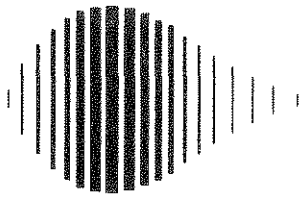

NATIONAL CENTER FOR EARTHQUAKE
ENGINEERING RESEARCH
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ITERATIVE SEISMIC ANALYSIS OF PRIMARY-SECONDARY SYSTEMS

by

A. Singhal, L.D. Lutes and P. Spanos
Department of Civil Engineering and Mechanical Engineering
Rice University
Houston, Texas 77251

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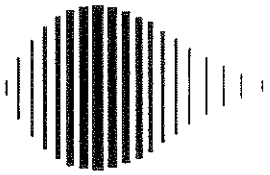

ITERATIVE SEISMIC ANALYSIS OF PRIMARY-SECONDARY SYSTEMS

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A. Singhal[1], L.D. Lutes[2] and P. Spanos[3]

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1 Research Assistant, Dept. of Civil Engineering, Rice University
2 Professor, Dept. of Civil Engineering, Texas A&M University
3 Professor, Dept. of Civil Engineering and Dept. of Mechanical Engineering, Rice University

NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
State University of New York at Buffalo
Red Jacket Quadrangle, Buffalo, NY 14261

ABSTRACT

The accuracy and efficiency of different methods to obtain time histories of response of coupled systems subjected to seismic excitations are investigated. Both linear and nonlinear systems are considered. The Duhamel integral approach and the Newmark method are investigated for the composite system (i.e., without any subsystem analysis). Three different formulations of primary-secondary subsystem analysis are considered, and compared with the composite analyses. In each of these subsystem analysis methods iteration is performed to find the solution to coupled subsystem equations. The mass coupled approach in which the subsystems are coupled only by inertia terms is an existing technique while the two stiffness coupled approaches are new to this study.

For linear systems, the Newmark method for the composite system seems generally to give better results than the primary-secondary methods, although the variations between methods are generally quite small. There is a significant variation between the computation times for different methods for nonlinear systems. The direct stiffness coupled subsystem approach is found to be more efficient than the other methods, especially if the nonlinearity is only in one of the smaller subsystems. It is also shown that only Rayleigh damping can simultaneously lead to classical normal modes for the composite and for each subsystem without imposing any restrictions on the stiffness or mass matrices.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1.0 INTRODUCTION

Dynamic analysis of structures is important for the safe and efficient design of structures in seismically active areas. Sometimes, complex structures are divided into substructures for the ease of analysis. Commonly, if a structure is divided into two substructures and one of them is small (small mass and stiffness ratio) compared to the other, the smaller substructure is referred to as the secondary and the larger as the primary. In this study, the term primary-secondary has been retained but no assumption about the relative values of the parameters of the primary and secondary is made.

The objective of this study is to compare the accuracy and the efficiency of the various computation schemes for earthquake response. It is also intended to study the effect of different mass and stiffness ratios on the methods. The other objective is to find out conditions which lead to the simultaneous diagonalization of the damping matrix for the composite system as well as all the subsystems. The final objective is to find out the relative merits of the different methods when applied to nonlinear systems.

In chapter 2, some of the techniques available for the analysis of primary-secondary systems are reviewed briefly. These include cascade analysis and perturbation techniques. Also reviewed briefly are some of the methods used in modeling earthquake ground motion and some of the techniques used in the integration of equations of motion.

In chapter 3, the methods available for the analysis of the composite system, i.e., without subdividing into primary and secondary systems, are discussed. These methods are the Duhamel integral approach for the composite and the Newmark method for the composite. The conditions which need be satisfied in order to uncouple the damping matrix for the composite system along with those of the subsystems are also discussed.

This topic seems to be misinterpreted by some investigators who assume that having the same modal damping for all the subsystems automatically gives rise to a damping matrix for the composite system which can be uncoupled. However, it is shown here that this is not true. It seems that the Rayleigh condition for the composite system is the only condition which can give rise to simultaneous diagonalization of the composite along with all the subsystems without imposing any restrictions on the stiffness or mass matrices.

After subdividing the composite, the various approaches for the primary-secondary analysis are discussed. These are the mass coupled, and two stiffness coupled approaches. The direct and the diagonalized stiffness coupled are new methods which have been proposed in this study.

In chapter 4, simulation results are presented for linear and nonlinear systems. A method for generating representative time histories is discussed. For the linear case, two structures are considered in which the relative sizes of the primary to the secondary are different. The effect of different mass and stiffness ratios and of different time steps is considered. For the nonlinear case, the second structure is considered to be attached with nonlinear interface springs. Two different magnitudes of nonlinearities are considered. Also considered are the relative merits of the the various methods when all subsystems are nonlinear as well as the case when only the interface is nonlinear.

Finally, in chapter 5, some conclusions are listed regarding the relative accuracy and efficiency of the different methods of analysis for linear and nonlinear systems. It seems that substructuring need not always be the more efficient way of obtaining time histories of response to earthquake excitation.

2.0 BACKGROUND

The dynamic analysis of structures is carried out in order to design structures subjected to forces varying with time. Dynamic analysis is complicated by the fact that inertia forces result from structural displacements which in turn depend on the magnitude of the inertia forces. To avoid using an iterative scheme the problem is formulated in terms of differential equations. If the mass of the structure is assumed to be distributed, then the response quantities must be defined at each point on the structure. In this case the problem can only be formulated in terms of partial differential equations because the response quantities will vary with respect to both space and time.

However, if the mass of the structure is assumed to be concentrated at some discrete points, then the analytical problem is simplified because inertia forces are developed only at these points. These structures are often known as lumped mass systems. For these structures the response quantities are computed only at these discrete points and the problem can be formulated in terms of an ordinary differential equation. In this study only lumped mass systems have been used.

2.1 Integrating Dynamic Equations of Motion

There are several techniques that can be used for the solution of any set of second order differential equations. Specifically, consider the equation

$$M\ddot{\vec{x}} + C\dot{\vec{x}} + K\vec{x} = \vec{f}(t) \tag{2.1}$$

which describes the motion of a lumped mass system with the coefficient matrices defined by the following energy relationships

$$\text{potential energy} = \frac{1}{2}\vec{x}^T K\vec{x} \tag{2.2}$$

$$\text{kinetic energy} = \frac{1}{2}\dot{\vec{x}}^T M \dot{\vec{x}} \tag{2.3}$$

$$\text{energy dissipated per unit time} = \dot{\vec{x}}^T C \dot{\vec{x}} \tag{2.4}$$

Mathematically eqn. (2.1) represents a system of linear differential equations of second order and the solution can be obtained by standard procedures for the solution of differential equations with constant coefficients. However, the procedures proposed for the solution of a general system of equations can become expensive if the order of the matrices is large - unless advantage is taken of the special characteristics of the coefficient matrices M, C and K. Defining these matrices using eqns. (2.2), (2.3), (2.4) assures the symmetry of these matrices. Generally two methods of integration are considered for solving eqn. (2.1). These are direct integration and modal superposition.

2.1.1 Direct Integration Methods

In direct integration eqn. (2.1) is integrated using a numerical step by step procedure[5, 31]. The term "direct" is used to imply that eqn. (2.1) is not transformed into a different form prior to the numerical integration. Direct integration methods are based on two ideas. First, eqn. (2.1) is not satisfied at every time t, but only at discrete times an interval Δt apart. This means that static equilibrium, which includes the effect of inertia and damping forces is satisfied at discrete time points within the interval of solution. The second idea is that a variation of displacement, velocity and acceleration within each time step Δt is assumed. The form of the assumption on the variation of displacement, velocity and acceleration within each time interval determines the accuracy, stability and the cost of the solution procedure.

It is assumed that the initial displacement, velocity and acceleration vectors denoted by $\vec{x}_0$, $\dot{\vec{x}}_0$, and $\ddot{\vec{x}}_0$ are known and the solution is required from time 0 to T. For the solution, the time range is subdivided into n equal time intervals Δt ($\Delta t = T/n$). The

integration scheme computes an approximate solution at times Δt, $2\Delta t$,....,T.

The Newmark method is one such direct integration method in which the displacement and velocity at the next time step are given in terms of the response parameters at the previous time step and the acceleration at the next time step. Since the acceleration at the next time step needs to be assumed, this method is an implicit procedure. This method has been used in this study and it will be described in detail later.

2.1.2 Stability and Accuracy of a Direct Integration Method

An integration scheme is said to be unconditionally stable if the solution for any initial conditions does not grow without bound for any time step, in particular when the time step is large compared to the shortest time period of the structure. The error in the computed value of the acceleration, velocity and displacement components should not grow due to round off. A method is conditionally stable if the errors do not grow only when the time step is below a certain critical level Δt_{cr}. The stability of any method can be evaluated using the concept of spectral radius[4].

The accuracy of a numerical integration scheme depends on the loading, the physical parameters of the system and the time step. In general, if the time step is large, accuracy might be obtained in the lower mode responses but not in the higher modes, in particular, the higher modes are effectively filtered out of the solution. When an unconditionally stable scheme is used, the time step is chosen with regard to accuracy only and not with regard to stability. This generally allows a larger time step to be used. In the dynamic analysis of most structures only frequencies within a specified range are of interest. The loading generally defines which frequencies are significant and how small a time step should be used. In the case of earthquake loading, excitation components with periods smaller than about 0.05 sec. generally are not accurately

recorded[31] and so it may not be justifiable to include the response in these higher frequencies in the analysis.

2.1.3 Modal Superposition

A coordinate transformation which changes a set of N coupled equations of motion of a multi degree of freedom system into a set of N uncoupled equations is the basis of the modal superposition method. If the damping is of a restricted form, where the coefficient matrices satisfy the relationship $C M^{-1} K = K M^{-1} C$, then the damping can be expressed by modal damping ratios. The system is then said to have classical normal modes and there exists a matrix Θ such that

$$\Theta^T M \Theta = I \tag{2.5}$$

$$\Theta^T C \Theta = \Gamma = \text{diagonal} \tag{2.6}$$

$$\Theta^T K \Theta = \Lambda = \text{diagonal} \tag{2.7}$$

and if one introduces variables $\vec{y}$ such that $\vec{x} = \Theta \vec{y}$ then eqn. (2.1) is replaced by

$$\ddot{\vec{y}} + \Gamma \dot{\vec{y}} + \Lambda \vec{y} = \Theta^T \vec{f} \tag{2.8}$$

This is a set of N independent equations of motion, one for each mode of vibration. These equations can now be solved using the Duhamel integral approach.

2.2 Earthquake Motions

The applied loads to civil engineering structures are in many cases of random nature. Due to the uncertainty about details of future earthquakes and due to the availability of only a few records of past earthquakes for a given site condition, earthquake ground motion is considered a random process. Analyses which use actual recorded data of particular earthquakes are equivalent to a deterministic approach and generally provide an inadequate approach for developing response statistics. However,

the deterministic aspects of such analyses become less significant when a large number of past earthquakes is considered. Simulation procedures have the advantage that they can produce as many sample records as desired.

A direct characterization of earthquake motion in the time domain is provided by accelerograms. An accelerogram is the time record of ground acceleration during an earthquake. Information about duration, frequency content and maximum acceleration of ground shaking can be obtained from an accelerogram.

A response spectrum is associated with the dynamic behavior of a single degree of freedom system resting on a base which is subjected to an acceleration history specified by a particular accelerogram under consideration. The response spectrum represents the maximum values of the response parameters as displacement, velocity or acceleration of the single degree of freedom system subjected to the given excitation. The maximum values of the response parameters depend on the ratio of critical damping ζ of the single degree of freedom system.

The concept of design spectrum has been described in references 26 and 27. The response spectrum corresponding to a particular accelerogram exhibits local irregularities in the frequency domain. When an ensemble of accelerograms produced on geologically and seismologically similar sites is used to determine the response spectra, then the response spectra are smooth functions of frequency. If an approach based on the concept of response spectrum is to be used for earthquake resistant design of structures, then a design spectrum which represents recorded and expected strong ground shaking at a certain location can be used. The design spectra smooth out the irregularities of any particular response spectrum and represent the repetitive characteristics of the ensemble of response spectra.

The two basic approaches for generating time histories consistent with design

spectra are deterministic and stochastic. The deterministic approach is based on a real or artificial acceleration record the spectrum of which approximates the design spectrum. This record is then modified using spectral raising or suppressing techniques. The stochastic approach is based on determining the power spectral density of a process, a realization of which can be an approximation of a record in a deterministic approach.

Several methods for generating artificial time histories have been developed by Housner and Jennings[9], Levy, Kozin and Moorman[14] and Shinozuka and Sato[21]. Some deterministic methods of generating artificial time histories also exist[29]. As pointed out earlier, the usefulness of the deterministic approaches is restricted due to a limited number of past earthquake records for a given site condition.

One process that has been used in modeling earthquakes is stationary white noise. The real earthquake motion is nonwhite because it does not contain equal energy at all frequencies. Housner and Jennings have suggested using a filtered white noise. These models are based on the belief that seismic disturbances generated in an underlying rock filter through the overlying soil media. The seismic disturbances at the bed rock might be considered to be white, but the motions at the surface also represent the vibratory characteristics of the soil strata.

The method of Housner and Jennings is based on the following approximate relationship between the velocity response spectrum $S_v(\omega,0)$ with zero damping and the power spectrum $G(\omega)$

$$G(\omega) = \frac{0.2304}{t_0} S_v^2(\omega, 0) \tag{2.9}$$

where t_0 is the duration of the earthquake. Eqn. (2.9) was obtained by Rosenblueth and Bustamante[18]. Housner and Jennings suggested that the power spectral density given

by eqn. (2.9) be fitted to a simple algebraic function

$$G(\omega) = \frac{a\,(1 + 4b^2\omega^2/c^2)}{(1 - \omega^2/c^2) + 4b^2\omega^2/c^2} \tag{2.10}$$

To match the average response spectra in the range $2.1 \leq \omega \leq 21$ the constants were found to be $a = 0.2196\ t_0$; $b^2 = 0.410$; $c^2 = 242$. The power spectrum given by eqn. (2.10) is identical to the response of a linear system to a white noise with a power spectral density of 1.

