NATIONAL CENTER FOR EARTHQUAKE
ENGINEERING RESEARCH
State University of New York at Buffalo

A NEW SOLUTION TECHNIQUE FOR RANDOMLY EXCITED HYSTERETIC STRUCTURES

by

G.Q. Cai and Y.K. Lin
Center for Applied Stochastics Research
Florida Atlantic University
Boca Raton, Florida 33431-0991

Technical Report NCEER-88-0012

May 16, 1988

This research was conducted at Florida Atlantic University and was partially supported by the National Science Foundation under Grant No. ECE 86-07591.

NOTICE

A NEW SOLUTION TECHNIQUE FOR RANDOMLY EXCITED HYSTERETIC STRUCTURES

by

G.Q. Cai[1] and Y.K. Lin[2]

May 16, 1988

Technical Report NCEER-88-0012

NCEER Contract Number 87-2010

NSF Master Contract Number ECE 86-07591

1 Research Assistant, Center for Applied Stochastics Reseach, College of Engineering, Florida Atlantic University
2 Schmidt Professor of Engineering and Director, Center for Applied Stochastics Research, Florida Atlantic University

NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
State University of New York at Buffalo
Red Jacket Quadrangle, Buffalo, NY 14261

PREFACE

The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. Initially, the emphasis is on structures and lifelines of the types that would be found in zones of moderate seismicity, such as the eastern and central United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

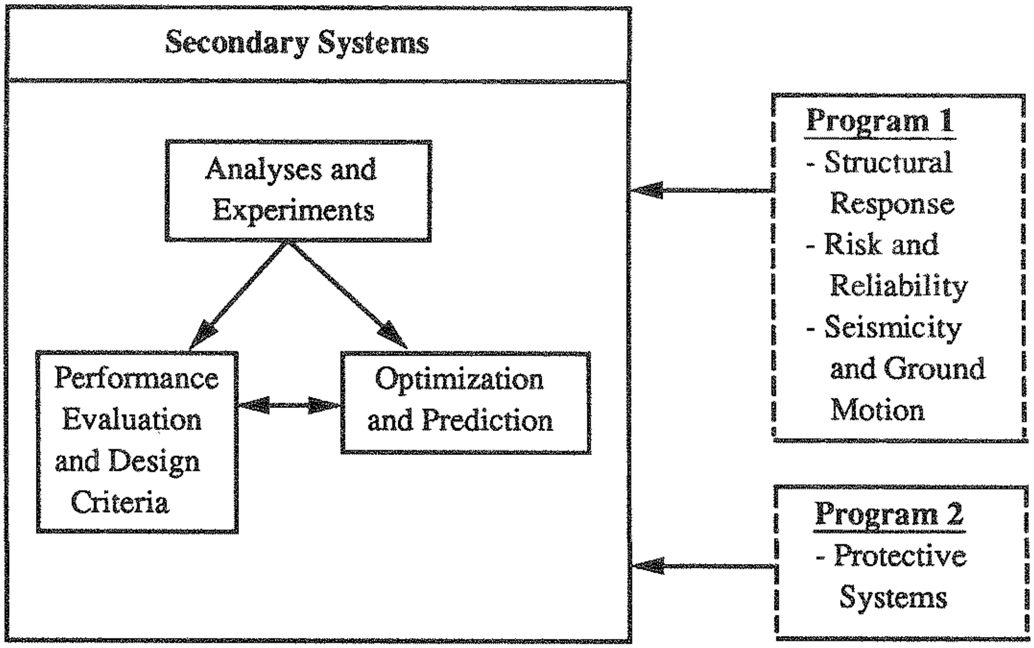
This technical report pertains to the second program area and, more specifically, to secondary systems.

In earthquake engineering research, an area of increasing concern is the performance of secondary systems which are anchored or attached to primary structural systems. Many secondary systems perform vital functions whose failure during an earthquake could be just as catastrophic as that of the primary structure itself.

The research goals in this area are to:

1. Develop greater understanding of the dynamic behavior of secondary systems in a seismic environment while realistically accounting for inherent dynamic complexities that exist in the underlying primary-secondary structural systems. These complexities include the problem of tuning, complex attachment configuration, nonproportional damping, parametric uncertainties, large number of degrees of freedom and nonlinearities in the primary structure.
2. Develop practical criteria and procedures for the analysis and design of secondary systems.
3. Investigate methods of mitigation of potential seismic damage to secondary systems through optimization or protection. The most direct route is to consider enhancing their performance through optimization in their dynamic characteristics, in their placement within a primary structure or in innovative design of their supports. From the point of view of protection, base isolation of the primary structure or the application of other passive or active protection devices can also be fruitful.

Current research in secondary systems involves activities in all three of these areas. Their interaction and interrelationships with other NCEER programs are illustrated in the accompanying figure.



Dynamics of a hysteretic structure is considered in this report. Under the action of a strong earthquake, the primary structure can deform into the hysteretic regime. This possibility must be taken into consideration in the analysis and design of secondary systems. Consequently, a thorough understanding of the hysteretic behavior of the primary structure is essential.

A new method of hysteretic structural analysis is developed in this report. In this procedure, a hysteretic system is replaced by another system belonging to a class of generalized stationary potential for which an exact solution is possible. It is shown that the method has broad applicability and it is accurate over a wide range of excitation levels and system parameters.

ABSTRACT

An approximate procedure is developed for computing the probability density and statistical properties for the response of a hysteretic system under Gaussian white-noise excitations. In this procedure, the original system is replaced by another system belonging to the class of generalized stationary potential for which an exact solution is possible. The replacement is based on a criterion that the statistical average of the energy dissipation remains the same for substituting and substituted systems. The method is applicable to either bilinear or smooth type hysteresis and for either hardening or softening hysteretic behaviors without the restriction that the response be a narrow-band process or the energy dissipation be small. Comparison of computed results with available simulation results indicates that the proposed method is accurate for wide ranges of excitation levels and system parameters.

ACKNOWLEDGEMENT

The research reported in this paper was supported by sub-contract No. NCEER 87-2010 under the auspices of the National Center for Earthquake Engineering Research under NSF Grant ECE-86-07591. Opinions, findings and conclusions expressed are those of the writers and do not necessarily reflect the views of our sponsors.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

SECTION 1

INTRODUCTION

In structural dynamics, the term hysteresis is used to describe a non-conservative system behavior, in which the restoring force depends not only on the instantaneous deformation but also the past history of deformation. If motion is cyclic, then a plot of restoring force versus deformation forms a closed loop, the area within the loop being the energy loss per cycle. Reinforced concrete structures are perhaps the most notable examples of hysteretic systems. Steel structures, while essentially linear within the elastic limit, also become hysteretic when loaded beyond that limit. Excellent reviews of hysteresis characteristics for various structural elements were given by Sozen [21] and Iwan [10].

The problem of hysteretic system under random excitation is difficult, and no mathematically exact solution is available at the present time. Most published approximate solutions have been restricted to single-degree-of-freedom systems, governed typically by the following equation of motion:

$$\frac{d^2X}{dt^2} + 2\zeta\omega_0\frac{dX}{dt} + F(t) = \Xi(t) \tag{1.1}$$

where $\Xi(t)$ is a stochastic process often assumed to be a Gaussian white noise, and

$$F(t) = \alpha\omega_0^2 X + (1-\alpha)\omega_0^2 Z(t), \qquad 0 < \alpha < 1 \tag{1.2}$$

Equation (1.2) describes a restoring force consisting of a linear component and a hysteretic component. In the limit $\alpha = 1$, the system reduces to a linear one, in which case ω_0 represents the natural frequency, and ζ represents the ratio of damping to the critical damping.

Equations (1.1) and (1.2) may be replaced by their non-dimensionized versions as follows

$$\ddot{x} + 2\zeta\dot{x} + f(\tau) = \xi(\tau) \tag{1.3}$$

$$f(\tau) = \alpha x + (1-\alpha)z(\tau) \tag{1.4}$$

using the transformations $x = \frac{X}{\Delta}$, $z = \frac{Z}{\Delta}$, $\tau = \omega_0 t$, $\zeta(\tau) = \Xi(\tau/\omega_0)/(\omega_0^2\Delta)$ and $f(\tau) = F(\tau/\omega_0)/(\omega_0^2\Delta)$ where Δ is a characteristic displacement. The hysteretic component z in equation (1.4) can often be modeled by a first order differential equation

$$\dot{z} = g(x,\dot{x},z) \tag{1.5}$$

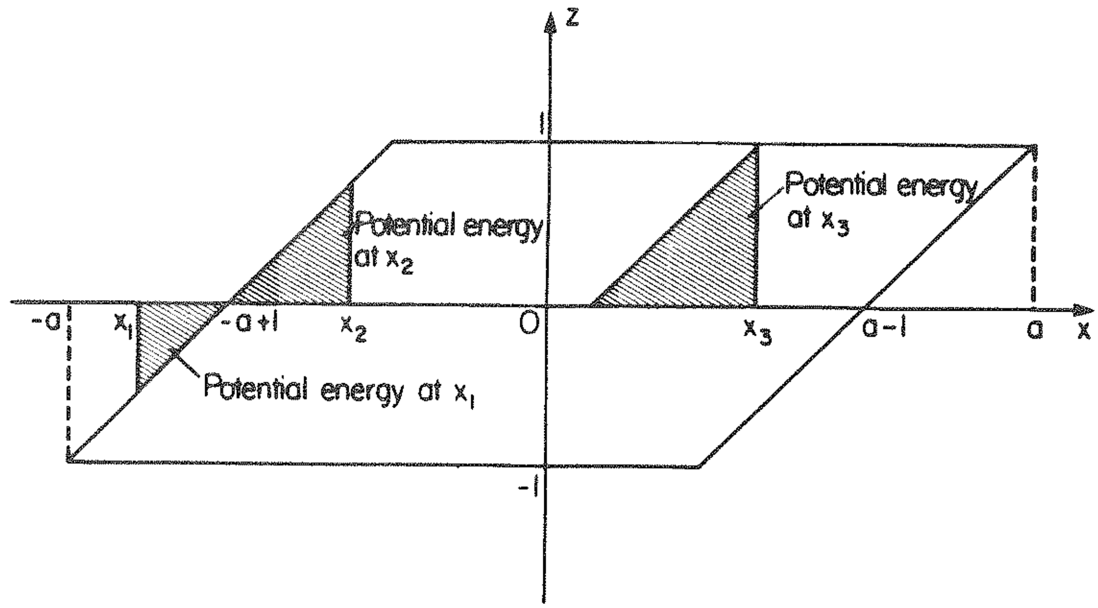
One simple example is the widely used elasto-plastic model,

$$\dot{z} = \begin{cases} 0 & |z| = 1 \\ \dot{x} & |z| < 1 \end{cases} \tag{1.6}$$

shown in Fig. 1-1(a) where the elastic limit has been chosen as the characteristic displacement Δ. The restoring force $f(\tau)$ obtained from combining this hysteresis component $z(\tau)$ and the linear component constitutes the well-known bilinear model shown in Fig. 1-1(b).

