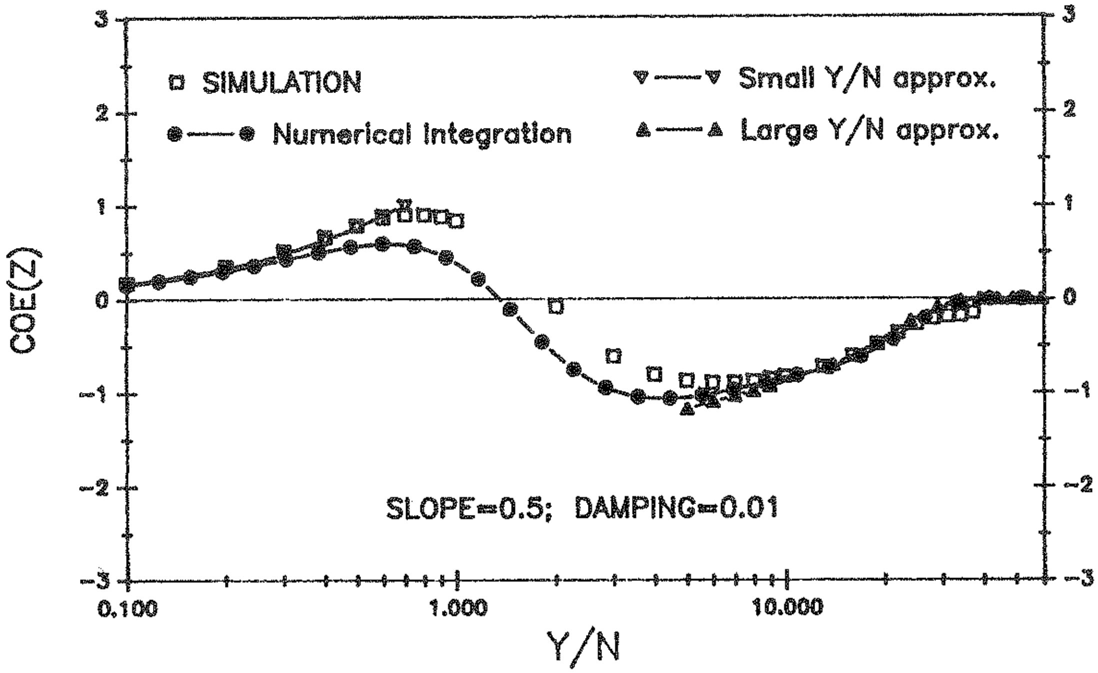


(a) damping = 1 %

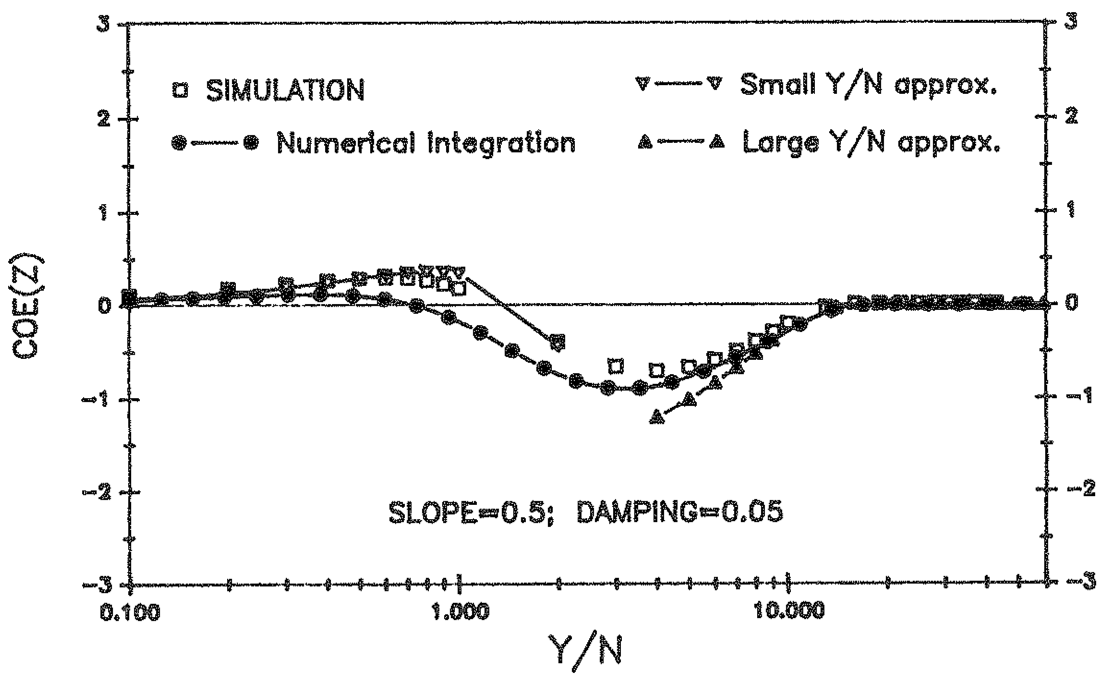


(b) damping = 5 %

Figure 5–6 COE of displacement for slope=1/21

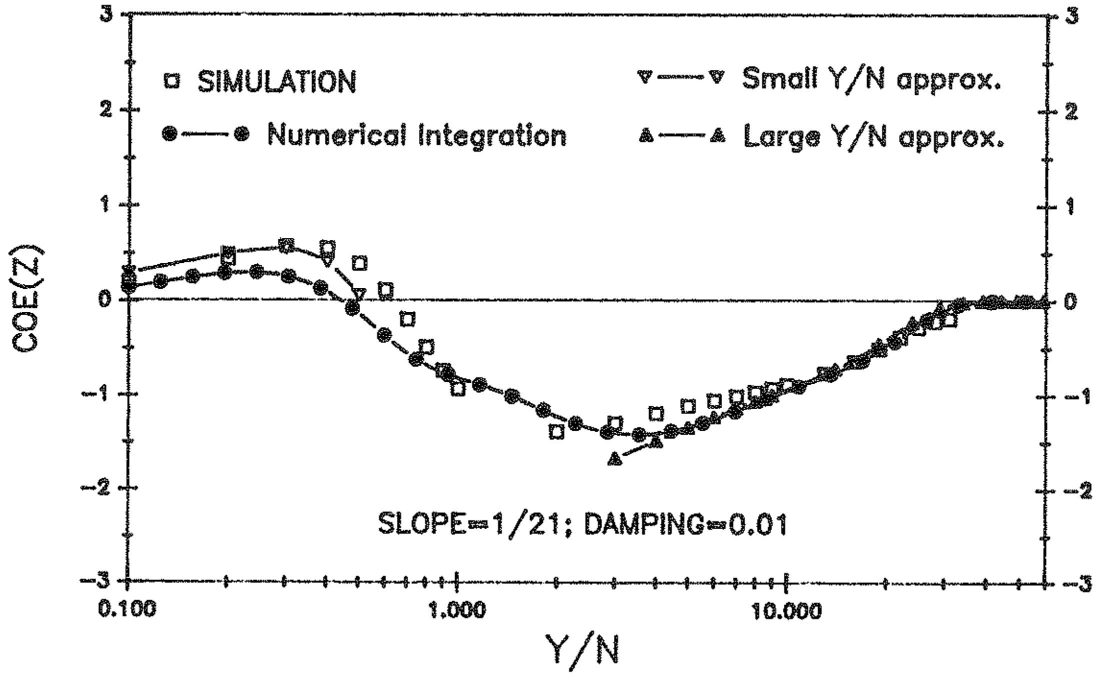


(a) damping = 1 %

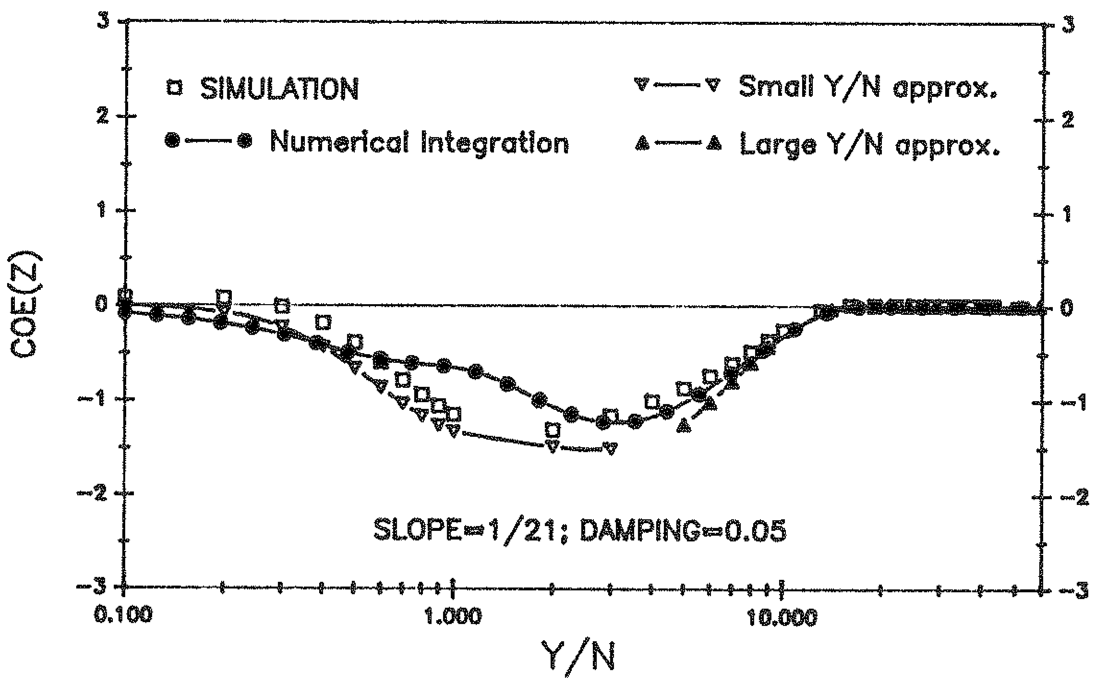


(b) damping = 5 %

Figure 5-7 COE of acceleration for slope=0.5



(a) damping = 1 %



(b) damping = 5 %

Figure 5–8 COE of acceleration for slope=1/21

SECTION VI

SUMMARY AND CONCLUSIONS

Response nonnormality has been investigated for a yielding primary structure subjected to a normally distributed ground acceleration. This is viewed as a preliminary step in the nonlinear investigation of primary-secondary systems, since this primary acceleration would be the base excitation of a light secondary system. The primary structure considered here has bilinear hysteretic yielding. Results have been presented both from simulation and from analysis of a simplified nonhysteretic substitute structure. Obtaining response moments (rms and coe) for the substitute structure generally requires simple numerical integration, although closed-form solutions have been obtained for simplifications appropriate to either large or small values of the yield level.

From the numerical results the following conclusions can be drawn:

1. nonnormality of the response of a secondary system should definitely be investigated, since the absolute acceleration of the primary structure is sometimes decidedly nonnormal.

2. The most nonnormal response acceleration found was in the direction of amplitude limiting (coe $\simeq$ -1.5). Nonnormality in the opposite sense (coe $\simeq$ 1.0) was also observed, though, for smaller values of the yield level.

3. The nonhysteretic substitute sustem used here gave quite good predictions of both the rms and coe of the acceleration response. It's incorporation into the investigation of coupled primary-secondary systems warrants investigation.

4. The substitute system also gave very good predictions of the rms values

of displacement response, but the coe values for displacement were significantly in error for a nearly elastoplastic structure.

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APPENDIX A
APPROXIMATION OF CERTAIN INTEGRALS

For use in eqs. 4.14 to 4.16 it is necessary to approximate the integral of $(cos\theta)^{1/2}$ from zero to some upper limit θ^* which is always between zero and $\frac{\pi}{2}$. This is easily accomplished if $(cos\theta)^{1/2}$ on $0 \leq \theta \leq \frac{\pi}{2}$ can be approximated by some simple polynomial. It is most important that this approximation be quite good for small θ values, since these values will contribute most to the integral. In order to obtain this good approximation for small θ it was decided to include the first two terms from the power series for $(cos\theta)^{1/2}$. Thus

$$\begin{aligned} (cos\theta)^{1/2} &= (1 - \frac{\theta^2}{2} + O(\theta^4))^{1/2} \\ &= 1 - \frac{\theta^2}{4} + O(\theta^4) \end{aligned} \qquad (A.1)$$