In reference 15 the response of a single degree of freedom system subjected to stationary excitation with different frequency contents is considered. The single degree of freedom system is considered with linear properties as well as bilinear hysteretic yielding behavior.

A fairly good approximation of an earthquake is obtained by using stationary models with appropriate modulating functions in order to introduce time nonstationarity. This approach is described in references 14 and 21. The model of Shinozuka and Sato[21] filters a white input, and a function of time is included to induce nonstationarity. A significant feature of this model is that it insures that the associated ground velocity, in addition to the ground acceleration, eventually tends toward zero. This model, which has been used in this study, will be described later.

2.3 Secondary Systems

Complex structures are often subdivided into primary and secondary systems. The primary systems generally represent the main structures and buildings which consist of beams, columns, trusses, floors and shear walls. These support the secondary systems. For example, the primary structure can be a civil engineering structure and the secondary can be the electrical and mechanic installations.

A secondary system is generally supported, on a single support or multiple supports, from the primary structure. The systems with single support are usually small and generally consist of small machinery or equipment. The multiply supported systems, like piping in industrial facilities, can be spatially large.

Traditionally, structures have been subdivided into primary and secondary systems because of the practical difficulty in carrying out a combined dynamic analysis where large matrices had to be stored. Primary-secondary analysis has been carried out considering the primary and secondary in cascade and by using perturbation techniques.

2.3.1 Cascade Analysis

During an earthquake, the secondary system is excited by the primary which supports it and its response is written in terms of the motion of the primary system[22]. The motion of the primary system can be written in terms of the responses of the individual modes. Thus the response of the secondary system is investigated by considering two single degree of freedom systems placed in cascade as shown in fig. 2.1. The first single degree of freedom system represents one of the modes of the primary system and the second represents a single degree of freedom secondary system or one of its modes.

A common method [23] used for calculating the response of a multiply supported secondary system is the floor response spectrum method. A floor response spectrum is the representation of the peak response of a single degree of freedom oscillator of variable frequency subjected to the motion of a support point ("floor") on the primary system. The response of the secondary system is then computed by modal combination in terms of the ordinates of the floor response spectra associated with the support points.

However, this approach neglects the interaction between the primary and the

secondary systems. The effect of interaction can be significant when tuning exists. Tuning is the coincidence of the frequencies of the two systems and may give rise to resonant responses of the secondary system. This approach also neglects the non-classical damping character of the composite primary-secondary system, which may result in inaccurate estimation of the response. It will be shown in chapter 3 that the composite system might have non-classical damping even if each individual subsystem has classical damping. The usual modal combination rules do not account for the cross-correlations between modal responses of the secondary system or the cross-correlations between the various support excitations resulting from spatial coupling. The net effect of these inaccuracies is an error in the computed response which can be on the conservative side or on the unconservative side. However, this error may not be significant[22] if the mass ratio of the secondary to primary system is less than 0.01.

It is possible to consider the cross-correlations between modal responses and between support motions by using the cross-cross floor spectrum method [2]. This spectrum is defined in terms of the cross-correlation between the responses of two oscillators attached to the support points on the primary system. The oscillators represent the modes of the secondary system.

2.3.2 Perturbation Techniques

In this method the dynamic properties of the combined primary-secondary system are computed in terms of those of the primary and the secondary alone. The secondary system is assumed to be light compared to the primary so that the dynamic properties of the combined system are not too different from those of the primary.

It is assumed[19,20] that for light secondary systems there will only be a slight change in the frequencies and mode shapes of the combined system. It has been established that first order errors in the mode shapes result in second order errors in the

frequencies. Thus, as an approximation it is assumed that the portions of the modal vectors corresponding to structural degrees of freedom retain their shape after the secondary system is attached. This leads to a second order approximation of the frequencies of the combined system. However, this is only appropriate when the major contribution to the modal matrix of the combined system comes from the primary. Perturbation methods have been extended to deal with cases when the secondary system is tuned to a primary mode.[11,12].

The techniques mentioned above assume the lightness of the secondary in comparison to the primary leading to certain simplifying mathematical approximations. Spanos et al. have presented a technique[28] which can be used to predict response time histories for any mass ratio. This involves a predictor-corrector scheme to converge iteratively to the interface accelerations of the combined system. A slight modification to this technique, wherein one converges iteratively to the interface displacements of the combined system instead of the interface accelerations is suggested here. These schemes will be described in detail later.

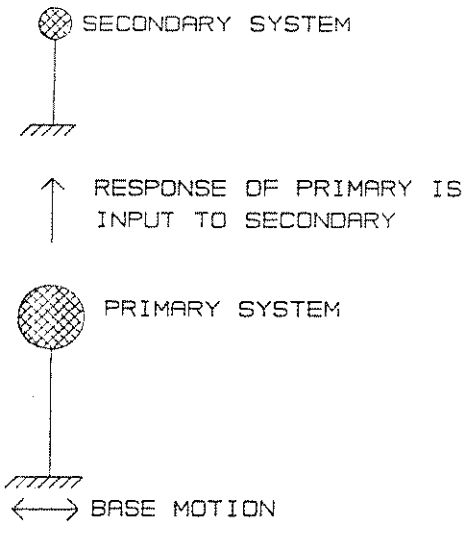


Fig. 2.1 Cascade Analysis

3.0 ANALYSIS OF SECONDARY SYSTEMS

In the numerical integration of eqn. (2.1) the time is discretized to a set $(t_1, t_2, \ldots, t_n)$ and the response parameters are evaluated at these times. The values of $\vec{x}_{n+1}$ and $\dot{\vec{x}}_{n+1}$ are found from the knowledge of $\vec{f}(t)$, $\vec{x}_n \equiv \vec{x}(t_n)$ and $\dot{\vec{x}}_n \equiv \dot{\vec{x}}(t_n)$. Generally only discrete values of the excitation $\vec{f}$ are used but any available information about the way in which $\vec{f}(t)$ varies between $\vec{f}_n \equiv \vec{f}(t_n)$ and $\vec{f}_{n+1}$ can be used.

3.1 Duhamel Integral Approach for Composite System

In the composite system approach the structure is not subdivided into primary and secondary systems but the whole structure is considered as one composite unit. Consider eqn. (2.1) for the composite system with the coefficient matrices as defined in eqns. (2.2), (2.3) and (2.4). Let a matrix $h^*(t)$ be defined such that $h^*_{jk}(t)$ represents the $x_j(t)$ homogeneous response when the initial conditions are velocity at kth degree of freedom $\dot{x}_k(0) = 1$, all other velocities being zero and all displacements being zero. Similarly let a matrix $g^*(t)$ be defined such that $g^*_{jk}(t)$ represents the homogeneous $x_j(t)$ when the initial conditions are displacement at kth degree of freedom $x_k(0) = 1$, all other displacements being zero and all velocities being zero. Then the response at the next time step is given by

$$\vec{x}_{n+1} = [g^*(\Delta t)]\,\vec{x}_n + [h^*(\Delta t)]\,\dot{\vec{x}}_n + \int_0^{\Delta t} [h^*(u)]\,M^{-1}\,\vec{f}(t_n + u)\,du \qquad (3.1)$$

$$\dot{\vec{x}}_{n+1} = [g^{*\prime}(\Delta t)]\,\vec{x}_n + [h^{*\prime}(\Delta t)]\,\dot{\vec{x}}_n + \int_0^{\Delta t} [h^{*\prime}(u)]\,M^{-1}\,\vec{f}(t_n + u)\,du \qquad (3.2)$$

The matrices M and K in eqn. (2.1) are real, symmetric and positive definite. There exists a matrix Θ such that the following relationships hold

$$\Theta^T M \Theta = I = \text{identity matrix} \tag{3.3}$$

$$\Theta^T K \Theta = \Lambda = \text{diag}(\omega_1^2, \omega_2^2, \ldots\ldots, \omega_n^2) \tag{3.4}$$

where $\omega_j > 0$, $j = 1,2,\ldots\ldots n$ are the real natural frequencies of the undamped system and diag( ) denotes a diagonal matrix.

If the matrices M, C and K satisfy the relationship $K M^{-1} C = C M^{-1} K$, then the system is said to have classical normal modes and the matrix C is also diagonalized by the matrix Θ.

$$\Theta^T C \Theta = \Gamma = \text{diag}(2\zeta_1\omega_1, 2\zeta_2\omega_2, \ldots\ldots, 2\zeta_n\omega_n) \tag{3.5}$$

where ζ_j is known as the modal damping for the jth mode. For this case eqn. (2.1) can be reduced to a system of n uncoupled second order differential equations by the transformation $\vec{x} = \Theta \vec{y}$. Substituting for $\vec{x}$ in eqn. (2.1)

$$M \Theta \ddot{\vec{y}} + C \Theta \dot{\vec{y}} + K \Theta \vec{y} = \vec{f}(t) \tag{3.6}$$

$$\Theta^T M \Theta \ddot{\vec{y}} + \Theta^T C \Theta \dot{\vec{y}} + \Theta^T K \Theta \vec{y} = \Theta^T \vec{f}(t)$$

or

$$I \ddot{\vec{y}} + \Gamma \dot{\vec{y}} + \Lambda \vec{y} = \vec{p}(t) \quad \text{where } \vec{p}(t) = \Theta^T \vec{f}(t) \tag{3.7}$$

Eqn. (3.7) represents a system of n uncoupled second order differential equations which can be expressed as

$$\ddot{y}_j + 2\zeta_j \omega_j \dot{y} + \omega_j^2 y_j = p_j \tag{3.8}$$

The matrices h^* and g^* for $\vec{y}$ are represented by by h and g respectively. The matrices h^* and g^* can be expressed in terms of h and g by

$$h^* = \Theta h \Theta^T \quad \text{and} \quad g^* = \Theta g \Theta^T \tag{3.9}$$

The general solution of eqn. (3.8) can be written as

$$y_{j_{n+1}} = g_{jj}(\Delta t) y_{j_n} + h_{jj}(\Delta t)\dot{y}_{j_n} + \int_0^{\Delta t} h_{jj}(u)\, p_j\,(t_n+u)\, du \qquad (3.10)$$

$$\dot{y}_{j_{n+1}} = g'_{jj}(\Delta t)\, y_{j_n} + h'_{jj}(\Delta t)\dot{y}_{j_n} + \int_0^{\Delta t} h'_{jj}(u)\, p_j\,(t_n+u)\, du \qquad (3.11)$$

Here it may be noted that the matrices g and h are diagonal. The diagonal elements of g and h are given by

$$g_{jj}(t) = e^{-\zeta_j \omega_j t}\left[\cos(\omega_{d_j} t) + \frac{\omega_j \zeta_j}{\omega_{d_j}} \sin(\omega_{d_j} t)\right] \qquad (3.12)$$

$$h_{jj}(t) = \frac{e^{-\zeta_j \omega_j t}}{\omega_{d_j}} \sin(\omega_{d_j} t) \quad \text{where } \omega_{d_j} = \omega_j \sqrt{1-\zeta_j^2} \qquad (3.13)$$

The matrices h' and g' are defined as

$$h'(t) = \frac{d}{dt}h(t) \quad \text{and} \quad g'(t) = \frac{d}{dt}g(t) \qquad (3.14)$$

However, it should be noted that the integrals in eqns. (3.1), (3.2), (3.10) and (3.11) require knowledge of the variation of $\vec{f}(t)$ from $\vec{f}_n$ to $\vec{f}_{n+1}$.

3.2 Newmark Method for Composite System

Here also the structure is not subdivided into primary and secondary systems but considered as one composite unit. In the Newmark method it is assumed that the values of displacement, velocity, and acceleration are known at time t_n and these quantities need to be evaluated at time t_{n+1}. The displacement $\vec{x}_{n+1}$ and the velocity $\dot{\vec{x}}_{n+1}$ at the next time step are given in terms of the acceleration $\ddot{\vec{x}}_{n+1}$ by the following relationships

$$\vec{x}_{n+1} = \vec{x}_n + \dot{\vec{x}}_n \Delta t + \left(\frac{1}{2} - \beta\right)\ddot{\vec{x}}_n (\Delta t)^2 + \beta\, \ddot{\vec{x}}_{n+1} (\Delta t)^2 \qquad (3.15)$$

$$\dot{\vec{x}}_{n+1} = \dot{\vec{x}}_n + (1-\gamma)\,\ddot{\vec{x}}_n\,\Delta t + \gamma\,\ddot{\vec{x}}_{n+1}\,\Delta t \qquad (3.16)$$

The parameters γ and β indicate how much of the acceleration at the end of the interval enters into relations for velocity and displacement at the end of the interval. It has been shown [16] that unless the quantity $\gamma = \frac{1}{2}$ there is a spurious damping introduced which is proportional to $(\gamma - \frac{1}{2})$.

There is a correspondence between β and the variation in acceleration during the time interval. A physical relationship is possible only for four values of β. It is seen that $\beta = 1/4$ corresponds to a uniform value of acceleration equal to the mean of the initial and final values of the acceleration during the time interval, $\beta = 1/6$ corresponds to a linear variation of the acceleration during the time interval, $\beta = 1/8$ corresponds to a step function with a value equal to the initial value for the first half and a value equal to the the final value for the second half of the interval and $\beta = 0$ corresponds to two pulses of acceleration at the beginning and the end of the interval with each of these pulses equal to one half of the acceleration times the time interval, one occurring just before the end of the preceding interval and the other just after the beginning of the next interval[16].

In Newmark's original procedure the value of $\ddot{\vec{x}}_{n+1}$ is assumed and the values of $\vec{x}_{n+1}$ and $\dot{\vec{x}}_{n+1}$ are computed using eqns. (3.15) and (3.16). Then using eqn. (2.1) at time t_{n+1}, the value of $\ddot{\vec{x}}_{n+1}$ is calculated by substituting the computed values of $\vec{x}_{n+1}$ and $\dot{\vec{x}}_{n+1}$. If the error between the calculated value of $\ddot{\vec{x}}_{n+1}$ and the assumed value of $\ddot{\vec{x}}_{n+1}$ is within a permissible limit, the solution is said to have converged. If the solution has not converged, then the value of $\ddot{\vec{x}}_{n+1}$ calculated using eqn. (2.1) is taken as the assumed value and the whole process is repeated until convergence occurs. The rate of convergence toward equality of the calculated and assumed accelerations is a function

of the time step Δt. The error in the Newmark method is proportional to $(\Delta t)^2$.