The first analysis of the response of a bilinear system to statistically stationary random excitation was due to Caughey [7], who used the methods of equivalent linearization and slowly varying parameters. Accurate results were obtained for the mean square response when the hysteretic component in the

(a)



(b)

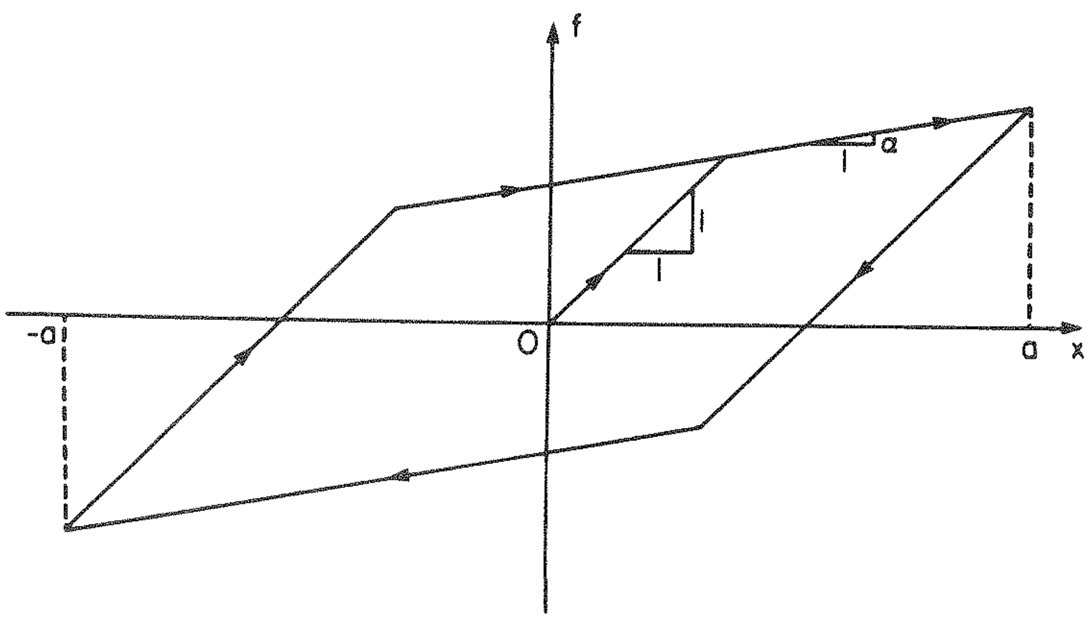


FIGURE 1-1 A bilinear hysteretic system: (a) elasto-plastic component; (b) restoring force-deformation relationship.

system is not dominant, namely, the coefficient α in (1.4) is not too small; significant errors were found when this is not the case [11].

For structural reliability estimations, the more detailed knowledge of the response, namely its probability distribution is required. Lutes [16] obtained an estimate of the response probability density for a bilinear hysteretic system using an equivalent non-hysteretic nonlinear system belonging to Caughey's class of solvable systems [8]. Selection of the equivalent system was based on equal energy dissipation. The applicability of his method requires that energy dissipation be small and the response be a narrow-band random process since a so-called resonant frequency must be calculated. However, the approximation does not appear to be more accurate than that of the equivalent linearization when values of the mean square response obtained from the two methods are compared. Subsequently, Lutes and Takemiya [17] devised a modified power balance method to improve the accuracy of the computed mean square response, especially for nearly elasto-plastic systems.

Roberts [18] and Zhu and Lei [28] have used another approach called stochastic averaging [23] in the treatment of bilinear hysteretic systems, and obtained the probability densities for the amplitude and energy envelope, respectively, for the stationary response by solving the so-called Fokker-Planck equations for the respective response variables. Again, applicability of the stochastic averaging method requires that the energy dissipation must be small. Kobori, Minai and Suzuki [e.g. 13] computed the statistical moments of the displacement and velocity using the Fokker-Planck equation for these variables and assumed forms for approximate probability densities.

The abrupt change of slope in the deformation-restoring force relationship in a bilinear model is obviously not realistic. To remedy this deficiency Iwan

[9] has proposed a distributed hysteretic model, composed of a number of Jenkin's elements in parallel, each of which consists of a linear spring in series with a Coulomb damper. This smooth hysteresis model was used by Takemiya and Lutes [24] in conjunction with the power balance method, and by Spanos [22] in conjunction with stochastic averaging and equivalent linearization in the investigation of system response.

Another smooth hysteresis model proposed initially by Bouc [3] and extended later by Wen [25] [26] is described by

$$\dot{z} = -\gamma |\dot{x}| z |z|^{n-1} - \beta\dot{x} |z|^{n} + A\dot{x} \tag{1.7}$$

Sketches of z versus x and the restoring force f versus x are shown in Figs. 1-2(a) and (b), respectively. By choosing different sets of values for γ, β, n and A the behaviors of real hysteretic systems can be approximated closely [2]. This model has become more widely used recently, since it is more amenable to analytical treatments. Further refinements of the model were proposed by Baber and Noori [1], and by Casciati [6]. The Galerkin method [25], Gaussian closure [12], equivalent linearization [26] and stochastic averaging [19] have all been applied to the analysis of this model with various degrees of success and limitations.

The references quoted above were limited to additive random excitations, namely excitations appearing as inhomogeneous terms in the equation of motion. One exception was the work by Shih and Lin [20] in which a multiplicative random excitation, appearing in a coefficient in the governing equation, was also included in the analysis. In this case, the multiplicative excitation arose due to vertical ground motions during an earthquake. Both additive and multiplicative random excitations were modeled as amplitude-modulated

(a)

(b)

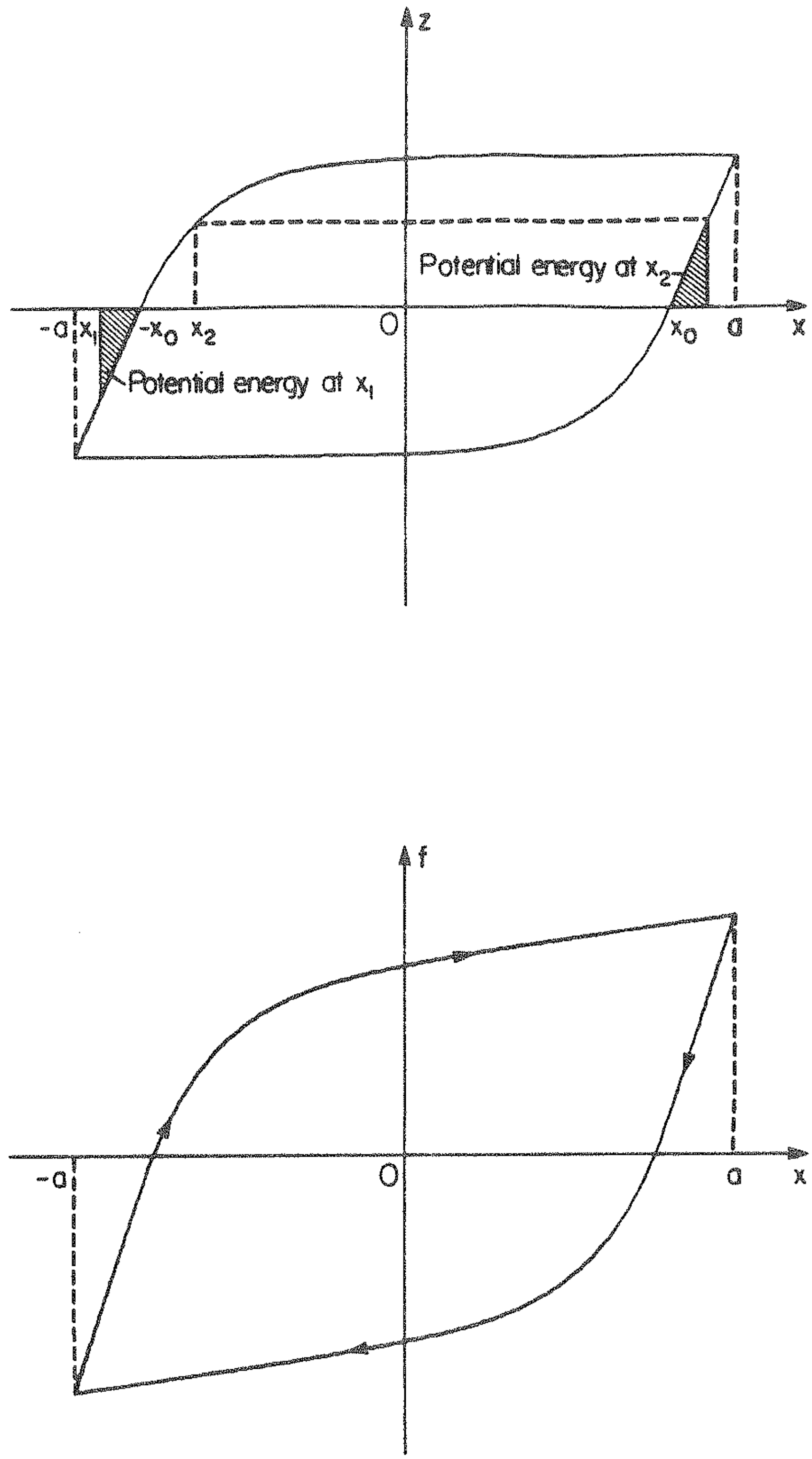


FIGURE 1-2 A smooth hysteretic system: (a) hysteretic component; (b) restoring force-deformation relationship.

nonstationary processes, and the Gaussian closure technique was employed to compute the statistics of the structural response.

In this paper, a new approximate procedure, developed recently by the authors [15,5] for computing the probability density of the response of a nonlinear oscillator to Gaussian white-noise excitations, is extended for application to hysteretic systems. The excitations can be either additive, or multiplicative, or both. The approximation is obtained from a class of exact solutions, called the class of generalized stationary potential [14,4], which includes Caughey's solutions [8] as special cases. The approximation is also based on balancing the statistical averages of energy dissipation, but the approach is more direct, and most importantly, it can be applied without the restrictions of small dissipated energy or being a narrow-band random process. The class of generalized stationary potential is richer than Caughey's class of exact solutions to allow for more accurate approximations, as to be demonstrated by comparison with earlier results and Monte Carlo simulations.

SECTION 2

METHOD OF ENERGY DISSIPATION BALANCING

Consider a general single-degree-of-freedom nonlinear system

$$\ddot{X} + h(x,\dot{x}) = f_i(x,\dot{x})\, W_i(\tau) \tag{2.1}$$

where $W_i(\tau)$ are Gaussian white noises with delta-type correlation functions

$$E[W_i(\tau)W_j(\tau + u)] = 2\pi K_{ij}\delta(u) \tag{2.2}$$

It has been shown [4] that the response of the stochastic system (2.1) has an exact stationary probability density

$$p_s(x,\dot{x}) = C\exp[-\phi(\lambda)] \tag{2.3}$$

where

$$\lambda = \frac{1}{2}\dot{x}^2 + \int g(x)dx = \frac{1}{2}\dot{x}^2 + G(x) \tag{2.4}$$

if function $h(x,\dot{x})$ in (2.1) can be expressed in the form of

$$h(x,\dot{x}) = \pi\dot{x}K_{ij}f_i(x,\dot{x})f_j(x,\dot{x})\phi'(\lambda) - \pi K_{ij}f_i(x,\dot{x})\frac{\partial}{\partial\dot{x}}f_j(x,\dot{x}) + g(x) \tag{2.5}$$

in which case the system is said to belong to the class of generalized stationary potential. In equations (2.3) and (2.5), C is a normalization constant, ϕ is an arbitrary function, and a prime denotes one differentiation with respect to the argument. Of course, equation (2.3) is a valid probability density only if it is normalizable, namely, integration on x and $\dot{x}$ over the state space is finite.

If the multiplicative random excitations are absent or if they do not appear in the coefficient of $\dot{x}$, then g(x) can be identified as the conservative spring force of the system; otherwise, it is the effective spring force which may include the well-known correction term of Wong and Zakai [27]. Therefore, λ given in equation (2.4) is either the total kinetic and potential energy or the effective total energy. The integral shown in (2.4) may be treated as an indefinite integral and the choice of an integration constant is unnecessary since it can be compensated when computing the normalization constant C in (2.3).

From the point of view of obtaining an exact solution, functions h, g and f_i and the spectral constants K_{ij} in equation (2.5) are given. The objective is to find a consistent $\phi'(\lambda)$ function which satisfies equation (2.5). When equation (2.5) can indeed be satisfied, the stochastic system is said to belong to the class of generalized stationary potential. To our knowledge, this class includes all the published exact solutions to date.

