NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
State University of New York at Buffalo

SEISMIC FLOOR RESPONSE SPECTRA FOR A COMBINED SYSTEM BY GREEN'S FUNCTIONS

by

F.M. Lavelle, L.A. Bergman and P.D. Spanos
Department of Civil Engineering and Mechanical Engineering
Rice University
Houston, Texas 77251

Technical Report NCEER-88-0011

May 1, 1988

This research was conducted at Rice University and was partially supported by the National Science Foundation under Grant No. ECE 86-07591.

NOTICE

SEISMIC FLOOR RESPONSE SPECTRA FOR A COMBINED SYSTEM BY GREEN'S FUNCTIONS

by

F.M. Lavelle[1], L.A. Bergman[2] and P.D. Spanos[3]

May 1, 1988

Technical Report NCEER-88-0011

NCEER Contract Number 87-2011

NSF Master Contract Number ECE 86-07591

1 Graduate Student, Dept. of Civil Engineering, Rice University
2 Associate Professor, Dept. of Aeronautical and Astronautical Engineering, University of Illinois at Urbana-Champaign
3 Professor, Depts. of Mechanical Engineering and Civil Engineering, Rice University

NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
State University of New York at Buffalo
Red Jacket Quadrangle, Buffalo, NY 14261

PREFACE

The National Center for Earthquake Engineering Research (NCEER) is devoted to the expansion of knowledge about earthquakes, the improvement of earthquake-resistant design, and the implementation of seismic hazard mitigation procedures to minimize loss of lives and property. Initially, the emphasis is on structures and lifelines of the types that would be found in zones of moderate seismicity, such as the eastern and central United States.

NCEER's research is being carried out in an integrated and coordinated manner following a structured program. The current research program comprises four main areas:

- Existing and New Structures
- Secondary and Protective Systems
- Lifeline Systems
- Disaster Research and Planning

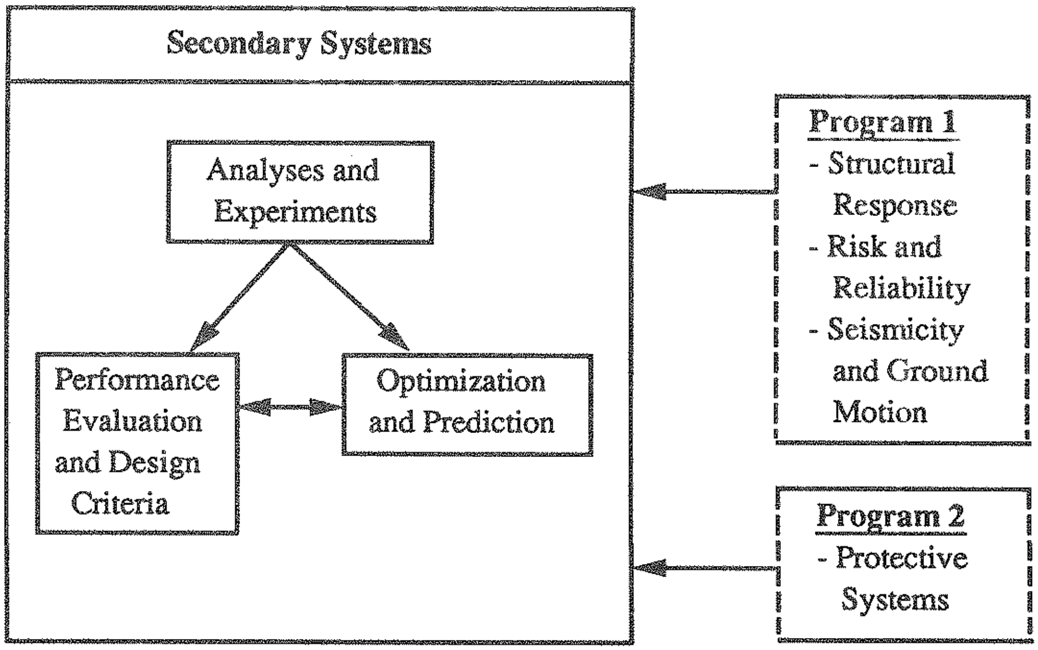
This technical report pertains to the second program area and, more specifically, to secondary systems.

In earthquake engineering research, an area of increasing concern is the performance of secondary systems which are anchored or attached to primary structural systems. Many secondary systems perform vital functions whose failure during an earthquake could be just as catastrophic as that of the primary structure itself.

The research goals in this area are to:

1. Develop greater understanding of the dynamic behavior of secondary systems in a seismic environment while realistically accounting for inherent dynamic complexities that exist in the underlying primary-secondary structural systems. These complexities include the problem of tuning, complex attachment configuration, nonproportional damping, parametric uncertainties, large number of degrees of freedom and nonlinearities in the primary structure.
2. Develop practical criteria and procedures for the analysis and design of secondary systems.
3. Investigate methods of mitigation of potential seismic damage to secondary systems through optimization or protection. The most direct route is to consider enhancing their performance through optimization in their dynamic characteristics, in their placement within a primary structure or in innovative design of their supports. From the point of view of protection, base isolation of the primary structure or the application of other passive or active protection devices can also be fruitful.

Current research in secondary systems involves activities in all three of these areas. Their interaction and interrelationships with other NCEER programs are illustrated in the accompanying figure.



The work described in this report deals with the combined secondary - primary system analysis. A simple case consisting of a continuous primary structure and a discrete secondary system is studied in order to develop an understanding of the combined system behavior. Two different solution techniques are used; the first is a truncated modal analysis and the second involves the cascade assumption in which the response of the primary system is assumed to be independent of the secondary system dynamics. Several general conclusions are drawn for this class of secondary systems based on the results obtained in this study.

ABSTRACT

A parametric study of the response of a class of linear secondary systems to earthquake excitations is made. The combined dynamical system studied consists of a single-degree-of-freedom oscillator attached to a continuous cantilever beam. The generalized differential equation governing free vibration is solved exactly for the modes of the combined system. Floor response spectra for records from the 1985 Mexico City and the 1940 El Centro earthquakes are presented in several formats. The floor response spectrum concept is an extension of widely used ground response spectrum concept and may provide useful insight into the behavior of linear secondary systems. The effects of each of the system parameters are examined. It is shown that the frequency characteristics of the excitation modify the relative importance of the system parameters. The study also demonstrates the effectiveness and limitations of the cascade approximation which is often applied as an approximate solution to the problem.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

SECTION 1
INTRODUCTION

In this investigation, a simple exact model of a combined dynamical system is used to develop an understanding of the response of a secondary system to earthquake excitations. The system considered consists of a viscously damped uniform cantilever beam with a damped single-degree-of-freedom oscillator attached at some point along the length of the beam (see Figure 1-1). The cantilever beam is frequently employed as a simple limiting case in dynamic analysis of some structures. The attached oscillator can be thought of as a piece of equipment, a passive control device, or some other secondary system at a given level of the structure. Both the beam and the oscillator are assumed to behave linearly throughout the response.

The aim of this study is to compute and interpret earthquake response spectra for the deformation of the oscillator relative to the beam when the system is subjected to deterministic earthquake excitations. In addition to giving insights into the behavior of linear secondary systems, the response spectra can be used in the analysis of multi-degree-of-freedom secondary systems in the same fashion as traditional ground response spectra are used to estimate maximum responses of multi-degrees-of-freedom primary systems (e.g. the square root of the sum of the squared maximum modal responses).

Recent papers by Bergman and Nicholson [1,8] have presented an exact solution technique for the forced vibration of combined systems such as the one considered here. While other recent approaches to secondary system problems have employed modal synthesis or perturbation techniques (e.g. [9,10]), the method presented in [1] uses separation of variables and the resulting generalized differential equation to compute the natural frequencies and modes of the combined system. The existence of an orthogonality relation allows the forced vibration problem to be solved by standard modal analysis techniques. Because the beam is continuous, results are given in terms of an infinite sum over the modes of the system. The results in [8] are exact for proportionally damped systems. However, when damping coefficients are specified for both the beam and the oscillator, the system is nonproportionally damped (i.e., the modal equations are coupled by the damping terms). In these cases, the modal series is truncated to N terms, and the resulting system of equations is written in

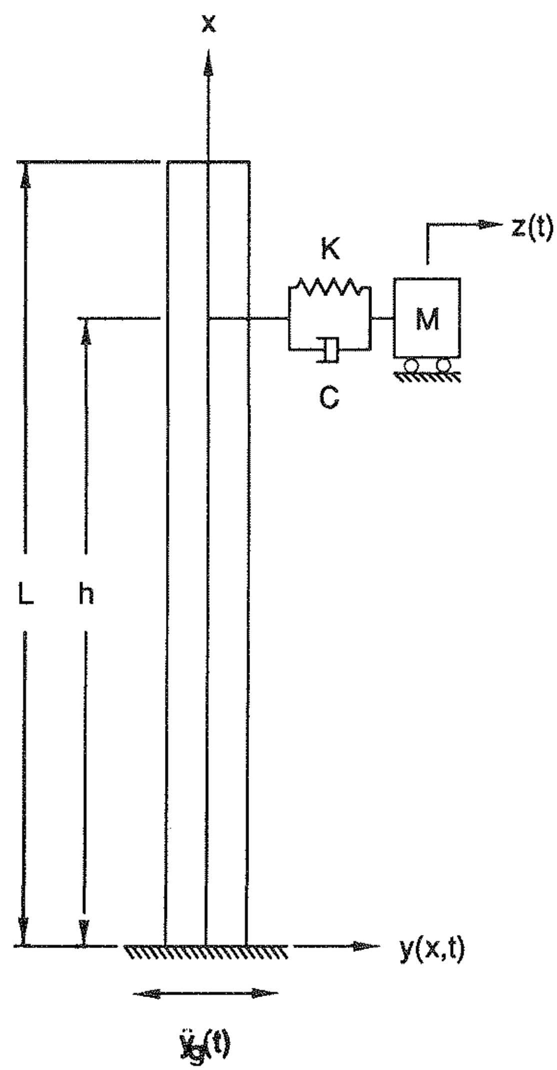


FIGURE 1-1 Combined Beam-Oscillator System

state space form. The 2N x 2N system of equations is then uncoupled by means of a complex eigenproblem. The solution resulting from this process converges to the exact solution as the number of modes considered goes to infinity. With this in mind, this technique will henceforth be referred to as a truncated modal analysis of the combined system.

