NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
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SUBSTRUCTURING TECHNIQUES IN THE TIME DOMAIN FOR PRIMARY-SECONDARY STRUCTURAL SYSTEMS

by

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NOTICE

SUBSTRUCTURING TECHNIQUES IN THE TIME DOMAIN FOR PRIMARY - SECONDARY STRUCTURAL SYSTEMS

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ABSTRACT

The question of how a primary structural system is affected by the presence of secondary structural and/or non-structural attachments and vice-versa during an intense dynamic event such as an earthquake can best be answered by recourse to numerical simulation coupled with experimental verification. Towards this goal, the present work develops a substructuring methodology in the time domain for the analysis of structures containing secondary systems. In particular, the structure can be broken into a primary substructure that in turn contains secondary substructures. There is no restriction on the size or configuration of the secondary substructures. The present methodology allows for a separate analysis of the primary and secondary substructures under the influence of the applied loads. Then, the response of these substructures to the interaction forces that are responsible for their coupling is also separately evaluated and acts as a correction to the response due to the presence of the applied loads. The size of both primary and secondary subsystem representations can be reduced by introducing modal condensation via Ritz vectors that do not form complete modal sets. Numerical implementation of the present methodology is done through the use of direct numerical integration. The presence of interaction forces that are unknown at the current time step, however, necessitates the introduction of a predictor-corrector scheme. If modal condensation is used, then the predictor-corrector scheme is applied at the modal equation level. Finally, a three-story shear building containing a two degree-of-freedom secondary system is carefully analyzed with and without modal condensation. Parametric studies are conducted by changing the properties of the secondary substructure and by considering different types of applied loads in the form of ground excitations.

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INTRODUCTION

1.1 Secondary Systems

The progress that has been made in the seismic analysis of structural systems during the past decades (Newmark and Rosenblueth, 1971) has resulted in improvements in the analysis, design and construction of buildings, bridges, dams, etc., under seismic excitations. More recently, an area of increasing concern is seismic performance of secondary systems that are attached in some way to a primary structural system. In general, there are two basic types of secondary systems, namely, non-structural and structural. Examples of the first case include computer and control systems, machinery, electrical panels, and storage tanks. Performance integrity of these systems under seismic loads transmitted through the primary structural system is important because they serve vital functions and their failure may have serious consequences. Examples of the second case include stairways, structural partitions, suspended ceilings, piping systems and ducts. Interaction of these systems with the primary system is again important because they are capable of modifying the structural behavior of the primary system in unanticipated ways.

Traditionally, the seismic analysis of secondary systems is done using the floor spectrum approach. In this approach, the response behavior of the primary structural system at the support points of a secondary system is first determined in the absence of the secondary system. The response spectra at the support points, or the 'floor response spectra', are then used as input to the secondary system. The response behavior of the secondary system is finally computed on the basis of this input, using one of several modal combination rules.

The use of this approach, however, leads to a number of deficiencies. The most serious one is the fact that it ignores the primary-secondary interaction. The floor spectrum method gives acceptable results for secondary systems with relatively small masses and with frequencies which are not tuned to a frequency of the primary structural system. When the two systems are tuned to each other, however, a gross error in estimation of the secondary system behavior may result (Crandall and Mark 1963, Singh 1975, Kapur and Shao 1973). Another problem has to do with the use of modal combination rules. Significant error again can arise since these commonly used rules do not account for such important effects as cross-correlations between support motions and between modal responses (Amin et al 1971 Chakravarti and Vanmarke 1973).

Recently, a number of analytical methods have been developed in an attempt to correct the deficiencies outlined above. All of these are based on an analysis of the combined primary-secondary (PS) system. In Sackman and Kelly (1979) and Igusa and Der Kiureghian (1983), perturbation methods are used by treating the parameters of secondary systems as small. This leads to better accuracies in analyzing tuned two degree-of-freedom (dof) systems. Modal analysis of the PS system has also been developed by Ruzicka and Robinson (1980), Villaverde and Newmark (1980), and Der Kiureghian et al (1983 a,b) who found the mode shapes and frequencies of the combined system by using perturbation techniques. The modal responses are subsequently combined to give the physical displacements by using a modal combination rule. However, the important non-classical damping characteristics are not adequately addressed. In addition, various kinds of simplifying assumptions are used such as (i) The primary and secondary structural systems are classically damped and linearly elastic, (ii) The mass of the secondary system is considerably smaller than that of the primary system, (iii) Input floor spectra at all support points are identical, and (iv) Interaction effects of foundation, cascaded tertiary or higher level subsystems, or other separate secondary systems are ignored.

In more recent work, Igusa and Kiureghian (1985 a,b) used pertrubation theory to determine the modal properties of a structure from the modal properties of its substructures and to evaluate the secondary substructure response using modal combination. The effects of spatial coupling that results because of the presence of multiply supported secondary substructures, of tuning and of non-classical damping are all taken into account. A simplified approach for computing the maximum response of a light secondary system with one point of attachment to a primary substructure under seismic motions by avoiding solution of a large eigenvalue problem was introduced by Villaverde (1986). His procedure uses the component mode synthesis technique and takes into account the interaction effect between the two substructures. A component mode synthesis technique was also introduced by Suarez and Singh (1987a) for obtaining the modal properties of a combined single dof secondary system attached to a primary substructure in terms of the modal properties of the individual systems. No assumption is made as to the size of the secondary substructure. In a series of papers, Singh and his co-workers (Suarez and Singh 1987b; Suarez and Singh 1987c; Burdisso and Singh 1987a; Singh and Burdisso 1987; Burdisso and Singh 1987b) describe how to synthesize the eigenproperties of the combined structure from knowledge of the eigenproperties of the primary and secondary substructures for cases where either the primary substructure or the combined system are nonclassicially damped. Two basic techniques are developed, namely pertrubation techniques for tuned or detuned secondary equipment and an eigensolution in terms of a nonlinear characteristic equation that is valid for both light and heavy secondary substructures. Based on these analyses, the aforementioned researchers proceed to construct floor response spectra that take into account the interaction effect between primary and secondary substructures. Also, the case of multiple supported secondary systems is examined, where it is necessary to consider the correlation between the motions of all points to which the secondary system is attached. Parametric studies on the frequency response of a single dof secondary system contained within a multiple dof primary system were conducted by Yong and Lin (1987) and by

Holung et al (1987). The latter investigators used the transfer matrix approach to find the mode shapes and natural frequencies of a twenty story building to which a single dof secondary system is attached.

1.2 Substructuring Approaches

The key idea behind substructuring is to reduce the size of the system under consideration. This can be accomplished by either dividing the structure into a number of substructures whose boundaries are arbitrarily specified or by transforming the physical coordinates of the structure into a smaller set of generalized coordinates. There are also substructuring techniques that combine both of these approaches.

Substructuring through partitioning

The structure is first partitioned into subsystems that may or may not correspond to physically identifiable substructures. Then each subsystem is treated as a complex structural element and finite elements or any other suitable numerical method can be used to model it. Once the displacements and forces at the interfaces between subsystems have been determined, each subsystem is analyzed separately under known boundary conditions.

Under static conditions, the problem is solved in two steps (Przemieniecki 1968). At first a substructure is isolated from its neighbors by setting the boundary displacements equal to zero, and the reactions necessary for maintaining fixity of the common boundaries are computed. Next, the boundaries are relaxed and the true displacements there, as well as the remaining interior displacements, are computed. The complete solution is synthesized by adding the results of these two steps. Under dynamic conditions, it is difficult to explicitly solve for the interaction forces at the common boundaries between substructures. The researchers that have addressed this problem (Kennedy et al 1981, Ordonez et al 1985) determine the interaction forces by imposing compatibility of accelerations at interface dof within the context of time stepping algorithms. In addition, information on the modal shapes of both primary and secondary substructures is required by their methods.

Substructuring through generalized coordinates - Ritz vectors

The basic idea here is to introduce a coordinate transformation in the equation of motion so as to transform it from order nxn to order mxm, m<n. Such a coordinate transformation can be accomplished using a set of eigenvectors or a set of Ritz vectors. In either case, the vector of physical coordinates {x} and the vector of generalized coordinates {q} are related through the transformation matrix [A] as

$$\{x\} = [A]\ \{q\} \tag{1.1}$$

Note that the transformation matrix [A] is of order nxm.

It is not efficient to use the complete set of eigenvectors of the structure to build [A], despite the advantage of uncoupled modal equations of motion that results for classically damping cases, because of the difficulties associated with solving the associated eigenvalue problem. Wilson et al (1982) and Wilson (1985) demonstrated that the use of the Ritz set of vectors is preferable to a reduced set of eigenvectors of the same order. Ritz vectors depend on the spatial distribution of the applied loads and the resulting modal stiffness matrix is not diagonal, as the case would be if the eigenvectors were used. Therefore, a subsequent modal analysis of the mxm system of equations of motion resulting in orthogonal Ritz vectors is necessary to rectify this problem.

Substructuring through generalized coordinates - Constraint equations

Another way of constructing matrix [A] of Eq. 1.1 is through specifying a relation between the vector or independent (or active) coordinates $\{x_a\}$

and the vector of dependent coordinates $\{x_d\}$, where $\{x\}^T = \{x_a, x_d\}$. This is done through the constraint equation

$$[R_{da}, R_{dd}] \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = 0 \tag{1.2}$$

Equation 1.2 is then used to solve for $\{x_d\}$ in terms of $\{x_a\}$ so that

$$\{x\} = \begin{bmatrix} I_{aa} \\ -R_{dd}^{-1}R_{da} \end{bmatrix} \{x_a\} = [A] \{x_a\} \tag{1.3}$$

where [I] is the identity matrix. A constraint equation such as Eq 1.2 arises when an elastic structure contains rigid attachments (Weaver 1966).

Another type of constraint equation can be obtained from static condensation. This case arises when certain dof in the equation of motion do not have any mass and force terms associated with them. These 'static' dof then become the dependent dof. Thus, if the stiffness matrix [K] is partitioned according to active and dependent dof, it is easy to show that [A] becomes a function of the submatrices of [K] as

$$[A] = \begin{bmatrix} I_{aa} \\ -K_{dd}^{-1}K_{da} \end{bmatrix} \tag{1.4}$$

In the Guyan reduction method (Guyan 1965), the transformation matrix is identified with Eq. 1.4, despite the fact that inertia is associated with all dof. The physical meaning of the columns of [A] is that each represents a static displacement pattern resulting from imposing a unit displacement at one active dof, while the remaining active coordinates are fixed and the dependent coordinates are released. Anderson et al (1968) discuss the effect that the choice of $\{x_a\}$ has on the accuracy of the eigensolution of typical problems. Finally, Johnson et al (1980) discuss a quadratic eigensolution procedure for improving the accuracy of the Guyan reduction method.