In many practical cases, equation (2.5) imposes severe restrictions which cannot be satisfied exactly. However, a substituting system can be found within the class of generalized stationary potential which is closest to the original system in some statistical sense. The criterion we propose to select such a substituting system is that the ensemble average of the dissipated energy remains the same for the substituting and the substituted systems. Application of this criterion yields

$$E[\dot{x}h(x,\dot{x}) - \pi\dot{x}^2 K_{ij}f_i f_j \phi'(\lambda) + \pi\dot{x}K_{ij}f_i \frac{\partial f_j}{\partial \dot{x}}] = 0 \qquad (2.6)$$

where E[ ] denotes ensemble averaging. It follows upon using the approximate probability density (2.3) to evaluate the above ensemble average [5],

$$\phi'(\lambda) = \frac{\int_{a_1(\lambda)}^{a_2(\lambda)} \{[h + \pi K_{ij} f_i \frac{\partial f_j}{\partial \dot{x}}]_{\dot{x} = \sqrt{2\lambda - 2G(x)}} - [h + \pi K_{ij} f_i \frac{\partial f_j}{\partial \dot{x}}]_{\dot{x} = -\sqrt{2\lambda - 2G(x)}}\} dx}{\pi \int_{a_1(\lambda)}^{a_2(\lambda)} \{[\dot{x} K_{ij} f_i f_j]_{\dot{x} = \sqrt{2\lambda - 2G(x)}} - [\dot{x} K_{ij} f_i f_j]_{\dot{x} = -\sqrt{2\lambda - 2G(x)}}\} dx} \tag{2.7}$$

where $a_1(\lambda)$ and $a_2(\lambda)$ are the two roots of the equation $G(x) = \lambda$; they are the minimum and the maximum values of x, respectively, corresponding to a given effective energy level λ, as depicted in Fig. 2-1.

Equation (2.7) can be greatly simplified if the hysteresis constitutive law is symmetric, so that $a_1(\lambda) = - a_2(\lambda) = a(\lambda)$ and if only an additive Gaussian white-noise excitation is present, in which case

$$\phi'(\lambda) = \frac{2\zeta}{\pi K} + \frac{A_r(\lambda)}{2\pi K \int_{-a(\lambda)}^{a(\lambda)} \sqrt{2\lambda - 2G(x)} dx} \tag{2.8}$$

where $A_r(\lambda)$ is the area of the hystersis loop corresponding to a given λ, and K is the spectral density of the additive random excitation.

Substituting (2.8) into (2.3), we obtain an approximate stationary probability density for the hysteretic system

$$p_s(x,\dot{x}) = C\exp\{- \frac{2\zeta}{\pi K}\lambda - \int \frac{A_r(\lambda)}{2\pi K \int_{-a(\lambda)}^{a(\lambda)} \sqrt{2\lambda - 2G(x)} dx} d\lambda\} = \rho(\lambda) \tag{2.9}$$

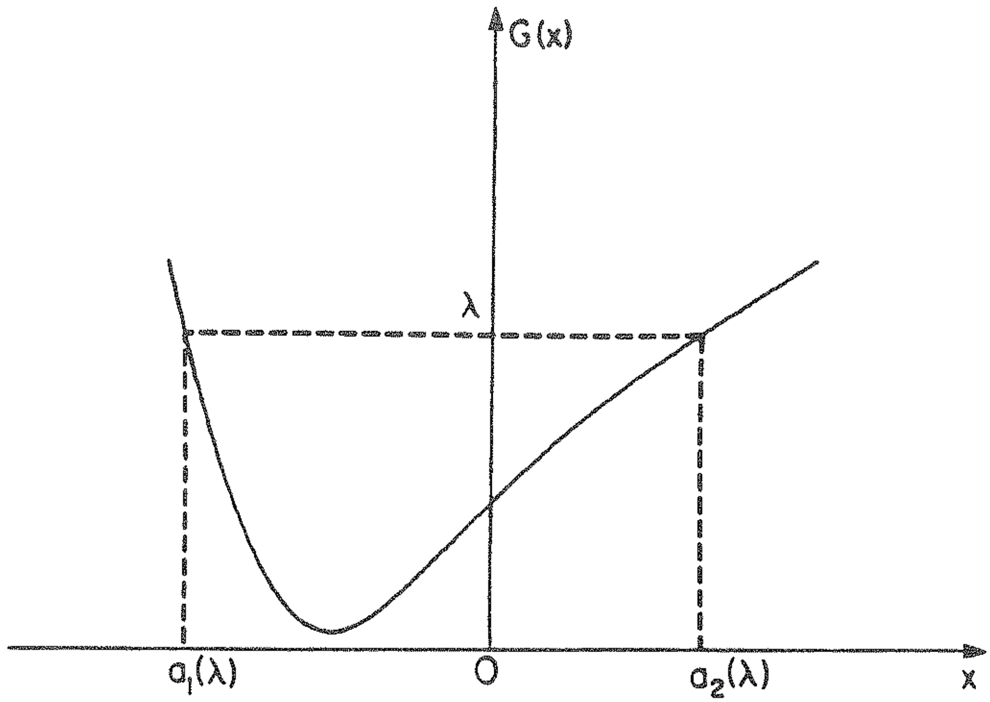


FIGURE 2-1 Potential energy function G(x).

which depends only on λ. We hasten to add that this expression is an approximate probability density for x and $\dot{x}$, but not for λ. The mean-square value of the displacement may be computed from

$$E[x^2] = \iint_{-\infty}^{+\infty} x^2 p_s(x,\dot{x})dxd\dot{x} = \int_0^{\infty} \rho(\lambda) \left[\int_{-a(\lambda)}^{a(\lambda)} \frac{2x^2}{\sqrt{2\lambda - 2G(x)}} dx \right] d\lambda \qquad (2.10)$$

The probability densities for the energy level λ and the amplitude a, respectively, can be found readily from equation (2.9). Specifically,

$$p_s(\lambda) = 2\rho(\lambda) \int_{-a(\lambda)}^{a(\lambda)} \frac{dx}{\sqrt{2\lambda - 2G(x)}} \qquad (2.11)$$

and

$$p_s(a) = 2 \mid g(a) \mid \rho[\lambda(a)] \int_{-a}^{a} \frac{dx}{\sqrt{2\lambda(a) - 2G(x)}} \qquad (2.12)$$

As indicated, λ must be expressed in terms of the amplitude a in equation (2.12). Once the area A_r within a hysteresis loop and the effective spring force $g(x)$ are determined, the probability densities (2.9), (2.11) and (2.12) and the mean-square displacement can be evaluated numerically.

SECTION 3

APPLICATION TO BILINEAR HYSTERETIC SYSTEM

A bilinear hysteretic system is described mathematically by equations (1.3), (1.4) and (1.6), depicted graphically in Figs. 1-1(a,b), and represented schematically in Fig. 3-1. The area of the hysteresis loop A_r, representing the dissipated energy per cycle at amplitude a, which is attributable solely to the hysteretic property can be obtained by inspection:

$$A_r = \begin{cases} 0, & a \le 1 \\ 4(a - 1), & a \ge 1 \end{cases} \tag{3.1}$$

The potential energy of the system accumulated in the two spring elements, shown in Fig. 3-1, represents the ability of the system to return to a local equilibrium upon removal of the external force. Its values for different ranges of x can be computed by referring to the shaded areas in Fig. 1-1(a). Specifically,

$$G(x) = \begin{cases} \frac{1}{2}x^2, & a \le 1 \\ \frac{1}{2}\alpha x^2 + \frac{1}{2}(1-\alpha)(x + a-1)^2; & a \ge 1,\ -a \le x \le -a + 2 \\ \frac{1}{2}\alpha x^2 + \frac{1}{2}(1-\alpha); & a \ge 1,\ -a + 2 \le x \le a \end{cases} \tag{3.2}$$

Plots of G(x) versus x for $\alpha = 1/21$ are shown in Fig. 3-2 for increasing $x(\dot{x}>0)$. Plots corresponding to decreasing $x(\dot{x} < 0)$ are mirror images of those shown in Fig. 3-2. The potential energy of a hysteretic system is seen to be a non-unique function of the displacement x, since it also depends on the amplitude a

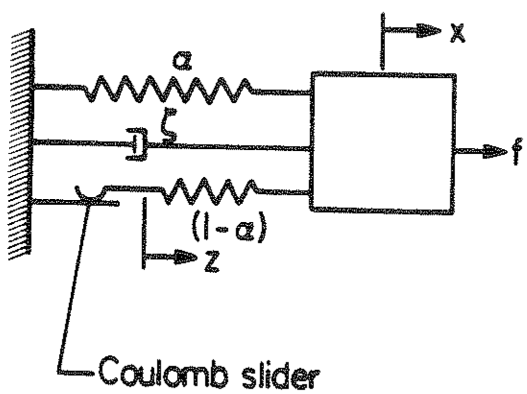


FIGURE 3-1 Schematic representation of a bilinear hysteretic system.

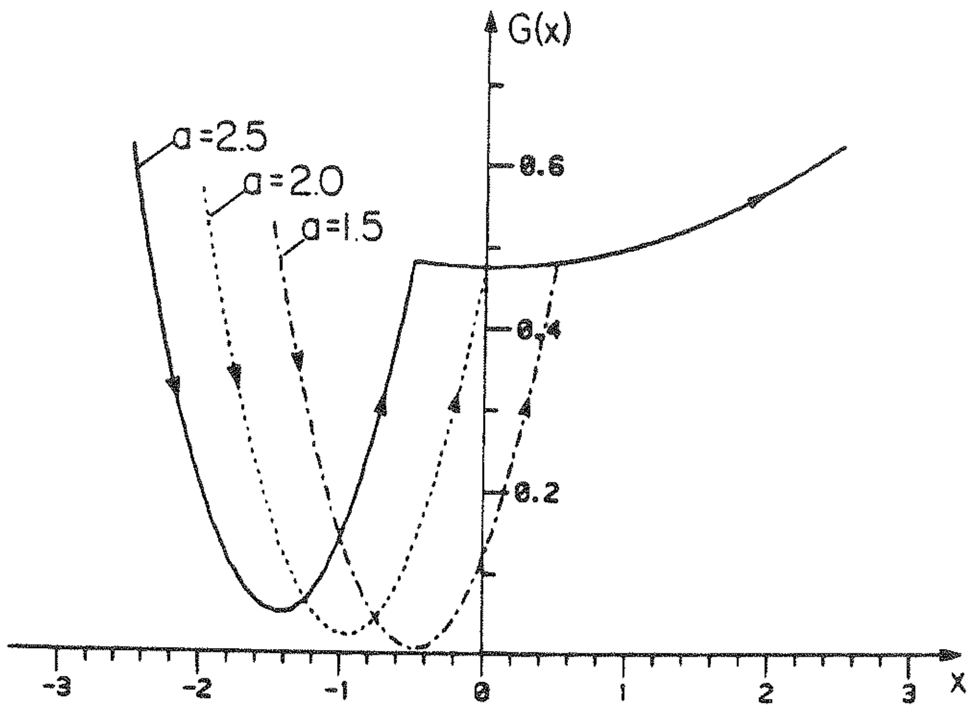


FIGURE 3-2 Potential energy of a bilinear hysteretic system with $\alpha = 1/21$, $\dot{x} \geq 0$.

and whether x is increasing or decreasing. Finally, the total energy λ is related to the amplitude as

$$\lambda = \begin{cases} \frac{1}{2}a^2, & a \le 1 \\ \frac{1}{2}(1-\alpha) + \frac{1}{2}\alpha a^2, & a > 1 \end{cases} \qquad (3.3)$$