Another approach which will be referred to as the cascade approximation can be used to compute approximate secondary system response spectra. In this approach, the effect of the secondary system on the beam is neglected which leaves the ground motion as the only remaining beam excitation. With this assumption, the response of the beam is readily computed. Given this approximation for the beam motion, the response of the oscillator involves a simple computation. The effectiveness of the cascade approximation is obviously dependent upon the mass of the oscillator relative to the mass of the beam. The influences of this and other parameters are revealed by a comparison of cascade response spectra and response spectra based on the truncated modal analysis of the combined system.

SECTION 2
SYSTEM ANALYSIS

2.1 Equations of Motion

The governing equations for linear response of the beam and oscillator to a base acceleration, $\ddot{y}_g(t)$, can be derived in terms of the constraint force, F(t), at the interface between the continuous and discrete elements of the system. The equation for the beam response, y(x,t), is

$$\rho\,\ddot{y}(x,t) + C_b\,\dot{y}(x,t) + EI\,y''''(x,t) \;=\; F(t)\,\delta(x-h) - \rho\,\ddot{y}_g(t) \tag{1}$$

where partial derivatives with respect to time and space are denoted by $\dot{(\,)} = \frac{\partial(\,)}{\partial t}$ and $(\,)' = \frac{\partial(\,)}{\partial x}$, respectively. Further, E is the elastic modulus; I is the beam moment of inertia; ρ is the mass per unit length; C_b is the viscous damping per unit length; and $\delta(x-h)$ is the Dirac delta function which indicates that the constraint force is a concentrated force applied at the attachment point, x = h.

The equation of motion for the oscillator in terms of the beam response at the attachment point, y(h,t), is

$$M\,\ddot{z}(t) + C\,\dot{z}(t) + K\,z(t) \;=\; C\,\dot{y}(h,t) + K\,y(h,t) - M\,\ddot{y}_g(t) \tag{2}$$

where M is the oscillator mass; C is the oscillator viscous damping coefficient; and K is the linear spring constant. Note that the response, z(t), is the displacement of the oscillator with respect to the base of the cantilever beam. The relative displacement or deformation of the oscillator is [z(t) - y(h,t)]. The oscillator equation can also be written in terms of the constraint force, F(t). In this case, the equation is

$$M\,\ddot{z}(t) \;=\; -F(t) - M\,\ddot{y}_g(t) \tag{3}$$

The constraint force is the sum of the forces generated by the oscillator spring and dashpot at time, t. Therefore,

$$F(t) \;=\; C\,[\,\dot{z}(t) - \dot{y}(h,t)\,] + K\,[\,z(t) - y(h,t)\,] \tag{4}$$

In order to isolate the important parameters of the combined system, it is useful to recast the equations of motion in dimensionless form before proceeding with the solution. To accomplish this the following terms are introduced:

$$\bar{y}(\xi,\tau) = \frac{y(x,t)}{L} \quad ; \quad \bar{z}(\tau) = \frac{z(t)}{L} \quad ; \quad \xi = \frac{x}{L} \quad ; \quad \eta = \frac{h}{L}$$

$$\Omega^2 = \frac{EI}{\rho L^4} \quad ; \quad \tau = \Omega t \quad ; \quad \bar{\delta}(\xi-\eta) = L\,\delta(x-h)$$

$$\ddot{\bar{y}}_g(\tau) = \frac{\ddot{y}_g(t)}{g} \quad ; \quad \gamma = \frac{\rho g L^3}{EI} = \frac{g}{L\Omega^2} \quad ; \quad \bar{F}(\tau) = \frac{L^2 F(t)}{EI}$$

$$\kappa = \frac{KL^3}{EI} \quad ; \quad \mu = \frac{M}{\rho L} \quad ; \quad \omega^2 = \frac{K}{M}$$

$$\Omega_0^2 = \frac{\omega^2}{\Omega^2} = \frac{\kappa}{\mu} \quad ; \quad \varepsilon_b = \frac{C_b}{\rho\Omega} = 2\,(3.516)\,\zeta_b \quad ; \quad \varepsilon = \frac{C}{M\Omega} = 2\,\zeta\,\Omega_0 \qquad (5)$$

In equation (5), g is the acceleration of gravity; ζ is the percent of critical damping in the oscillator when uncoupled from beam; and ζ_b is the percent of critical damping in the first mode of the beam disregarding the oscillator. The parameters κ and μ are the oscillator to beam ratios of stiffness and mass respectively. The parameter Ω_0 will be referred to as the dimensionless frequency ratio of the system. The natural frequencies of a cantilever beam are $\lambda_i \Omega$ (where λ_i = 3.516, 22.03, ...). Therefore, Ω_0 is a constant multiple of the ratio of oscillator natural frequency to beam natural frequency.

After substituting the above terms into the equations of motion, equations (1), (2), and (4) can be rewritten as

$$\ddot{\bar{y}}(\xi,\tau) + \varepsilon_b \dot{\bar{y}}(\xi,\tau) + \bar{y}''''(\xi,\tau) = \bar{F}(\tau)\,\bar{\delta}(\xi-\eta) - \gamma \ddot{\bar{y}}_g(\tau) \qquad (6)$$

$$\ddot{\bar{z}}(\tau) + \varepsilon\,\dot{\bar{z}}(\tau) + \Omega_0^2\,\bar{z}(\tau) = \varepsilon\,\dot{\bar{y}}(\eta,\tau) + \Omega_0^2\,\bar{y}(\eta,\tau) - \gamma \ddot{\bar{y}}_g(\tau) \qquad (7)$$

$$\bar{F}(\tau) = \mu\,\varepsilon\,[\,\dot{\bar{z}}(\tau) - \dot{\bar{y}}(\eta,\tau)\,] + \kappa\,[\,\bar{z}(\tau) - \bar{y}(\eta,\tau)\,] = -\mu\,[\,\ddot{\bar{z}}(\tau) + \gamma \ddot{\bar{y}}_g(\tau)\,] \qquad (8)$$

where the temporal and spatial partial derivatives are now $\dot{(\;)} = \frac{\partial(\;)}{\partial\tau}$ and $(\;)' = \frac{\partial(\;)}{\partial\xi}$, respectively.

2.2 Free Vibration

In order to compute the natural frequencies and modes, and to establish an orthogonality relation for the modes of the combined system, the approach developed by Nicholson and Bergman [8] is applied to the dimensionless equations (6), (7), and (8). Free vibration requires that the damping coefficients and external forces be set equal to zero. This reduces equation (6) to

$$\ddot{\bar{y}}(\xi,\tau) + \bar{y}''''(\xi,\tau) = \bar{F}(\tau)\,\bar{\delta}(\xi-\eta) \tag{9}$$

where

$$\bar{F}(\tau) = \kappa\,[\,\bar{z}(\tau) - \bar{y}(\eta,\tau)\,] = -\mu\,\ddot{\bar{z}}(\tau) \tag{10}$$

Assuming that the space and time dependencies of the response can be separated, let

$$\bar{y}(\xi,\tau) = Y(\xi)\,Q(\tau) \tag{11}$$

and

$$\bar{z}(\tau) = A\,Y(\eta)\,Q(\tau) = Z\,Q(\tau) \tag{12}$$

where A is an amplification factor defining the free vibration displacement of the oscillator as a multiple of the beam displacement at $\xi = \eta$. Substituting equations (10), (11) and (12) into equation (9) and separating variables leads to

$$-\frac{\ddot{Q}(\tau)}{Q(\tau)} = \frac{Y''''(\xi)}{Y(\xi)\,[\,1 + \mu A\,\bar{\delta}(\xi-\eta)\,]} = \alpha^4 \tag{13}$$

where α^4 is the separation constant. Equation (13) can be rewritten as

$$\ddot{Q}(\tau) + \alpha^4 Q(\tau) = 0 \tag{14}$$

and

$$Y''''(\xi) - \alpha^4 Y(\xi) = \alpha^4 \mu A\,\bar{\delta}(\xi-\eta)\,Y(\eta) \tag{15}$$

Substituting equations (11), (12), and (14) into equation (10) and solving for A yields

$$A = \frac{\Omega_0^2}{\Omega_0^2 - \alpha^4} \tag{16}$$

Equation (14) demonstrates that the time dependent portion of the response, $Q(\tau)$, is a harmonic function with dimensionless frequency α^2.

Equation (15) can be solved in terms of the Green's function that satisfies the generalized differential equation

$$G''''(\xi,\eta;\alpha) - \alpha^4 G(\xi,\eta;\alpha) = \bar{\delta}(\xi - \eta) \tag{17}$$

and the homogeneous boundary conditions of a cantilever beam. The definition of $G(\xi,\eta;\alpha)$ for this case is given in [8]. Since equation (15) is a linear differential equation with the same boundary conditions as equation (17), the solution to equation (15) is simply

$$Y(\xi) = \mu A \alpha^4 Y(\eta) G(\xi,\eta;\alpha) \tag{18}$$

The natural frequencies of the non-dimensional system $(\alpha_n^2 \ ; \ n = 1,2,3,....)$ are obtained by letting $\xi \rightarrow \eta$ and solving for the roots of the homogeneous equation

$$[\, 1 - \mu \frac{\Omega_0^2 \alpha^4}{\Omega_0^2 - \alpha^4} G(\eta,\eta;\alpha) \,] Y(\eta) = 0 \tag{19}$$

The eigenvalues, α_n, satisfying equation (19) can be shown to interlace the sequence {0, 1.875.., 4.694.., 7.854.., ...} where the positive numbers are the eigenvalues of a non-dimensional cantilever beam [3]. These bounds can be used along with a root finder to efficiently compute the α_n's. The natural frequencies of the original, dimensional system are $\Omega \alpha_n$ where Ω is the temporal scaling factor defined in equation (5).