Substructuring through component mode synthesis

A more general way of substructuring for dynamic problems is the component mode synthesis method that, in a way, combines both structural partitioning aspects as well as the generalized coordinate concepts. As before, we distinguish between the interior and the boundary of a given substructure. We also retain the representation

$$\{x\} = [B]\ \{Q\} \tag{1.5}$$

where {Q} are the component generalized coordinates and [B] is a matrix of preselected component modes. Equation 1.5 can be viewed as an approximation of the displacement vector {x} no different than the one in Eq. 1.1. Matrix [B] may contain (a) the normal modes of the free vibration problem, (b) constraint modes and (c) attachment modes. In the first case, the normal modes are obtained by solving the classical eigenvalue problem for one of the following three boundary conditions: fixed boundary, free boundary, and partially fixed boundary. In addition, some substructuring methods consider the boundary to be loaded, whereby mass and stiffness coefficients are added to the original mass and stiffness matrices, respectively. The modal matrix $[\Psi_n]$, whose columns are the normal modes, is normalized with respect to the mass matrix. It is quite standard to use in Eq. 1.5 a truncated set of normal modes $[B] = [\Psi_m]$ where $m<n$. In the second case, the constraint modes are the same as those used in the Guyan reduction technique. In the third case, an attachment mode is defined as the static deflection at an active dof due to a unit force at that dof, while the remaining active coordinates are force-free.

When the component synthesis method is applied to free vibrations, the original equation of motion of the system reduces to solution of uncoupled modal equations for the component generalized coordinates {Q} of each substructure. A number of variations on the component mode synthesis technique exist (Craig and Bampton 1968), most of which are modifications of the original method of Hurty (1965). The component mode synthesis can be extended to damped structures (Klein and Dowell 1974) and to forced vibrations. It is also possible to include rigid-body modes, which appear in aerospace structures, in the formulation.

More current research (Leung 1979, Arora and Nguyen 1980, Bathe and Gracewski 1981) on the component mode synthesis method focuses on improving the accuracy as well as the efficiency of the methodology, on investigating normal mode truncation, and on implementing the technique in general purpose finite element programs. Furthermore, error analyses have been done and attempts have been made to extend the method to systems with nonporportional damping and complex eigenvalues/vectors. A comprehensive review article on the component mode synthesis method can be found in Craig (1985).

Other methods of substructuring

Other methods of substructuring include: (i) The use of ordinary admissible function representation for substructures (Hale and Meirovitch 1980, Meirovitch and Hale 1981). These admissible functions are low order polynomials that simplify computations, and the method of weighted residuals is used to approximately enforce the geometric compatibility conditions. Convergence of the method is achieved as the number of admissible functions increases: (ii) The transfer matrix method automatically achieves a reduction in the size of the matrices without having to truncate any dof (Chiatti and Sestieri 1981). This method, however, is restricted to structures with a chain-like topology. The accuracy of the transfer matrix method for large systems can be improved by use of the brancing concept; and (iii) The finite strip method, which is tailored for plates and shells, can be thought of as equivalent to a substructuring technique (Dawe and Morris 1982). Substructuring can also be used in conjunction with statistical concepts to analyze the random response of coupled subsystems. Finally, substructuring can be combined with analytical-experimental techniques, and Goyder (1980) describes how to model components from experimentally measured data. Two review articles (Greif and Wu 1983; Greif 1986) summarize the work done on substructure analysis of vibrating systems from the early 1980's onwards and reference over a hundred papers.

1.3 Overview of Present Work

This work is arranged as follows: Section 2 discusses the basic theory of substructuring in the time domain as well as numerical implementation aspects, including the predictor-corrector scheme developed to handle the interaction forces between primary and secondary substructures. Section 3 introduces the concept of modal condensation through the use of Ritz vectors. It also discusses the predictor-corrector scheme of the previous section for the case of modal coordinates. Then, Section 4 contains numerical studies for a three-dof shear building containing a two-dof secondary substructure whose properties are varied and a number of excitations are considered. Finally, Section 5 draws a list of conclusions and discusses further work along this area.

SECTION 2

SUBSTRUCTURING IN THE TIME DOMAIN

2.1 Basic Equations

The equations of motion of a structure such as the one shown in Fig. 1 are of the form

$$\begin{bmatrix} M_p & 0 \\ 0 & M_s \end{bmatrix} \begin{Bmatrix} \ddot{x}_p \\ \ddot{x}_s \end{Bmatrix} + \begin{bmatrix} C_p & 0 \\ 0 & C_s \end{bmatrix} \begin{Bmatrix} \dot{x}_p \\ \dot{x}_s \end{Bmatrix} + \begin{bmatrix} K_p & 0 \\ 0 & K_s \end{bmatrix} \begin{Bmatrix} x_p \\ x_s \end{Bmatrix} = \begin{Bmatrix} F_p \\ F_s \end{Bmatrix} + \begin{Bmatrix} R_p \\ R_s \end{Bmatrix} \qquad (2.1)$$

if a lumped parameter representation (e.g., Fig. 2) is assumed. In the above, [M], [C] and [K] are mass, damping and stiffness matrices, respectively, and {x} are displacements. Furthermore, subscripts p and s denote primary and secondary substructure, respectively, and dots indicate time derivatives. Finally, {F} and {R} respectively contain the applied loads and the interaction forces that are manifested at the points of attachment between the primary and secondary substructures. In general, there is coupling in Eq. 2.1 between the primary and secondary dof because

$$\{R_p\} = \{f(\{x_p\},\{x_s\},\{\dot{x}_p\},\{\dot{x}_s\})\} \qquad (2.2)$$

and similarly for $\{R_s\}$. If {R} is brought over to the left-hand side of Eq. 2.1, we then have a coupled nxn system of equations whose solution is referred to as the base solution. If not, then Eq. 2.1 uncouples into an mxm system of equations governing the motions of the primary substructure and an lxl system (m+l=n) for the secondary substructure, provided the interaction forces can be evaluated in some way.

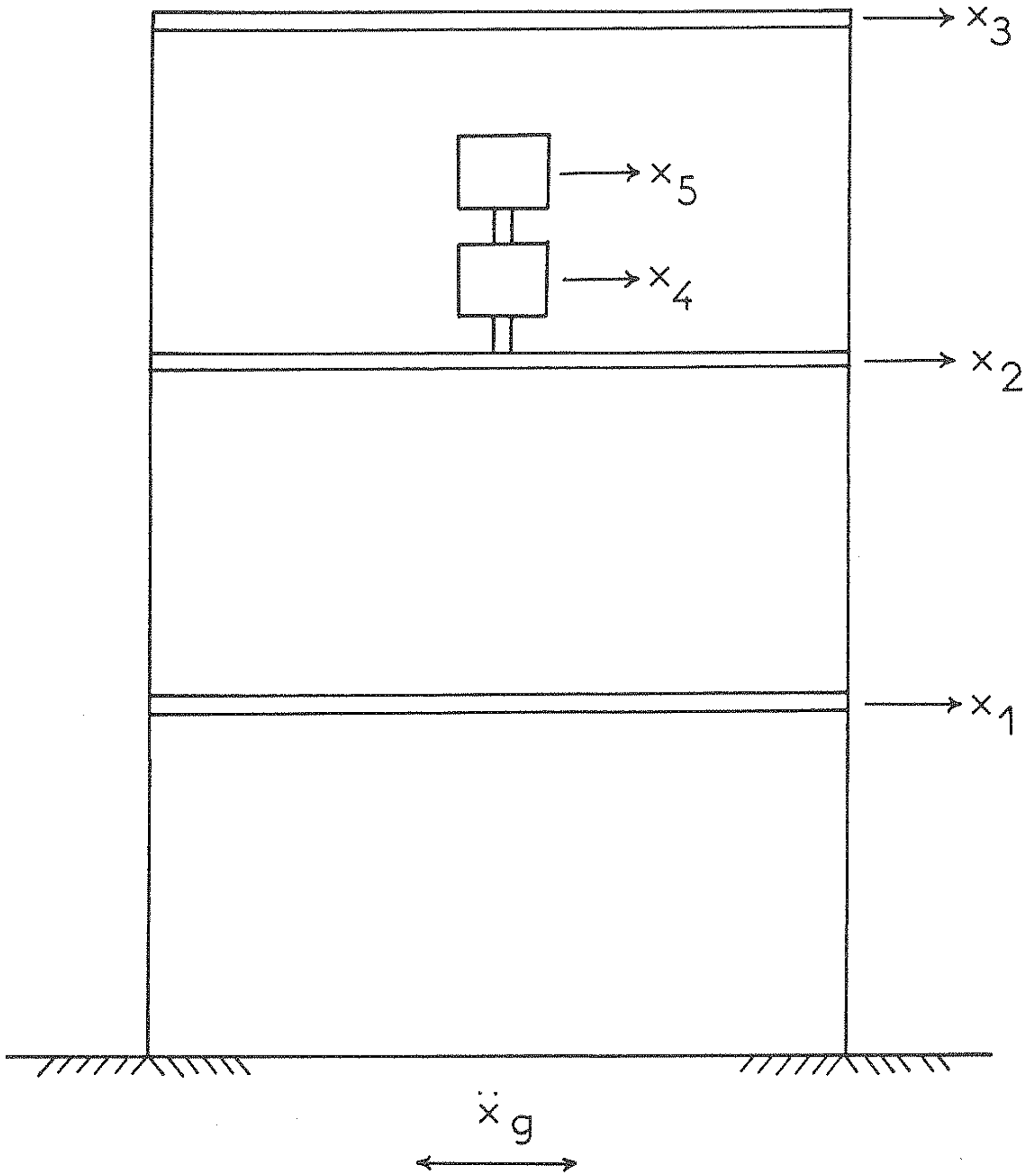

Figure 1 - Structural system composed of primary system (dof 1,2,3) and secondary system (dof 4, 5).