Another approach [5] is to solve eqns. (2.1), (3.15) and (3.16) simultaneously. These give a total of 3n simultaneous scalar equations relating the 3n unknown components of $\vec{x}_{n+1}$, $\dot{\vec{x}}_{n+1}$ and $\ddot{\vec{x}}_{n+1}$. Thus the solution can be found by inverting a matrix and performing matrix multiplication.

In reference 29 a review of the solution techniques used in the direct integration of the equations of motion has been presented. It has been concluded that the Newmark - $\beta = 1/4$ method is more efficient than the Park, Wilson θ - $\theta = 1.4$ and Houbolt methods for linear dynamic analysis. In nonlinear dynamic analysis the Newmark - $\beta = 1/4$ method should be preferred if equilibrium iterations are performed at each time step.

3.3 Conditions for Classical Normal Modes

In this section conditions which lead to uncoupled equations for the composite system as well as the primary, secondary and interface systems are investigated. Consider the primary and the secondary systems represented by eqns. (3.17) and (3.18) respectively.

$$\begin{bmatrix} M_{p_{bb}} & 0 \\ 0 & M_{p_{pp}} \end{bmatrix} \begin{Bmatrix} \ddot{X}_{p_b} \\ \ddot{X}_p \end{Bmatrix} + \begin{bmatrix} C_{p_{bb}} & C_{p_{bp}} \\ C_{p_{pb}} & C_{p_{pp}} \end{bmatrix} \begin{Bmatrix} \dot{X}_{p_b} \\ \dot{X}_p \end{Bmatrix} + \begin{bmatrix} K_{p_{bb}} & K_{p_{bp}} \\ K_{p_{pb}} & K_{p_{pp}} \end{bmatrix} \begin{Bmatrix} X_{p_b} \\ X_p \end{Bmatrix} = \begin{Bmatrix} F_{p_b} + F_{p_i} \\ F_p \end{Bmatrix} \quad (3.17)$$

$$\begin{bmatrix} M_{s_{bb}} & 0 \\ 0 & M_{s_{ss}} \end{bmatrix} \begin{Bmatrix} \ddot{X}_{s_b} \\ \ddot{X}_s \end{Bmatrix} + \begin{bmatrix} C_{s_{bb}} & C_{s_{bs}} \\ C_{s_{sb}} & C_{s_{ss}} \end{bmatrix} \begin{Bmatrix} \dot{X}_{s_b} \\ \dot{X}_s \end{Bmatrix} + \begin{bmatrix} K_{s_{bb}} & K_{s_{bs}} \\ K_{s_{sb}} & K_{s_{ss}} \end{bmatrix} \begin{Bmatrix} X_{s_b} \\ X_s \end{Bmatrix} = \begin{Bmatrix} F_{s_b} + F_{s_i} \\ F_s \end{Bmatrix} \quad (3.18)$$

The subscripts p, s and b refer to the primary, secondary and interface degrees of freedom respectively. It may be noted that $F_{p_i} + F_{s_i} = 0$. Eqns. (3.17) and (3.18) can be combined together to yield the following equation for the composite system.

$$\begin{bmatrix} M_{11} & 0 & 0 \\ 0 & M_{22} & 0 \\ 0 & 0 & M_{33} \end{bmatrix} \begin{Bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_3 \end{Bmatrix} + \begin{bmatrix} C_{11} & 0 & C_{13} \\ 0 & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{Bmatrix} + \begin{bmatrix} K_{11} & 0 & K_{13} \\ 0 & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \quad (3.19)$$

The coordinates have been chosen such that X_1 and X_2 are coupled only by virtue of being coupled to X_3. The coordinates X_1 and X_2 represent the primary and secondary respectively whereas the coordinates X_3 represent the interface. It has been assumed that X_1 and X_2 are not inertially coupled with X_3. A special case of this is when X's represent the displacements of mass points making M diagonal. The submatrices in eqn. (3.19) can be defined in terms of the matrices in eqns. (3.17) and (3.18) as follows

$$M_{11} = M_{p_{pp}}, \quad M_{22} = M_{s_{ss}}, \quad M_{33} = M_{p_{bb}} + M_{s_{bb}}$$

$$C_{11} = C_{p_{pp}}, \quad C_{13} = C_{p_{pb}}, \quad C_{22} = C_{s_{ss}}, \quad C_{23} = C_{s_{sb}}$$

$$C_{31} = C_{p_{bp}}, \quad C_{32} = C_{s_{bs}}, \quad C_{33} = C_{p_{bb}} + C_{s_{bb}}$$

$$F_1 = F_p, \quad F_2 = F_s, \quad F_3 = F_{p_b} + F_{s_b} \quad (3.20)$$

The relationships for the K submatrices can be obtained from the relationships for C submatrices with K replaced for C.

Eqn. (2.1) (or 3.19)can be uncoupled in the following cases.

1. Rayleigh Condition: It is assumed that C is a linear combination of M and K given by

$$C = \alpha M + \gamma K \quad (3.21)$$

Here eqn (2.1) can always be uncoupled by the matrix Θ defined in eqns. (3.4), (3.5) and (3.6). It may be noted that the modal damping ratios satisfy the following

relationship

$$\alpha + \gamma \omega_j^2 = 2 \zeta_j \omega_j \tag{3.22}$$

2. A more general condition for the equations to be uncoupled is when the matrices satisfy the relationship $C M^{-1} K = K M^{-1} C$. This can also be expressed in a different form given by [6]

$$C = M \sum_b a_b [M^{-1} K]^b \tag{3.23}$$

It can be seen that the Rayleigh condition is a special case of this.

3. For all the modes to have equal modal damping ζ, K can be found as follows

$$\Theta^T C \Theta = 2 \zeta \, \mathrm{diag}(\omega_j)$$

$$\mathrm{diag}(\omega_j) = \frac{1}{2\zeta} \Theta^T C \Theta$$

$$\text{and} \qquad \mathrm{diag}(\omega_j^2) = \frac{1}{4\zeta^2} \Theta^T C \Theta \Theta^T C \Theta$$

$$\text{But} \qquad \Theta^T M \Theta = I \qquad \text{gives} \quad M = \Theta^{T^{-1}} \Theta^{-1}$$

$$\text{so that} \qquad \Theta \Theta^T = M^{-1}$$

K can also be expressed as

$$K = \Theta^{T^{-1}} \mathrm{diag}(\omega_j^2) \Theta^{-1}$$

Now substituting for $\mathrm{diag}(\omega_j^2)$ in the expression for K, we get

$$K = \frac{1}{4\zeta^2} C M^{-1} C \tag{3.24}$$

This condition is distinctly different from condition 1 and both cannot be simultaneously satisfied for dimension greater than 2.

The three parts of the composite system, eqn. (3.19), when analyzed separately, can be represented as

$$M_{11}\ddot{X}_1 + C_{11}\dot{X}_1 + K_{11}X_1 = F_1 - C_{13}\dot{X}_3 - K_{13}X_3$$

$$M_{22}\ddot{X}_2 + C_{22}\dot{X}_2 + K_{22}X_2 = F_2 - C_{23}\dot{X}_3 - K_{23}X_3$$

$$M_{33}\ddot{X}_3 + C_{33}\dot{X}_3 + K_{33}X_3 = F_3 - C_{31}\dot{X}_1 - C_{32}\dot{X}_2 - K_{31}X_1 - K_{32}X_2 \qquad (3.25)$$

The uncoupling of eqns. (3.25) involves diagonalization of the matrices M_{jj}, C_{jj} and K_{jj} which are the submatrices on the diagonal of eqn. (3.19). The conditions 1, 2 and 3 as given above, but applied to individual subsystems, ensure the diagonalization of these matrices.

It is obvious that no diagonalization condition for eqns. (3.25) is sufficient to ensure diagonalization of the composite system. This is because diagonalization of the left hand sides of eqns. (3.25) does not impose any restraints on C_{13} and C_{23} and such restraints are required for the diagonalization of the composite system.

However it seems possible that a diagonalization condition for the composite system will result in the diagonalization of eqns. (3.25) also. The conditions mentioned above will be considered.

1. Rayleigh Condition: If $C = \alpha M + \gamma K$, then $C_{jj} = \alpha M_{jj} + \gamma K_{jj}$ and therefore each subset also satisfies the Rayleigh condition. Therefore, when the composite system satisfies the Rayleigh condition, eqns. (3.25) can also be uncoupled.

2. This condition for classical normal modes is as expressed by eqn. (3.23). For some K matrices it may be possible to choose a C matrix which satisfies this condition for the composite structure as well as the substructures whereas for some other K matrices it may be impossible. For example, consider

$$C = a_0 M + a_1 K + a_2 K M^{-1} K \qquad \text{with } a_2 \neq 0 \tag{3.26}$$

This assures classical normal modes of the composite structure but not of the substructures. The first diagonal submatrix of C is

$$C_{11} = a_0 M_{11} + a_1 K_{11} + a_2 (K_{11} M_{11}^{-1} K_{11} + K_{13} M_{33}^{-1} K_{31}) \tag{3.27}$$

Thus the primary system has classical normal modes if and only if

$$C_{11} M_{11}^{-1} K_{11} = K_{11} M_{11}^{-1} C_{11}$$

or

$$a_0 M_{11} M_{11}^{-1} K_{11} + a_1 K_{11} M_{11}^{-1} K_{11} + a_2 (K_{11} M_{11}^{-1} M_{11}^{-1} K_{11} + K_{13} M_{33}^{-1} K_{31} M_{11}^{-1} K_{11}) =$$

$$a_0 K_{11} M_{11}^{-1} M_{11} + a_1 K_{11} M_{11}^{-1} K_{11} + a_2 (K_{11} M_{11}^{-1} K_{11} M_{11}^{-1} K_{11} + K_{11} M_{11}^{-1} K_{13} M_{33}^{-1} K_{31})$$

or $$K_{13} M_{33}^{-1} K_{31} M_{11}^{-1} K_{11} = K_{11} M_{11}^{-1} K_{13} M_{33}^{-1} K_{31} \tag{3.28}$$

in particular $\left[K_{13} M_{33}^{-1} K_{31} M_{11}^{-1} K_{11}\right]$ should be a symmetric matrix. This is a restriction which must be imposed on K (or M) in order that both the composite system and the primary system have classical normal modes, when the damping of the composite system is as defined by eqn. (3.26). Similar restrictions must be placed on the secondary and the interface if they are to have classical normal modes. If terms beyond a_2 are included in eqn. (3.26), then more complicated restrictions need to be placed on K in order to have classical normal modes for the composite as well as the substructures.

3. In this case equal modal damping in all the modes of the composite system as well as each subsystem is required. Here K is defined by

$$K = \begin{bmatrix} K_{11} & 0 & K_{13} \\ 0 & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} = \frac{1}{4\zeta^2} C M^{-1} C$$

$$K = \frac{1}{4\zeta^2} \begin{bmatrix} C_{11} & 0 & C_{13} \\ 0 & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} M_{11}^{-1} & 0 & C_{11} \\ 0 & M_{22}^{-1} & 0 \\ 0 & 0 & C_{31} \end{bmatrix} \begin{bmatrix} C_{11} & 0 & C_{13} \\ 0 & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \qquad (3.29)$$

$$= \frac{1}{4\zeta^2} \begin{bmatrix} C_{11}M_{11}^{-1}C_{11}+C_{13}M_{33}^{-1}C_{31} & C_{13}M_{33}^{-1}C_{32} & C_{11}M_{11}^{-1}C_{13}+C_{13}M_{33}^{-1}C_{33} \\ C_{23}M_{33}^{-1}C_{31} & C_{22}M_{22}^{-1}C_{22}+C_{23}M_{33}^{-1}C_{32} & C_{22}M_{22}^{-1}C_{23}+C_{23}M_{33}^{-1}C_{33} \\ C_{31}M_{11}^{-1}C_{11}+C_{33}M_{33}^{-1}C_{31} & C_{32}M_{22}^{-1}C_{22}+C_{33}M_{33}^{-1}C_{32} & C_{31}M_{11}^{-1}C_{13}+C_{32}M_{22}^{-1}C_{23}+C_{33}M_{33}^{-1}C_{33} \end{bmatrix}$$

The following relationships can be seen from eqn. (3.29)

$$K_{11} = \frac{1}{4\zeta^2} (C_{11} M_{11}^{-1} C_{11} + C_{13} M_{33}^{-1} C_{31})$$

$$K_{22} = \frac{1}{4\zeta^2} (C_{22} M_{22}^{-1} C_{22} + C_{23} M_{33}^{-1} C_{32})$$

$$K_{33} = \frac{1}{4\zeta^2} (C_{33} M_{33}^{-1} C_{33} + C_{31} M_{11}^{-1} C_{13} + C_{32} M_{22}^{-1} C_{23}) \qquad (3.30)$$

However the primary system (X_1) would uncouple with damping ζ in each mode if and only if

$$K_{11} = \frac{1}{4\zeta^2} C_{11} M_{11}^{-1} C_{11} \qquad (3.31)$$

The simultaneous satisfaction of eqns. (3.30) and (3.31) requires that

$$C_{13} M_{33}^{-1} C_{31} = 0 \quad \text{or} \quad (C_{13} M_{33}^{-1/2}) (C_{13} M_{33}^{-1/2})^T = 0 \qquad (3.32)$$

But this is possible if and only if $C_{13} = 0$. Similarly uncoupling of the secondary with modal damping ζ in each mode is possible only if $C_{23} = 0$. Further if $C_{13} = 0$ and $C_{23} = 0$ then K_{33} is given by

$$K_{33} = \frac{1}{4\zeta^2} C_{33} M_{33}^{-1} C_{33} \tag{3.33}$$

i.e., the interface equations also uncouple with damping ζ.