Since the magnitude of a free vibration mode is arbitrary, it is convenient to define the magnitude of each mode by $Y_n(\eta) = 1$. Thus, the n'th mode shape is

$$Y_n(\xi) = \mu A_n \alpha_n^4 G(\xi,\eta;\alpha_n) = \mu \frac{\Omega_0^2 \alpha_n^4}{\Omega_0^2 - \alpha_n^4} G(\xi,\eta,\alpha_n) \tag{20a}$$

$$Z_n = A_n \tag{20b}$$

In order to perform a traditional forced vibration analysis of the system, the final tool that is necessary is an orthogonality relation. By applying the operator

$$\int_0^1 Y_m(\xi) [\ \cdots\ ] d\xi \tag{21}$$

to equation (15), and integrating by parts (using the homogeneous boundary conditions), Nicholson and Bergman [8] have shown that the orthogonality relation is

$$\int_0^1 \left[1 + \mu A_n A_m \bar{\delta}(\xi - \eta) \right] Y_m(\xi)\, Y_n(\xi)\, d\xi \;=\; d_n\, \delta_{mn} \tag{22}$$

where δ_{mn} is the Kronecker delta, and

$$d_n \;=\; \int_0^1 Y_n^2(\xi)\, d\xi + \mu A_n^2 \tag{23}$$

is the n'th modal mass.

2.3 Response to Base Excitation

With a complete set of orthogonal modes now in hand, the forced vibration analysis follows directly from traditional techniques. The response is assumed to be of the form

$$\bar{y}(\xi,\tau) \;=\; \sum_{n=1}^{\infty} a_n(\tau)\, Y_n(\xi) \tag{24}$$

$$\bar{z}(\tau) \;=\; \sum_{n=1}^{\infty} a_n(\tau)\, A_n Y_n(\eta) \;=\; \sum_{n=1}^{\infty} a_n(\tau)\, A_n \tag{25}$$

Substituting equations (24), (25), (8), and (15) into equation (6) yields

$$\sum_{n=1}^{\infty} \left[\ddot{a}_n(\tau) + \varepsilon_b\, \dot{a}_n(\tau) + \alpha_n^4\, a_n(\tau) \right] Y_n(\xi) \;=\; -\gamma \ddot{\bar{y}}_g(\tau) + \mu\, \varepsilon \sum_{n=1}^{\infty} \frac{\alpha_n^4}{\Omega_0^2} A_n\, \dot{a}_n(\tau) \tag{26}$$

Similarly, substituting equations (24), (25), and (16) into equation (7) leads to

$$\sum_{n=1}^{\infty} A_n\, \ddot{a}_n(\tau) + \varepsilon \frac{\alpha_n^4}{\Omega_0^2} A_n\, \dot{a}_n(\tau) + \alpha_n^4 A_n\, a_n(\tau) \;=\; -\gamma \ddot{\bar{y}}_g(\tau) \tag{27}$$

Operating on equation (26) by $\int_0^1 Y_m(\xi)\, [\ \cdots\ ]\, d\xi$, adding equation (27) multiplied by μA_m, and applying equation (22) yields the infinite dimensional system of equations

$$\sum_{n=1}^{\infty} \left\{ d_n\, \delta_{mn}\, \ddot{a}_n(\tau) + [\, \varepsilon_b\, d_n\, \delta_{mn} + \mu A_n A_m (\, \varepsilon \frac{\alpha_n^4\, \alpha_m^4}{\Omega_0^4} - \varepsilon_b\,)\,]\, \dot{a}_n(\tau) + \alpha_n^4\, d_n\, \delta_{mn}\, a_n(\tau) \right\} =$$

$$-\gamma \ddot{y}_g(\tau) \left[\int_0^1 Y_m(\xi)\, d\xi + \mu A_m \right] \qquad m = 1, 2, 3, \cdots \tag{28}$$

The equations uncouple if

$$\varepsilon \frac{\alpha_n^4 \alpha_m^4}{\Omega_0^4} = \varepsilon_b \text{ for } all \ m \neq n. \tag{29}$$

Bergman and Nicholson [1] define this to be the condition for proportional damping in the combined system. However, if ε and ε_b (or, equivalently, ζ and ζ_b) are assigned fixed values, the equations remain coupled by the damping terms. In this case, the infinite system of equations represented in equation (28) can be truncated to N equations and approximated in 2N x 2N state space [6,7] as

$$[A]\{\dot{b}(\tau)\} + [B]\{b(\tau)\} = -\gamma \ddot{y}_g(\tau)\{\tilde{f}\} \tag{30}$$

where

$$[A] = \begin{bmatrix} [C] & [M] \\ [M] & [0] \end{bmatrix} \quad ; \quad [B] = \begin{bmatrix} [K] & [0] \\ [0] & -[M] \end{bmatrix}$$

$$\{b(\tau)\} = \begin{Bmatrix} \{a(\tau)\} \\ \{\dot{a}(\tau)\} \end{Bmatrix} \quad ; \quad \{\tilde{f}\} = \begin{Bmatrix} \{f\} \\ \{0\} \end{Bmatrix} \tag{31}$$

The elements of the submatrices M, C, K, and f are:

$$M_{mn} = d_n \delta_{mn} \quad ; \quad C_{mn} = \varepsilon_b d_n \delta_{mn} + \mu A_n A_m \left(\varepsilon \frac{\alpha_n^4 \alpha_m^4}{\Omega_0^4} - \varepsilon_b \right)$$

$$K_{mn} = \alpha_n^4 d_n \delta_{mn} \quad ; \quad f_m = \int_0^1 Y_m(\xi)\, d\xi + \mu A_m \qquad m,n = 1,\dots,N \tag{32}$$

The equations in (30) can be uncoupled by a set of N pairs of complex conjugate eigenvectors. The associated eigenproblem is

$$[B]\{\phi_i\} = -p_i [A]\{\phi_i\} \qquad i = 1,\dots,2N \tag{33}$$

The modal matrix $[\Phi] = [\{\phi_1\} \cdots \{\phi_{2N}\}]$ is orthogonal with respect to both [A] and [B]. Therefore,

$$[\Phi]^T [A] [\Phi] = diag[\hat{a}_i] \tag{34}$$

$$[\Phi]^T [B] [\Phi] = diag[\hat{b}_i] \tag{35}$$

and

$$p_i = -\frac{\hat{b}_i}{\hat{a}_i} \qquad i = 1,...,2N \tag{36}$$

Letting

$$\{b(\tau)\} = [\Phi] \{u(\tau)\} \tag{37}$$

and premultiplying equation (30) by $[\Phi]^T$ uncouples the system of state space equations. The i'th equation is

$$\hat{a}_i \dot{u}_i(\tau) + \hat{b}_i u_i(\tau) = -\gamma \ddot{\bar{y}}_g(\tau) \hat{f}_i \qquad i = 1,...,2N \tag{38}$$

where

$$\{\hat{f}\} = [\Phi]^T \{\tilde{f}\} \tag{39}$$

Using equations (5) and (36), equation (38) can be rewritten as

$$\dot{u}_i(\tau) - p_i u_i(\tau) = -\frac{\hat{f}_i g}{\hat{a}_i L \Omega^2} \ddot{\bar{y}}_g(\tau) \qquad i = 1,...,2N \tag{40}$$

Let

$$\Delta\tau = \Omega \Delta t \tag{41}$$

where Δt is the constant time step used in the digitized earthquake record. Assuming that the system is initially at rest (i.e., u_i (0) = 0 for all i) and that the ground acceleration is piecewise linear between specified values [4], the solution to equation (40) can be expressed as

$$u_i(\tau + \Delta\tau) = u_i(\tau) e^{p_i \Delta\tau} - \frac{\hat{f}_i g}{\hat{a}_i L \Omega^2} \{ \ddot{\bar{y}}_g(\tau) [(p_i \Delta\tau - 1) e^{p_i \Delta\tau} + 1] +$$

$$\ddot{\bar{y}}_g(\tau + \Delta\tau) [e^{p_i \Delta\tau} - (p_i \Delta\tau - 1)] \} \qquad i = 1,...,2N \tag{42}$$

The transformation back to state space coordinates from the uncoupled state space coordinates is accomplished through equation (37). The temporal components of the response,

$a_n(\tau)$, are contained in the upper half of the state vector $\{b(\tau)\}$. Finally, the beam and oscillator motions are computed by summing the first N terms in equations (24) and (25) respectively.

SECTION 3
FLOOR RESPONSE SPECTRA

Using digitized acceleration records from the 1940 El Centro (S00E comp.) [5] and 1985 Mexico City (N00E comp. of the CDAF record) [11] earthquakes, a series of floor response spectra are computed for the combined cantilever beam - oscillator system. These two earthquakes are chosen because they represent two very different types of ground motion. The El Centro record is a broad-band excitation and is more likely to excite higher modes in the structure. On the other hand, the Mexico City earthquake is basically a narrow-band, low frequency ground acceleration. Ground response spectra for the earthquakes are shown in Figure 3-1. The dashed lines represent the maximum ground acceleration, velocity, and displacement for each of the earthquakes.

In addition to using the the truncated modal analysis method discussed in the previous section, response spectra curves are also computed by using the simpler cascade approach. Under the cascade assumption, the beam response is not influenced by the presence of the oscillator. Equations (1) and (2) are uncoupled by setting the interaction force, F(t), equal to zero in equation (1). The solution to equation (1) can then be easily computed by standard dynamic analysis of continuous beams [2,5]. The beam response is then included in the right hand side of equation (2), and the oscillator equation is reduced to a simple single degree of freedom problem.