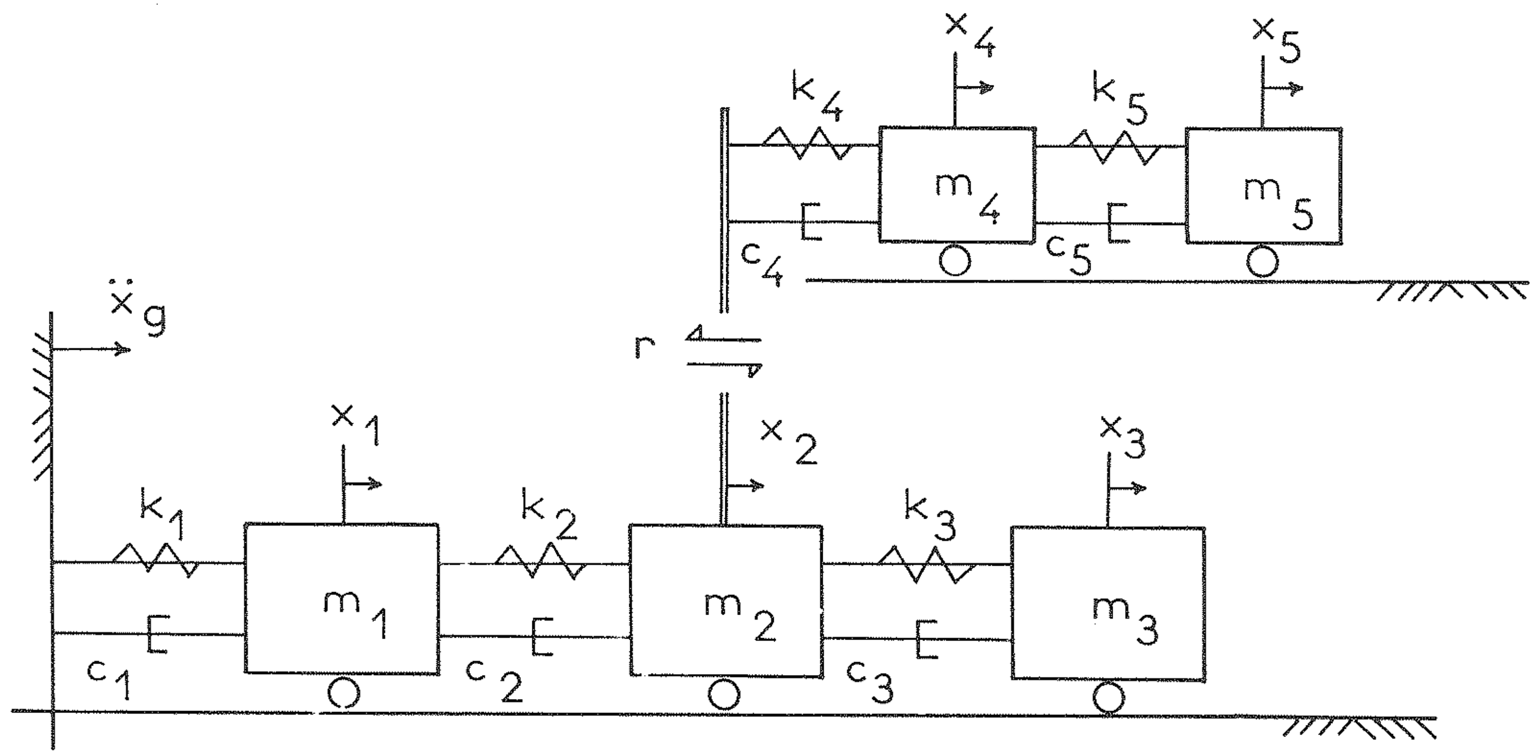

Figure 2 - Lumped parameter representation of the structural system of Figure 1.

If this last approach is followed, then it is convenient to decompose the displacement as

$$x_j(t) = x_j'(t) + x_j''(t) \tag{2.3}$$

and similarly for the velocity and acceleration. In the above, $x_j'(t)$ is the response of dof j belonging to either primary or secondary substructure under the influence of applied loads only. Also, $x_j''(t)$ is the response of dof j when the substructure is under the influence of the interaction forces only. Obviously, this type of decomposition is possible for linearized substructure behavior. It should be noted that solution for the primed response is accomplished independently of the double-primed response, and the latter solution acts as a correction to the former by accounting for the coupling effect between primary and secondary substructures. Compared to the base solution where the entire nxn system of equations has to be solved, the substructuring approach reduces the problem to solution of two smaller subsystems with the possibility of condensing the dof of either or both subsystems.

2.2 Numerical Implementation

One way of solving the uncoupled form of Eq. 2.1 with an mxm system governing primary substructure response and an lxl system governing secondary substructure response is by numerically integrating each of these two systems. The only problem remaining is to evaluate the interaction force {R} at time step t_{i+1}, with t_i being the current time step. The algorithm designed for this purpose is a predictor-corrector type method. In particular, the velocity and acceleration can be written in terms of the displacement using central differences as

$$\{\dot{x}(t_i)\} = (-\{x(t_{i-1})\} + \{x(t_{i+1})\})\ /2\Delta t \tag{2.4}$$

and

$$\{\ddot{x}(t_i)\} = (\{x(t_{i-1})\} - 2\{x(t_i)\} +\{ x(t_{i+1})\})\ /\Delta t^2 \tag{2.5}$$

respectively, with $\Delta t = t_{i+1}-t_i$ being the time increment. When the above two expressions are substituted into the equations of motion (2.1) written at t_i, the resulting expression for the displacement vector at t_{i+1} is

$$\{x(t_{i+1})\} = ([M]/\Delta t^2 + [C]/2\Delta t)^{-1}\ (\{R(t_i)\} - ([K] -(2/\Delta t^2)[M])\{x(t_i)\}-([M]/\Delta t^2-[C]/2\Delta t)\{x(t_{i-1})\}) \tag{2.6}$$

where [M], [C] and [K] are the appropriate mass, damping and stiffness matrices and {R} is the interaction force. Equations 2.4-2.6 form the predictor branch, i.e., $\{x(t_{i+1})\}$ is computed from Eq. 2.6 and substituted in Eqs. 2.4 and 2.5 to give $\{\dot{x}(t_i)\}$ and $\{\ddot{x}(t_i)\}$, respectively.

For the corrector branch, Newmark method expansions are used to get updated values for the acceleration and velocity and a corrected value for the displacement based on the information supplied by the predictor branch as

$$\{\ddot{x}(t_{i+1})\} = (\{x(t_{i+1})\}-\{x(t_i)\})\ /\alpha\Delta t^2-\{\dot{x}(t_i)\}\ /\alpha\Delta t+\{\ddot{x}(t_i)\}(1-1/2\alpha) \tag{2.7}$$

and then

$$\{x(t_{i+1})\} = \{x(t_i)\} + \{\dot{x}(t_i)\}\Delta t + ((0.5-\alpha)\{\ddot{x}(t_i)\} + \alpha\{\ddot{x}(t_{i+1})\})\Delta t^2 \tag{2.8}$$

$$\{\dot{x}(t_{i+1})\} = \{\dot{x}(t_i)\} + ((1-\delta)\{\ddot{x}(t_i)\} + \delta\{\ddot{x}(t_{i+1})\})\Delta t \tag{2.9}$$

Note that $\delta \geq 0.5$ and $\alpha \geq 0.25(0.5+\delta)^2$ are the usual Newmark method parameters. In the above algorithm, as with all numerical integration methods, fresh values of the displacement, velocity and acceleration replace old values as soon as they become available.

The final task is to compute an updated value for the interaction force based on updated values for the displacement and velocity, Eqs. 2.8 and 2.9, respectively, at the dof adjoining the attachment point. In view of Eq. 2.2, the updated interaction force component r(t) at an attachment point is given by

$$r(t_{i+1}) = k(x_p(t_{i+1}) - x_s(t_{i+1})) + c(\dot{x}_p(t_{i+1}) - \dot{x}_s(t_{i+1})) =$$

$$= k(x_p'(t_{i+1}) - x_s'(t_{i+1}) + x_p''(t_{i+1}) - x_s''(t_{i+1})) + c(\dot{x}_p'(t_{i+1}) - \dot{x}_s'(t_{i+1}) + \dot{x}_p''(t_{i+1}) - \dot{x}_s''(t_{i+1})) \tag{2.10}$$

In the above, the primed quantities are the response of the dofs of the primary and secondary substructures adjoining the attachment point to the applied loads. They can be computed separately from the double primed quantities, which are the response of the aformentioned dof to the interaction force. These last quantities are known at t_i since $r(t_i)$ has already been evaluated. Subsequently, these quantities are computed at t_{i+1} using the predictor-corrector scheme previously outlined.

SECTION 3

SUBSTRUCTURING WITH MODAL CONDENSATION

3.1 Ritz Vectors

The use of modal analysis for solution of primary and secondary substructures with coupling accounted for by virtue of the interaction force allows for condensation of the modal dof. This is achieved by simply using a subset of the complete set of mode shapes, although care must be excercised to assure that enough modes are retained in order to obtain an accurate solution. The Ritz vectors are chosen here because they yield more accurate results than an equal number of mode shapes (Wilson et al, 1982). The only drawbacks are that Ritz vectors depend on the spatial distribution of applied dynamic loads and that they do not form an orthogonal set of vectors. The first drawback is of little consequence because for the case of ground motions, the overall spatial distribution of the loads does not change for a given structure. The second drawback is remedied by a subsequent orthogalization of the original Ritz vectors.

Although Ritz vectors have been treated elsewhere (Wilson et al, 1982), the algorithm for their generation is included here for completeness. The first Ritz vector $\{\phi_1\}$ is found by solving

$$[K]\{\phi_1^*\} = \{f\} \tag{3.1}$$

where $\{f\}$ encompases the spatial variation of $\{F\}$. Then, the first Ritz vector is normalized using

$$\{\phi_1\}^T[M]\{\phi_1\} = 1 \quad (3.2)$$

Aditional Ritz vectors are found through the recursive relation

$$[K]\{\phi_i^*\} = [M]\{\phi_{i-1}\} \quad (3.3)$$

with orthogonality correction as

$$\{\phi^{**}{}_i\} = \{\phi_i^*\} - \sum_{j=1}^{i-1} C_j\{\phi_j\}, \; C_j=\{\phi_j\}^T[M]\{\phi_i^*\} \quad (3.4)$$

and normalization as

$$\{\phi_i\}^T[M]\{\phi_i\} = 1 \quad (3.5)$$

Pre- and post- multiplication of the stiffness [K] and mass [M] matrices by matrix $[\phi]$ containing the Ritz vectors $\{\phi_i\}$ gives the modal stiffness $[K^*]$ and mass $[M^*]$ matrices, respectively, with only the latter one being diagonal. A set of Ritz vectors $\{\psi_i\}$ orthogonal to the stiffness matrix as well is obtained by solving the eigenvalue problem

$$([K^*]-\omega_i^2[M^*])\{\psi_i\} = 0 \quad (3.6)$$

where ω_i are corresponding frequencies. It should be noted that the above eigenvalue problem is solved after a decision on how many Ritz vectors will be used is reached.

The matrix containing the final Ritz vectors is obtained as

$$[A] = [\phi][\psi] \tag{3.7}$$

where $[\psi]$ contains the $\{\psi_i\}$ vectors. Final modal stiffness and mass matrices that are diagonal can now be obtained by pre- and post-multiplication of [K] and [M] with matrix [A].