Numerical comparison shows that using only these first two terms gives an approximation of $(cos\theta)^{1/2}$ which is quite good for a surprisingly large range of θ values. In particular the error is less than 0.04 for $\theta \leq 1.2$ rad. Beyond that point the errors become more significant because this parabola only drops to 0.383 at $\theta = \frac{\pi}{2}$, where the target function is zero. To give a more accurate approximation it was rather arbitrarily decided to add one more term to eq. A.1 which would give the proper value for $\theta = \frac{\pi}{2}$ without significantly changing the values from eq. A.1 for smaller θ values. By trial and error the approximation was chosen as

$$(cos\theta)^{1/2} \simeq 1 - (\frac{\theta}{2})^2 - (0.5784\ \theta)^{10} \qquad (A.2)$$

The maximum error in this expression is about 0.12 for $\theta \simeq 1.35$, but the error in the integral of $(cos\theta)^{1/2}$ must be considerably less than this. One further test of eq. A.2 was made by squaring both sides of the relationship and integrating this expression from zero to θ^*. The left-hand-side, of course, gives $sin\theta^*$ and the right-hand-side approximates this within 1% over the entire range from zero to $\frac{\pi}{2}$. Thus, eq. A.2 was judged to be an adequate approximation, and its integral is exactly eq. 4.17.

For the integral of $(cos\theta)^{3/2}$ a slight modification of the above procedure was used. Obviously one could simply take the third power of eq. A.2 and integrate that expression, but a slightly simpler form was considered preferable. It is not very efficient to use the the first two terms in the power series expansion for $(cos\theta)^{3/2}$ since $1-\frac{3}{4}\theta^2$ decreases too rapidly, becoming negative for $\theta > 1.155$. The alternative used here was to approximate $(cos\theta)^{3/4}$ in much the same way as was done for $(cos\theta)^{1/2}$. Thus,

$$(cos\theta)^{3/4} \simeq 1 - \frac{3}{8}\theta^2 - (0.4912\ \theta)^{10} \qquad (A.3)$$

This is, in fact, a better approximation than eq. A.2, being within 0.015 for all pertinent θ values. This expression was then squared to give

$$\begin{aligned}(cos\theta)^{3/2} \simeq 1 &- \frac{3}{4}\theta^2 + \frac{9}{64}\theta^4 - 2(0.4912\ \theta)^{10} \\ &+ \frac{3}{4}\theta^2(0.4912\ \theta)^{10} + (0.4912\ \theta)^{20}\end{aligned} \qquad (A.4)$$

Integration then gives eq. 4.18.

APPENDIX B

SOME LIMITING APPROXIMATIONS

In reference 15, simplified approximations for $g(x)$ and $f(H)$ were presented for both very large and very small Y/N values. These approximations were both shown to give analytical approximations for $E(x^2)$, requiring no numerical integration. The following paragraphs present slight modification of these approximations and show that the other desired response moments, $E(x^4), E(\ddot{z}^2)$ and $E(\ddot{z}^4)$, can all be evaluated analytically as well.

B.1 Large Y/N

In order to model accurately the abrupt increase in damping at the level $x = A$, the $f(H)$ function is taken as

$$
\begin{aligned}
f(H) &= 2\beta_0 w_0 \qquad \text{for } H \le w_0^2 Y^2/2 \\
&= 2\beta_0 w_0 + (2w_0/\pi)\sqrt{\alpha}(1-\alpha)(2H/w_0^2Y^2 - 1) \qquad \text{for } H > w_0^2Y^2/2
\end{aligned}
\tag{B.1}
$$