Although the cascade solution is independent of the mass ratio, μ, the technique produces reliable results for the limiting case in which the mass ratio approaches zero. On the other hand, modal analysis of the combined system is valid for all mass ratios and is exact as the number of modes considered goes to infinity. For the figures presented in this section, five combined system modes are kept in the truncated modal analysis, and five cantilever beam modes are kept in the cascade analysis. Truncated modal analysis allows a check on the accuracy of the cascade solution over a wide range of system parameters such as mass ratio, natural period of the oscillator, fundamental period of the beam, and oscillator location and damping. In addition to investigating the limitations of the cascade solution, the parametric study also provides information required to make some generalizations about the behavior of this class of combined systems when exposed to earthquake loadings.

damping is most effective when the frequency ratio is in the range of dominant earthquake frequencies. This is similar to the behavior observed in ground response spectra shown in Figure 3-1.

The effect of the beam parameter Ω on the deformation of the secondary system is demonstrated in Figure 3-4. In this pair of figures, response spectra are plotted for various fundamental periods of the beam. The dashed curves in the plots were taken from the ground response spectra with the same oscillator parameters and correspond to an infinitely stiff beam ($T_b = 0$). For oscillator periods greater than the largest fundamental beam period considered, the response curves converge monotonically towards the dashed ground response curve as T_b approaches zero.

The most significant curve shown in Figure 3-4 appears in the Mexico City plot when $T_b = 2$ seconds. In this case, the fundamental frequency of the beam falls within the relatively narrow band of dominant frequencies in the Mexico City earthquake. The earthquake excitation is amplified by the beam and the oscillator experiences accelerations on the order of ten times the maximum ground acceleration over a wide range of oscillator natural periods. Because of the broad-band characteristics of the El Centro record, the oscillator deformation spectra in Figure 3-4(b) do not exhibit extreme amplifications for any particular fundamental beam period.

Figures 3-5 and 3-6 display maximum deformation versus mass ratio and oscillator location respectively. Several values of oscillator natural period are assumed in each of the two figures while the two damping parameters and the fundamental beam period are held constant. The values plotted for $\mu = 0$ in Figure 3-5 are obtained by using the cascade solution technique, and the values plotted for $\eta = 0$ in Figure 3-6 are taken from the appropriate ground response spectra.

Increasing the mass ratio while holding the frequency ratio and other parameters constant generally results in smaller maximum deformations. This pattern appears to be followed more closely for broad-band excitations (Figure 3-5(b)) than for narrow-band excitations (Figure 3-5(a)). The natural frequencies of the combined system change as the mass ratio is varied. Shifting of natural frequencies appears to be more important when the excitation is narrow-banded because the system natural frequencies are more likely to move into or out of the range of dominant excitation frequencies. This is the cause for the irregularity of

Typical floor response spectra for maximum relative deformation, pseudo relative velocity, and pseudo absolute acceleration of the oscillator are shown in Figures 3-2 through 3-4. In each of these figures, three dashed lines representing the maximum acceleration, velocity, and displacement of the ground for the earthquake record under consideration are provided as reference lines.

In Figure 3-2, the oscillator location (η) , the fundamental period of the beam (T_b) , and the damping ratios (ζ, ζ_b) are held constant, and response spectra are plotted for the two earthquakes as a function of the uncoupled natural period of the oscillator (T). Instead of giving the maximum response values for several ratios of oscillator damping as is normally done in ground response spectra, the curves in Figure 3-2 represent different mass ratios (μ). The dashed curve corresponds to a mass ratio of zero and is computed using the cascade solution technique; the solid curves are computed using the truncated modal analysis approach.

Peaks in the cascade response occur when the oscillator natural period is equal to one of the natural periods of the beam. The peaks due to "tuned" secondary systems are sharply reduced when interaction between the beam and oscillator is taken into consideration. In general, interaction increases as the mass ratio gets larger. Except for oscillators with natural frequencies near a narrow band of dominant excitation frequencies, deformation spectra generated using the cascade assumption tend to give upper bounds on the actual response. The bound is most conservative for tuned oscillators. Therefore, for secondary systems that are not closely tuned to a natural frequency of the beam, it appears that the cascade approximation may be acceptable for a fairly wide range of parameters.

The limiting values of the deformation spectra in Figure 3-2 are a predictable extension of the limiting values observed in ground response spectra. The maximum absolute acceleration of very stiff oscillators (short period) tends towards the maximum acceleration of the beam at the connection point. The maximum acceleration of the beam depends on the mass ratio because such oscillators behave as an additional concentrated mass on the beam. The mass ratio is particularly important if higher modes of the beam are significantly excited, as is the case in Figure 3-2(b) for the El Centro earthquake.

In Figure 3-3 a mass ratio of 0.05 is assumed, and the oscillator deformation is plotted for four oscillator damping ratios. For almost any combination of system parameters, an increase in oscillator damping causes a reduction in the maximum deformation. Oscillator the T = 2 seconds curve in Figure 3-5(a). Another distinct feature in both figures is the rapid increase in deformation at low mass ratios when the oscillator period is equal to the fundamental beam period of one second. This is a result of decreased interaction between the beam and oscillator as the mass ratio goes to zero.

Figure 3-6 confirms that the maximum deformation of the secondary system generally increases as the oscillator is moved away from the base of the cantilever beam. Exceptions to this rule tend to occur when the oscillator is at or near a node of a highly excited beam mode and for systems with large mass ratios. For most cases, ground response spectra with the appropriate oscillator damping serve as a lower bound for the deformation spectra of oscillators located above the base of the beam.

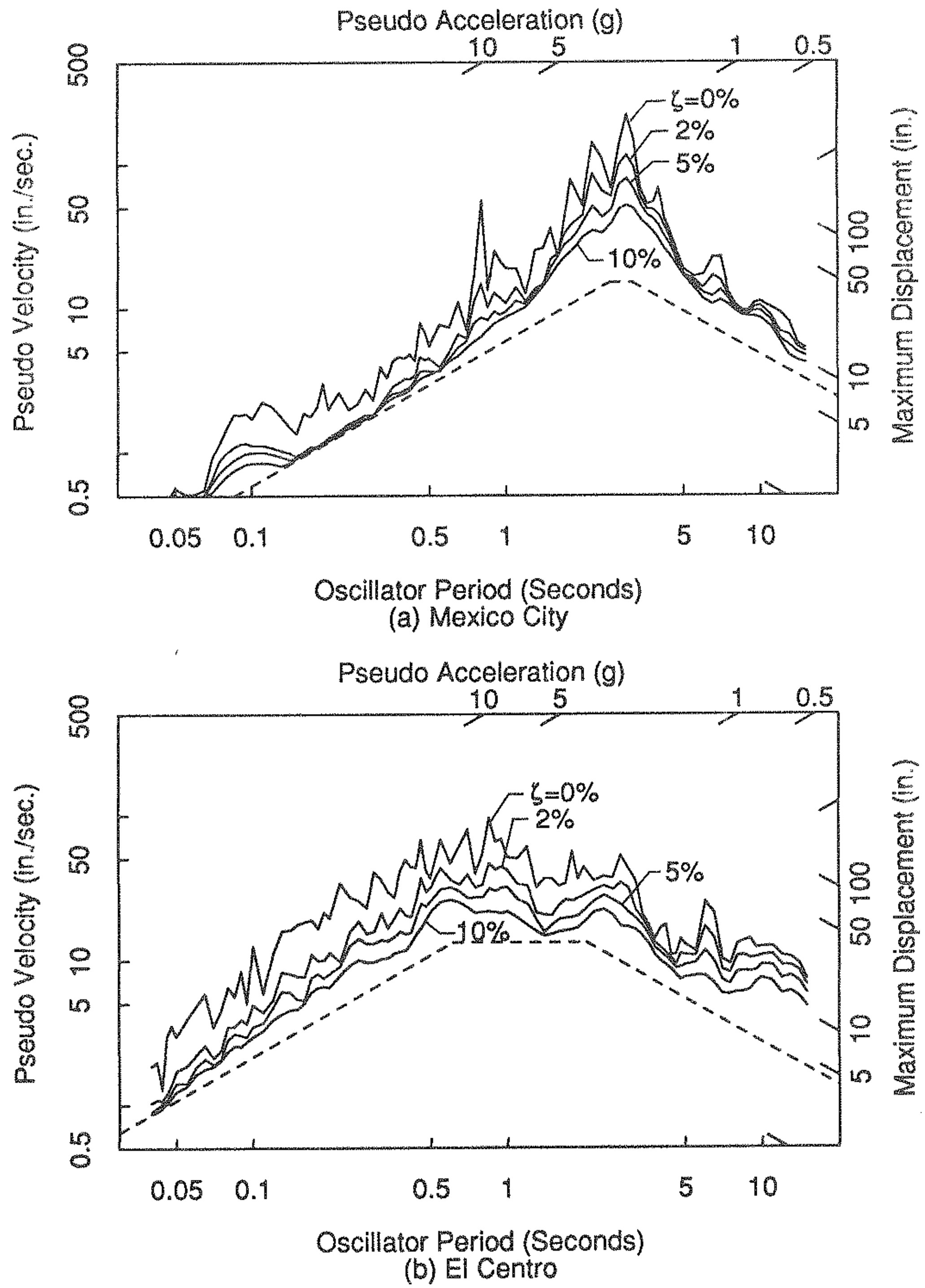


FIGURE 3-1 Ground Response Spectra for the 1985 Mexico City and 1940 El Centro Earthquakes