3.2 Numerical Implementation

The algorithm constructed for solving problems using the substructuring methodology outlined in Section 2.1 in conjunctin with modal condensation is as follows: At first, the orthogonal Ritz vectors and corresponding frequencies of the primary and secondary substructures are computed as indicated in Section 3.1. Since the spatial distribution of the applied loads is different from the spatial distribution of the interaction forces, two sets of orthogonal Ritz vectors are computed for each substructure, as shown in Fig. 3. In practice, these two sets of orthogonal Ritz vectors for each substructure will often turn out to be the same because of the similarity in the spatial distribution of the applied loads and of the interaction force. Second, this modal type of information is used to uncouple the equations of motion of each of the two substructures. Depending on the number of modes used, a set of equations less than or equal to m and less than or equal to ℓ results for the primary and secondary substructures, respectively. These modal equations are of the form

$$\ddot{q}_k(t) + 2\,\xi_k\omega_k\dot{q}(t) + \omega_k^2\,q_k(t) = (\bar{f}_k+\bar{r}_k)\,/\bar{m}_k \tag{3.8}$$

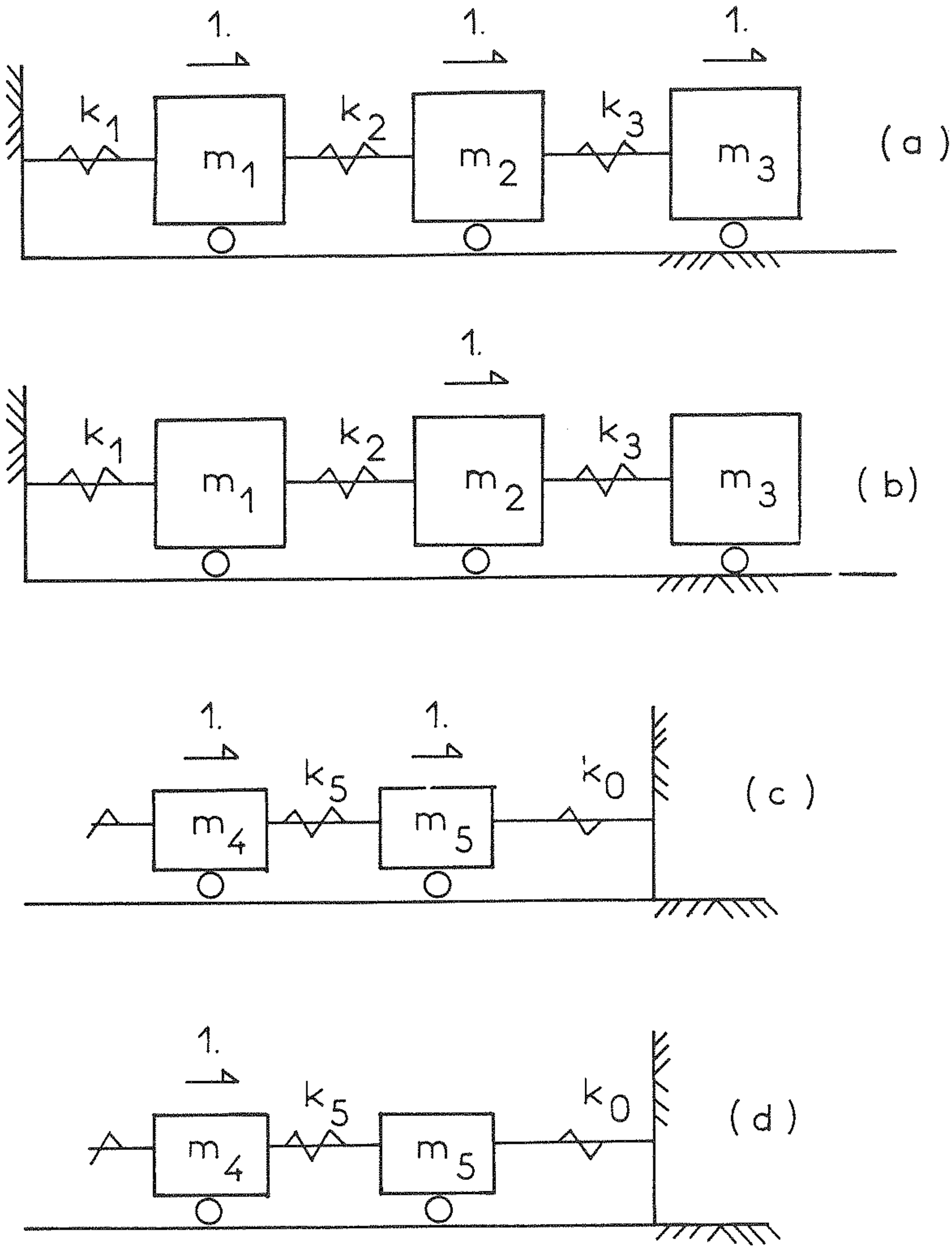

Figure 3 - Ritz vectors for (a) primary substructure, applied loads (P-L); (b) primary substructure, interaction force (P-I); (c) secondary substructure, applied loads (S-L) and (d) secondary substructure, interaction force, (S-I).

where $q_k(t)$ is the generalized coordinate, ξ_k is the modal damping factor, ω_k is the frequency, $\bar{f}_k(t)$ is the generalized load, $\bar{r}_k(t)$ is the generalized interaction force and $\bar{m}_k$ the modal mass, all corresponding to the kth Ritz vector. Third, the modal response of the primary and secondary substructures to the applied loads (primed solution) and to the interaction forces (double-primed solution) is computed by numerically integrating equations such as Eq. 3.8. Fourth, the solution for the displacements $\{x\}$ is recovered from the modal response $\{q\}$ through the transformation

$$\{x\} = \begin{Bmatrix} x_p \\ x_s \end{Bmatrix} = \begin{Bmatrix} x'_p \\ x'_s \end{Bmatrix} + \begin{Bmatrix} x''_p \\ x''_s \end{Bmatrix} = [A]\left(\begin{Bmatrix} q'_p \\ q'_s \end{Bmatrix} + \begin{Bmatrix} q''_p \\ q''_s \end{Bmatrix}\right) = [A]\{q\} \qquad (3.9)$$

It should be noted that although the applied loads are known at time step t_{i+1}, the interaction forces are not. Thus, the methodology described in Section 2.2 for predicting the interaction forces at t_{i+1} from their values at t_i must be re-introduced. This is done at the generalized coordinate level by first expressing $\dot{q}_k(t_i)$ and $\ddot{q}_k(t_i)$ in terms of $q_k(t_{i+1})$ using central differences as in Eqs. 2.4 and 2.5, respectively. Subsequently, by substituting these expressions in Eq. 3.8 written at t_i and re-arranging, the generalized displacement at t_{i+1} is obtained as

$$q_k(t_{i+1}) = (1+\xi_k\omega_k\Delta t)^{-1}\left((2-\omega_k^2\Delta t^2)q_k(t_i) - (1-\xi_k\omega_k\Delta t)\, q_k(t_{i-1}) + \Delta t^2\, \bar{r}_k(t_i)\right) \qquad (3.10)$$

where $\bar{r}_k(t_i)$ is the interaction force expressed in modal coordinates. Eq. 3.10 is used to predict $q_k(t_{i+1})$. Corrected values of $q_k(t_{i+1})$ and update values of $\dot{q}_k(t_{i+1})$ and $q_k(t_{i+1})$ are found in conjunction with Eqs. 2.7-2.9 written in terms of the generalized coordinates. Next, Eq. 3.9 is used to recover the displacements and velocities at t_{i+1} from knowledge of the corresponding generalized coordinates at t_{i+1}. Finally, the interaction force at t_{i+1} is computed by recourse to Eq. 2.10.

SECTION 4
NUMERICAL EXAMPLES

4.1 Five DOF Structure

The concept of substructuring is now explained through an example. Consider the structure that was shown in Fig. 1, which contains a secondary substructure within the primary substructure. A lumped mass idealization utilizing 3 dof for the primary substructure and 2 dof for the secondary substructure was also shown in Fig. 2. The structure is subjected to ground accelerations $\ddot{x}_g(t)$ and the relative displacement at dof j is $x_j(t)$. The equations of motion of the structure can easily be obtained as

$$\begin{bmatrix} m_1 & & & & \\ & m_2 & & & \\ & & m_3 & & \\ & & & m_4 & \\ & & & & m_5 \end{bmatrix} \cdot \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \\ \ddot{x}_5 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 & & & \\ -k_2 & k_2 + k_3 & -k_3 & & \\ & -k_3 & k_3 & & \\ & & & k_5 & -k_5 \\ & & & -k_5 & k_5 \end{bmatrix} \cdot \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} =$$

$$= - \begin{Bmatrix} m_1 \\ m_2 \\ m_2 \\ m_4 \\ m_5 \end{Bmatrix} \cdot \ddot{x}_g - \begin{Bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0 \end{Bmatrix} r \qquad (4.1)$$

where

$$r = k_4(x_4 - x_2) + c_4 (\dot{x}_4 - \dot{x}_2) \qquad (4.2)$$

is the interaction force at the point of attachment between primary and secondary substructures. The usual notation is employed, with m_j and k_j denoting mass and stiffness coefficients associated with dof j, respectively. Damping is included in this example, but since the damping matrix has the same form as the stiffness matrix, it is omitted from Eq. 4.1 for tidiness.

If the interaction force r in Eq. 4.1 is brought to the left-hand side and the coupled 5 x5 system of equations is solved, the solution obtained is called the base solution. By keeping r on the right-hand side, Eq. 4.1 uncouples as

$$\begin{bmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & -k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = -\begin{Bmatrix} m_1 \\ m_2 \\ m_3 \end{Bmatrix} \ddot{x}_g - \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} r \qquad (4.3)$$

for the primary substructure and

$$\begin{bmatrix} m_4 & \\ & m_5 \end{bmatrix} \cdot \begin{Bmatrix} \ddot{x}_4 \\ \ddot{x}_5 \end{Bmatrix} + \begin{bmatrix} k_5 & -k_5 \\ -k_5 & k_5 + k_0 \end{bmatrix} \cdot \begin{Bmatrix} x_4 \\ x_5 \end{Bmatrix} = -\begin{Bmatrix} m_4 \\ m_5 \end{Bmatrix} \ddot{x}_g + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} r \qquad (4.4)$$

for the secondary substructure. Note that in Eq. 4.4, a soft spring k_0 has been added to the stiffness matrix to avoid rigid body motion associated with the unassembled secondary system. Numerical studies show that values of k_0 less than 0.001 k_5 are appropriate. Solution of the the primary and secondary substructures can now proceed separately, provided the interaction force can be evaluated.

The following properties are now chosen for the uncoupled primary substructure:

$$\left.\begin{array}{l} k_1 = k_2 = k_3 = k_p = 10{,}000 \text{ lb/in} \\ \text{and} \\ m_1 = m_2 = m_3 = m_p = 10 \text{ lbs}^2\text{/in} \end{array}\right\} \qquad (4.5)$$

The above numerical values result in three natural frequencies for the uncoupled primary substructure that are listed in Table 1. A damping matrix $[C_p]$ is constructed following standard procedure by assuming modal viscous damping factors of

$$\xi_1 = \xi_2 = 0.05, \xi_3 = 0.06 \qquad (4.6)$$

A number of cases for the uncoupled secondary substructure are now considered. In all cases,

$$\left.\begin{array}{l} k_4 = k_5 = k_s \\ m_4 = m_5 = m_s \end{array}\right\} \qquad (4.7)$$

and damping matrix $[C_s]$ is constructed using the following modal viscous damping factors

$$\xi_4 = \xi_5 = 0.02 \qquad (4.8)$$

Also, the value of dashpot c_4 in Eq. 4.2 is taken as equal to zero. For the first three cases we have (a) $m_s/m_p = 0.1$, (b) $m_s/m_p = 0.01$ and (c) $m_s/m_p = 0.001$. Stiffness ratio k_s/k_p is the same as the mass ratio for each particular case so that the natural frequencies listed in Table 1 are all the same. Note that a zero natural frequency reveals the rigid body motion

Table 1

Natural frequencies (in rad/s) of primary substructure, secondary substructures and combined structure.