This is somewhat misleading as it can be seen from eqn. (3.29) that for $C_{13} = 0$ and $C_{23} = 0$, K_{13} and K_{23} are also null matrices. Thus, this is a trivial system in which the primary and the secondary are initially uncoupled from the interface, i.e., three uncoupled systems.

Thus it can be seen that there is no nontrivial system in which the composite system along with the primary, secondary, and interface systems uncouples with damping ζ in each mode. It may be noted that the conditions determined here for the existence of classical normal modes seem to be more restrictive than what other authors have sometimes presumed. In reference 2 it is stated that the composite system will have classical normal modes when C is chosen to give the same percent of critical damping in each mode of the primary and secondary systems. For more than two degrees of freedom in either the primary or the secondary this requires a non-Rayleigh damping and so it appears that K as well as C must be restricted if the composite system is to have classical normal modes. Therefore it seems that the Rayleigh condition is the only condition which can result in the simultaneous diagonalization of the composite, primary, secondary and interface systems without imposing any restrictions on K.

3.4 Inertia Coupled Approach

3.4.1 Transformation of the Equations of Motion

In this approach[7] eqns. (3.19) is modified so that there is no stiffness coupling between the various subsystems. Let a transformation matrix Θ be defined such that

$$\vec{X} = \Theta \vec{Y} \qquad \text{where } \Theta = \begin{bmatrix} \Theta_{11} & 0 & \Theta_{13} \\ 0 & \Theta_{22} & \Theta_{23} \\ 0 & 0 & \Theta_{33} \end{bmatrix} \tag{3.34}$$

The submatrix Θ_{13} is chosen such that the stiffness forces in the primary depend only on Y_1 and the submatrix Θ_{23} is similarly chosen so that the forces in the secondary depend only on Y_2. The stiffness forces for the primary can be written as

$$K_{11} X_1 + K_{13} X_3 = K_{11} (\Theta_{11} Y_1 + \Theta_{13} Y_3) + K_{13} \Theta_{33} Y_3 \tag{3.35}$$

It can be seen that Y_3 can be eliminated from eqn. (3.35) if

$$\Theta_{13} = - K_{11}^{-1} K_{13} \Theta_{33} \tag{3.36}$$

Similarly Θ_{23} should be defined as

$$\Theta_{23} = - K_{22}^{-1} K_{23} \Theta_{33} \tag{3.37}$$

so that the secondary forces depend only on Y_2. The stiffness and mass matrices in the Y space can now be written as

$$\Theta^T K \Theta = \begin{bmatrix} \Theta_{11}^T K_{11} \Theta_{11} & 0 & 0 \\ 0 & \Theta_{22}^T K_{22} \Theta_{22} & 0 \\ 0 & 0 & \Theta_{33}^T K_{33}^* \Theta_{33} \end{bmatrix} \tag{3.38}$$

$$\text{where} \qquad K_{33}^* = K_{33} - K_{31} K_{11}^{-1} K_{13} - K_{32} K_{22}^{-1} K_{23}$$

$$\Theta^T M \Theta = \begin{bmatrix} \Theta_{11}^T M_{11} \Theta_{11} & 0 & \Theta_{11}^T M_{11} \Theta_{13} \\ 0 & \Theta_{22}^T M_{22} \Theta_{22} & \Theta_{22}^T M_{22} \Theta_{23} \\ \Theta_{13}^T M_{11} \Theta_{11} & \Theta_{23}^T M_{22} \Theta_{22} & \Theta_{33}^T M_{33}^* \Theta_{33} \end{bmatrix} \tag{3.39}$$

$$\text{where} \qquad M_{33}^* = K_{31} K_{11}^{-1} M_{11} K_{11}^{-1} K_{13} + K_{32} K_{22}^{-1} M_{22} K_{22}^{-1} K_{23} + M_{33}$$

It may be noted that the subsystems and interface do not have any stiffness coupling in the Y space. Further Θ_{11}, Θ_{22} and Θ_{33} could be chosen so as to give diagonal submatrices on the diagonal of the transformed stiffness matrix. For example, choosing

Θ_{11} to be made up of eigen vectors of $M_{11}^{-1} K_{11}$, Θ_{22} to be eigen vectors of $M_{22}^{-1}K_{22}$, and Θ_{33} to be eigen vectors of $M_{33}^{*-1}K_{33}^{*}$ results in diagonal submatrices.

Assuming that the damping is of the Rayleigh type, the equations of motion in the Y space can be written as

$$\begin{bmatrix} I & 0 & M_{pb} \\ 0 & I & M_{sb} \\ M_{pb}^T & M_{sb}^T & I \end{bmatrix} \begin{Bmatrix} \ddot{Y}_1 \\ \ddot{Y}_2 \\ \ddot{Y}_3 \end{Bmatrix} + \begin{bmatrix} 2\zeta_p\omega_p & 0 & C_{pb} \\ 0 & 2\zeta_s\omega_s & C_{sb} \\ C_{pb}^T & C_{sb}^T & 2\zeta_b\omega_b \end{bmatrix} \begin{Bmatrix} \dot{Y}_1 \\ \dot{Y}_2 \\ \dot{Y}_3 \end{Bmatrix} + \begin{bmatrix} \omega_p^2 & 0 & 0 \\ 0 & \omega_s^2 & 0 \\ 0 & 0 & \omega_b^2 \end{bmatrix} \begin{Bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} \quad (3.40)$$

where

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} \Theta_{11}^T & 0 & 0 \\ 0 & \Theta_{22}^T & 0 \\ \Theta_{13}^T & \Theta_{23}^T & \Theta_{33}^T \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \quad (3.41)$$

The equations of motion for the primary, secondary and the interface systems can now be written as

$$\ddot{Y}_1 + 2\zeta_p\omega_p\dot{Y}_1 + \omega_p^2 Y_1 = P_1 - M_{pb}\ddot{Y}_3 - C_{pb}\dot{Y}_3 \quad (3.42)$$

$$\ddot{Y}_2 + 2\zeta_s\omega_s\dot{Y}_2 + \omega_s^2 Y_2 = P_2 - M_{sb}\ddot{Y}_3 - C_{sb}\dot{Y}_3 \quad (3.43)$$

$$\ddot{Y}_3 + 2\zeta_b\omega_b\dot{Y}_3 + \omega_b^2 Y_3 = P_3 - M_{pb}^T\ddot{Y}_1 - M_{sb}^T\ddot{Y}_2 - C_{pb}^T\dot{Y}_1 - C_{sb}^T\dot{Y}_2 \quad (3.44)$$

The term "inertia coupled approach" for this formulation refers to the right-hand-sides of these equations. That is, each Y_j vector appears to be governed by an uncoupled set of modal equations for that subsystem, but these equations are coupled to other Y vectors in the right hand side of the equations.

3.4.2 Method of Solution

The dynamic response of the primary-secondary system (as suggested in reference

28) is obtained through an iterative solution of eqns. (3.42), (3.43) and (3.44). It is assumed that the values of $\ddot{Y}_{1_n}$, $\ddot{Y}_{2_n}$ and $\ddot{Y}_{3_n}$ at time t_n are known and these quantities need to be evaluated at time t_{n+1}. In this approach the values of $\ddot{Y}_{3_{n+1}}$ and $\dot{Y}_{3_{n+1}}$ need to be predicted in order to start the computation. These predicted values of $\ddot{Y}_{3_{n+1}}$ and $\dot{Y}_{3_n}$ are substituted into eqns. (3.42) and (3.43) to obtain the values of $\ddot{Y}_{1_{n+1}}$, $\ddot{Y}_{2_{n+1}}$, $\dot{Y}_{1_{n+1}}$ and $\dot{Y}_{2_{n+1}}$.

For predicting the interface acceleration, the temporal slope of the interface acceleration is used. To determine the temporal slope, eqns. (3.42), (3.43) and (3.44) are differentiated to yield

$$\dddot{Y}_{1_n} + 2\,\zeta_p\,\omega_p\,\ddot{Y}_{1_n} + \omega_p^2\,\dot{Y}_{1_n} = \dot{P}_1 - M_{pb}\,\dddot{Y}_{3_n} - C_{pb}\,\ddot{Y}_{3_n} \tag{3.45}$$

$$\dddot{Y}_{2_n} + 2\,\zeta_s\,\omega_s\,\ddot{Y}_{2_n} + \omega_2^2\,\dot{Y}_{2_n} = \dot{P}_2 - M_{sb}\,\dddot{Y}_{3_n} - C_{sb}\,\ddot{Y}_{3_n} \tag{3.46}$$

$$\dddot{Y}_{3_n} + 2\,\zeta_b\,\omega_b\,\ddot{Y}_{3_n} + \omega_b^2\dot{Y}_3 = \dot{P}_{3_n} - M_{pb}^T\,\dddot{Y}_{1_n} - M_{sb}^T\,\dddot{Y}_{2_n} - C_{pb}^T\,\ddot{Y}_{1_n} - C_{sb}^T\,\ddot{Y}_{2_n} \tag{3.47}$$

Now substituting for $\dddot{Y}_{1_n}$ and $\dddot{Y}_{2_n}$ from eqns. (3.46) and (3.47) into eqn. (3.48) gives

$$\left\{I - M_{pb}^T M_{pb} - M_{sb}^T M_{sb}\right\}\dddot{Y}_{3_n} = \dot{P}_{3_n} - C_{pb}^T\ddot{Y}_{1_n} - C_{sb}^T\ddot{Y}_{2_n} - M_{pb}^T\left\{\dot{P}_{1_n} - C_{pb}\ddot{Y}_{3_n} - 2\zeta_p\omega_p\ddot{Y}_{1_n}\right.$$

$$\left. - \omega_p^2\dot{Y}_{1_n}\right\} - M_{sb}^T\left\{\dot{P}_{2_n} - C_{sb}\ddot{Y}_{3_n} - 2\zeta_s\omega_s\ddot{Y}_{2_n} - \omega_s^2\dot{Y}_{2_n}\right\} - 2\zeta_b\omega_b\ddot{Y}_{3_n} - \omega_b^2\dot{Y}_{3_n} \tag{3.48}$$

The values of $\dot{P}_{1_n}$, $\dot{P}_{2_n}$ and $\dot{P}_{3_n}$ are determined numerically. The value of $\dddot{Y}_{3_n}$ can now be determined from eqn. (3.48). It may be noted that the matrix on the left hand side of eqn. (3.48) need be inverted only once and that it can then be used at all times. The interface modal accelerations and velocities are predicted using a truncated Taylor series

$$\ddot{Y}^{P}_{3_{n+1}} = \ddot{Y}_{3_n} + \Delta t\, \dddot{Y}_{3_n} \tag{3.49}$$

$$\dot{Y}^{P}_{3_{n+1}} = \dot{Y}_{3_n} + \Delta t\, \ddot{Y}_{3_n} + \frac{(\Delta t)^2}{2} \dddot{Y}_{3_n} \tag{3.50}$$

The predicted interface acceleration $\ddot{Y}^{P}_{3_{n+1}}$ and interface velocity $\dot{Y}^{P}_{3_{n+1}}$ are substituted into eqns. (3.42) and (3.43) to determine the response of the interior degrees of freedom. An unconditionally stable Newmark step by step integration scheme is used to integrate eqns. (3.42) and (3.43). The calculated values of $\ddot{Y}^{P}_{1_{n+1}}$ and $\ddot{Y}^{P}_{2_{n+1}}$ are used to update the values of $\ddot{Y}^{P}_{3_{n+1}}$ and $\dot{Y}^{P}_{3_{n+1}}$ using Newmark's method to integrate eqn. (3.44). The iteration is terminated when the error between two successive estimates of the interface acceleration and the interface velocity is less than that required for a specified accuracy. One measure of accuracy ε is defined as the ratio of the difference between the calculated and the assumed values to the assumed value. To initiate the solution scheme, the dynamic equilibrium equations can be solved at the time origin.

3.5 Stiffness Coupled Approach

In this method eqn. (3.19) is considered as such without any transformation. Here also the response of the primary-secondary system can be obtained by solving the equations of motion iteratively. The iteration can be carried out without uncoupling these equations or the equations can be uncoupled and the iteration carried out.

3.5.1 Direct Integration

Here the equations are not transformed in any way prior to the iteration. The equations of motion for the primary, secondary and interface systems can be written as

$$M_{11}\ddot{X}_1 + C_{11}\dot{X}_1 + K_{11}X_1 = F_1 - C_{13}\dot{X}_3 - K_{13}X_3 \tag{3.51}$$

$$M_{22}\ddot{X}_2 + C_{22}\dot{X}_2 + K_{22}X_2 = F_2 - C_{23}\dot{X}_3 - K_{23}X_3 \tag{3.52}$$

$$M_{33}\ddot{X}_3 + C_{33}\dot{X}_3 + K_{33}X_3 = F_3 - C_{31}\dot{X}_1 - C_{32}\dot{X}_2 - K_{31}X_1 - K_{32}X_2 \qquad (3.53)$$

These subsystems are considered to be "stiffness coupled" because of the $K_{ij}X_j$ terms on the right-hand-side of the equations.

It is assumed that the response parameters at time t_n are known and these parameters need to be evaluated at time t_{n+1}. However the values of $X_{3_{n+1}}$ and $\dot{X}_{3_{n+1}}$ need to be predicted in order to start the computation. These quantities are predicted using a truncated Taylor series

$$\dot{X}^P_{3_{n+1}} = \dot{X}_{3_n} + \Delta t\,\ddot{X}_{3_n} \qquad (3.54)$$

$$X^P_{3_{n+1}} = X_{3_n} + \Delta t\,\dot{X}_{3_n} + \frac{(\Delta t)^2}{2}\ddot{X}_{3_n} \qquad (3.55)$$

The predicted velocity $\dot{X}^P_{3_{n+1}}$ and displacement $X^P_{3_{n+1}}$ are substituted into eqns. (3.51) and (3.52) to determine the response of the interior degrees of freedom. Newmark's method is used for integrating eqns. (3.51) and (3.52). The calculated values of $X^P_{1_{n+1}}$, $\dot{X}^P_{1_{n+1}}$, $X^P_{2_{n+1}}$, $\dot{X}^P_{2_{n+1}}$ are used to calculate the new values of $X^P_{3_{n+1}}$ and $\dot{X}^P_{3_{n+1}}$ using Newmark's method to integrate eqn. (3.53). The iteration is terminated when the error between two successive estimates of the interface displacement and the interface velocity is less than a prescribed limit ε. It may be noted that a matrix inversion is required for Newmark's method here whereas in the mass coupled approach no such inversion was required as the equations had been uncoupled.