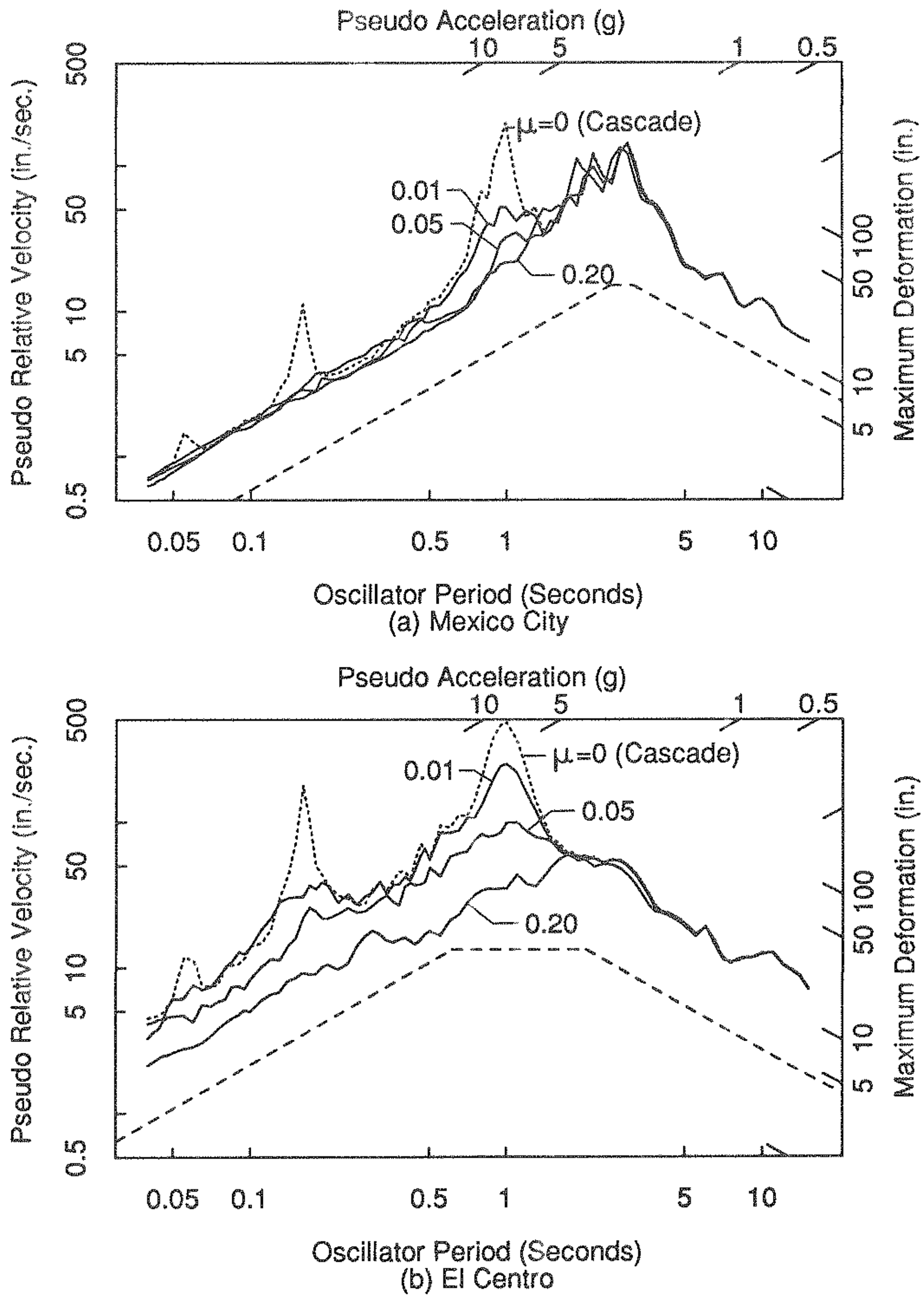
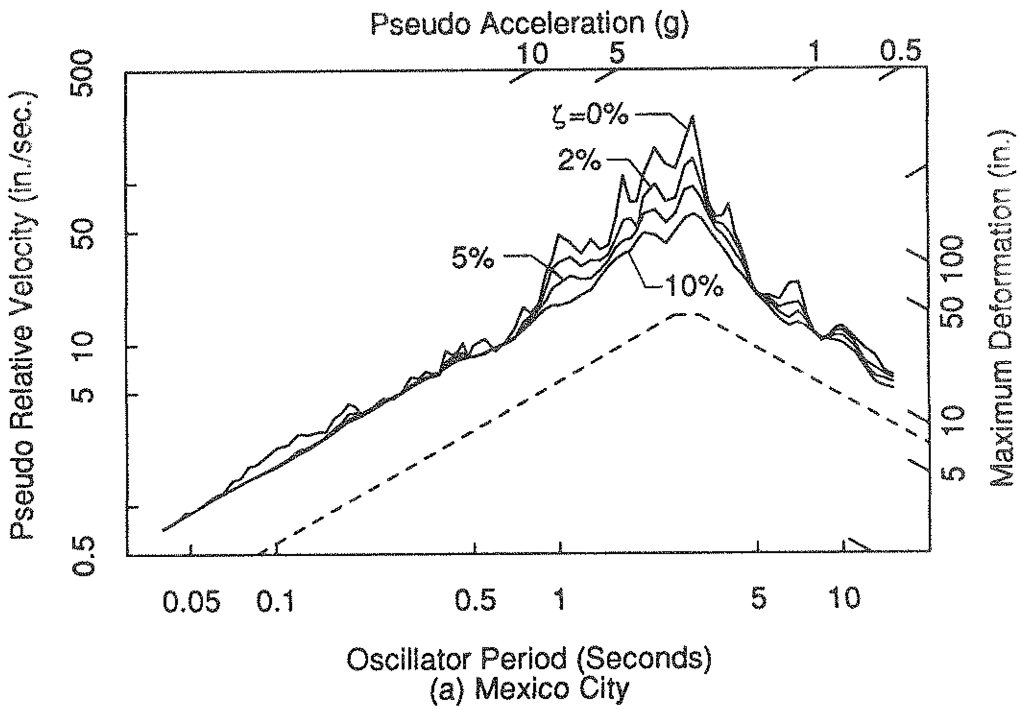
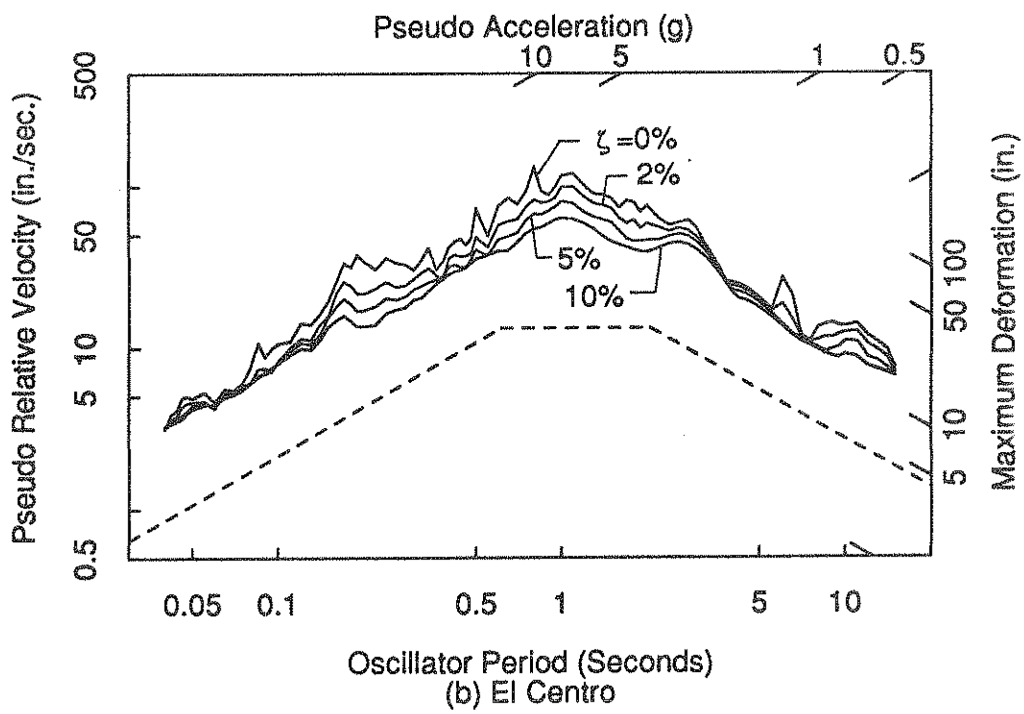


FIGURE 3-2 Oscillator Deformation Spectra: Modal Analysis vs. Cascade Analysis (Constant Parameters: $T_b = 1$ second, $\zeta = \zeta_b = 2\%$, $\eta = 1$)



(a) Mexico City



(b) El Centro

FIGURE 3-3 Oscillator Deformation Spectra for Various Oscillator Damping Ratios (Constant Parameters: $T_b = 1$ second, $\zeta_b = 2\%$, $\mu = 0.05$, $\eta = 1$)