I. Primary Substructure			
ω_1	ω_2	ω_3	
14.07	39.43	56.98	

II. Secondary Substructures							
Cases a,b,c		Case d		Case e		Case f	
ω_1	ω_2	ω_1	ω_2	ω_1	ω_2	ω_1	ω_2
0.0	44.72	14.07	36.83	39.43	103.23	56.98	149.2

III. Combined Structure (Primary substructure plus secondary substructure, case d)					
ω_1	ω_2	ω_3	ω_4	ω_5	
12.34	15.94	36.71	39.63	57.29	

problem associated with the unassembled secondary substructure. For cases (d), (e), and (f) we have m_s/m_p + 0.1 but the stiffness ratio k_s/k_p changes to produce tuning with respect to the first, second and third natural frequencies of the primary substructure. The natural frequencies corresponding to these last three cases are listed in Table 1. According to the definition of tuning, the secondary substructure must be anchored at the attachment point with the primary substructure, and the attachement point remains immobile. Hence the presence of two natural frequencies in Table 1 for these last three cases.

Each of the six possible combinations of the primary substructure with the secondary substructures is a complete structure with five natural frequencies whose solution is labeled as the base solution. As shown in Table 1 for the combination of the primary substructure with the secondary substructure of case (d), the natural frequencies of the assembled structure do not correspond to the natural frequencies of either substructure. This is so because the mass ratio of secondary to the primary substructure is not negligible.

Three types of ground motions are considered here: a low frequency sine excitation with forcing frequency $\Omega = 0.628$ rad/s, a high frequency sine excitation with forcing frequency $\Omega = 31.42$ rad/s and a suddenly applied and maintained shock. All three types of accelerograms have a peak ground acceleration of 0.1g, g being the gravity constant.

4.2 Substructuring without Modal Condensation

All results presented in this section are obtained via the methodology outlined in Section 2.2 and for the cases described in the previous section. The applied load is the ground motion with sinusoidal time variation and low frequency of excitation. The predictor-corrector alorithm is a conditionally stable one and a time step as large as one-fifth times the lowest natural period of the primary substructure yields very accurate results. All the results obtained in this and the next section are for time steps between one-fifth and one-tenth times the lowest natural period of the primary substurcture.

At first, the base solution (solution of the combined system) is compared with the solution obtained using the substructuring concept for the structural system consisting of the primary substructure with the secondary substructure of case (a). These results are shown in Fig. 4 in terms of the displacement time history at the 5th dof (see Fig. 2) and are identical. The substructuring concept yields the same results as the ones obtained for the base solution at other dof as well and for the other five combinations of a secondary substructure with the primary substructure.

Having thus established the validity of the substructuring approach, the next results depict the effect of the mass ratio, cases (a) - (c), on the displacements at the 4th dof (Fig. 5) and on the interaction force (Fig. 6). Since cases (a) - (c) all result in the same detuned natural frequencies, the displacements experienced by these secondary substructures are all the same. The interaction forces experienced in the attachment spring, however, decrease as the secondary substructure becomes lighter in comparison to the primary substructure.

Finally, Figs. 7 and 8 investigate the tuning effect. In particular, Fig. 7 shows the displacement time history at dof 4 for the secondary substructure tuned to the first (case(d)), the second (case (e)) and the third (case (f)) natural frequencies of the primary substructure. Figure 8 shows similar results for the interaction force time history. Since the frequency of the applied load is low, case (d) is the secondary substructure whose motions are affected the most. The interaction force time history is the same for these three cases because they all have the same mass. Finally, the frequency content of both displacements and interaction force is proportional to the frequency content of the excitation at this low level of forcing frequency.

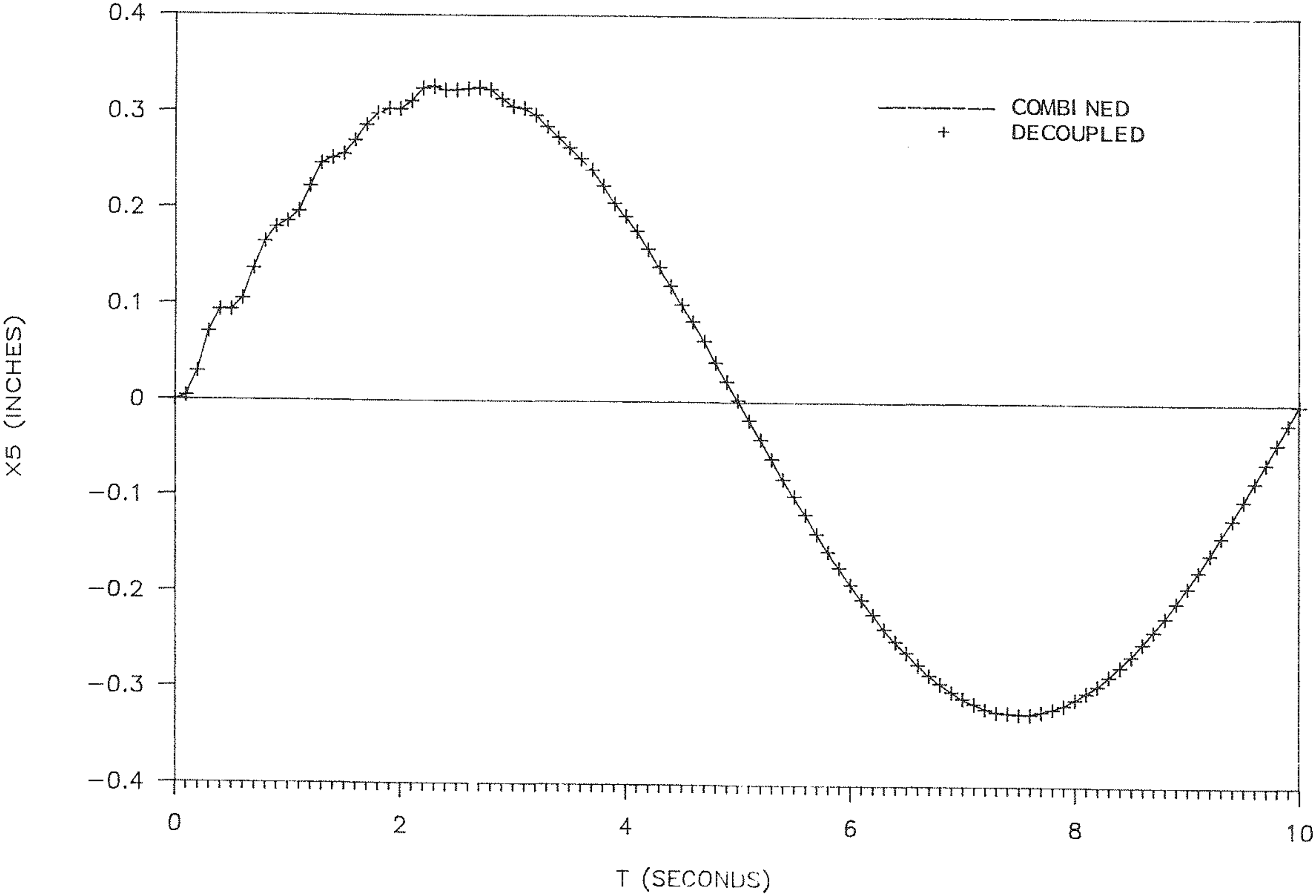

Figure 4 - Comparison between base solution and solution obtained using primary-secondary substructure case (a) under low frequency sine excitation.

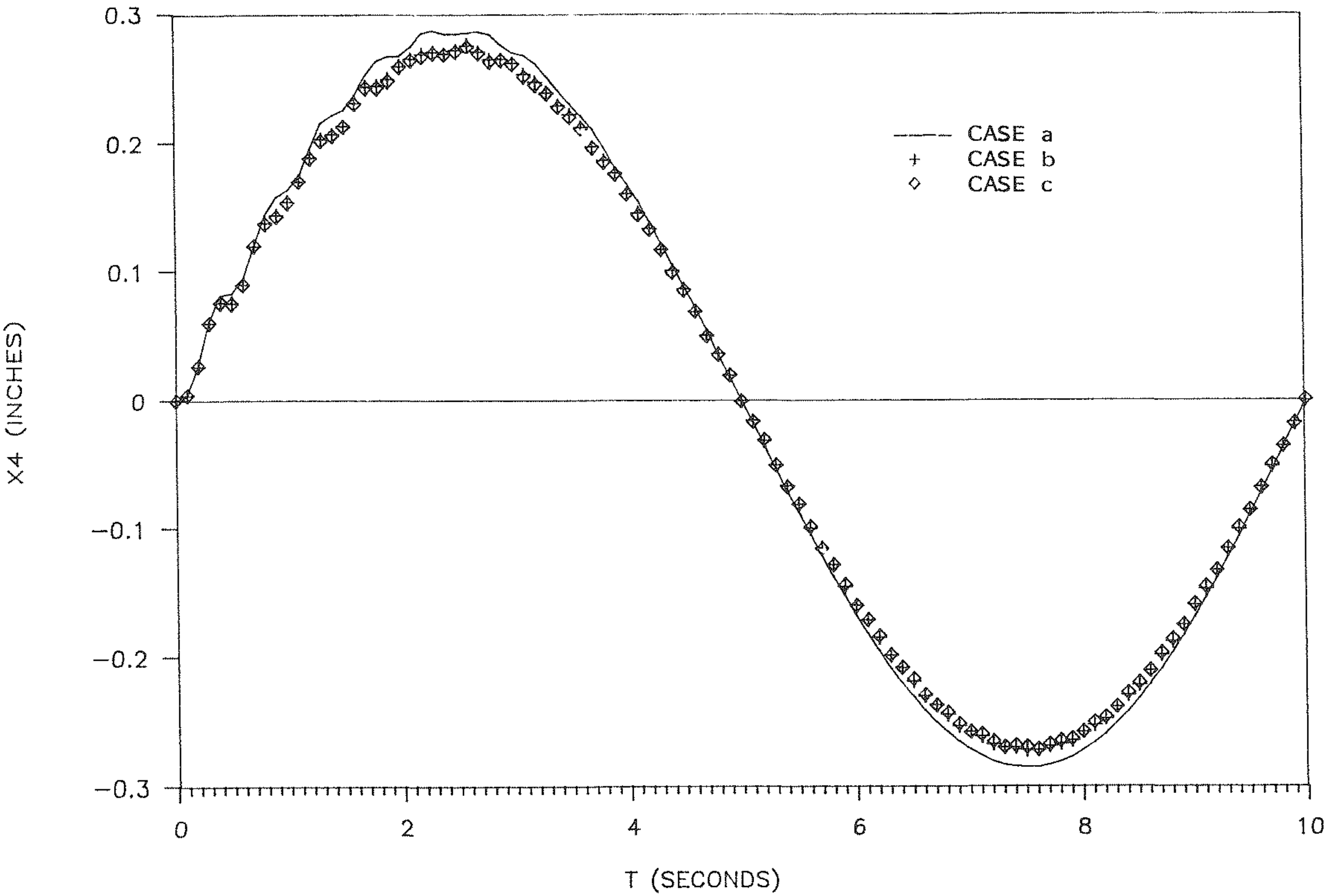

Figure 5 - Displacements at dof 4 for primary-secondary substructure cases (a) - (c) under low frequency sine excitation.