3.5.2 Diagonalized Approach

In order to avoid a matrix inversion as required in the previous method it may be desirable to uncouple the left-hand sides of eqns. (3.51), (3.52) and (3.53). Let a transformation matrix Φ be defined as follows

$$\vec{X} = \Phi \vec{Y} \qquad \text{where} \quad \Phi = \begin{bmatrix} \Phi_{11} & 0 & 0 \\ 0 & \Phi_{22} & 0 \\ 0 & 0 & \Phi_{33} \end{bmatrix} \tag{3.56}$$

The submatrices Φ_{11}, Φ_{22} and Φ_{33} can be chosen to be composed of the eigen vectors of $M_{11}^{-1} K_{11}$, $M_{22}^{-1} K_{22}$ and $M_{33}^{-1} K_{33}$ so as to uncouple eqns. (3.51), (3.52) and (3.53). The transformed mass and stiffness matrix in the Y space can be written as

$$\Phi^T M \Phi = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \tag{3.57}$$

$$\Phi^T K \Phi = \begin{bmatrix} \omega_p^2 & 0 & K_{pb} \\ 0 & \omega_s^2 & K_{sb} \\ K_{pb}^T & K_{sb}^T & \omega_b^2 \end{bmatrix} = \begin{bmatrix} \omega_p^2 & 0 & \Phi_{11}^T K_{13} \Phi_{33} \\ 0 & \omega_s^2 & \Phi_{22}^T K_{23} \Phi_{33} \\ \Phi_{33}^T K_{31} \Phi_{11} & \Phi_{33}^T K_{32} \Phi_{22} & \omega_b^2 \end{bmatrix} \tag{3.58}$$

Assuming that the damping is of the Rayleigh type, the equations of motion in the Y space can now be written as

$$\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{Bmatrix} \ddot{Y}_1 \\ \ddot{Y}_2 \\ \ddot{Y}_3 \end{Bmatrix} + \begin{bmatrix} 2\zeta_p \omega_p & 0 & C_{pb} \\ 0 & 2\zeta_s \omega_s & C_{sb} \\ C_{pb}^T & C_{sb}^T & 2\zeta_b \omega_b \end{bmatrix} \begin{Bmatrix} \dot{Y}_1 \\ \dot{Y}_2 \\ \dot{Y}_3 \end{Bmatrix} + \begin{bmatrix} \omega_p^2 & 0 & K_{pb} \\ 0 & \omega_s^2 & K_{sb} \\ K_{pb}^T & K_{sb}^T & \omega_b^2 \end{bmatrix} \begin{Bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} \tag{3.59}$$

where

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} \Phi_{11}^T & 0 & 0 \\ 0 & \Phi_{22}^T & 0 \\ 0 & 0 & \Phi_{33}^T \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \tag{3.60}$$

The equations of motion for the primary, secondary and the interface in the Y space can be written as

$$\ddot{Y}_1 + 2\zeta_p \omega_p \dot{Y}_1 + \omega_p^2 Y_1 = P_1 - C_{pb} \dot{Y}_3 - K_{pb} Y_3 \tag{3.61}$$

$$\ddot{Y}_2 + 2\zeta_s \omega_s \dot{Y}_2 + \omega_s^2 Y_2 = P_2 - C_{sb} \dot{Y}_3 - K_{sb} Y_3 \tag{3.62}$$

$$\ddot{Y}_3 + 2\zeta_b\omega_b\dot{Y}_3 + \omega_b^2 Y_3 = P_3 - C_{pb}^T\dot{Y}_1 - C_{sb}^T\dot{Y}_2 - K_{pb}^T Y_1 - K_{sb}^T Y_2 \qquad (3.63)$$

Note that this formulation is a direct parallel to eqns. (3.42), (3.43) and (3.44) for the inertia coupled approach. In both situations the left-hand sides of the equations are diagonalized while the terms on the right-hand sides still couple the subsystems.

It may also be noted that when the mass matrix is not originally diagonal, an approach similar to that used in the inertia coupled approach can be used to uncouple the mass matrix instead of the stiffness matrix.

For solving eqns. (3.61), (3.62) and (3.63) the values of $\dot{Y}_{3_{n+1}}$ and $Y_{3_{n+1}}$ are predicted using a truncated Taylor series.

$$\dot{Y}^P_{3_{n+1}} = \dot{Y}_{3_n} + \Delta t\ddot{Y}_{3_n} \qquad (3.64)$$

$$Y^P_{3_{n+1}} = Y_{3_{n+1}} + \Delta t\dot{Y}_{3_{n+1}} + \frac{(\Delta t)^2}{2}\ddot{Y}_{3_{n+1}} \qquad (3.65)$$

These predicted values of the interface velocity $\dot{Y}^P_{3_{n+1}}$ and the interface displacement $Y^P_{3_{n+1}}$ are substituted into eqns. (3.61) and (3.62) to obtain the values of $\dot{Y}^P_{1_{n+1}}$, $Y^P_{1_{n+1}}$, $\dot{Y}^P_{2_{n+1}}$ and $Y^P_{2_{n+1}}$ which are then substituted into eqn. (3.63) to obtain the updated values of $\dot{Y}^P_{3_{n+1}}$ and $Y^P_{3_{n+1}}$. The Newmark integration scheme is used here also. The iteration is continued until the error between two successive estimates of the interface velocity and the interface displacement is below a certain prescribed level ε.

4.0 NUMERICAL RESULTS

The various methods mentioned in chapter 3 were used for the analysis of both linear and nonlinear primary-secondary systems,
in order to compare the accuracy and efficiency of the methods.

4.1 Structures Considered

4.1.1 Linear Structures

The two linear structures that were used are shown in fig. 4.1. In the first structure both the primary and the secondary have five degrees of freedom, while the second structure has ten degrees of freedom in the primary and only two in the secondary. The first structure was used to study the methods when the primary and the secondary have about the same number of degrees of freedom and the second structure depicts a structure in which the secondary is relatively small compared to the primary.

Recall that the substructure analysis procedures require consideration of three subsystems: primary, secondary, and interface. The substructuring for the two structures is shown in fig. 4.2. This figure also shows the modal frequencies both for the composite structures and for each substructure. These frequencies have been normalized by $(K/M)^{1/2}$. For the first structure (part a of the figure) the values outside the parentheses (for the composite system and the interface subsystem) are for a mass and stiffness ratio of 0.1, while the values in parentheses are for $m/M = k/K = 0.5$. For the second structure the mass and stiffness ratio is 0.2.

The damping for the linear structures was taken to be stiffness proportional, at a level giving a damping ratio of 0.05 in the first mode of the composite structure.

4.1.2 Nonlinear Structures

Here the second structure with nonlinear interface springs has been considered. The forces in these springs vary as a function of deformation as

$$F = k_1 d + k_3 d^3 \tag{4.1}$$

where d is the deformation of the spring and k_1 and k_3 are positive constants. It may be noted that this gives a hardening spring since the resistance(stiffness) of the spring increases with increasing deformation.

Here the parameter k_1 was chosen to be 80% and 50% of the corresponding linear stiffness. Since the linear stiffness was assumed to be 0.2K, k_1 was calculated as 0.16K and 0.10K. The parameter k_3 was computed by equating the maximum force in the nonlinear spring to that in the linear case under a deformation equal to the mean of the maximum deformation of the spring in the linear situation, the mean being taken over the ensemble. The choice of k_3 was based on the concept of secant stiffness and a desire to have the nonlinear systems have maximum response values of the same order of magnitude as those of the linear system. For $k_1 = 0.16K$, k_3 for the first (top) spring was computed as 0.000363K and for the second (bottom) spring as 0.000080K. For $k_1 = 0.10K$, k_3 for the first spring was computed as 0.000909K and for the second spring as 0.000200K.

If the response levels of the nonlinear systems had been identical to those of the linear system, the the cubic portion of eqn. 4.1 would have contributed 20% and 80% to the maximum restoring forces for the two different choices of k_1 and k_3 (based on the ensemble average). In fact, the maximum responses for $k_1 = 0.16K$ were somewhat smaller than for the linear system and those for $k_1 = 0.10K$ were somewhat larger. The effect of this was that the mean maximum contribution of the cubic term was 13% for the first spring and 15% for the second spring when $k_1 = 0.16K$. For $k_1 = 0.10K$ these numbers were 69% and 54%, respectively.

4.2 Type of Excitation Used

The purpose of this study was to compare the accuracy and efficiency of the various computation schemes for earthquake response. Stationary white noise has been used by some investigators to represent an earthquake, but it cannot be considered as the most realistic representation of an earthquake. Ground motion during an earthquake is obviously nonwhite and it is not stationary as it builds up and then decays with time.

Random process simulation procedure has been used in this study because of the convenience in generating representative time histories. The external excitation has been chosen to be filtered Gaussian white noise modulated by a time function. The excitation at the bedrock has been assumed, by some investigators, to be white noise which is then filtered by the overlying soil strata. The effect of the soil strata has been represented by a single degree of freedom system in this study. The white noise is defined to be a process having a constant power spectral density of intensity S_{FF} over the entire range of frequencies. The autocorrelation function for white noise is given by

$$E[F(t_1)\,F(t_2)] = R_{FF}(t_2 - t_1) = 2\pi\, S_{FF}\, \delta(t_2 - t_1) \tag{4.2}$$

where δ is the Dirac delta function. White noise can be visualized as a shot noise defined by

$$F(t) = \sum_{i=1}^{N} A_i\, \delta(t - t_i) \tag{4.3}$$

where $t_i - t_{i-1} = \Delta t$, A_i are independent normal random variables the mean of which is zero and the mean square is a constant. The arrival time of the first of this train of pulses is uniformly distributed in the interval $(0, \Delta t)$. It can be shown that

$$E[A^2] = 2\pi\, S_{FF}\, \Delta t \tag{4.4}$$

The level of excitation is often characterized by a measure with units of length.

The definition of one such constant which has been used in this study is

$$D = \sqrt{\frac{2S_{FF}}{\omega_0^3}} = \sqrt{\frac{E[A^2]}{\pi \omega_0^2 \Delta t}} \tag{4.5}$$

where ω_0 has been taken to be the fundamental frequency of the structure. It may be noted that the normalized time step used in this study is defined as $\overline{\Delta t} = \omega_0 \Delta t$

The sequence of random numbers generated is then modulated to introduce nonstationarity. Here the modulating function $\psi(t)$ has been chosen to be [21]

$$\psi(t) = (e^{-\alpha t} - e^{-\beta t})\, U(t) \qquad \text{with } \beta > \alpha > 0 \tag{4.6}$$

where $\alpha = 0.02/\omega_0$, $\beta = 0.04/\omega_0$ and $U(t)$ is a unit step function.

The equation of motion for the filter can be written as

$$m\ddot{z} + c\dot{z} + kz = F \tag{4.7}$$

This equation of motion was normalized with respect to ω_0. The ratio of the frequency of the filter to ω_0 has been assumed to be 1.5 and the damping ratio of the filter has been assumed to be 0.6 [32].

It may be noted that if normal random numbers are generated with zero mean and variance $E[A^2]$, the sequence of the independent variables $A_i\psi(t_i)$ may be divided by $D\omega_0$ and added instantaneously to the normalized filter velocity $\dot{\bar{z}}_n$ ($\dot{\bar{z}} = \dot{z}/D\omega_0$) at time t_i. This can be expressed as

$$\dot{\bar{z}}_n^+ = \dot{\bar{z}}_n^- + A_i\psi(t_i) / (E[A^2]/\pi\overline{\Delta t})^{1/2} \tag{4.8}$$

The normalized absolute acceleration of the filter $\ddot{\bar{z}}$, where $\ddot{\bar{z}} = (\ddot{z} + \ddot{w})/D\omega_0^2$ and $\ddot{w} = -F/m$ is the acceleration at the bed rock, is obtained at each time step and that is taken to be the excitation at the base of the structure. The analytical expressions for the response statistics of the filter have been derived in reference 21.