Using equations (3.2) and (3.3), we obtain various integrals which appear in equations (2.9) - (2.11) as follows

$$\int_{-a}^{a} \frac{dx}{\sqrt{2\lambda - 2G(x)}} = \begin{cases} \pi, & a \le 1 \\ \frac{\pi}{2}\left(1 + \frac{1}{\sqrt{\alpha}}\right) + \sin^{-1} \frac{1+\alpha-a\alpha}{1-\alpha+a\alpha} + \frac{1}{\sqrt{\alpha}} \sin^{-1} \frac{a-2}{a}, & a \ge 1 \end{cases} \qquad (3.4)$$

$$\int_{-a}^{a} \sqrt{2\lambda-2G(x)}\,dx = \begin{cases} \frac{1}{2}\pi a^2, & a \le 1 \\ (a-1)(1-\alpha)\sqrt{\alpha(a-1)} + \frac{(1-\alpha+a\alpha)^2}{2}\left(\frac{\pi}{2} + \sin^{-1} \frac{1+\alpha-a\alpha}{1-\alpha+a\alpha}\right) \\ \quad + \frac{\sqrt{\alpha}}{2} a^2\left(\frac{\pi}{2} + \sin^{-1} \frac{a-2}{a}\right), & a \ge 1 \end{cases} \qquad (3.5)$$

$$\int_{-a}^{a} \frac{x^2}{\sqrt{2\lambda-2G(x)}}dx = \begin{cases} \frac{1}{2}\pi a^2, & \alpha \le 1 \\ (4a-5-3a\alpha + 3\alpha + \frac{2-a}{\alpha})\sqrt{\alpha(a-1)} + \\ + \frac{1}{2}[3(a-1)^2(1-\alpha)^2+a(2-2\alpha-a+2a\alpha)](\frac{\pi}{2}+\sin^{-1}\frac{1+\alpha-a\alpha}{1-\alpha+a\alpha}) \\ + \frac{a^2}{2\sqrt{\alpha}}(\frac{\pi}{2} + \sin^{-1}\frac{a-2}{a}), & \alpha \ge 1 \end{cases} \quad (3.6)$$

Substituting (3.1), (3.4), (3.5) and (3.6) into (2.9) through (2.12) and integrating with respect to λ numerically, we can obtain the stationary probability density $p_S(x,\dot{x})$, $p_S(\lambda)$ and $p_S(a)$, as well as the mean square response $E[x^2]$.

A nearly elasto-plastic system (α=1/21) under white noise excitation was studied numerically. Different values for the viscous damping coefficient (ζ=0, 0.01, 0.05) were assumed for the investigation. The root-mean-square response σ_X computed for different ζ values and normalized with respect to $D=\sqrt{2K}$ is shown in Fig. 3-3(a)-(c) where D is a measure of the excitation strength. The results appear to be in excellent agreement with known analog simulations [11] at all response levels and for all three values of ζ. Also shown in Fig. 3-3(a)-(c) are results obtained from equivalent linearization [11], which do not agree as well with the simulation results, particularly in the range of intermediate excitation levels. Yet, it is in this range that the hysteretic component in the restoring force plays a dominant role. The comparison suggests that the present procedure is much better suited for the analysis of bilinear hysteretic systems.

Figs. 3-4(a)-(c) depict the computed stationary probability density for the total response energy λ for the same system. It is seen that when the excitation strength is either weak (D=0.05) or strong (D=3), the total energy λ follows approximately an exponential probability distribution, similar to that of a linear system. In such cases, hysteresis effect is unimportant, and the linear viscous effect has a much greater influence on the probabilistic structure of the response. This is also borne out in the computed mean-square properties, shown in Figs. 3-3(a) - (c), and explains why linearization technique can lead to somewhat more acceptable results in these ranges. At an intermediate excitation strength (D=0.5), however, the energy dissipated by the hysteresis component is important. Therefore, the probability distribution of the total energy deviates greatly from being exponential, as shown in Fig. 3-4(b).

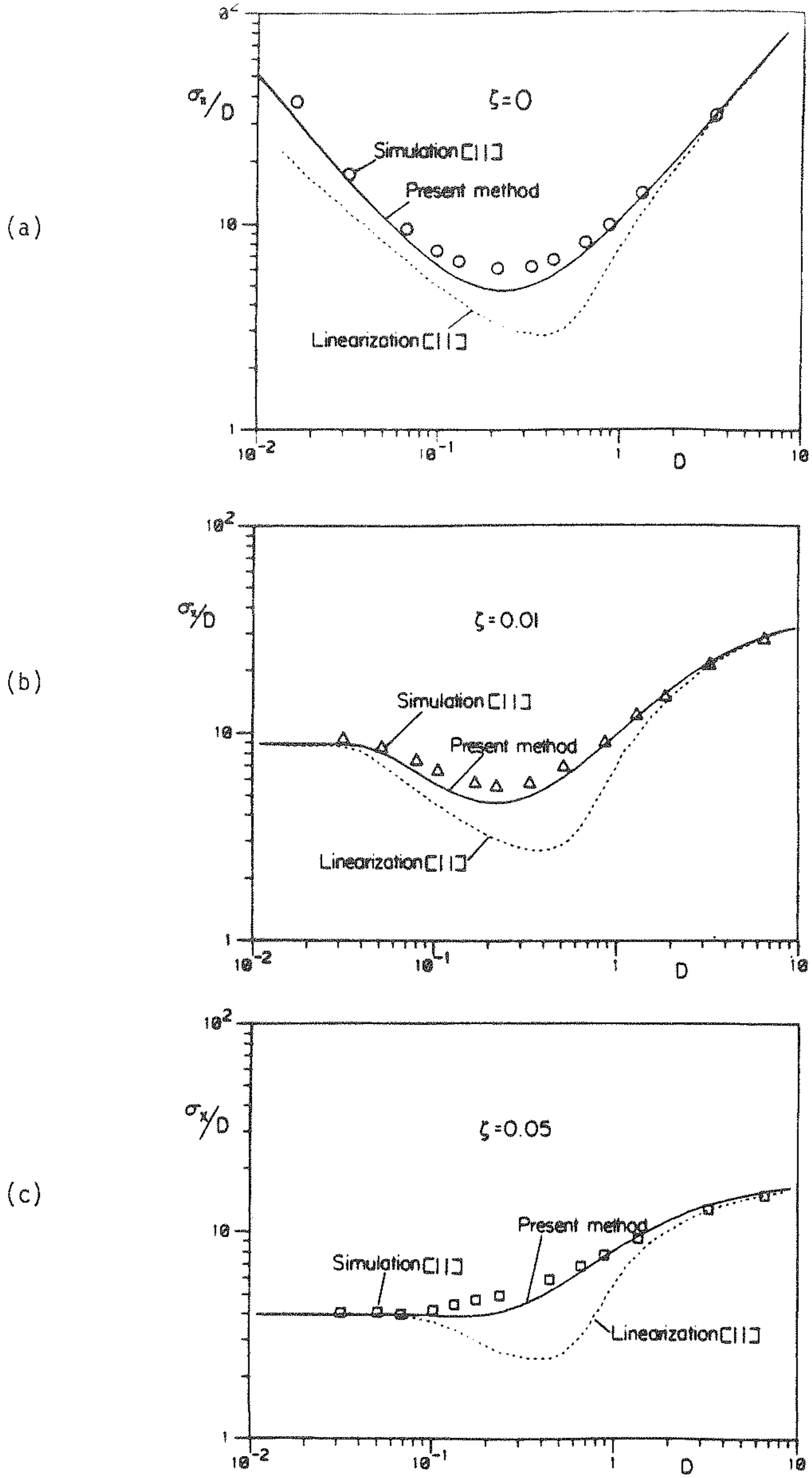


FIGURE 3-3 RMS response of a bilinear hysteretic system with $\alpha = 1/21$: (a) $\zeta = 0$, (b) $\zeta = 0.01$, (c) $\zeta = 0.05$.

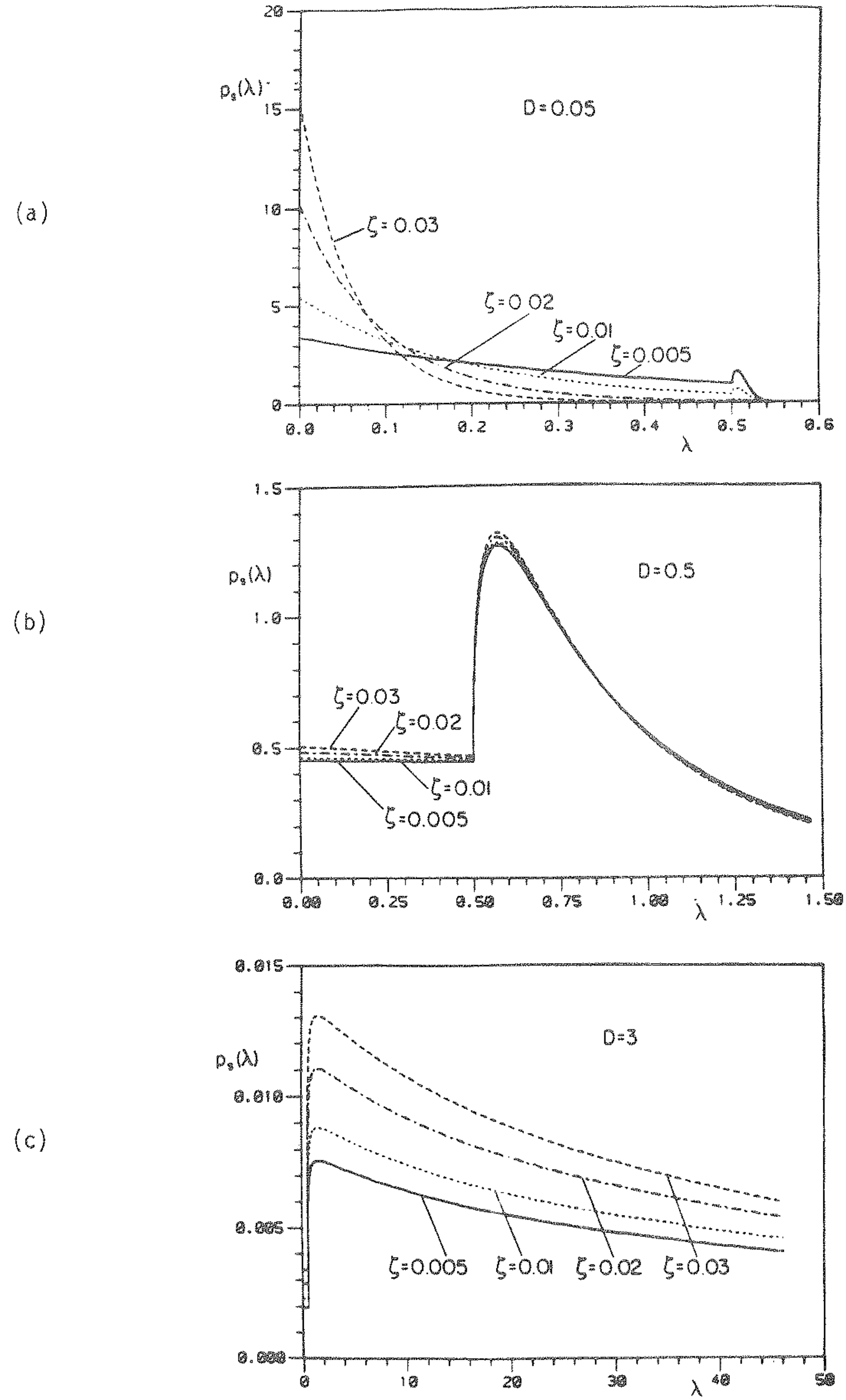


FIGURE 3-4 Stationary probability density of total energy for a bilinear hysteretic system with $\alpha = 1/21$: (a) weak excitation $D = 0.05$, (b) intermediate excitation $D = 0.5$, (c) strong excitation $D = 3$.