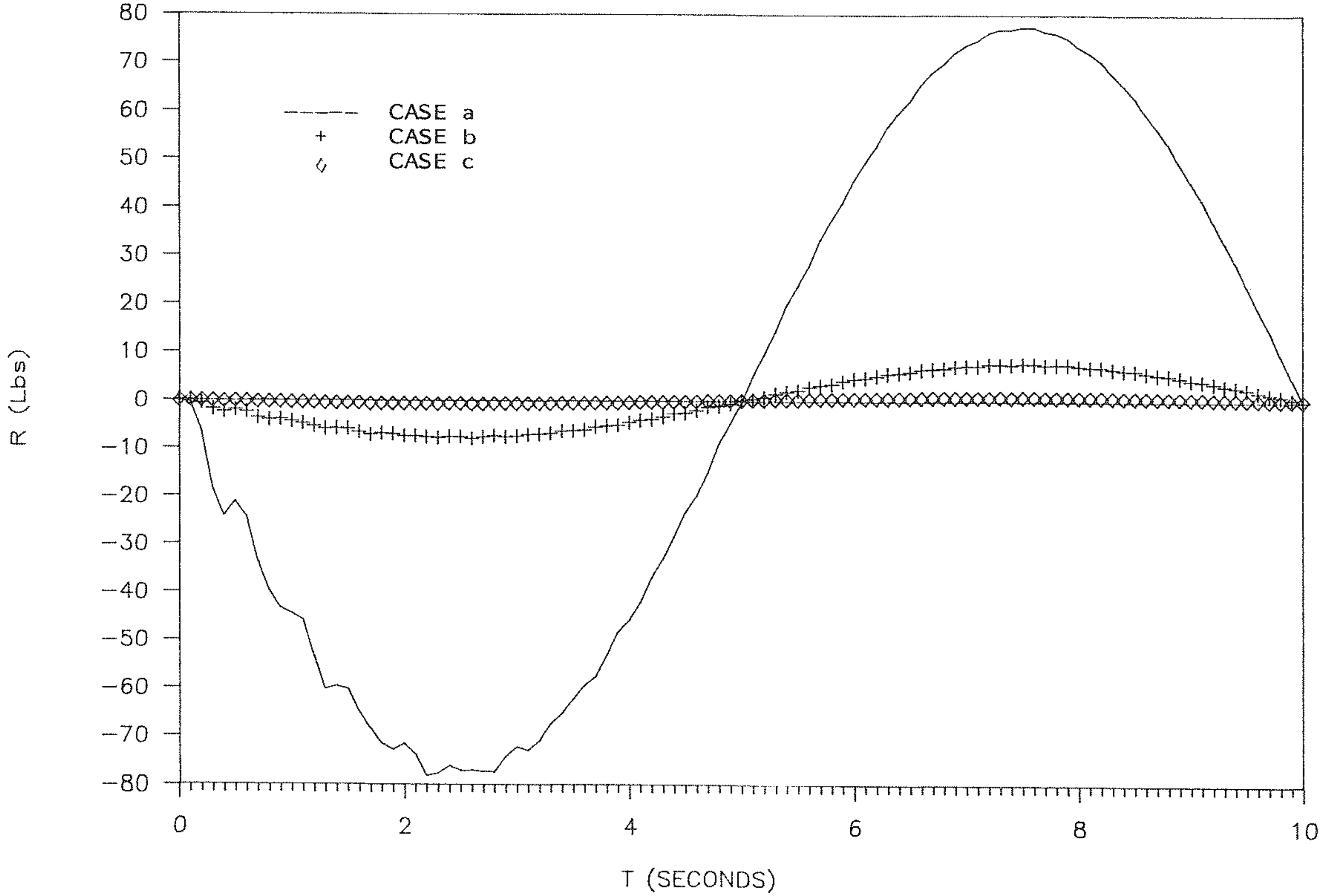

Figure 6 - Interaction force for primary-secondary substructure cases (a) - (c) under low frequency sine excitation.

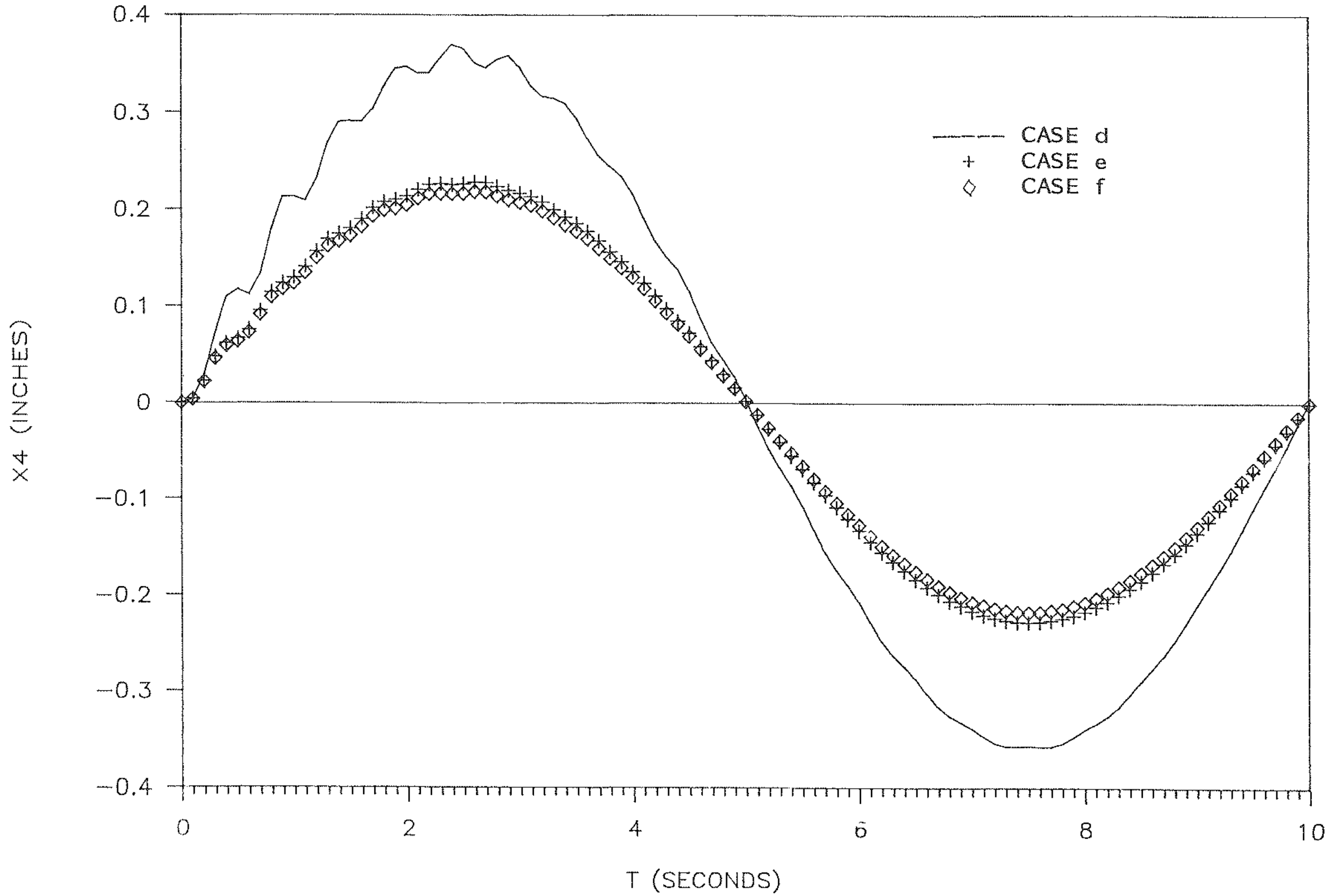

Figure 7 - Displacements at dof 4 for primary-secondary substructure cases (d) - (f) under low frequency sine excitation.