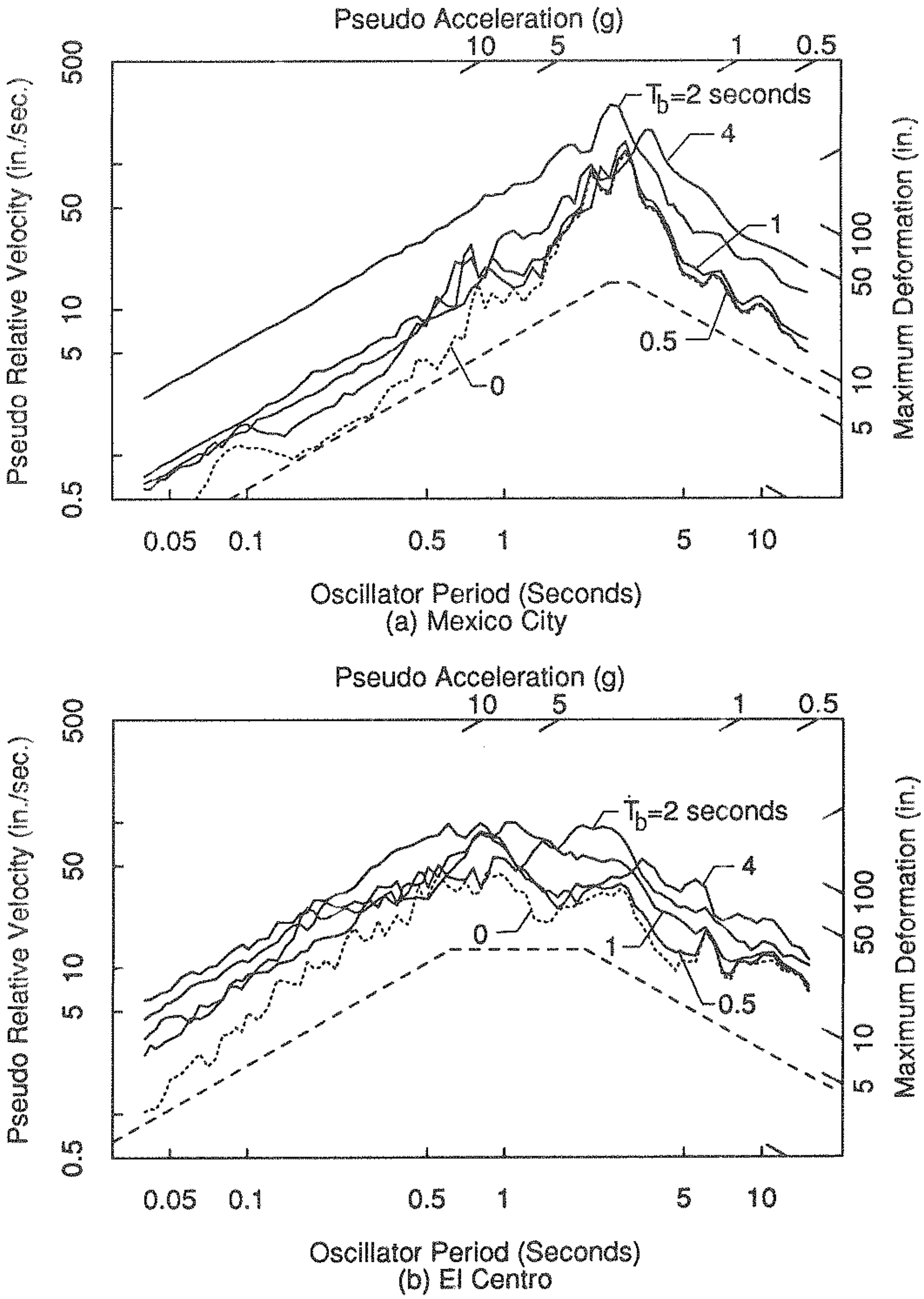


FIGURE 3-4 Oscillator Deformation Spectra for Various Fundamental Beam Periods (Constant Parameters: $\zeta = \zeta_b = 2\%$, $\mu = 0.05$, $\eta = 1$)

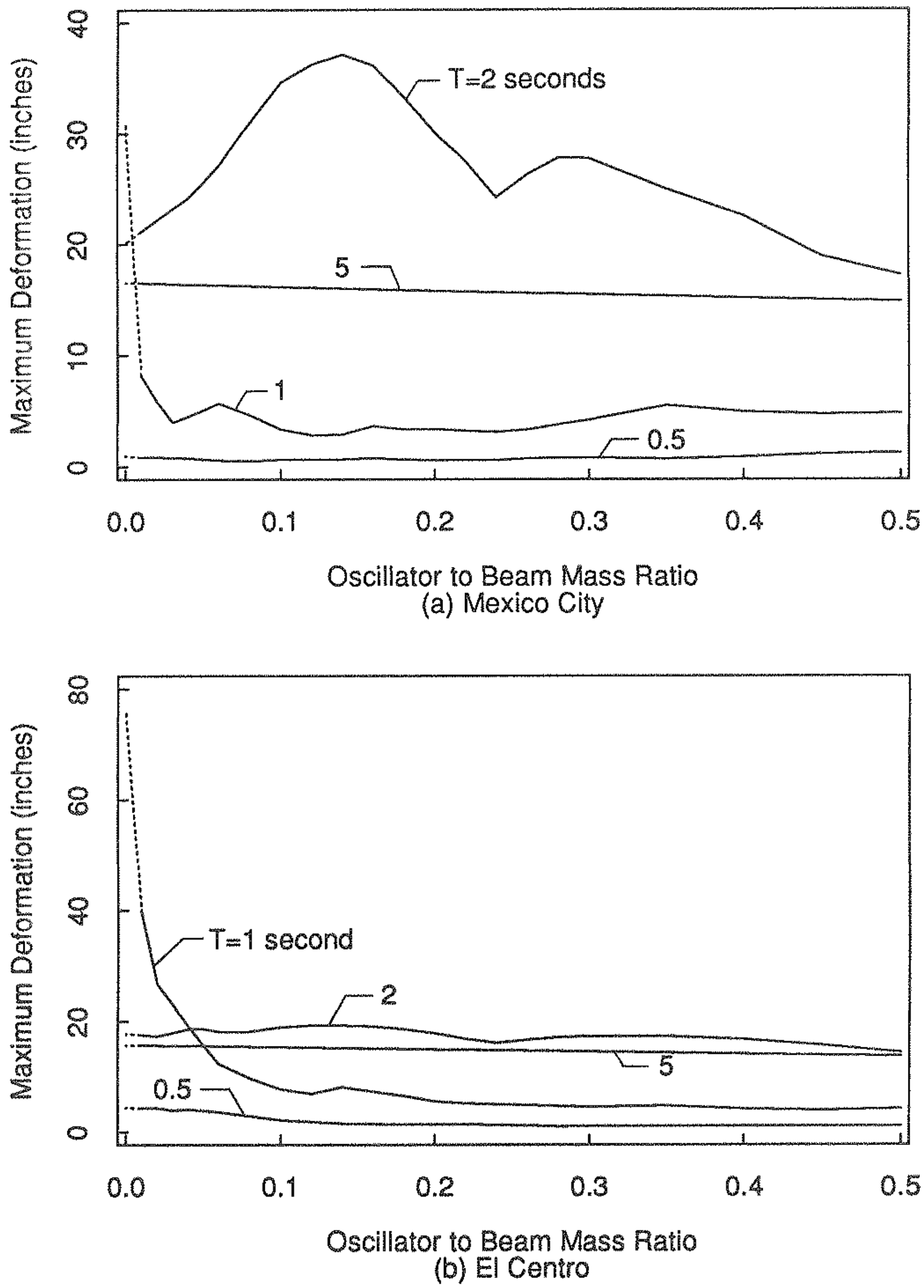


FIGURE 3-5 Oscillator Deformation as a Function of Mass Ratio (Constant Parameters: $T_b = 1$ second, $\zeta = \zeta_b = 2\%$, $\eta = 1$)

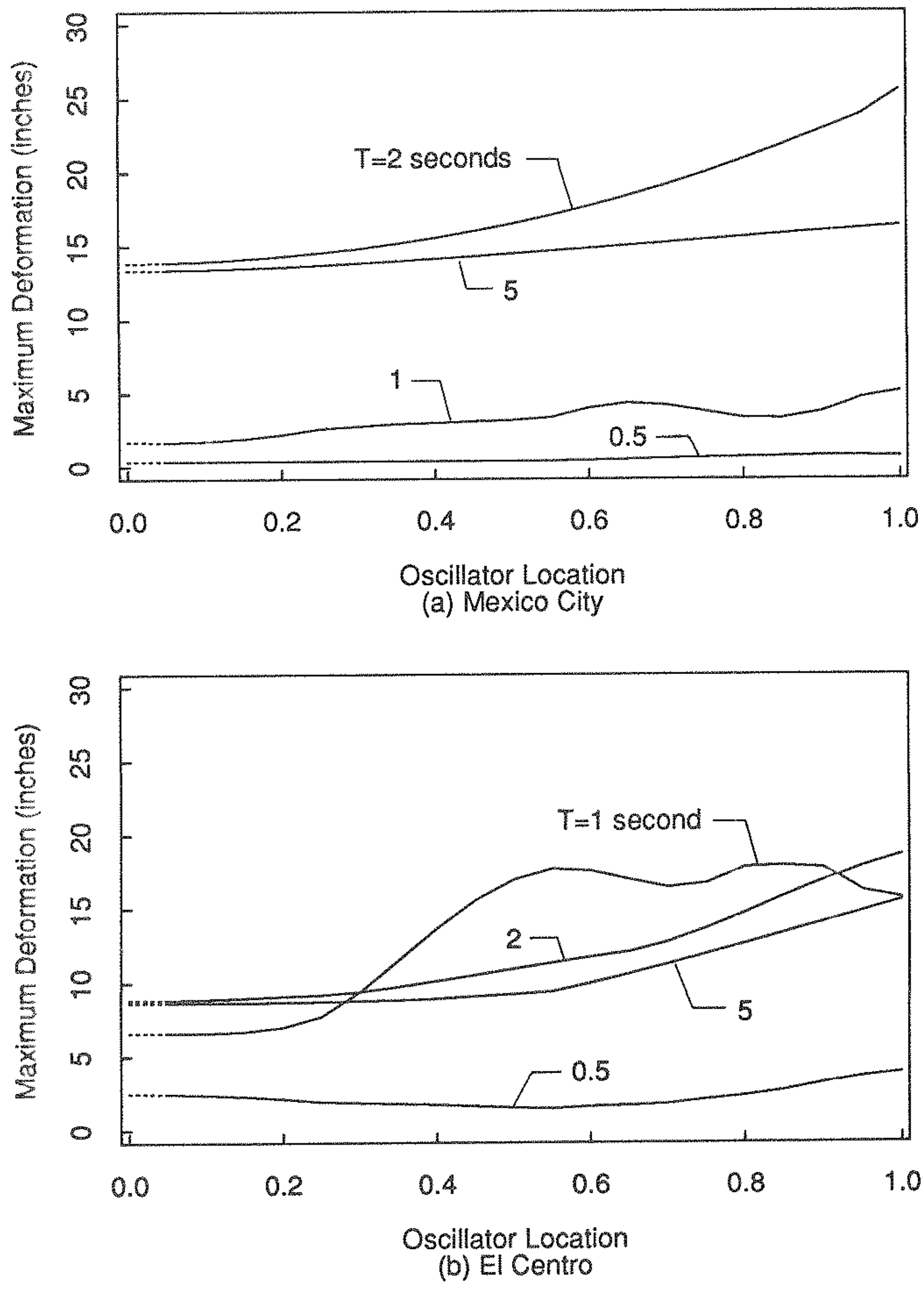


FIGURE 3-6 Oscillator Deformation as a Function of Location (Constant Parameters: T_b = 1 second, $\zeta = \zeta_b = 2\%$, $\mu = 0.05$)

SECTION 4
SUMMARY

The system considered in this paper consists of a continuous primary structure and a discrete secondary system. The primary structure is a linear, viscously damped, cantilever beam, and the secondary system is a linear, damped, single degree of freedom oscillator. The combined dynamical system is subjected to translational ground excitations. Two specific records are considered. The first is a narrow-banded, low frequency record from the 1985 Mexico City earthquake, and the second is the widely used 1940 El Centro accelerogram which has a relatively broad range of frequency components.