For all values of α, the function $f(H)$ in eq. 4.10 tends to eq. B.1 as H tends to $w_0^2Y^2/2$. The approximation of $g(x)$ for this limiting case is simply taken as the expression for an unyielding system;

$$g(x) = w_0^2 x \tag{B.2}$$

For this system with nonlinear damping and linear spring the probability distribution of A can be written as

$$p_A(a) = C_1 a \exp(-r_1 a^2) \qquad for\ a \leq Y$$
$$= C_1 a \exp[-r_2 - r_3(a^2 + r_4)^2] \qquad for\ a > Y$$

with

$$r_1 = \frac{2\beta_0}{\pi N^2}$$
$$r_2 = \frac{2\beta_0}{\pi}\frac{Y^2}{N^2} - \frac{\beta_0^2 Y^2}{\sqrt{\alpha}(1-\alpha)N^2}$$
$$r_3 = \frac{\sqrt{\alpha}(1-\alpha)}{\pi^2 N^2 Y^2}$$
$$r_4 = (\frac{\pi\beta_0}{\sqrt{\alpha}(1-\alpha)} - 1)Y^2$$

In order to evaluate C_1 and the desired moments of x and $\ddot{z}$ (as in section 4) it is necessary to obtain the integrals of a^k times $p_A(a)$ for various even k values. To simplify the notation let

$$Q_k = \int_0^Y a^k \exp(-r_1 a^2) da$$

and

$$R_k = \int_Y^\infty a^k \exp[-r_2 - r_3(a^2 + r_4)^2] da$$

The Q_k terms can be evaluated from

$$Q_1 = \frac{1}{2r_1}[1 - \exp(-r_1 Y^2)]$$

and the recursive relationship

$$Q_k = -\frac{1}{2r_1} Y^{k-1} \exp(-r_1 Y^2) + \frac{k-1}{2r_1} Q_{k-2} \qquad for\ k \geq 2$$

The evaluation of R_k is simplified by using a change of variables $u = a^2 + r_4$. Then

$$R_k = \frac{1}{2}\exp(-r_2)\int_{Y^2+r_4}^{\infty}(u - r_4)^{(k-1)/2}\exp(-r_3u^2)du$$
$$= \frac{1}{2}\exp(-r_2)\sum_{j=0}^{\frac{(k-1)}{2}}\binom{\frac{(k-1)}{2}}{j}(-r_4)^{-j+(k-1)/2}S_j$$

in which

$$S_j = \int_{Y^2+r_4}^{\infty} u^j \exp(-r_3u^2)du$$

can be found from

$$S_0 = \frac{1}{2}\left(\frac{\pi}{r_3}\right)^{1/2} erfc[\sqrt{r_3}(Y^2 + r_4)]$$
$$S_1 = \frac{1}{2r_3}\exp[-r_3(Y^2 + r_4)^2]$$

and

$$S_j = \frac{1}{2r_3}(Y^2 + r_4)^{j-1}\exp[-r_3(Y^2 + r_4)^2] + \frac{j-1}{2r_3}S_{j-2} \qquad for\ j \geq 2$$

Using this notation one finds that

$$C_1^{-1} = Q_1 + R_1$$

$$E(x^2) = \frac{C_1}{2}(Q_3 + R_3)$$

$$E(x^4) = \frac{3C_1}{8}(Q_5 + R_5)$$

$$E(\ddot{z}^2) = \frac{C_1 w_0^4}{2}[(1+4\beta_0^2)(Q_3+R_3) + \frac{8\beta_0}{\pi Y^2}\sqrt{\alpha}(1-\alpha)(R_5 - Y^2 R_3)$$
$$+ \frac{4}{\pi^2 Y^4}\alpha(1-\alpha)^2(R_7 - 2Y^2 R_5 + Y^4 R_3)]$$