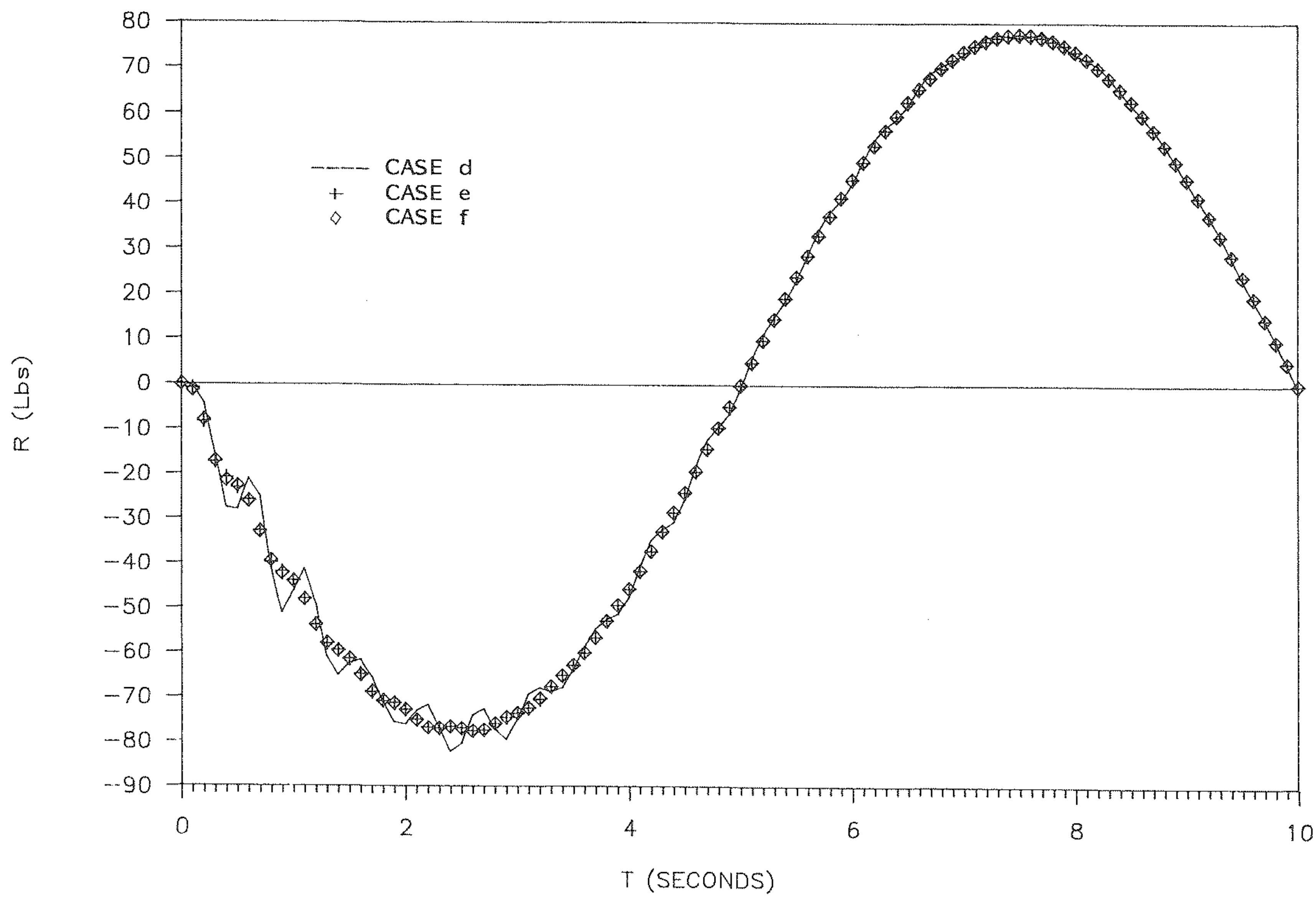

Figure 8 - Interaction force for primary-secondary substructure cases (d) - (f) under low frequency sine excitation.

4.3 Substructuring with Modal Condensation

At first, orthogonal Ritz vectors and their associated frequencies are computed for the primary (P) and secondary (S) substructures and for static distributions of the applied loads (L) and the interaction force (I), as was depicted in Fig. 3. Table 2 shows this information for the primary substructure and the secondary substructures that are tuned to the first, second and third natural frequencies of the primary substructure, i.e., cases (d) - (f), respectively. It should first be noted that the full set of orthogonal Ritz vectors and their associated frequencies are identical to their modal counterparts, i.e., the mode shapes and natural frequencies. Subsets of the full orthogonal Ritz vector set, however, differ from their corresponding modal counterparts. Next, the orthogonal Ritz vector sets obtained for applied load cases (L) and (I) are identical since the static distributors of (L) and (I) are quite similar. Finally, the artificial spring k_o that inhibits the rigid body motion of a secondary substructure affects the corresponding mode shape $\{\phi_1\}^T = \lfloor 1,1 \rfloor$ and natural frequency $\omega_1 = 0$ just enough to allow for a modal decomposition of the secondary substructure.

Figures 9 - 11 are for the low frequency sinusoidal ground excitation and primary-secondary substructure combination (d). In particular, they respectively depict the displacement time history of the 4 dof, the interation force time history, and the displacement time history of the 2nd dof obtained for two cases: (i) the full set of Ritz vectors and (ii) only one Ritz vector for each of the primary (P) and secondary (S) substuctures and for loading cases (L) and (I). It is observed that a truncated set of only one Ritz vector for each substructure will give exact results. This spectacular reduction is due to the fact that the excitation has a low forcing frequency compared to the natural frequencies of the substuctures.

Table 2

Orthogonal Ritz vectors and associated frequencies (in rad/s) of primary and secondary substructures.

I. Primary Substructure				
No. of Ritz vectors	Frequency	Ritz Vector		
1 2 3	14.07 39.43 56.98	0.1037 -0.2331 0.1870	0.1870 -0.1037 -0.2331	0.2331 0.1870 0.1037
1 2	14.07 44.19	0.1038 0.1132	0.1869 0.2037	0.2330 -0.2138
1	14.08	0.1049	0.1889	0.2309

II. Secondary Substructure				
Case	No. of Ritz vectors	Frequency	Ritz Vector	
d	1 2	0.007 32.22	0.7071 -0.7071	0.7071 0.7071
e	1 2	0.007 90.23	0.7071 -0.7071	0.7071 0.7071
f	1 2	0.007 130.4	0.7071 -0.7071	0.7071 0.7071
d,e,f	1	0.007	0.7071	0.7071

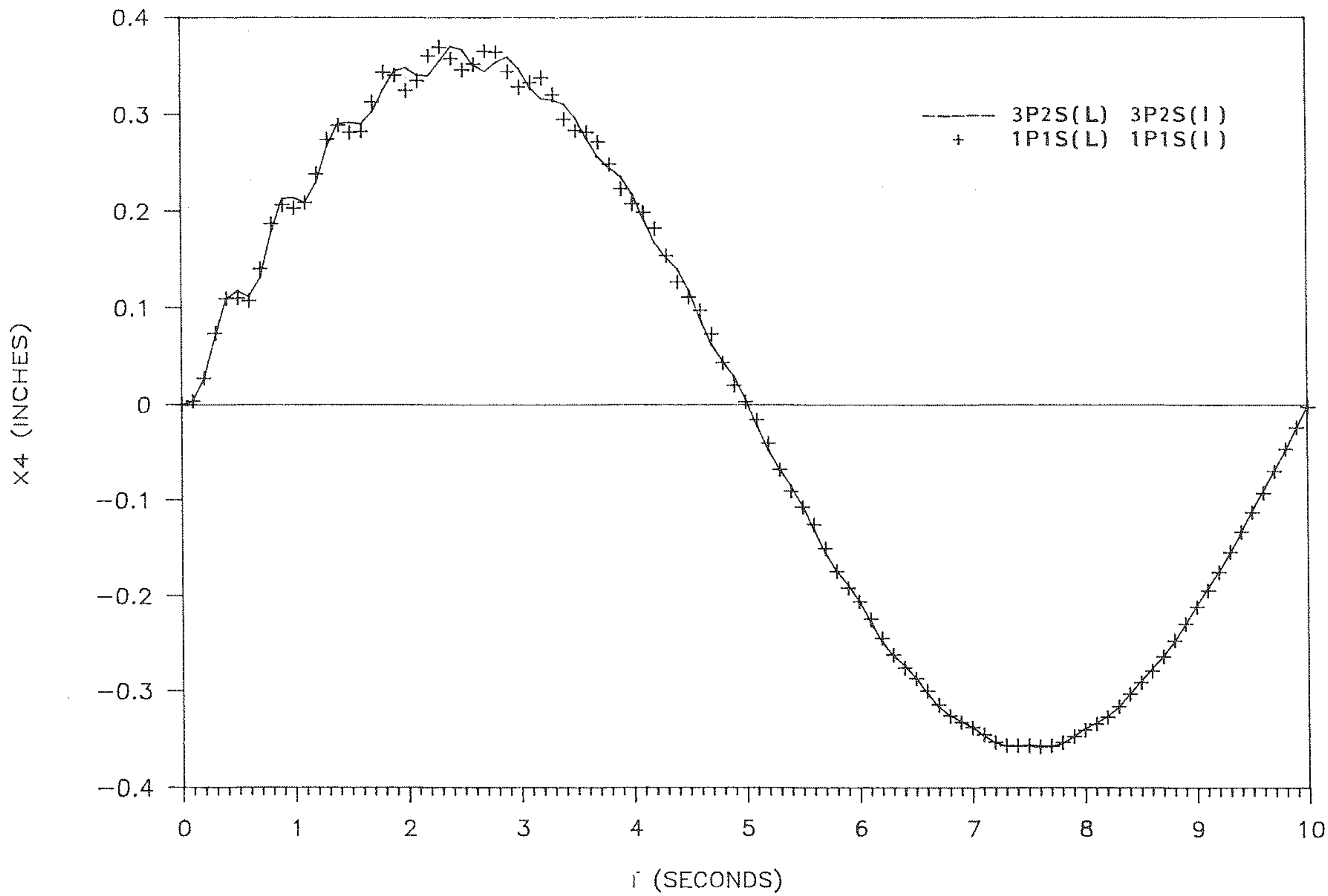

Figure 9 - Effect of mode condensation on the displacements at dof 4 of primary-secondary substructure case (d) under low frequency sine excitation.

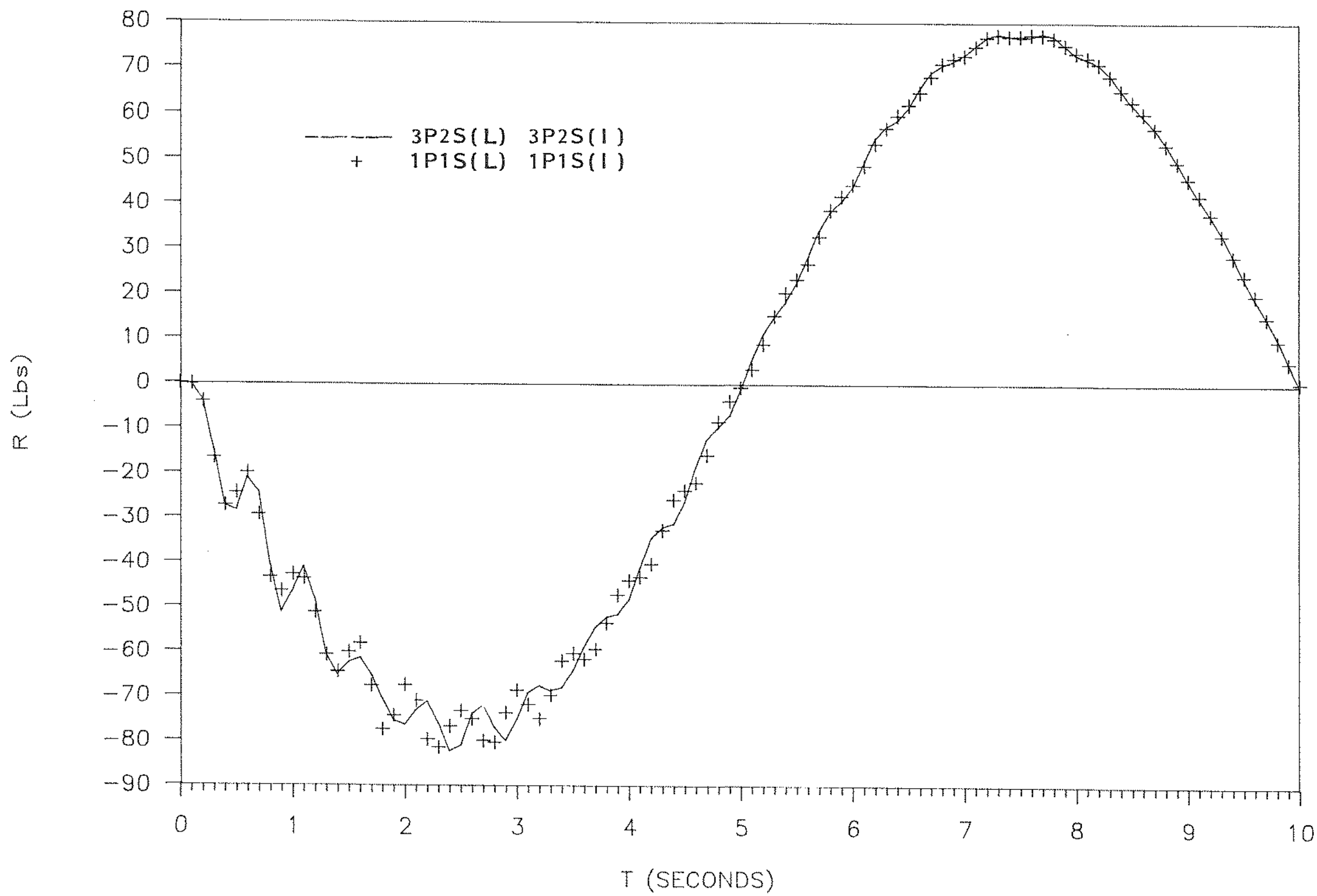

Figure 10 - Effect of mode condensation on the interaction force of primary-secondary substructure case (d) under low frequency sine excitation.