SECTION 4

APPLICATION TO SMOOTH HYSTERETIC SYSTEMS

To illustrate how the present procedure can be applied to smooth hysteretic systems, consider the Bouc-Wen model for the hysteretic component governed by equation (1.7). The smoothness of the force-displacement curve for this model is controlled by n, the general slopes of the curve controlled by $\gamma + \beta$ and the slimness of the hysteresis loop by γ.

Equation (1.7) can be transformed to

$$\frac{dz}{dx} = \begin{cases} A + (\gamma - \beta)\,|z|^n, & \dot{x} \geq 0, \quad z \leq 0 \\ A - (\gamma + \beta) z^n, & \dot{x} \geq 0, \quad z \geq 0 \\ A + (\gamma - \beta) z^n, & \dot{x} \leq 0, \quad z \geq 0 \\ A - (\gamma + \beta)\,|z|^n, & \dot{x} \leq 0, \quad z \leq 0 \end{cases} \tag{4.1}$$

The area of hysteresis loop corresponding to a given amplitude a is computed from the integral

$$A_r = 2 \int_{-a}^{a} z(x)dx \tag{4.2}$$

The total potential energy, representing the ability of the system to return to a local equilibrium, again consists of two parts: one stored in the linear element, another in the hysteretic element. The latter may be computed by referring to the shaded areas shown in Fig. 1-2(a) for different values of x. For a specific system, the functional relationship between z and x can be obtained by integrating (4.1). Then the hysteresis loop area A_r as a function of amplitude a, and the total potential energy G(x) can also be obtained in closed forms.

In what follows, the case of $n = 1$ and $A = 1$ will be considered in more detail. The assumption of $A = 1$ does not result in a loss of generality, because it can always be accomplished by adjusting the characteristic length Δ when normalizing equations (1.1) and (1.2). For this special case, the hysteretic component of the restoring force is found to be

$$z(x) = \begin{cases} \dfrac{1}{\gamma-\beta}[1-e^{-(\gamma-\beta)(x+x_0)}]; & -a \le x \le -x_0, \quad \gamma \ne \pm\beta \\ \\ \dfrac{1}{\gamma+\beta}[1-e^{-(\gamma+\beta)(x+x_0)}]; & -x_0 \le x \le a, \quad \gamma \ne \pm\beta \end{cases}$$

$$z(x) = \begin{cases} x+x_0; & -a \le x \le -x_0, \ \gamma = \beta \\ \\ \dfrac{1}{2\gamma}[1-e^{-2\gamma(x+x_0)}]; & -x_0 \le x \le a, \quad \gamma = \beta \end{cases} \tag{4.3}$$

$$z(x) = \begin{cases} \dfrac{1}{2\gamma}[1-e^{-2\gamma(x+x_0)}]; & -a \le x \le -x_0, \ \gamma = -\beta \\ \\ x+x_0; & -x_0 \le x \le a, \quad \gamma = -\beta \end{cases}$$

where x_0, shown in Fig. 1-2(b) is uniquely determined for a given amplitude a by solving $z(\pm x_0)=0$. The area of the hysteresis loop corresponding to a given amplitude is given by

$$A_r = \frac{4}{\gamma^2-\beta^2}\{\gamma a - \beta x_0 + \frac{\gamma}{\gamma+\beta}[e^{-(\gamma+\beta)(a+x_0)}-1]\}, \quad \gamma \neq \pm\beta$$

$$A_r = -(a-x_0)^2 + \frac{2x_0}{\gamma}, \quad \gamma = \beta \qquad (4.4)$$

$$A_r = (a+x_0)^2 - \frac{2x_0}{\gamma}, \quad \gamma = -\beta$$

and the potential energy by

$$G(x) = \begin{cases} \frac{1}{2}\alpha x^2 + \frac{1-\alpha}{\gamma-\beta}\{x+x_0+ \frac{1}{\gamma-\beta}[e^{-(\gamma-\beta)(x+x_0)}-1]\}; & -a \leq x \leq -x_0, \quad \gamma \neq \pm\beta \\ \frac{1}{2}\alpha x^2 + \frac{1-\alpha}{\gamma^2-\beta^2}\{1-e^{-(\gamma+\beta)(x+x_0)} - \frac{\gamma+\beta}{\gamma-\beta}\ell n[1+ \frac{\gamma+\beta}{\gamma+\beta}(1-e^{-(\gamma+\beta)(x+x_0)})]\}; & -x_0 \leq x \leq a, \ \gamma \neq \pm\beta \end{cases}$$

$$G(x) = \begin{cases} \frac{1}{2}\alpha x^2 + \frac{1-\alpha}{2}(x+x_0)^2; & -a \leq x \leq -x_0, \quad \gamma = \beta \\ \frac{1}{2}\alpha x^2 + \frac{1-\alpha}{8\gamma^2}[1-e^{-2\gamma(x+x_0)}]^2; & -x_0 \leq x \leq a, \quad \gamma = \beta \end{cases}$$

$$G(x) = \begin{cases} \frac{1}{2}\alpha x^2 + \frac{1-\alpha}{2\gamma}(x+x_0) + \frac{1-\alpha}{4\gamma^2}[e^{-2\gamma(x+x_0)}-1]; & -a \leq x \leq -x_0, \ \gamma = -\beta \\ \frac{1}{2}\alpha x^2 + \frac{1-\alpha}{2\gamma}\{x+x_0 - \frac{1}{2\gamma}\ell n[1+2\gamma(x+x_0)]\}; & -x_0 \leq x \leq a, \quad \gamma = -\beta \end{cases} \qquad (4.5)$$

The potential energy of such a smooth hysteretic system is illustrated in Fig. 4-1 for α=1/21, $\gamma=\beta$=0.5 and different amplitudes and for increasing x. Plots for decreasing x are mirror images of those shown in Fig. 4-1. With the above results, it is rather simple to compute the stationary probability densities for the state variables x and $\dot{x}$, the total energy λ, and the amplitude a, as well as the mean square response using equations (2.9) - (2.12).

A smooth hysteretic system with α=1/21, $\gamma=\beta$=0.5 was studies numerically in

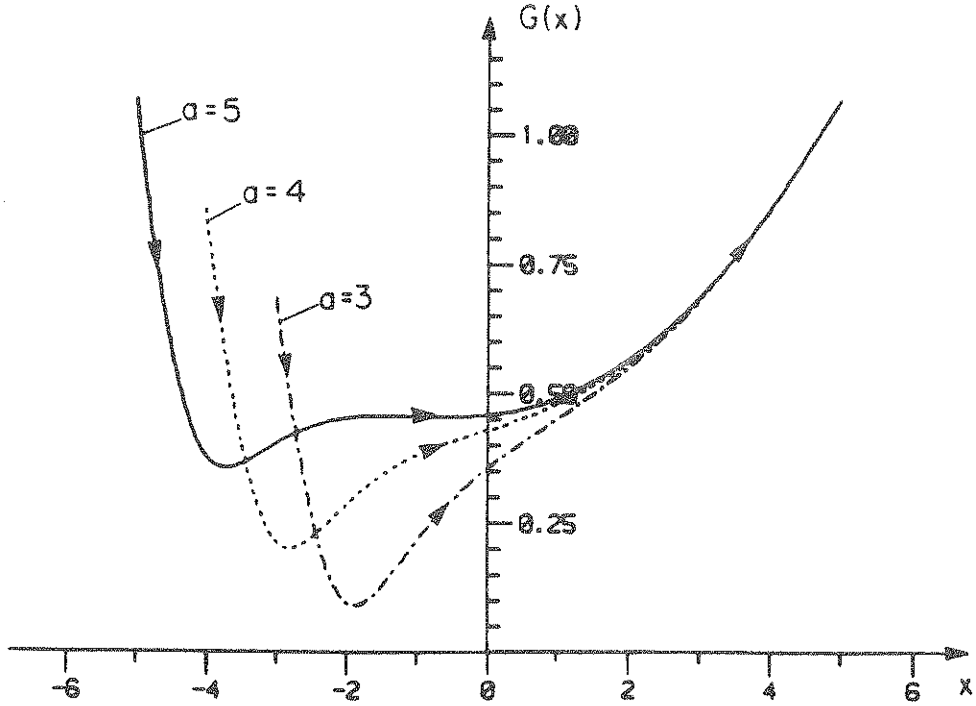


FIGURE 4-1 Potential energy of a smooth hysteretic system with α = 1/21 and $\gamma = \beta = 0.5$, $\dot{x} \geq 0$.

detail using the present dissipation-energy-balancing procedure. The computed mean-square displacements are shown in Figs. 4-2(a) and (b) for $\zeta=0$ and $\zeta=0.05$, respectively. Earlier results obtained by Wen [26] using equivalent linearization and digital simulation, and by Iyenger and Dash [12] using Gaussian closure are shown also in the figures for comparison. Results from both equivalent linearization and dissipation energy balancing agree with the simulation results.

The computed probability densities for the response amplitude in the absence of linear viscous damping are shown in Figs. 4-3 (a)-(c) with the excitation strength D as a varying parameter. As D increases, the change of the general shape of the probability distribution is again extremely interesting. Under either weak ($D < 0.25$) or strong ($D > 2$) excitation, the amplitude distribution is closer to Rayleigh, indicating that the linear component of the restoring force has the dominant effect. Under excitations of intermediate range ($0.5 < D < 2$), both the linear and hysteretic components contribute importantly, giving rise to two peaks in the amplitude probability density, as shown in Fig. 4-3(b).

The effect of linear viscous damping on the probability distribution of the response amplitude is illustrated in Fig. 4-4(a)-(c). The added damping does not appear to change the general shape of the distribution which is primarily controlled by the strength of excitation. Thus, the shape is closer to the single-peak Rayleigh distribution when the excitation strength is either weak (D=0.2) or very strong (D=5). It becomes a bi-modal distribution in the intermediate range of excitation strength (D=1). As expected, the additional viscous damping shifts a peak of the probability density to the left and shortens the tail of the distribution. This effect, however, is much greater