4.3 Algorithm for the Newmark Method

As mentioned earlier, the Newmark method has been used in this study. The displacement $\vec{x}_{n+1}$ and the velocity $\dot{\vec{x}}_{n+1}$ at the next time step are given in terms of the acceleration $\ddot{\vec{x}}_{n+1}$ by eqns. (3.15) and (3.16). The equilibrium equation (2.1) is also considered at time t_{n+1}. Here the approach where the matrix is inverted has been used. Eqns. (3.15) and (3.16) are used to calculate $\dot{\vec{x}}_{n+1}$ and $\ddot{\vec{x}}_{n+1}$ in terms of $\vec{x}_{n+1}$. These two relations for $\dot{\vec{x}}_{n+1}$ and $\ddot{\vec{x}}_{n+1}$ are substituted into eqn. (2.1) to solve for $\vec{x}_{n+1}$. The values of $\dot{\vec{x}}_{n+1}$ and $\ddot{\vec{x}}_{n+1}$ can then be computed using eqns. (3.15) and (3.16). The algorithm for the Newmark method involves calculating a set of constants as shown below

$$a_0 = \frac{1}{\beta \Delta t^2} \quad ; \quad a_1 = \frac{\gamma}{\beta \Delta t} \quad ; \quad a_2 = \frac{1}{\beta \Delta t} \quad ; \quad a_3 = \frac{1}{2\beta} - 1$$

$$a_4 = \frac{\gamma}{\beta} - 1 \quad ; \quad a_5 = \frac{\overline{\Delta t}}{2}\left(\frac{\gamma}{\beta} - 2\right) \quad ; \quad a_6 = \overline{\Delta t}(1-\gamma) \quad ; \quad a_7 = \gamma \overline{\Delta t} \tag{4.9}$$

The effective stiffness matrix $\overline{K}$ is formed as

$$\overline{K} = K + a_0 M + a_1 C \tag{4.10}$$

The matrix $\overline{K}$ is inverted only once for the linear case and that inverse can be used at all times. The effective loads need to be calculated at each time step as

$$\overline{f}_{n+1} = f_{n+1} + M\,(a_0 \vec{x}_n + a_2 \dot{\vec{x}}_n + a_3 \ddot{\vec{x}}_n) + C\,(a_1 \vec{x}_n + a_4 \dot{\vec{x}}_n + a_5 \ddot{\vec{x}}_n) \tag{4.11}$$

The displacement at the next time t_{n+1} is then given by

$$\vec{x}_{n+1} = \overline{K}^{-1} \overline{f}_{n+1} \tag{4.12}$$

The acceleration and the velocity at t_{n+1} are calculated using the following equations

$$\ddot{\vec{x}}_{n+1} = a_0\,(\vec{x}_{n+1} - x_n) - a_2 \dot{\vec{x}}_n - a_3 \ddot{\vec{x}}_n \tag{4.13}$$

$$\dot{\vec{x}}_{n+1} = \dot{\vec{x}}_n + a_6 \ddot{\vec{x}}_n + a_7 \ddot{\vec{x}}_{n+1} \tag{4.14}$$

This method is more efficient than the iterative approach in the Newmark method. In

this study the parameter β has been chosen to be 1/4 because the Newmark method is unconditionally stable for this value of β. The effect of different time steps has been studied for one of the linear systems, using normalized time steps of magnitude 0.1, 0.2 and 0.4.

4.4 Simulation Procedure

4.4.1 Linear System

Here the response of the systems obtained by the Duhamel integral approach for the composite system, as mentioned in sec. 3.1, has been referred to as the exact solution. The other methods of solution like the Newmark for composite, mass(inertia) coupled, direct stiffness and the uncoupled stiffness, all of which have been described in chapter 3, were also used to obtain the response of the systems.

An ensemble of 20 samples was used and the same ensemble was used for all the methods. Here the mean square error in the maximum absolute displacement attained until a specific time for each degree of freedom was considered for each sample. A normalized form of the error consisting of the ratio of of the error at each time to the exact solution at that time, has been used. The mean square was calculated by taking the mean over time of the square of the error in the maximum absolute displacement until that time. Then the rms value of this mean square error for each degree of freedom was computed across the ensemble.

Instead of using the actual displacements at each time, the maximum absolute displacement attained by a particular degree of freedom until that time was considered in computing the errors. When the actual displacements were considered for computing the errors, the percentage errors grew by enormous amounts. This is because the methods do not calculate the displacements very accurately near zero crossings.

Further, the index used in calculating the error gives too much importance to these low values of the displacement.

Three different ensembles of 20 samples each were considered. The ensembles were chosen to have 20 samples so as to make sure that representative errors were obtained. Similarly, three ensembles were chosen in order to verify that the errors obtained were not specific to any particular ensemble. The normalized duration of each sample was taken to be 60 rad.[21]. The mean square value of the normalized base acceleration $\ddot{z}$ for one of the ensembles is shown in figs. 4.3.

4.4.2 Nonlinear System

The nonlinear equations of motion can be linearized at any time t to give

$$M\ddot{\vec{x}} + C\dot{\vec{x}} + K_t\vec{x} = \vec{f} + \vec{f}_s \tag{4.15}$$

where K_t is the tangent stiffness matrix formed by assuming the stiffness of the nonlinear springs to be given by the slope of the tangent to the force deformation curve when the deformation of the spring is taken to be the deformation of the spring at the previous time step. The vector $\vec{f}_s$ has elements f_{s_i} which are nonzero when i corresponds to a node where a nonlinear spring is attached, all other elements being zero.

The element f_{s_i}, when i = a node where a nonlinear spring is attached, is defined as

$$f_{s_i} = k_t \, . \, d_x \tag{4.16}$$

where k_t is the slope of the tangent as defined above and d_x is the x-intercept of the above defined tangent. Fig. 4.11 shows how to obtain the values of k_t and d_x when the point d_{i-1}, which is the deformation of the spring at the previous time step, is known on the force deformation curve.

This tangent stiffness approach was used in applying each of the analysis methods to the nonlinear systems. The stiffnesses were not varied within any time step. Recall that the "exact" method for the linear systems involved Duhamel integration of the modal equations of motion for the composite system. In a direct (although cumbersome) extension of this idea, the composite modes for the appropriate tangent stiffness system were evaluated for the nonlinear system at the beginning of each time step, and Duhamel integration was then carried out. This procedure was considered to give the best comparison of error levels between linear and nonlinear systems.

4.5 Simulation Results

4.5.1 Linear System

Figs. 4.5 and 4.6 show representative results for two of the linear structures considered. The error results shown are for the first floor of the appropriate primary subsystem, but they are also representative of the errors at other locations. In part (a) of the figures, ε is plotted on the horizontal scale with % error on the left vertical scale and execution time in seconds for the various methods on the right vertical scale. Plotting the data in this way gives one some idea of how the error and the execution time for the different methods for primary-secondary analysis vary with ε. It may be recalled that ε has been defined to be the ratio of the difference between the computed and the assumed values to the assumed value of the relevant interface response measures in the iterative solutions of the coupled subsystem equations.

It can be seen from figs. 4.5 and 4.6 that for small values of ε the error in all the methods is approximately the same. In fact, this level of error corresponds to that in the Newmark method for the composite system. Reducing ε cannot reduce the computational error below this level. For the stiffness coupled approaches there exists an optimum ε which results in the least error and the lowest execution time for that

error. This value of ε is around 0.01. If ε is increased slightly above this limit, the error increases appreciably without any significant decrease in execution time, while reducing ε increases the computation time with no increase in accuracy. It may be noted that both the direct stiffness coupled and the diagonalized stiffness coupled approaches give almost the same error for the same ε but the diagonalized stiffness appears to be slightly more efficient especially for small ε. The error of the mass coupled approach appears to be almost independent of ε for small time steps. The execution time for all these three methods decreases with increasing ε and approaches the same value for larger ε. The error in the stiffness coupled methods also tends to remain constant after a certain value of ε. This value of ε is around 0.1. These characteristics can be attributed to the fact that the methods tend to become independent of the convergence parameter ε when it is so large that since no iteration is required to achieve the specified convergence of the interface response levels.

It has been observed that the ratio of the number of iterations per time step required in the inertia coupled approach to that required in the stiffness coupled approaches is about 2.8:1 for $\varepsilon = 0.001$ and 0.01. Both the stiffness coupled approaches required the same number of iterations per time step.

In part (b) of figs. 4.5 and 4.6, % error is plotted versus the execution time, so that one can easily see the "cost" that must be paid to achieve a given accuracy for different methods. On this plot the methods for primary-secondary analysis along with the methods for composite analysis are shown. The points for the methods used in the primary-secondary analysis on part (b) of the figures have been derived from the corresponding part (a). From figs. 4.5 and 4.6, it appears that the Newmark method for the composite system gives a smaller execution time than any of the other methods for the same amount of error. Another advantage in using either the Newmark method for the composite system or the direct stiffness coupled method is that the damping does

not have to be restricted to the Rayleigh form as the equations do not have to be uncoupled.

Comparing figs. 4.5 with 4.6 shows that the results for structures 1 and 2 are very consistent. For linear structures of this magnitude, the relative number of degrees of freedom in the primary and the secondary has little effect on the relative merits of the various schemes. This result might be quite different for very large structures in which the computer time required for eigen analysis or matrix inversion would become more significant in comparison to the time spent in actually obtaining time histories.

Plots similar to figs. 4.5 and 4.6 are given in ref. 25 for the other cases considered: the first structure with m/M = k/K = 0.5 (time step = 0.1), and the second structure with time step = 0.2 and 0.4 (m/M = k/K = 0.2). These plots are not reproduced here, since they are very similar in form to figs. 4.5 and 4.6. Table 4.1 summarizes a few of the values from those figures at key points identified by letters A through H on figs. 4.5 and 4.6.

It was found that the mass and stiffness ratios do not appear to affect the error in any of the methods. When the normalized time step is increased from 0.1 to 0.2 and 0.4, the error inherent to the Newmark method tends to grow appreciably. This increase in error could partially be due to the fact that the assumption on the variation of the external forces within each time step becomes more important as the time step is increased. In this study, a linear variation of the external forces between each time step is assumed when calculating the exact solution. The increase in the time step seems to increase the error in both the stiffness approaches much more appreciably than in the mass coupled approach. Further, the errors in these methods did not seem to become independent of ε when $\overline{\Delta t}$ was taken to be 0.4. The relative size of the primary to the secondary, i.e., comparing the first and second structures, does not not seem to affect

the accuracy and the efficiency of any of the methods for any time step.

4.5.2 Nonlinear System

Here also the rms value of the mean square error for each sample was calculated. Fig. 4.7 shows representative accuracy and computation time results in a form similar to the (a) parts of figs. 4.5 and 4.6. As before, the error results shown are for the first floor or the primary structure. In addition to the methods for the primary-secondary analysis, the results for the methods for composite analysis are also shown on the same figure. The % error for the Newmark method for the composite is shown on the left vertical scale and the execution times for the Newmark method and the exact analysis for the composite are shown on the right vertical scale.

The computation times in part (a) of fig. 4.7 are based on computations taking advantage of the fact that only the interface was nonlinear. In other situations all subsystems may be nonlinear and the computation times for this situation were determined by also doing all the computations that would be required if all subsystems were nonlinear, even though only the interface is nonlinear. Results for this situation are shown on part (b) of the figure. Part (a) represents the situation when $k_1 = 0.10K$ and part (b) is for $k_1 = 0.16K$.

The fact that only the interface springs were actually nonlinear for both parts of fig. 4.7 explains the fact that the % error vs. epsilon is almost the same on the two plots. However, the execution time is different on the two plots because eigen analysis or matrix inversion had to be carried out at each time step for all subsystems for part (b) whereas for part (a) the repeated matrix inversion or eigen analysis had to be carried out only for the interface and the secondary subsystems..

As in the linear situation, the error in all the methods approaches that in the

Newmark method for the composite system for low values of ε. Here, again, there is an optimum ε for the two stiffness coupled methods. This optimum value of ε is again around 0.01. The mass coupled approach, as before, is more or less independent of ε. As in the linear situation, the errors in the stiffness coupled methods do not increase after a certain value of ε. This value ε is also around 0.1. In contrast to the linear case, the execution times for the different methods for primary-secondary analysis do not decrease appreciably with an increase in ε. This is because the time required for matrix inversion or for eigen analysis becomes as important or more important than the time required for iteration at each time step.

However, it may be noted that the curves for the execution time in part (b) are almost parallel to the corresponding curves in part (a). This is because of the fact that eigen analysis or matrix inversion has to be carried out once for each time step irrespective of the number of iterations required to converge to the interface response parameters. The shift between the corresponding curves on the two figures is the additional time required for repeated eigen analysis or matrix inversion for the primary system. It was found that the magnitude of nonlinearity does not seem to affect any of the methods.

The greatest advantage of the primary-secondary analysis seems to be in the case when one of the subsystems is nonlinear. The primary-secondary analysis seems to be highly efficient especially if the nonlinearity is in one of the smaller subsystems. In primary-secondary analysis only the matrices for that subsystem and interface need be inverted at each time step or the eigen analysis for that subsystem and the interface alone need be carried out.

Even in the case when all subsystems were considered nonlinear, primary-secondary analysis appears relatively more efficient than in the linear case. When only

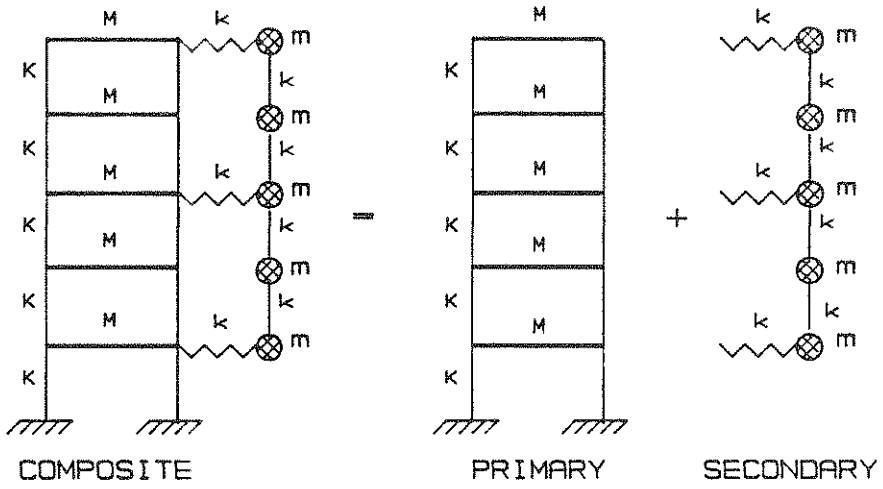
the interface was considered nonlinear, the mass coupled approach appears to be 5.5 times more efficient, diagonalized stiffness coupled appears to be 11.5 times more efficient and direct stiffness coupled 19 times more efficient than the exact procedure. When all subsystems were considered nonlinear, these values are 2 for mass coupled, 2.5 for diagonalized stiffness coupled and 11.5 for direct stiffness coupled. These values for efficiency have been calculated at the optimum value of ε.

The Newmark method for the composite appears to be about 8 times more efficient than the exact. However, the Newmark method for the composite system is not necessarily always the better approach for nonlinear analysis especially if the nonlinearity is in one of the smaller subsystems.