$$E(\ddot{z}^4) = \frac{3C_1 w_0^8}{8}\{(1+4\beta_0^2)^2(Q_5+R_5)$$
$$+ 8[\frac{2\beta_0}{\pi Y^2}\sqrt{\alpha}(1-\alpha)(R_7 - Y^2 R_5) + \frac{\alpha(1-\alpha)^2}{\pi^2 Y^4}(R_9 - 2Y^2 R_7 + Y^4 R_5)]$$
$$+ 16[\frac{4\beta_0^3}{\pi Y^2}\sqrt{\alpha}(1-\alpha)(R_7 - Y^2 R_5) + \frac{6\beta_0^2}{\pi^2 Y^4}\alpha(1-\alpha)^2(R_9 - 2Y^2 R_7 + Y^4 R_5)$$
$$+ \frac{4\beta_0}{\pi^3 Y^6}\alpha^{3/2}(1-\alpha)^3(R_{11} - 3Y^2 R_9 + 3Y^4 R_7 - Y^6 R_5)$$
$$+ \frac{\alpha^2(1-\alpha)^4}{\pi^4 Y^8}(R_{13} - 4Y^2 R_{11} + 6Y^4 R_9 - 4Y^6 R_7 + Y^8 R_5)]\}$$

B.2 Small Y/N

For Y sufficiently small the stiffness can be modelled by a linear spring with the post yielding stiffness;

$$g(x) = \alpha w_0^2 x$$

The appropriate nonlinear damping in this situation can be approximated from eq. 4.7 for A $\gg$ Y as

$$f[G(A)] \equiv f[\alpha w_0^2 A^2/2] = 2\beta_0 w_0 + \frac{4}{\pi} w_0 \frac{(1-\alpha)}{\sqrt{\alpha}} \frac{Y}{A}$$

The probability density can then be written as

$$p_A(a) = C_2 a \exp[-s_1(a+s_2)^2]$$

with

$$s_1 = \frac{2\alpha\beta_0}{\pi N^2}$$
$$s_2 = \frac{2(1-\alpha)Y}{\sqrt{\alpha}\beta_0\pi}$$

In this situation it is not necessary to divide the integrals from $a = 0$ to $a = \infty$ into two separate parts, so it is convenient to write the desired expressions directely in terms of moments of A. Thus, C_2 is found from $E(A^0) = 1$ and the response moments are

$$E(x^2) = \frac{1}{2}E(A^2)$$

$$E(x^4) = \frac{3}{8}E(A^4)$$

$$E(\ddot{z}^2) = \frac{\alpha w_0^4}{2}[4\beta_0^2 E(A^2) + \frac{16\beta_0}{\pi}\frac{1-\alpha}{\sqrt{\alpha}}YE(A) + \frac{16}{\pi^2}\frac{(1-\alpha)^2}{\alpha}Y^2] + \frac{\alpha^2 w_0^4}{2}E(A^2)$$

$$\begin{aligned} E(\ddot{z}^4) = {} & \frac{3\alpha^2 w_0^8}{8}[16\beta_0^4 E(A^4) + \frac{128\beta_0^3}{\pi}\frac{1-\alpha}{\sqrt{\alpha}}YE(A^3) \\ & + \frac{384\beta_0^2}{\pi^2}\frac{(1-\alpha)^2}{\alpha}Y^2E(A^2) + \frac{512\beta_0}{\pi^3}\frac{(1-\alpha)^3}{\alpha^{3/2}}Y^3E(A) + \frac{256}{\pi^4}\frac{(1-\alpha)^4}{\alpha^2}Y^4] \\ & + \frac{6\alpha^3 w_0^8}{8}[4\beta_0^2 E(A^4) + \frac{16\beta_0}{\pi}\frac{(1-\alpha)}{\sqrt{\alpha}}YE(A^3) + \frac{16}{\pi^2}\frac{(1-\alpha)^2}{\alpha}Y^2E(A^2)] \\ & + \frac{3\alpha^4 w_0^8}{8}E(A^4) \end{aligned}$$

The moments of A can be written as

$$E(A^k) = C_2\sum_{j=0}^{k+1}\binom{k+1}{j}(-s_2)^{k+1-j}T_j$$

with

$$T_j = \int_{s_2}^{\infty} u^j \exp(-s_1 u^2) du$$

for which

$$T_0 = \frac{1}{2}(\frac{\pi}{s_1})^{1/2} erfc(s_1^{1/2} s_2)$$

$$T_1 = \frac{1}{2s_1} \exp(-s_1 s_2^2)$$

and

$$T_j = \frac{(s_2)^{j-1}}{2s_1} \exp(-s_1 s_2^2) + \frac{j-1}{2s_1} T_{j-2} \qquad for\ j \geq 2$$

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