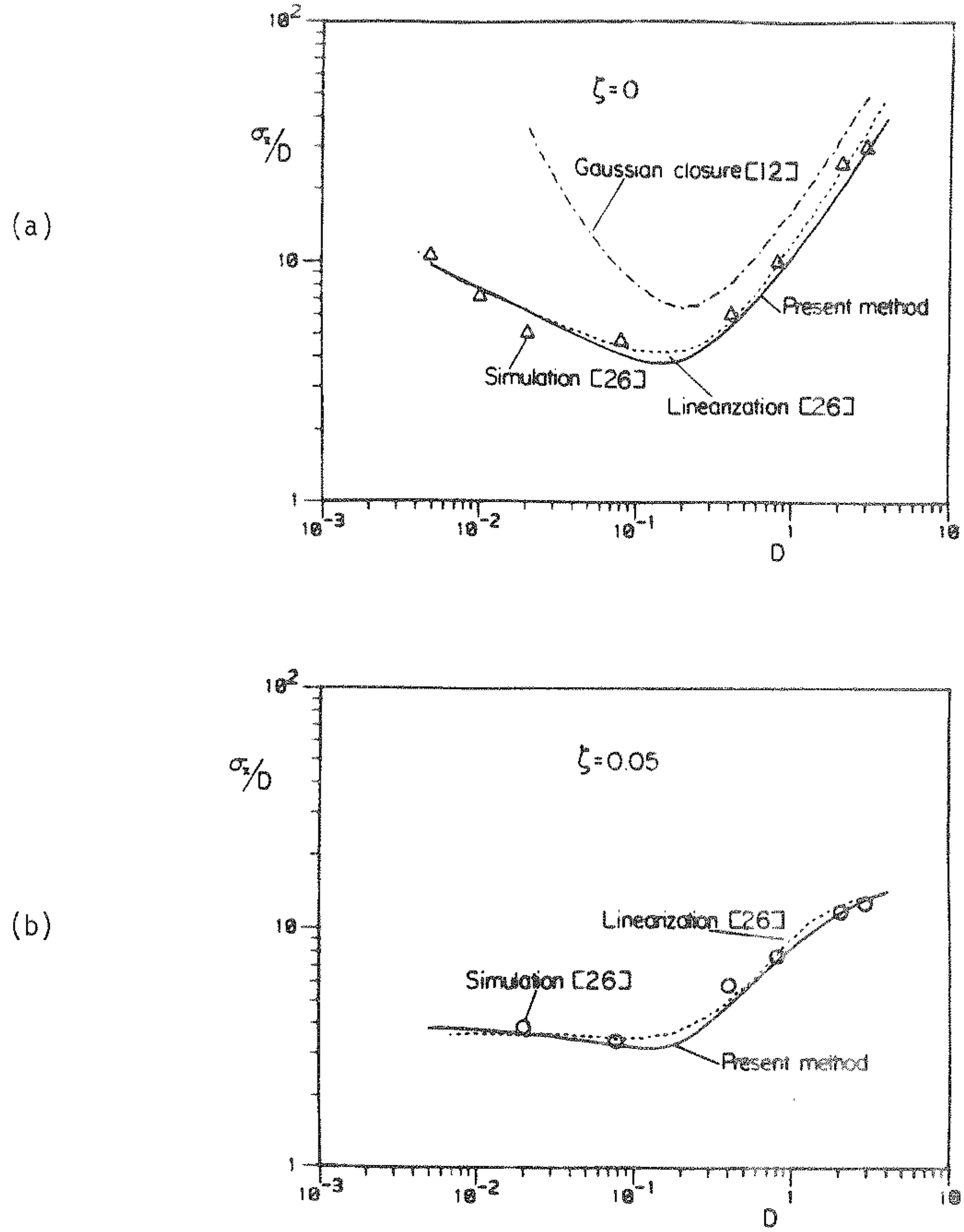


FIGURE 4-2 RMS response of a smooth hysteretic system with $\alpha = 1/21$ and $\gamma = \beta = 0.5$: (a) $\zeta = 0$, (b) $\zeta = 0.05$.

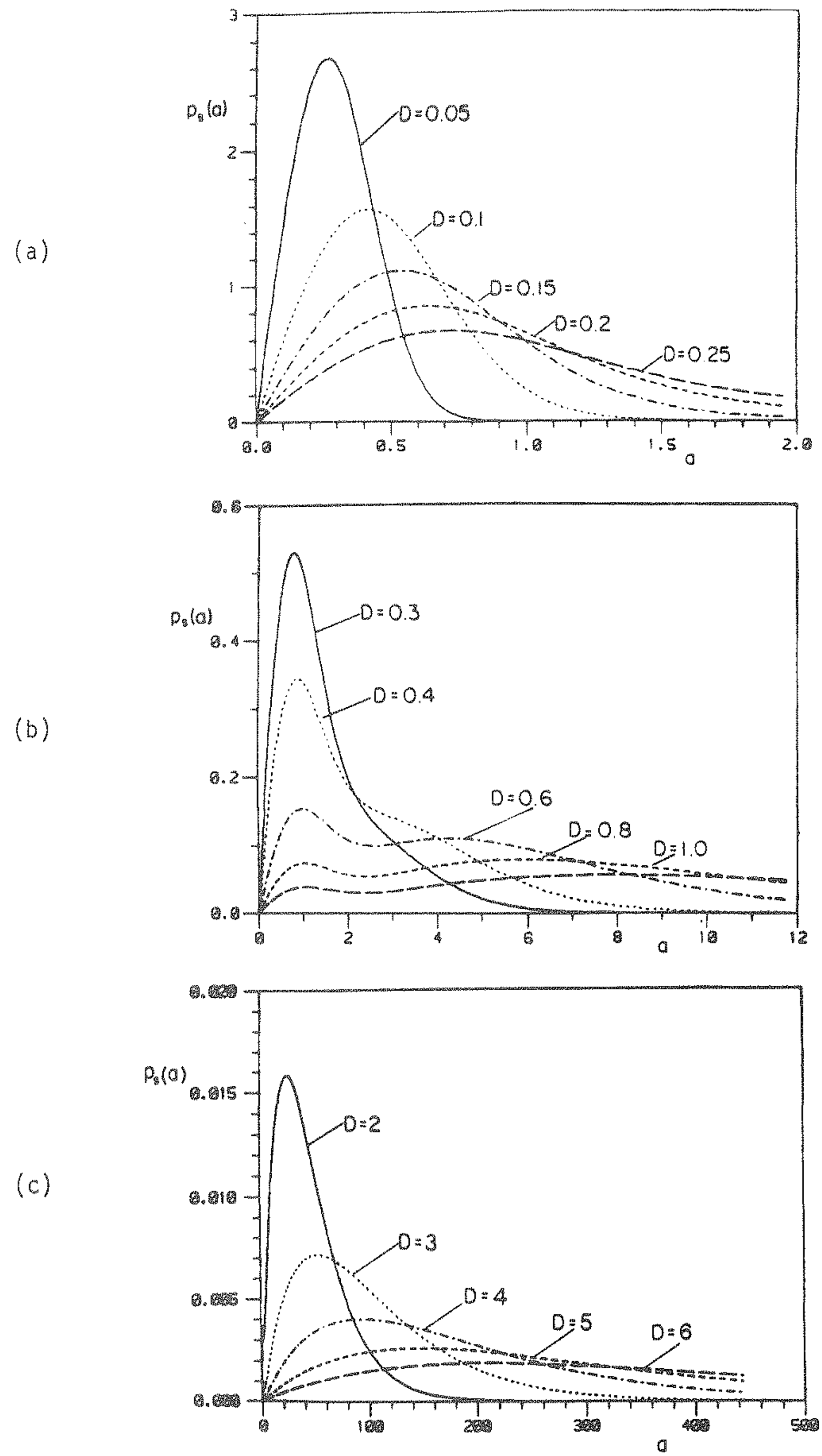


FIGURE 4-3 Stationary probability density of response amplitude of a smooth hysteretic system with $\alpha = 1/21$, $\gamma = \beta = 0.5$ and $\zeta = 0$: (a) weak excitations D = 0.05 to 0.25, (b) intermediate excitations D = 0.3 to 1.0, (c) strong excitations D = 2 to 6.

(a)

(b)

(c)

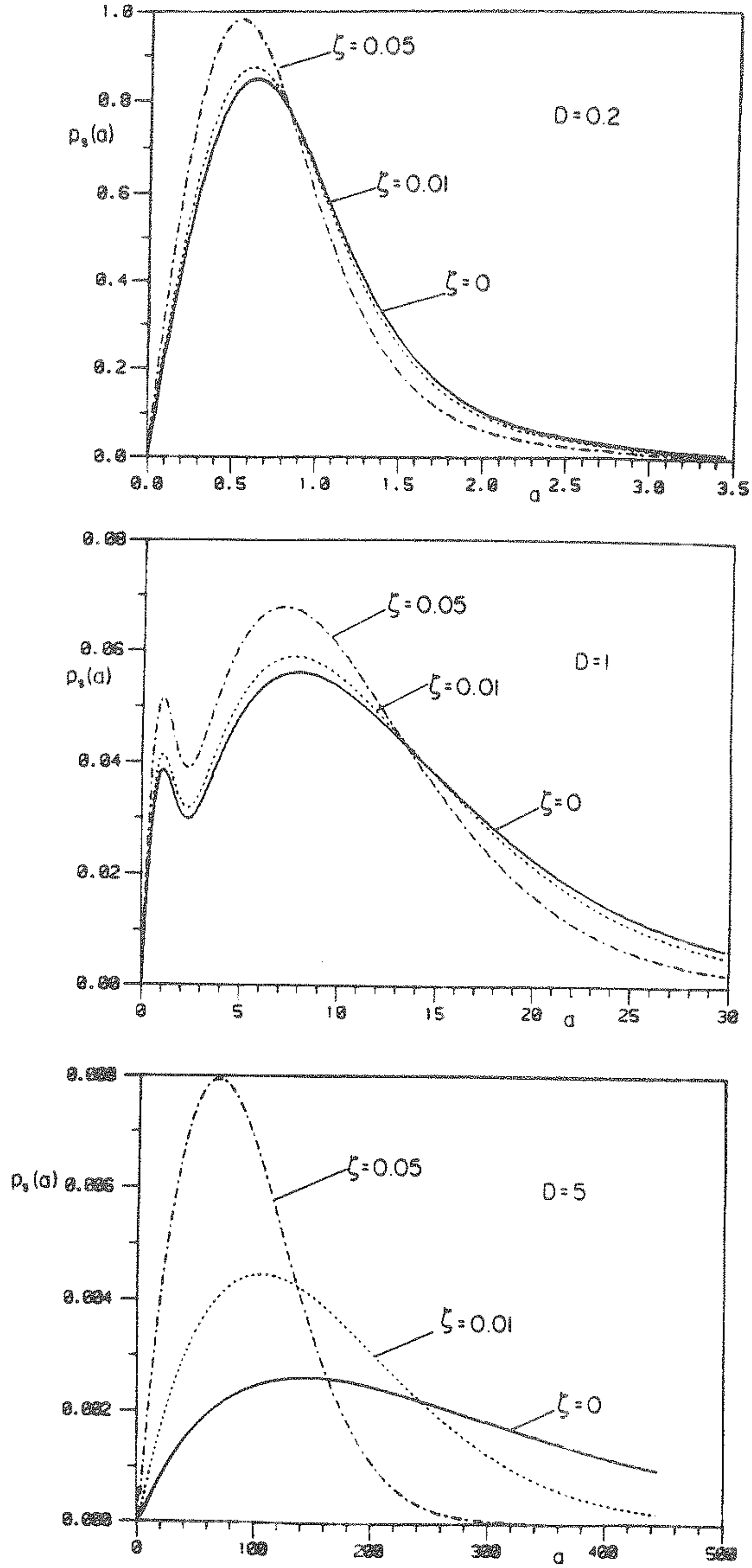


FIGURE 4-4 Stationary probability density of response amplitude of a smooth hysteretic system with $\alpha = 1/21$, $\gamma = \beta = 0.5$ and different values of damping: (a) weak excitation $D = 0.2$, (b) intermediate excitation $D = 1$, (c) strong excitation $D = 5$.

when the excitation is strong.

Finally, the root-mean-square responses were calculated for systems without the linear terms, namely, $\alpha=\zeta=0$. Two γ values were selected $\gamma = 0.02$ and 0.1, the smaller γ corresponding to a smaller hysteresis loop. Three β values were selected $\beta=0$, -1 and -5. The computed results are plotted in Figs. 4-5(a)-(c) and 4-6(a)-(c). Also shown in these figures for comparison are the results obtained using stochastic averaging and from simulations [19]. It is seen that both sets of analytical results essentially agree with each other, and with the simulation results when $\gamma+\beta<0$, namely, when the system exhibits a "hardening" tendency. However, when $\gamma+\beta>0$, corresponding to a "softening" system, results obtained from the two analytical procedures begin to diverge as the excitation strength D increases. In this case, there appears to be a reversing trend in the plot of σ_X/D versus D when D increases beyond a certain value which is predicted in our solution but not predicted in the stochastic averaging solution [19]. The simulation results also seem to suggest such a reversing trend, particularly those results shown in Fig. 4-6(a). It is of interest to note that the applicability of the stochastic averaging method requires that γ be small and that an approximate "back-bone" of the hysteretic system be computed, both of which may entail additional errors in the results.