Two different solution techniques are employed in the response calculations. The first approach is a truncated modal analysis in which exact free vibration modes of the combined system are obtained in terms of the Green's function of the vibrating cantilever beam. An orthogonality relation allows the uncoupling of the forced vibration problem in the case of a proportionally damped system. In general, the system is nonproportionally damped if damping coefficients are specified is terms of the beam and oscillator separately. To uncouple the damping terms, the infinite dimensional system of equations is truncated to the first N equations, and the problem is reformulated in state space. This step represents the only approximation involved in the modal analysis approach. The state space equations are uncoupled by means of a complex eigenproblem which leads to 2N complex-valued, first order differential equations. Assuming that the ground response behaves linearly between the discrete points given in the earthquake records, the equations can be solved exactly. Inverse transformations are then applied to obtain the response of the system in the original physical coordinates.

The second approach to the solution involves the cascade assumption in which the response of the beam is assumed to be independent of the oscillator motion. This uncouples the problem and allows a straightforward analysis of the cantilever beam subjected to base excitation. The beam response is then combined with the ground acceleration to form the forcing function for the ordinary differential equation governing the oscillator. Although the cascade solution is relatively simple to compute, it implicitly assumes that the ratio of

oscillator mass to beam mass is zero. The results are correct only for this limiting case.

Oscillator deformation spectra are presented in several different formats. The spectra are based on the same relationships used to plot ground response spectra on four way log paper. A direct comparison between the truncated modal analysis solution and the cascade solution is obtained in Figures 3-2 and 3-5 by varying the mass ratio of the system. Since the cascade solution is independent of mass ratio, the modal analysis response curves approach the cascade curve as the mass ratio tends towards zero. The spectra in Figure 3-3 are a direct extension of ground response spectra in which the results are presented for various percentages of critical damping in the oscillator. The effect of the beam stiffness is shown in Figure 3-4 by varying the fundamental period of the primary structure. In this case, a comparison with ground response spectra is made for the limiting case in which the beam becomes infinitely stiff. Finally, the results presented in Figure 3-6 demonstrate the effect of oscillator location. As should be expected, the oscillator deformation spectra tend towards the values obtained in ground response spectra as the oscillator is moved toward the base of the beam.

The results obtained in this study lead to several general conclusions about the response of this class of secondary systems. For most cases considered, the cascade solution produces a conservative upper bound on the response of the oscillator while ground response spectra usually provide a lower bound on the response. There are two instances where the cascade solution provides undesirable results. Extremely conservative response levels are obtained in cases where the oscillator is tuned to a natural frequency of the primary system. On the other hand, cascade results may be unconservative in cases where the natural frequency of the oscillator falls within or near a relatively narrow band of dominant earthquake frequencies. In situations other than these, particularly for small to moderate mass ratios, the cascade results are generally quite acceptable.

SECTION 5
ACKNOWLEDGMENT

A portion of this work was completed while one of the authors (Bergman) was on sabbatical leave in the Department of Mechanical Engineering and Material Science at Rice University. He wishes to express his appreciation to the Department, and especially to Professor Spanos, for their hospitality and support.

SECTION 6
REFERENCES

1. Bergman, L. A., and Nicholson, J. W., "Forced Vibration of a Damped Combined Linear System," **Journal of Vibration, Acoustics, Stress and Reliability in Design,** ASME, Vol. 107, No. 3, July 1985, pp. 275-281.

2. Clough, R. W., and Penzien, J., **Dynamics of Structures,** McGraw-Hill, Inc., New York, 1975.

3. Dowell, E. H., "On Some General Properties of Combined Dynamical Systems," **Journal of Applied Mechanics,** ASME, Vol. 46, March 1979, pp. 206-209.

4. Earthquake Engineering Research Laboratory, **Analysis of Strong Motion Earthquake Accelerograms,** Vol III, Part A, EERL 72-80, California Institute of Technology, Pasedena, 1972.

5. Earthquake Engineering Research Laboratory, **Strong Motion Earthquake Accelerograms: Digitized and Plotted Data,** Vol II, Part A, EERL 71-50, California Institute of Technology, Pasedena, 1971.

6. Foss, K. A., "Co-Ordinates Which Uncouple the Equation of Motion of Damped Linear Dynamic Systems," **Journal of Applied Mechanics,** ASME, Vol. 25, 1958, pp. 361-364.

7. Meirovitch, L., **Analytical Methods in Vibrations,** The MacMillan Co., New York, 1967.

8. Nicholson, J. W., and Bergman, L. A., "Free Vibration of Combined Dynamical Systems," **Journal of Engineering Mechanics,** ASCE, Vol. 112, No. 1, Jan. 1986, pp. 1-13.

9. Nour-Omid, B., Sackman, J. L., and Der Kiureghian, A., "Modal Characterization of Equipment Continuous Structure Systems," **Journal of Sound and Vibration,** Vol. 88, 1983, pp. 459-472.

10. Suarez, L. E., and Singh, M. P., "Floor Response Spectra with Structure-Equipment Interaction Effects by a Mode Synthesis Approach," **Earthquake Engineering and Structural Dynamics,** Vol. 15, 1987, pp. 141-158.

11. Universidad Nacional Autonoma de Mexico, Instituto de Ingenieria, **Mexican Earthquake: September 19, 1985,** Report No. 497.

APPENDIX A

NOTATION

$\hat{a}_i$	=	Elements of the diagonal matrix $[\Phi]^T [A] [\Phi]$.
$a_n(\tau)$, $\dot{a}_n(\tau)$, $\ddot{a}_n(\tau)$	=	Temporal component of the response in the n'th mode and its derivatives.
A_n	=	Oscillator amplification factor for the n'th mode.
[A]	=	State space coefficient matrix.
$\{b(\tau)\},\{\dot{b}(\tau)\}$	=	State vector and its derivative.
$\hat{b}_i$	=	Elements of the diagonal matrix $[\Phi]^T [B] [\Phi]$.
[B]	=	State space coefficient matrix.
C	=	Oscillator damping constant.
C_b	=	Beam viscous damping constant.
C_{mn}	=	Element of the matrix [C].
[C]	=	Full damping matrix of the truncated system of equations.
d_n	=	n'th generalized modal mass.
E	=	Modulus of elasticity of the beam.
f_m	=	Element of the vector {f}.
$\hat{f}_i$	=	Element of the vector $\{\hat{f}\}$.
$\{f\}$	=	Force vector of the truncated system of equations.
$\{\tilde{f}\}$	=	Force vector of the state space equations.
$\{\hat{f}\}$	=	Force vector of the uncoupled state space equations.
F(t)	=	Constraint force between the beam and oscillator.
$\bar{F}(\tau)$	=	Dimensionless constraint force.
g	=	Acceleration due to gravity.
$G(\xi,\eta;\alpha)$	=	Green's function of a cantilever beam.
h	=	Location of the oscillator with respect to the base of the beam.
I	=	Moment of inertia of the beam.
K	=	Oscillator spring constant.
K_{mn}	=	Element of the matrix [K].
[K]	=	Diagonal stiffness matrix of the truncated system of equations.
L	=	Length of the beam.
M	=	Mass of the oscillator.
M_{mn}	=	Element of the matrix [M].
[M]	=	Diagonal mass matrix of the truncated system of equations.
N	=	Number of modes kept in truncated system of equations.
p_i	=	Complex eigenvalue of the state space equations.