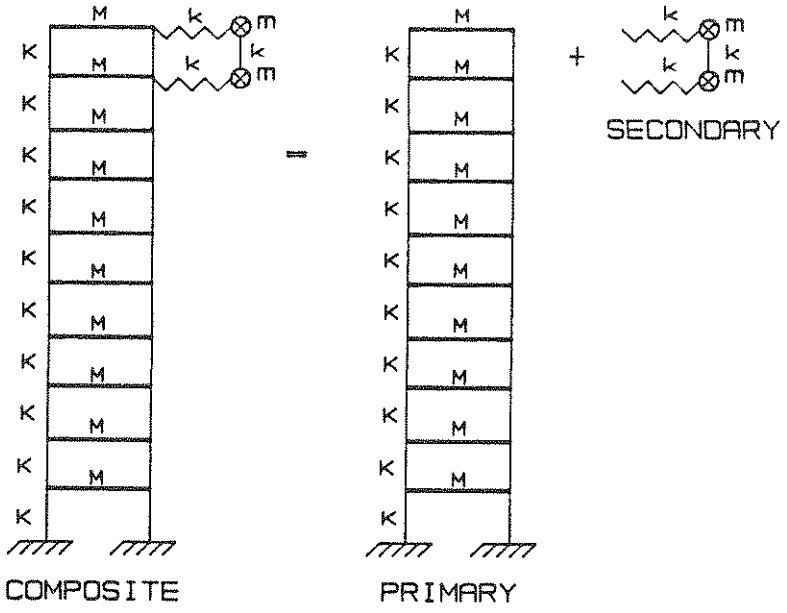
Table 4.1 Execution Times and Errors of Linear System Computation

(See figs. 4.5 and 4.6 for Coding)

	A	B	C	D	E	F		G		H
	sec	sec	sec	sec	%	%	sec	%	sec	sec
First Structure Time Step = 0.1										
m/M = k/K = 0.1	200	240	250	100	1.9	0.46	140	0.33	92	190
0.5	170	190	290	100	1.8	0.32	130	0.31	92	190
Second Structure m/M = k/K = 0.2										
Time Step = 0.1	250	300	450	100	2	0.52	150	0.52	120	230
0.2	180	230	210	50	13	2.0	90	2.0	60	116
0.4	170	220	80	40	-	7.6	81	7.6	30	57

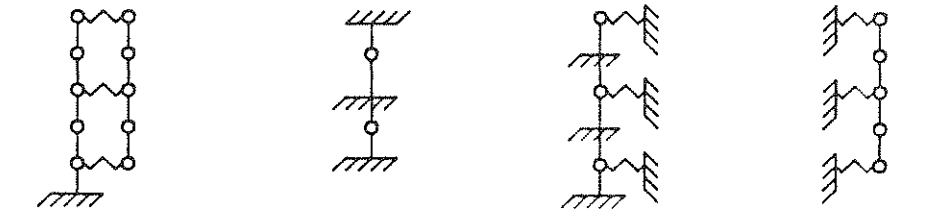


(a) First Structure



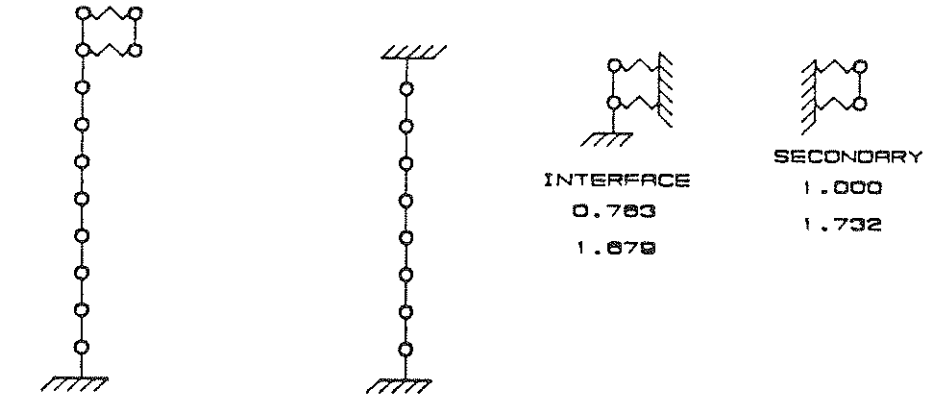
(b) Second Structure

Fig. 4.1 Structural Configurations Considered



COMPOSITE	PRIMARY	INTERFACE	SECONDARY
0.278 (0.251)	1.414	1.040 (1.225)	0.720
0.741 (0.743)	1.414	1.449 (1.581)	1.000
0.821 (0.873)		1.449 (1.581)	1.520
1.033 (1.119)			1.732
1.296 (1.266)			2.042
1.559 (1.633)			
1.674 (1.714)			
1.782 (1.856)			
1.918 (1.922)			
2.059 (2.126)			

(a) First Structure



COMPOSITE		PRIMARY		INTERFACE	SECONDARY
0.144	1.484	0.347	1.532	0.783	1.000
0.431	1.631	0.684	1.732	1.879	1.732
0.713	1.739	1.000	1.879		
0.977	1.843	1.288	1.970		
1.122	1.933				
1.255	1.986				

(b) Second Structure

Fig. 4.2 Frequencies for Linear Structures

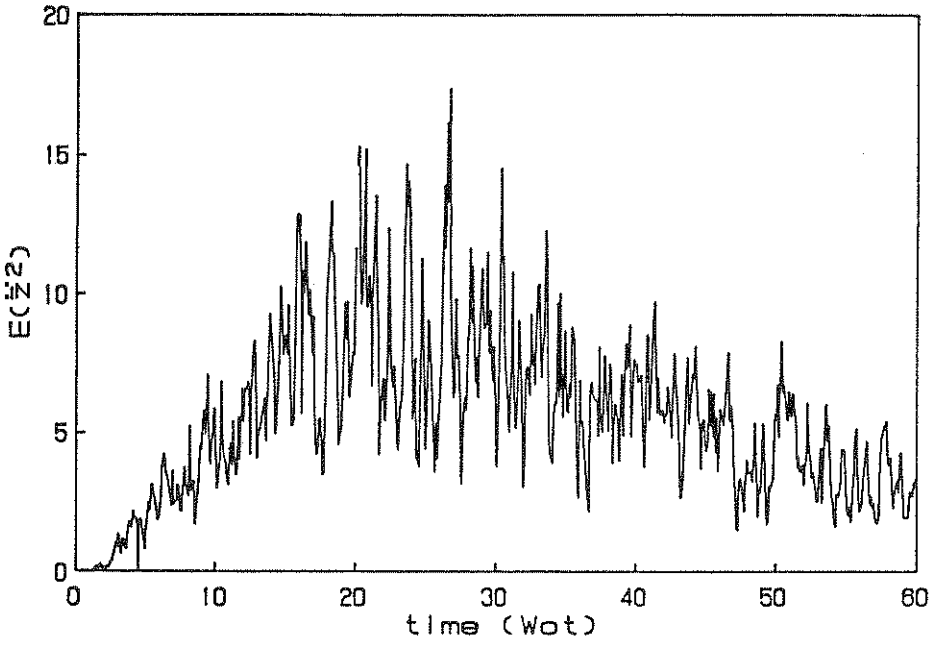


Fig. 4.3 Ensemble Average Ground Acceleration

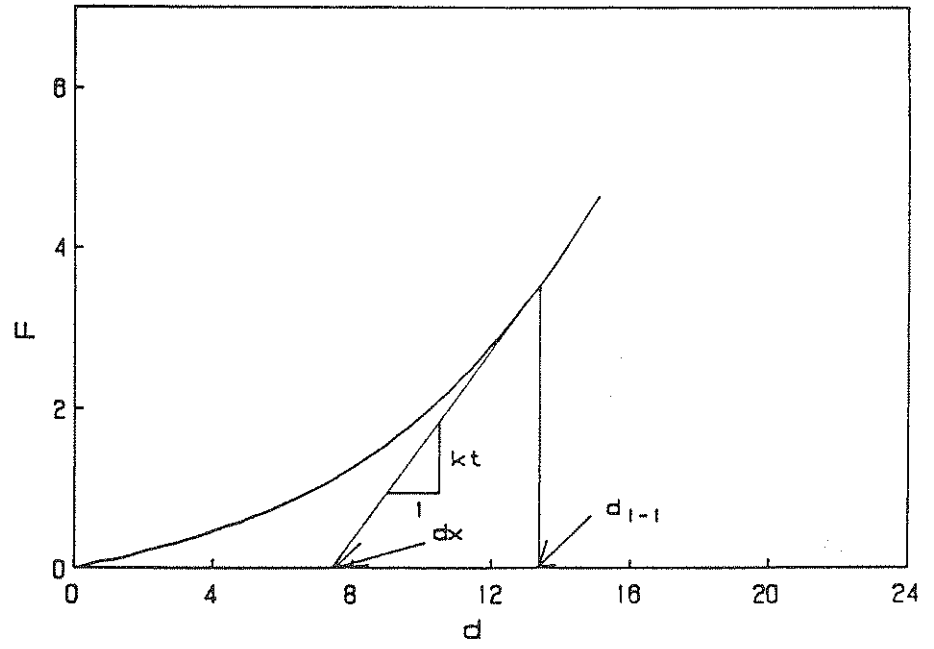
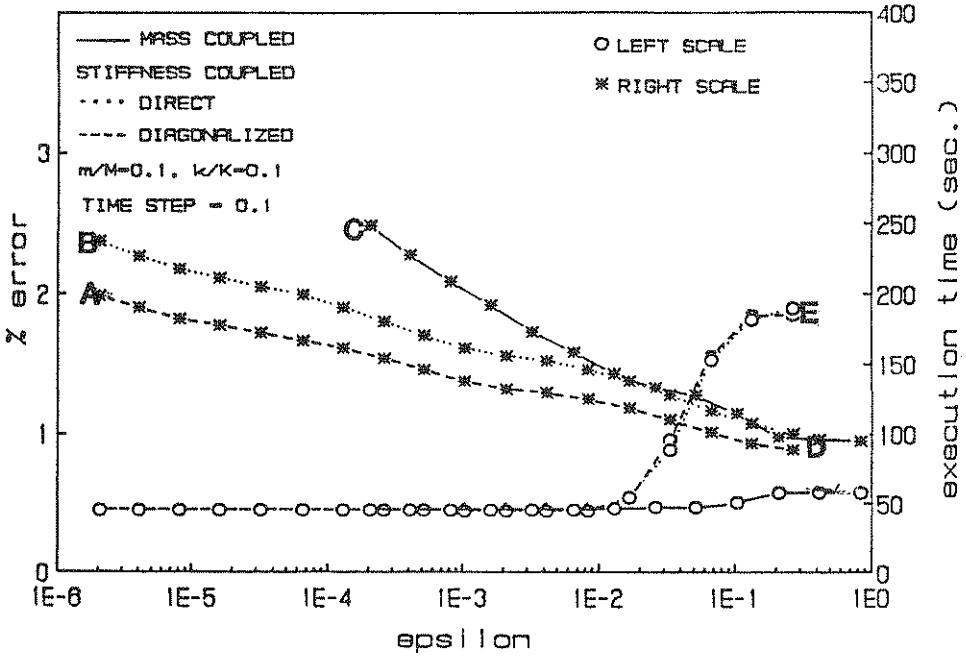
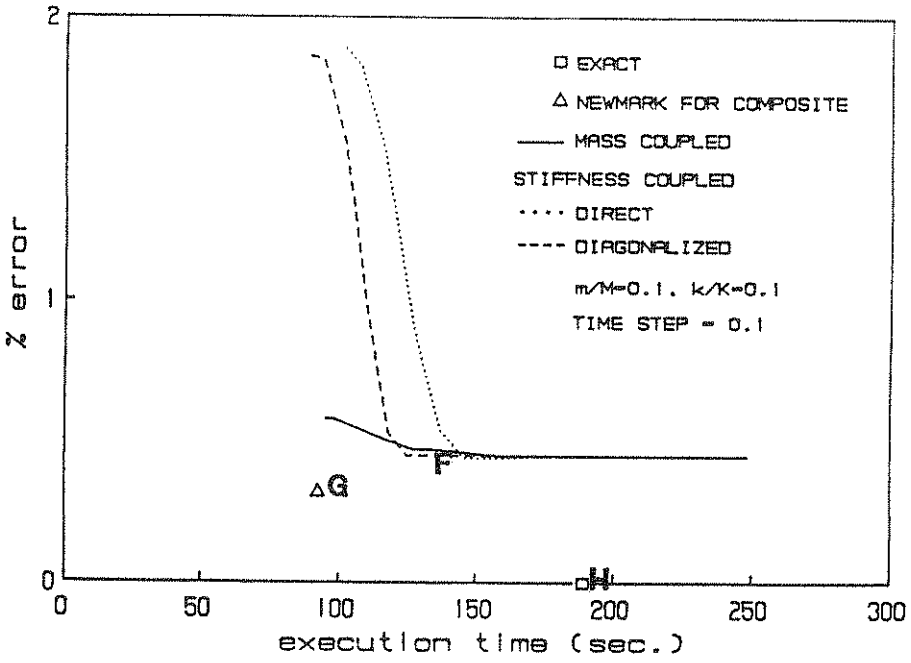