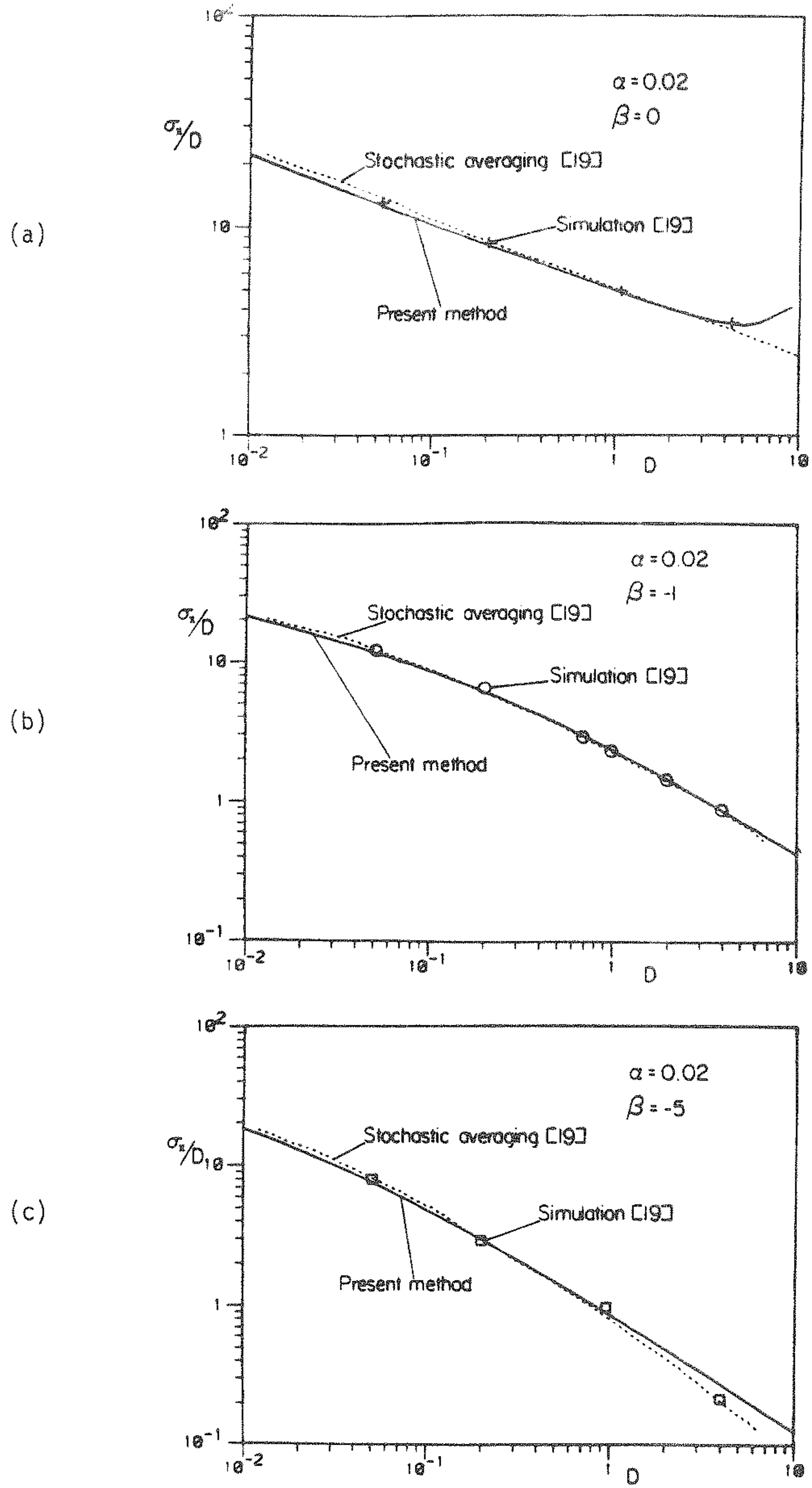


FIGURE 4-5 RMS response of a smooth hysteretic system with $\alpha = \zeta = 0$ and $\gamma = 0.02$: (a) $\beta = 0$, (b) $\beta = -1$, (c) $\beta = -5$.

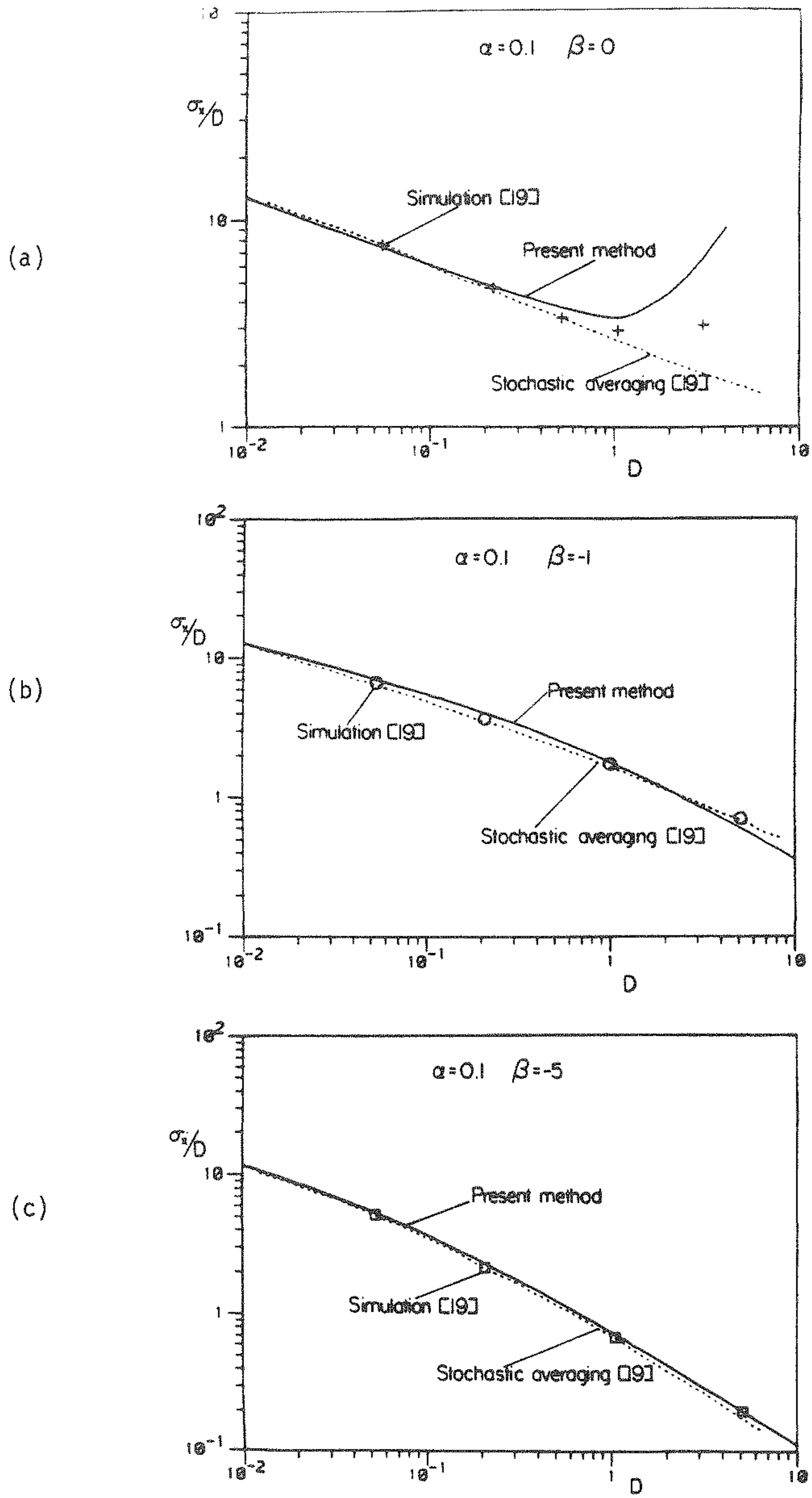


FIGURE 4-6 RMS response of a smooth hysteretic system with $\alpha = \zeta = 0$ and $\gamma = 0.1$: (a) $\beta = 0$, (b) $\beta = -1$, (c) $\beta = -5$.

SECTION 5

CONCLUSIONS

The method of energy dissipation balancing proposed herein to analyze randomly excited hysteretic systems is quite versatile. The deduced formulas for obtaining the stationary probability density and the mean square value of the system response are applicable to either piece-wise linear or smooth type hysteresis with neither the restriction that the response be a narrow-band process, nor the restriction that energy dissipation be small. Comparison of computed results with available simulation results indicates that the proposed method is accurate for wide ranges of excitation levels and for either hardening or softening type of hysteretic behaviors.

Two basic steps are required in the application of the proposed procedure. First, the area of the hysteresis loop must be calculated, and second, expressions for the potential energy stored in the linear spring and the hysteretic elements, representing the ability of the system to return to a local equilibrium, must be obtained for various ranges of deformation. Once these are accomplished, the remaining steps are quite straight-forward.

Although attention has been focused on hysterestic systems with a symmetric constitutive law and on purely additive random excitations, the method of analysis is applicable to asymmetric hysteresis and when multiplicative random excitations are also present. Multiplicative excitations can occur, for example, when a column is subjected to vertical seismic motion, which will be dealt with in a forthcoming paper.

SECTION 6

REFERENCES

1. Baber, T. T. and Noori, M. N., 1985, "Random Vibration of Degrading Pinching Systems", Journal of Engineeing Mechanics, ASCE, Vol. 111, EM8, pp. 1010-1026.

2. Baber, T. T. and Wen, Y. K., 1980, "Stochastic Equivalent Linearization for Hysteretic, Degrading, Multistory Structures", Civil Engineering Studies SRS No. 471, Department of Civil Engineering, University of Illinois, Urbana, Illinois.

3. Bouc, R., 1967, "Forced Vibration of Mechanical System with Hysteresis", Abstract, Proceedings of 4th Conference on Nonlinear Oscillation, Prague, Czechoslovakia.

4. Cai, G. Q. and Lin, Y. K., 1988, "On Exact Stationary Solutions of Equivalent Nonlinear Stochastic Systems", International Journal of Nonlinear Mechanics, to appear.

5. Cai, G. Q. and Lin, Y. K., 1988, "A New Approximate Solution Technique for Randomly Excited Nonlinear Oscillators", International Journal of Nonlinear Mechanics, to appear.

6. Casciati, F., 1987, "Nonlinear Stochastic Dynamics of Large Structural Systems by Equivalent Linearization", Proceedings, the 5th International Conference on Applications of Statistics and Probability in Soil and Structural Engineering, Vancouver, pp. 1165-1172.

7. Caughey, T. K., 1960, "Random Excitation of a System with Bilinear Hysteresis", Journal of Applied Mechanics, Marseille, Vol. 27, pp. 649-652.

8. Caughey, T. K., 1964, "On the Response of a Class of Nonlinear Oscillators to Stochastic Excitation", Proc. Colloq. Intern. du Centre National de la Recherche Scientifique, No. 148, pp. 393-402.

9. Iwan, W. D., 1966, "A Distributed-Element Model for Hysteresis and its Steady-State Dynamic Response", Journal of Applied Mechanics, Vol. 33, pp. 393-900.

10. Iwan, W. D., 1974, "Application of Nonlinear Analysis Techniques", Applied Mechanics in Earthquake Engineering, ASME Annual Meeting, AMD, Vol. 8, pp. 135-162.

11. Iwan, W. D. and Lutes, L. D., 1968, "Response of the Bilinear Hysteretic System to Stationary Random Excitation", Journal of the Acoustic Society of America, Vol. 43, pp. 545-552.

12. Iyengar, R. N. and Dash, P. K., 1978, "Study of the Random Vibration of Nonlinear Systems by the Gaussian Closure Technique", Journal of Applied Mechancis, Vol. 45, pp. 393-399.