$Q(\tau)$	=	Temporal component of the free vibration response.
t	=	Time.
T	=	Natural period of the oscillator when uncoupled from the beam.
T_b	=	Fundamental natural period of the beam without the oscillator.
$u_i(\tau)$, $\dot{u}_i(\tau)$	=	Coordinates in uncoupled state space.
x	=	Longitudinal axis of the beam.
$\{u(\tau)\}$	=	Vector of coordinates in uncoupled state space.
$y(x,t)$, $\dot{y}(x,t)$, $\ddot{y}(x,t)$	=	Displacement, velocity, and acceleration of the beam.
$\bar{y}(\xi,\tau)$, $\dot{\bar{y}}(\xi,\tau)$, $\ddot{\bar{y}}(\xi,\tau)$	=	Dimensionless displacement, velocity, and acceleration of the beam.
$\ddot{y}_g(t)$	=	Ground acceleration.
$\ddot{\bar{y}}_g(\tau)$	=	Dimensionless ground acceleration.
$Y_n(\xi)$, $Y_m(\xi)$	=	Free vibration mode shape of the beam.
$Y_n(\eta)$	=	Dimensionless beam displacement in the n'th mode at the oscillator attachment point.
$z(t)$, $\dot{z}(t)$, $\ddot{z}(t)$	=	Absolute displacement, velocity, and acceleration of the oscillator.
$\bar{z}(\tau)$, $\dot{\bar{z}}(\tau)$, $\ddot{\bar{z}}(\tau)$	=	Absolute dimensionless displacement, velocity, and acceleration of the oscillator.
Z_n	=	Free vibration mode shape of the oscillator.
α_n	=	n'th eigenvalue of the combined system.
$\delta(x-h)$	=	Dirac delta function at x=h.
$\bar{\delta}(\xi-\eta)$	=	Dimensionless Dirac delta function at $\xi = \eta$.
δ_{mn}	=	Kronecker delta function.
Δt	=	Time step in digitized earthquake record.
$\Delta\tau$	=	Dimensionless time step.
ε	=	Dimensionless damping coefficient of the oscillator.
ε_b	=	Dimensionless damping coefficient of the beam.
η	=	Dimensionless location of the oscillator.
γ	=	Dimensionless force parameter.
κ	=	Oscillator to beam stiffness ratio.
λ_i	=	Dimensionless part of the i'th natural frequency of a cantilever beam.
μ	=	Oscillator to beam mass ratio.
ω	=	Oscillator natural frequency.
Ω	=	Dimensional part of the natural frequencies of a cantilever beam.
Ω_0	=	Dimensionless frequency parameter of the combined system.
$\{\phi_i\}$	=	i'th complex mode of the state space equations.
$[\Phi]$	=	Complex modal matrix of the state space equations.
ρ	=	Mass per unit length of the beam.

τ	=	Dimensionless time.
ξ	=	Dimensionless longitudinal axis of the beam.
ζ	=	Ratio of damping to critical damping in the oscillator.
ζ_b	=	Ratio of damping to critical damping in the first beam mode.

NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
LIST OF PUBLISHED TECHNICAL REPORTS

The National Center for Earthquake Engineering Research (NCEER) publishes technical reports on a variety of subjects related to earthquake engineering written by authors funded through NCEER. These reports are available from both NCEER's Publications Department and the National Technical Information Service (NTIS). Requests for reports should be directed to the Publications Department, National Center for Earthquake Engineering Research, State University of New York at Buffalo, Red Jacket Quadrangle, Buffalo, New York 14261. Reports can also be requested through NTIS, 5285 Port Royal Road, Springfield, Virginia 22161. NTIS accession numbers are shown in parenthesis, if available.

NCEER-87-0001 "First-Year Program in Research, Education and Technology Transfer," 3/5/87, (PB88-134275/AS).

NCEER-87-0002 "Experimental Evaluation of Instantaneous Optimal Algorithms for Structural Control," by R.C. Lin, T.T. Soong and A.M. Reinhorn, 4/20/87, (PB88-134341/AS).

NCEER-87-0003 "Experimentation Using the Earthquake Simulation Facilities at University at Buffalo," by A.M. Reinhorn and R.L. Ketter, to be published.

NCEER-87-0004 "The System Characteristics and Performance of a Shaking Table," by J.S. Hwang, K.C. Chang and G.C. Lee, 6/1/87, (PB88-134259/AS).

NCEER-87-0005 "A Finite Element Formulation for Nonlinear Viscoplastic Material Using a Q Model," by O. Gyebi and G. Dasgupta, 11/2/87, (PB88-213764/AS).

NCEER-87-0006 "Symbolic Manipulation Program (SMP) - Algebraic Codes for Two and Three Dimensional Finite Element Formulations," by X. Lee and G. Dasgupta, 11/9/87.

NCEER-87-0007 "Instantaneous Optimal Control Laws for Tall Buildings Under Seismic Excitations," by J.N. Yang, A. Akbarpour and P. Ghaemmaghami, 6/10/87, (PB88-134333/AS).

NCEER-87-0008 "IDARC: Inelastic Damage Analysis of Reinforced Concrete-Frame Shear-Wall Structures," by Y.J. Park, A.M. Reinhorn and S.K. Kunnath, 7/20/87, (PB88-134325/AS).

NCEER-87-0009 "Liquefaction Potential for New York State: A Preliminary Report on Sites in Manhattan and Buffalo," by M. Budhu, V. Vijayakumar, R.F. Giese and L. Baumgras, 8/31/87, (PB88-163704/AS).

NCEER-87-0010 "Vertical and Torsional Vibration of Foundations in Inhomogeneous Media," by A.S. Veletsos and K.W. Dotson, 6/1/87, (PB88-134291/AS).

NCEER-87-0011 "Seismic Probabilistic Risk Assessment and Seismic Margin Studies for Nuclear Power Plants," by Howard H.M. Hwang, 6/15/87, (PB88-134267/AS).

NCEER-87-0012 "Parametric Studies of Frequency Response of Secondary Systems Under Ground-Acceleration Excitations," by Y. Yong and Y.K. Lin, 6/10/87, (PB88-134309/AS).

NCEER-87-0013 "Frequency Response of Secondary Systems Under Seismic Excitations," by J.A. HoLung, J. Cai and Y.K. Lin, 7/31/87, (PB88-134317/AS).

NCEER-87-0014 "Modelling Earthquake Ground Motions in Seismically Active Regions Using Parametric Time Series Methods," G.W. Ellis and A.S. Cakmak, 8/25/87, (PB88-134283/AS).

NCEER-87-0015 "Detection and Assessment of Seismic Structural Damage," by E. DiPasquale and A.S. Cakmak, 8/25/87, (PB88-163712/AS).

NCEER-87-0016 "Pipeline Experiment at Parkfield, California," by J. Isenberg and E. Richardson, 9/15/87, (PB88-163720/AS).

NCEER-87-0017 "Digital Simulations of Seismic Ground Motion," by M. Shinozuka, G. Deodatis and T. Harada, 8/31/87, (PB88-155197/AS).

NCEER-87-0018 "Practical Considerations for Structural Control: System Uncertainty, System Time Delay and Truncation of Small Forces," J. Yang and A. Akbarpour, 8/10/87, (PB88-163738/AS).

NCEER-87-0019 "Modal Analysis of Nonclassically Damped Structural Systems Using Canonical Transformation," by J.N. Yang, S. Sarkani and F.X. Long, 9/27/87, (PB88-187851/AS).

NCEER-87-0020 "A Nonstationary Solution in Random Vibration Theory," by J.R. Red-Horse and P.D. Spanos, 11/3/87, (PB88-163746/AS).

NCEER-87-0021 "Horizontal Impedances for Radially Inhomogeneous Viscoelastic Soil Layers," by A.S. Veletsos and K.W. Dotson, 10/15/87, (PB88-150859/AS).

NCEER-87-0022 "Seismic Damage Assessment of Reinforced Concrete Members," by Y.S. Chung, C. Meyer and M. Shinozuka, 10/9/87, (PB88-150867/AS).

NCEER-87-0023 "Active Structural Control in Civil Engineering," by T.T. Soong, 11/11/87, (PB88-187778/AS).

NCEER-87-0024 "Vertical and Torsional Impedances for Radially Inhomogeneous Viscoelastic Soil Layers," by K.W. Dotson and A.S. Veletsos, 12/87, (PB88-187786/AS).

NCEER-87-0025 "Proceedings from the Symposium on Seismic Hazards, Ground Motions, Soil-Liquefaction and Engineering Practice in Eastern North America, October 20-22, 1987, edited by K.H. Jacob, 12/87, (PB88-188115/AS).

NCEER-87-0026 "Report on the Whittier-Narrows, California, Earthquake of October 1, 1987," by J. Pantelic and A. Reinhorn, 11/87, (PB88-187752/AS).

NCEER-87-0027 "Design of a Modular Program for Transient Nonlinear Analysis of Large 3-D Building Structures," by S. Srivastav and J.F. Abel, 12/30/87, (PB88-187950/AS).

NCEER-87-0028 "Second-Year Program in Research, Education and Technology Transfer," 3/8/88.

NCEER-88-0001 "Workshop on Seismic Computer Analysis and Design With Interactive Graphics," by J.F. Abel and C.H. Conley, 1/18/88, (PB88-187760/AS).

NCEER-88-0002 "Optimal Control of Nonlinear Structures," J.N. Yang, F.X. Long and D. Wong, 1/22/88, (PB88-213772/AS).

NCEER-88-0003 "Substructuring Techniques in the Time Domain for Primary-Secondary Structural Systems," by G. D. Manolis and G. Juhn, 2/10/88, (PB88-213780/AS).

NCEER-88-0004 "Iterative Seismic Analysis of Primary-Secondary Systems," by A. Singhal, L.D. Lutes and P. Spanos, 2/23/88, (PB88-213798/AS).

NCEER-88-0005 "Stochastic Finite Element Expansion for Random Media," P. D. Spanos and R. Ghanem, 3/14/88, (PB88-213806/AS).

NCEER-88-0006 "Combining Structural Optimization and Structural Control," F. Y. Cheng and C. P. Pantelides, 1/10/88, (PB88-213814/AS).

NCEER-88-0007 "Seismic Performance Assessment of Code-Designed Structures," H.H-M. Hwang, J. Jaw and H. Shau, 3/20/88.

NCEER-88-0008 "Reliability Analysis of Code-Designed Structures Under Natural Hazards," H.H-M. Hwang, H. Ushiba and M. Shinozuka, 2/29/88.

NCEER-88-0009 "Seismic Fragility Analysis of Shear Wall Structures," J-W Jaw and H.H-M. Hwang, 4/30/88.

NCEER-88-0010 "Base Isolation of a Multi-Story Building Under a Harmonic Ground Motion - A Comparison of Performances of Various Systems," F-G Fan, G. Ahmadi and I.G. Tadjbakhsh, to be published.

NCEER-88-0011 "Seismic Floor Response Spectra for a Combined System by Green's Functions," F.M. Lavelle, L.A. Bergman and P.D. Spanos, 5/1/88.

National Center for Earthquake Engineering Research
State University of New York at Buffalo