Fig. 4.4 Tangent Stiffness Approach for Nonlinear Structure

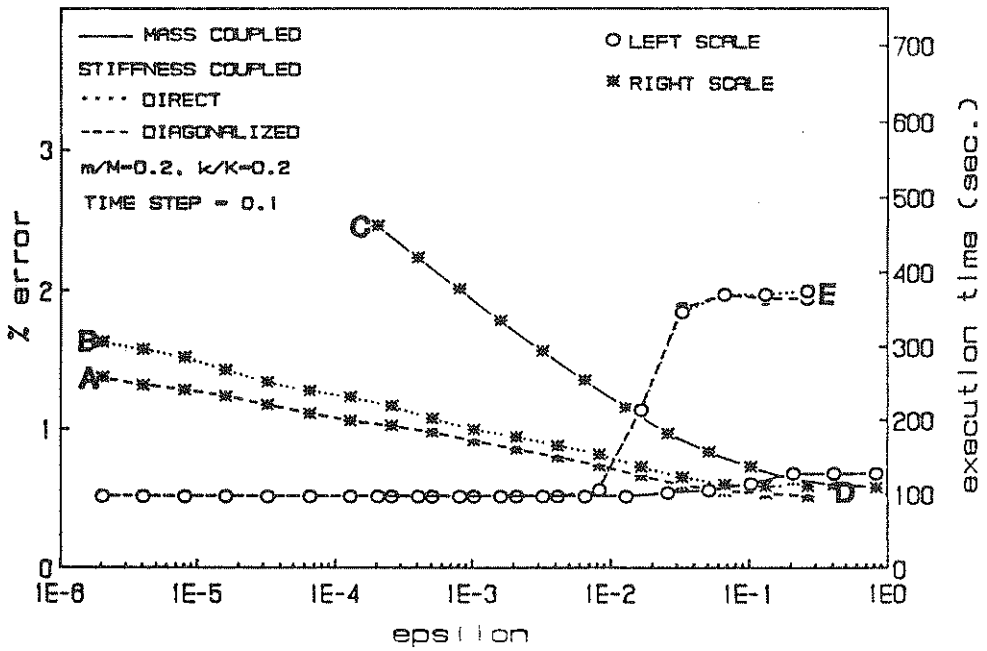


(a) Effect of Epsilon

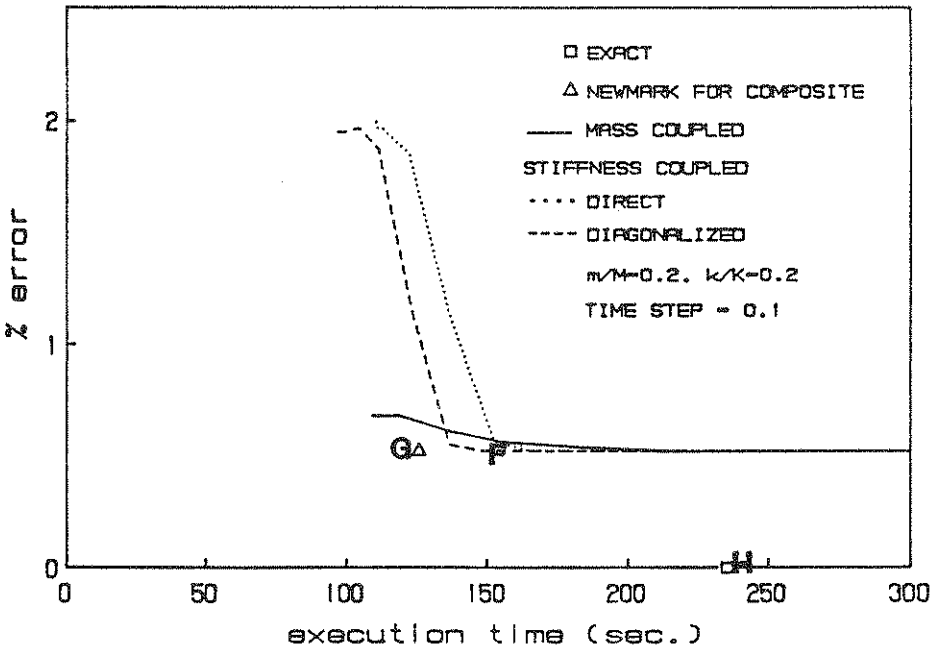


(b) Accuracy vs. Efficiency

Fig. 4.5 First Structure - Linear

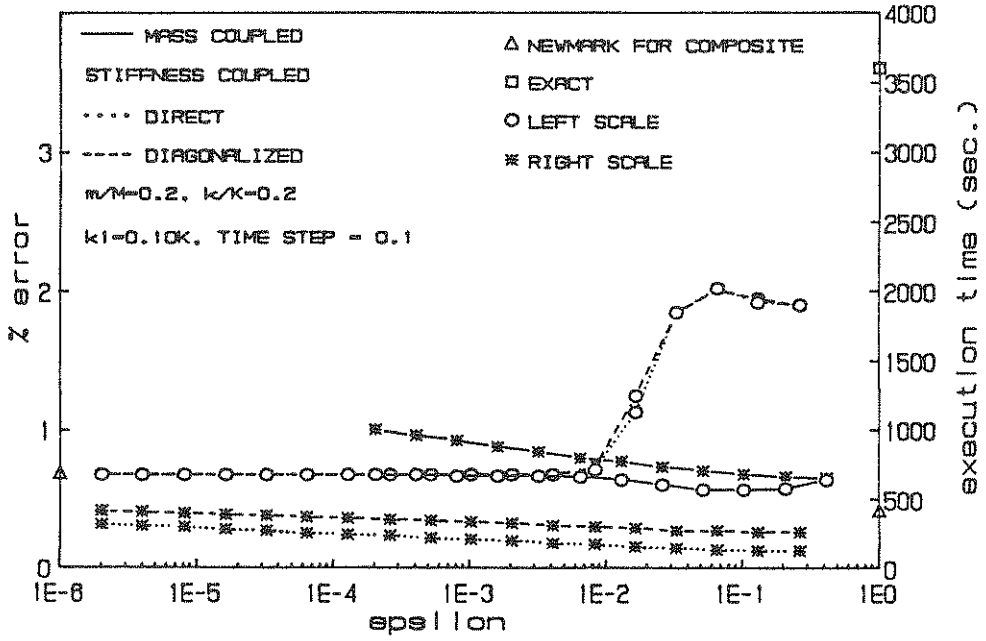


(a) Effect of Epsilon

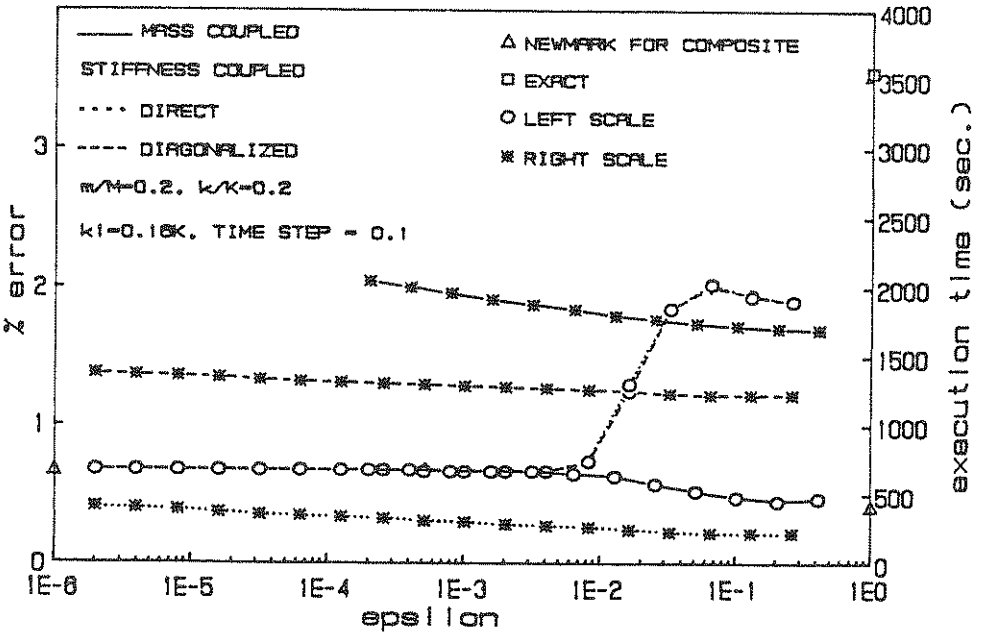


(b) Accuracy vs. Efficiency

Fig. 4.6 Second Structure - Linear



(a) Nonlinear Interface Only



(b) All Subsystems Nonlinear

Fig. 4.7 Second Structure - Nonlinear

5.0 SUMMARY AND CONCLUSIONS

5.1 Summary

Some of the commonly used methods for the analysis of primary-secondary systems like the cascade analysis and the perturbation techniques have been discussed. Cascade analysis uses separate subsystem analyses but ignores the effect of interaction. Thus, the presence of the secondary has no influence on the response of the primary. Perturbation analysis uses subsystem analyses to approximate the dynamic characteristics of the composite system and then uses composite analysis to obtain time histories of response. The limitations of these methods have also been briefly discussed. Some of the techniques for the simulation of representative time histories have also been reviewed.

Some techniques for obtaining response time histories of coupled systems to seismic excitations have been examined. One of these, the mass coupled approach, is an existing technique while the two stiffness coupled approaches are new variations. These methods use subsystem analyses in an iterative way to directly obtain response time histories. In each of these methods, a parameter ε denotes the degree of convergence required for termination of iteration. Thus, ε affects both accuracy and computation time. The Newmark method has been used to integrate the equations of motion for all the subsystems. For the mass coupled approach and the diagonalized stiffness coupled approach, the matrices on the left side of the equations are diagonalized and so no matrix inversion is needed in the Newmark method. For the direct stiffness coupled approach matrix inversion has to be carried out.

The conditions for simultaneous diagonalization of the damping matrices for the composite system and all the subsystems have been examined in some depth. The situations considered include the general case wherein the various damping matrices

satisfy the condition $K M^{-1} C = C M^{-1} K$, and certain specific cases like the Rayleigh condition and the condition for equal modal damping ratios.

The methods for obtaining time histories of response have been studied both for linear and nonlinear situations. In the linear case, two structures have been examined. The total numbers of degrees of freedom are similar (ten and twelve) but they have different ratios of secondary to primary degrees of freedom (5:5 and 2:10). The effects of different mass and stiffness ratios and of time step have also been examined. In the nonlinear case, the effect of changing the magnitude of the nonlinearities has been examined. The case when only one of the subsystems is nonlinear has been studied along with the case when all subsystems are treated as nonlinear. The emphases in these studies is on the relative accuracy and efficiency of the various techniques for computing response time histories.

5.2 Conclusions

5.2.1 Simultaneous Diagonalization of Damping Matrices

If the composite system satisfies the Rayleigh condition, then one can use a change of variables to simultaneously diagonalize the damping matrix for the composite system and one for each subsystem. This seems to be the only condition which can result in simultaneous diagonalization without imposing any restrictions on the stiffness or mass matrices. An assumption apparently made by some earlier investigators, that equal modal damping ratios for all subsystems also results in diagonalization of the composite, is shown to be in error.

Based on the study of the linear and nonlinear systems, the following conclusions can be arrived at for the two systems.

5.2.2 Linear System

It appears that the Newmark method for the composite system gives the smallest execution time compared to the other methods for the same amount of error. Among the three types of iterative subsystem analysis methods, it is difficult to conclude that any one is clearly better than the others. Each can give accurate results with reasonable computation times, but there are some differences between them.

For the stiffness coupled approaches there exists an optimum ε which results in the least error and the lowest execution time for that error. This value of ε is around 0.01. If ε is increased slightly above this limit, the error increases appreciably without any significant decrease in execution time. Both the direct and the diagonalized stiffness coupled approaches give almost the same error for the same ε but the diagonalized stiffness appears to be slightly more efficient especially for small ε. The error for the mass coupled approach appears to be almost independent of ε for small time steps. The execution time for all these three methods decreases with increasing ε and all three converge to the same value for larger ε. The error in the stiffness coupled methods also tends to remain constant after a certain value of ε. This value of ε is around 0.1.

The secondary-primary mass and stiffness ratios do not appear to affect the error in any of the methods. When the normalized time step is increased from 0.1 to 0.2 and 0.4, the error in the Newmark method itself tends to grow appreciably. The increase in the time step seems to increase the error in both the stiffness coupled approaches much more appreciably than in the mass coupled approach. Further, these methods did not seem to become independent of ε when $\overline{\Delta t}$ was taken to be 0.4. The relative number of degrees of freedom in the primary and the secondary does not not seem to affect the accuracy and the efficiency of any of the methods for any time step.

Another advantage in using the Newmark method for either the composite system or in the direct stiffness coupled analysis is that the damping does not have to be

restricted to the Rayleigh form since the equations do not have to be uncoupled.

5.2.3 Nonlinear System

As in the linear, case there is an optimum ε for the stiffness coupled approaches. This optimum value of ε is also around 0.01. The mass coupled approach, as before, is more or less independent of ε. As in the linear situation, the errors in the stiffness coupled methods do not increase after a certain value of ε. This value ε is also around 0.1. In contrast to the linear case, the execution times for the different methods for primary-secondary analysis do not decrease appreciably with an increase in ε. However, it may be noted that the curves for the execution time when all subsystems are considered nonlinear are almost parallel to the corresponding curves when only the interface is considered nonlinear. This is because of the fact that eigen analysis or matrix inversion has to be carried out once for each time step irrespective of the number of iterations required to converge to the interface response parameters. The shift between the corresponding curves on the two figures is the additional time required for eigen analysis or matrix inversion.

The greatest advantage of the primary-secondary analysis seems to be in the case when only one small subsystem is nonlinear. In primary-secondary analysis only the matrices for that subsystem and the interface need be inverted at each time step or the eigen analysis for that subsystem and interface alone need be carried out.

Even in the case when all subsystems were considered nonlinear, primary-secondary analysis appears more efficient than in the linear case. When only the interface was considered nonlinear, the mass coupled approach appears to be 5.5 times more efficient, diagonalized stiffness appears to be 11.5 times more efficient and direct stiffness 19 times more efficient than the exact procedure. When all subsystems were considered nonlinear, these values are 2 for mass coupled, 2.5 for diagonalized stiffness

and 11.5 for direct stiffness. These values for efficiency have been calculated at the optimum value of ε.

The Newmark method for the composite appears to be about 8 times more efficient than the exact. However, the Newmark method for the composite system is not necessarily always the better approach for nonlinear analysis especially if the nonlinearity is in one of the smaller subsystems. In particular, the direct stiffness coupled method was about 2.4 times as efficient as the Newmark for composite system when only one small subsystem was nonlinear, and the ratio was about 1.5 when all subsystems were treated as nonlinear.

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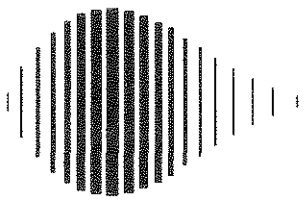

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