13. Kobori, T., Minai, R. and Suzuki, Y., 1976, "Stochastic Seismic Response of Hysteretic Structures", Bulletin of Disaster Prevention Research Institute, Kyoto University, Vol. 26, Part 1, No. 236, pp. 57-70.

14. Lin, Y. K. and Cai, G. Q., 1988, "Exact Stationary-Response Solution for Second Order Nonlinear Systems under Parametric and External White-Noise Excitations, Part II", Journal of Applied Mechanics, to appear.

15. Lin, Y. K. and Cai, G. Q., 1988, "Energy Dissipation Balancing - An Approximate Solution Technique for Nonlinearly Damped System under Random Excitations", in Developments in Theoretical and Applied Mechanics, Vol. XIV, Proceedings, 14th Southeastern Conference on Theoretical and Applied Mechanics, pp. 381-388.

16. Lutes, L. D., 1970, "Approximate Technique for Treating Random Vibration of Hysteretic Systems", Journal of the Acoustic Society of America, Vol. 48, pp. 299-306.

17. Lutes, L. D. and Takemiya, M., 1974, "Random Vibraion of Yielding Oscillator", Journal of Engineering Mechanics, ASCE, Vol. 100, EM2, pp. 343-357.

18. Roberts, J. B., 1978, "The Response of an Oscillator with Bilinar Hysteresis to Sttionary Random Excitation", Journal of Applied Mechanics, Vol. 45, pp. 923-928

19. Roberts, J. B., 1987 "Application of Averaging Methods to Randomly Excited Hysteretic Systems", Proceedings IUTAM Symposium on Nonlinear Stochastic Dynamic Engineering Systems, Springer-Verlag, Berlin, pp. 361-374.

20. Shih, T. Y. and Lin, Y. K., 1982, "Vertical Seismic Load Effect on Hysteretic Columns", Journal of Engineering Mechanics, ASCE, Vol. 88, EM2, pp. 242-254.

21. Sozen, M. A., 1974, "Hysteresis in Structural Elements", Applied Mechanics in Earthquake Engineering, ASME Annual meeting, AMD, Vol. 8, pp. 63-68.

22. Spanos, P. D., 1979, "Hysteretic Structural Vibrations under Random Load", Journal of the Acoustic Society of America, Vol. 65, pp. 404-410.

23. Stratonovich, R. L., 1963, Topics in the Theory of Random Noise, Vol. 1, Gordon and Breach, New York.

24. Takemiya, H. and Lutes, L. D., 1977, "Stationary Random Vibration of Hysteretic Systems", Journal of Engineering Mechancis, ASCE, Vol. 103, EM4, pp. 673-687.

25. Wen, Y. K., 1976, "Method for Random Vibration of Hysteretic Systems", Journal of Engineering Mechancis, ASCE, Vol. 102, EM2, pp. 249-263.

26. Wen, Y. K., 1980, "Equivalent Linearization for Hysteretic Systems under Random Excitation", Journal of Applied Mechanics, Vol. 47, pp. 150-154.

27. Wong, E. and Zakai, M., 1965, "On the Relation Between Ordinary and Stochastic Equations", Int. J. Eng. Sci., Vol. 3(2), pp. 213-229.

28. Zhu, W. Q. and Lei, Y., 1987, "Stochastic Averaging of Energy Envelope of Bilinear Hysteretic Systems", IUTAM Symposium on Nonlinear Stochastic Dynamics Engineering Systems, Springer-Verlag, Berlin, pp. 381-391.

NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
LIST OF PUBLISHED TECHNICAL REPORTS

The National Center for Earthquake Engineering Research (NCEER) publishes technical reports on a variety of subjects related to earthquake engineering written by authors funded through NCEER. These reports are available from both NCEER's Publications Department and the National Technical Information Service (NTIS). Requests for reports should be directed to the Publications Department, National Center for Earthquake Engineering Research, State University of New York at Buffalo, Red Jacket Quadrangle, Buffalo, New York 14261. Reports can also be requested through NTIS, 5285 Port Royal Road, Springfield, Virginia 22161. NTIS accession numbers are shown in parenthesis, if available.

NCEER-87-0001 "First-Year Program in Research, Education and Technology Transfer," 3/5/87, (PB88-134275/AS).

NCEER-87-0002 "Experimental Evaluation of Instantaneous Optimal Algorithms for Structural Control," by R.C. Lin, T.T. Soong and A.M. Reinhorn, 4/20/87, (PB88-134341/AS).

NCEER-87-0003 "Experimentation Using the Earthquake Simulation Facilities at University at Buffalo," by A.M. Reinhorn and R.L. Ketter, to be published.

NCEER-87-0004 "The System Characteristics and Performance of a Shaking Table," by J.S. Hwang, K.C. Chang and G.C. Lee, 6/1/87, (PB88-134259/AS).

NCEER-87-0005 "A Finite Element Formulation for Nonlinear Viscoplastic Material Using a Q Model," by O. Gyebi and G. Dasgupta, 11/2/87, (PB88-213764/AS).

NCEER-87-0006 "Symbolic Manipulation Program (SMP) - Algebraic Codes for Two and Three Dimensional Finite Element Formulations," by X. Lee and G. Dasgupta, 11/9/87.

NCEER-87-0007 "Instantaneous Optimal Control Laws for Tall Buildings Under Seismic Excitations," by J.N. Yang, A. Akbarpour and P. Ghaemmaghami, 6/10/87, (PB88-134333/AS).

NCEER-87-0008 "IDARC: Inelastic Damage Analysis of Reinforced Concrete-Frame Shear-Wall Structures," by Y.J. Park, A.M. Reinhorn and S.K. Kunnath, 7/20/87, (PB88-134325/AS).

NCEER-87-0009 "Liquefaction Potential for New York State: A Preliminary Report on Sites in Manhattan and Buffalo," by M. Budhu, V. Vijayakumar, R.F. Giese and L. Baumgras, 8/31/87, (PB88-163704/AS).

NCEER-87-0010 "Vertical and Torsional Vibration of Foundations in Inhomogeneous Media," by A.S. Veletsos and K.W. Dotson, 6/1/87, (PB88-134291/AS).

NCEER-87-0011 "Seismic Probabilistic Risk Assessment and Seismic Margin Studies for Nuclear Power Plants," by Howard H.M. Hwang, 6/15/87, (PB88-134267/AS).

NCEER-87-0012 "Parametric Studies of Frequency Response of Secondary Systems Under Ground-Acceleration Excitations," by Y. Yong and Y.K. Lin, 6/10/87, (PB88-134309/AS).

NCEER-87-0013 "Frequency Response of Secondary Systems Under Seismic Excitations," by J.A. HoLung, J. Cai and Y.K. Lin, 7/31/87, (PB88-134317/AS).

NCEER-87-0014 "Modelling Earthquake Ground Motions in Seismically Active Regions Using Parametric Time Series Methods," G.W. Ellis and A.S. Cakmak, 8/25/87, (PB88-134283/AS).

NCEER-87-0015 "Detection and Assessment of Seismic Structural Damage," by E. DiPasquale and A.S. Cakmak, 8/25/87, (PB88-163712/AS).

NCEER-87-0016 "Pipeline Experiment at Parkfield, California," by J. Isenberg and E. Richardson, 9/15/87, (PB88-163720/AS).

NCEER-87-0017 "Digital Simulations of Seismic Ground Motion," by M. Shinozuka, G. Deodatis and T. Harada, 8/31/87, (PB88-155197/AS).

NCEER-87-0018 "Practical Considerations for Structural Control: System Uncertainty, System Time Delay and Truncation of Small Forces," J. Yang and A. Akbarpour, 8/10/87, (PB88-163738/AS).

NCEER-87-0019 "Modal Analysis of Nonclassically Damped Structural Systems Using Canonical Transformation," by J.N. Yang, S. Sarkani and F.X. Long, 9/27/87, (PB88-187851/AS).

NCEER-87-0020 "A Nonstationary Solution in Random Vibration Theory," by J.R. Red-Horse and P.D. Spanos, 11/3/87, (PB88-163746/AS).

NCEER-87-0021 "Horizontal Impedances for Radially Inhomogeneous Viscoelastic Soil Layers," by A.S. Veletsos and K.W. Dotson, 10/15/87, (PB88-150859/AS).

NCEER-87-0022 "Seismic Damage Assessment of Reinforced Concrete Members," by Y.S. Chung, C. Meyer and M. Shinozuka, 10/9/87, (PB88-150867/AS).

NCEER-87-0023 "Active Structural Control in Civil Engineering," by T.T. Soong, 11/11/87, (PB88-187778/AS).

NCEER-87-0024 "Vertical and Torsional Impedances for Radially Inhomogeneous Viscoelastic Soil Layers," by K.W. Dotson and A.S. Veletsos, 12/87, (PB88-187786/AS).

NCEER-87-0025 "Proceedings from the Symposium on Seismic Hazards, Ground Motions, Soil-Liquefaction and Engineering Practice in Eastern North America, October 20-22, 1987, edited by K.H. Jacob, 12/87, (PB88-188115/AS).

NCEER-87-0026 "Report on the Whittier-Narrows, California, Earthquake of October 1, 1987," by J. Pantelic and A. Reinhorn, 11/87, (PB88-187752/AS).

NCEER-87-0027 "Design of a Modular Program for Transient Nonlinear Analysis of Large 3-D Building Structures," by S. Srivastav and J.F. Abel, 12/30/87, (PB88-187950/AS).

NCEER-87-0028 "Second-Year Program in Research, Education and Technology Transfer," 3/8/88.

NCEER-88-0001 "Workshop on Seismic Computer Analysis and Design With Interactive Graphics," by J.F. Abel and C.H. Conley, 1/18/88, (PB88-187760/AS).

NCEER-88-0002 "Optimal Control of Nonlinear Structures," J.N. Yang, F.X. Long and D. Wong, 1/22/88, (PB88-213772/AS).

NCEER-88-0003 "Substructuring Techniques in the Time Domain for Primary-Secondary Structural Systems," by G. D. Manolis and G. Juhn, 2/10/88, (PB88-213780/AS).

NCEER-88-0004 "Iterative Seismic Analysis of Primary-Secondary Systems," by A. Singhal, L.D. Lutes and P. Spanos, 2/23/88, (PB88-213798/AS).

NCEER-88-0005 "Stochastic Finite Element Expansion for Random Media," P. D. Spanos and R. Ghanem, 3/14/88, (PB88-213806/AS).

NCEER-88-0006 "Combining Structural Optimization and Structural Control," F. Y. Cheng and C. P. Pantelides, 1/10/88, (PB88-213814/AS).

NCEER-88-0007 "Seismic Performance Assessment of Code-Designed Structures," H.H-M. Hwang, J. Jaw and H. Shau, 3/20/88.

NCEER-88-0008 "Reliability Analysis of Code-Designed Structures Under Natural Hazards," H.H-M. Hwang, H. Ushiba and M. Shinozuka, 2/29/88.

NCEER-88-0009 "Seismic Fragility Analysis of Shear Wall Structures," J-W Jaw and H.H-M. Hwang, 4/30/88.

NCEER-88-0010 "Base Isolation of a Multi-Story Building Under a Harmonic Ground Motion - A Comparison of Performances of Various Systems," F-G Fan, G. Ahmadi and I.G. Tadjbakhsh, to be published.

NCEER-88-0011 "Seismic Floor Response Spectra for a Combined System by Green's Functions," F.M. Lavelle, L.A. Bergman and P.D. Spanos, 5/1/88.

NCEER-88-0012 "A New Solution Technique for Randomly Excited Hysteretic Structures," G.Q. Cai and Y.K. Lin, 5/16/88.

National Center for Earthquake Engineering Research
State University of New York at Buffalo