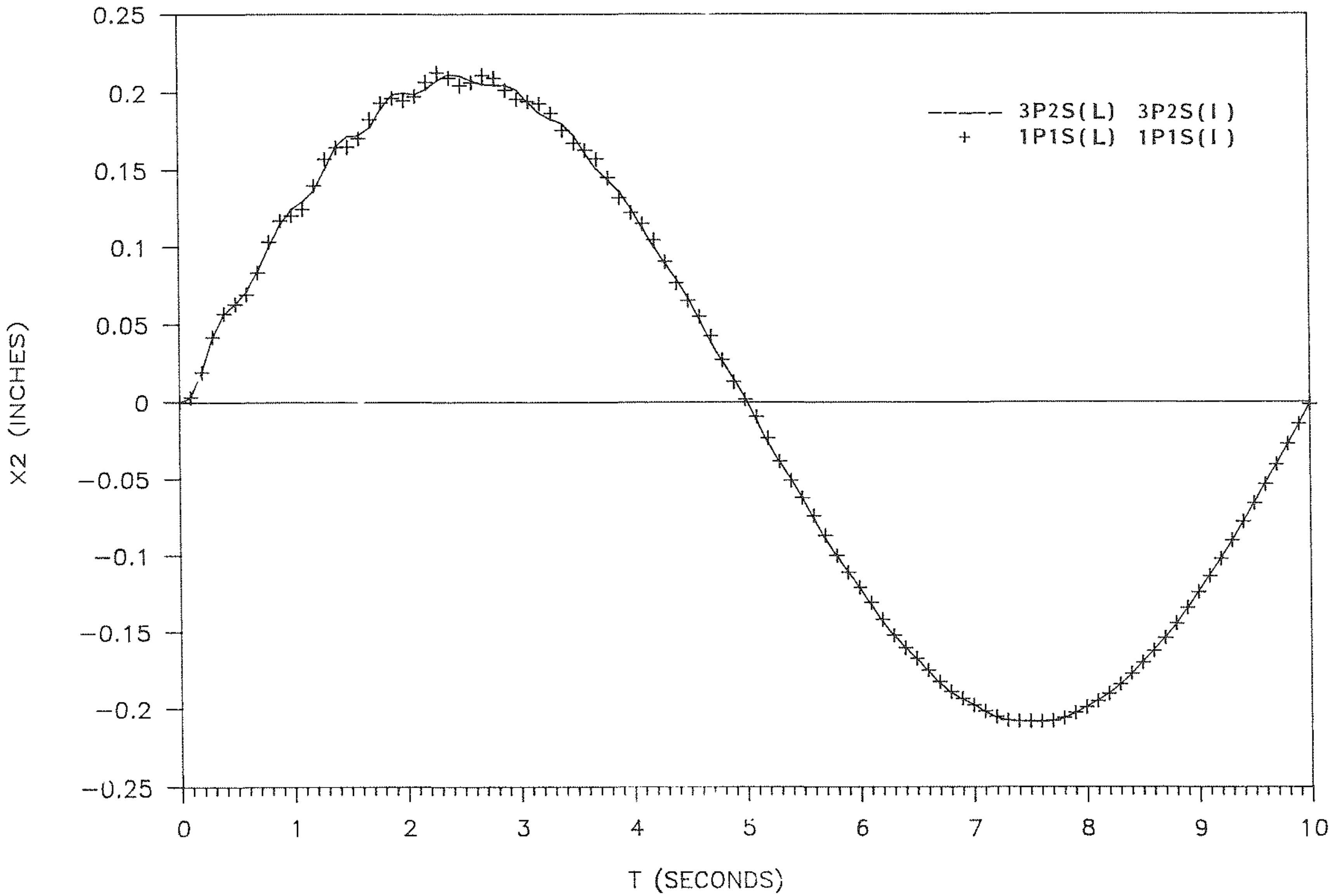

Figure 11 - Effect of mode condensation on the displacements at dof 2 of primary-secondary substructure case (d) under low frequency sine excitation.

Figures 12 - 16 are for the high frequency sinusoidal ground excitation and for primary - secondary combinations (d)-(f). At first, Fig. 12 demonstrates that the same displacements at dof 5 are obtained using substructuring with complete orthogonal Ritz vector sets as when the entire structure is solved using numerical integration. Figures 13 and 14 investigate the effect of modal trunctation on the interaction force for primary - secondary combination (d). In Fig. 13, a full representation is retained for loading case (I) and the reduced modal representations are in conjunction with loading case (L). The reverse holds true in Fig. 14. It is observed that it is preferable to maintain a full representation for loading case (I) than for loading case (L). Figure 15 plots the interaction force for primary - secondary combination (e) with a full modal representation retained for loading case (I). Comparison of Figs. 13 - 15 shows that it makes little difference whether it is the modal representation of the primary system or of the secondary system that is reduced. To be sure, less accurate results are obtained in Fig. 14 if one mode is used for (S) than if two modes are used for (P), but that corresponds to a 50% reduction in the modal representation of the secondary substructure compared to a 33.3% reduction in the modal representation of the primary substructure. Primary - secondary combination (f) has natural frequencies sufficiently removed from the forcing frequency of the excitation to allow for good quality results to be obtained with a reduced (P) substructure and a full (S) substructure representation, irrespective of the loading condition (L) or (I), as shown in Fig. 16.

Finally, Figs. 17 - 19 investigate the effect of a suddenly applied and maintained ground excitation on the displacements at the 4th dof for the primary substructure with a secondary substructures cases (d) or (e). This type of excitation contains a wide band of forcing frequencies that are bound to include the structure s natural frequencies. Figure 17 is for case (e) and again shows the excellent agreement of results obtained using substructuring with complete modal representation with results obtained by solving the entire structure in one try. Figures 18 and 19 are for case (d)

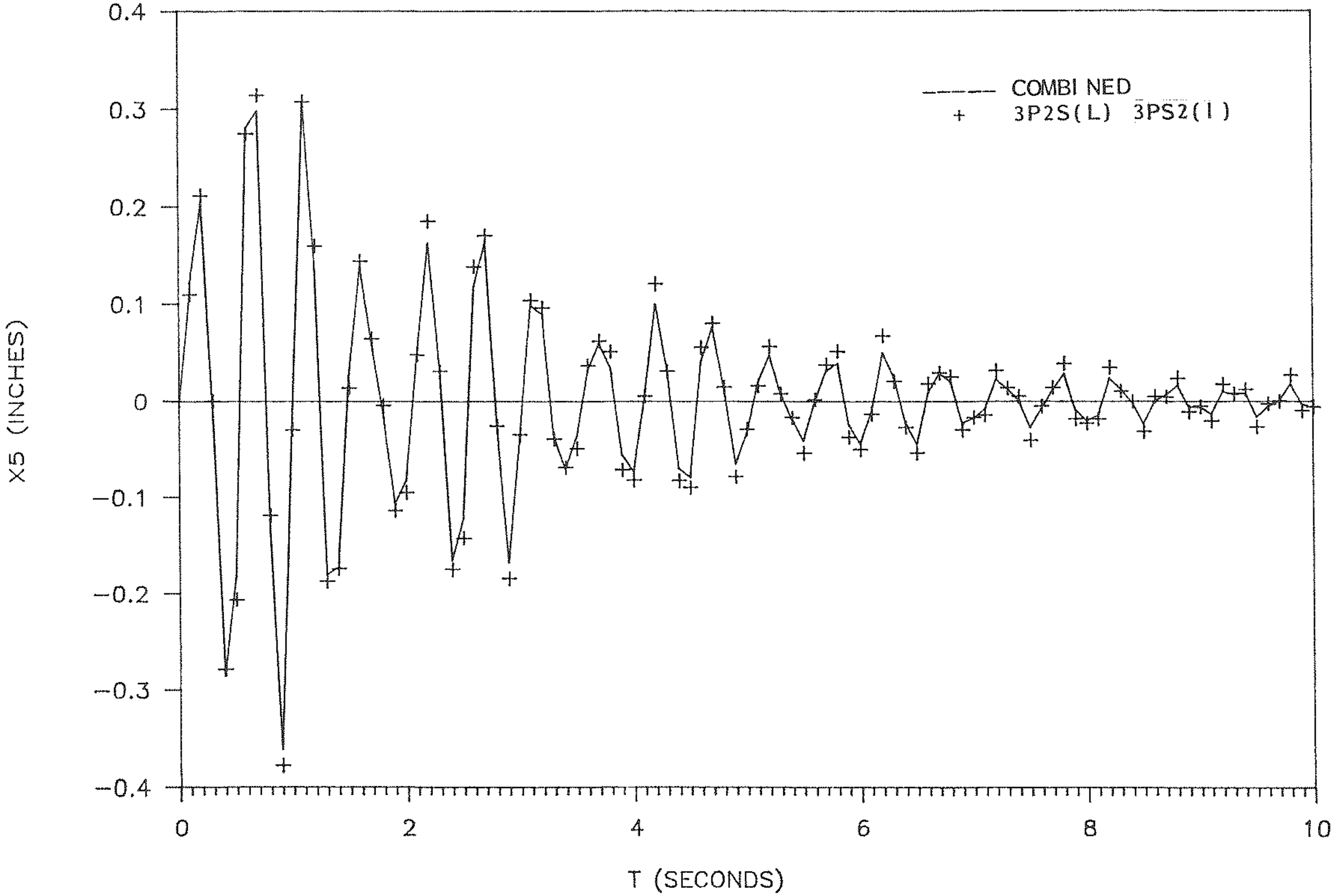

Figure 12 - Comparison between base solution and solution obtained using all the modes of the primary-secondary substructure case (d) under high frequency sine excitation.

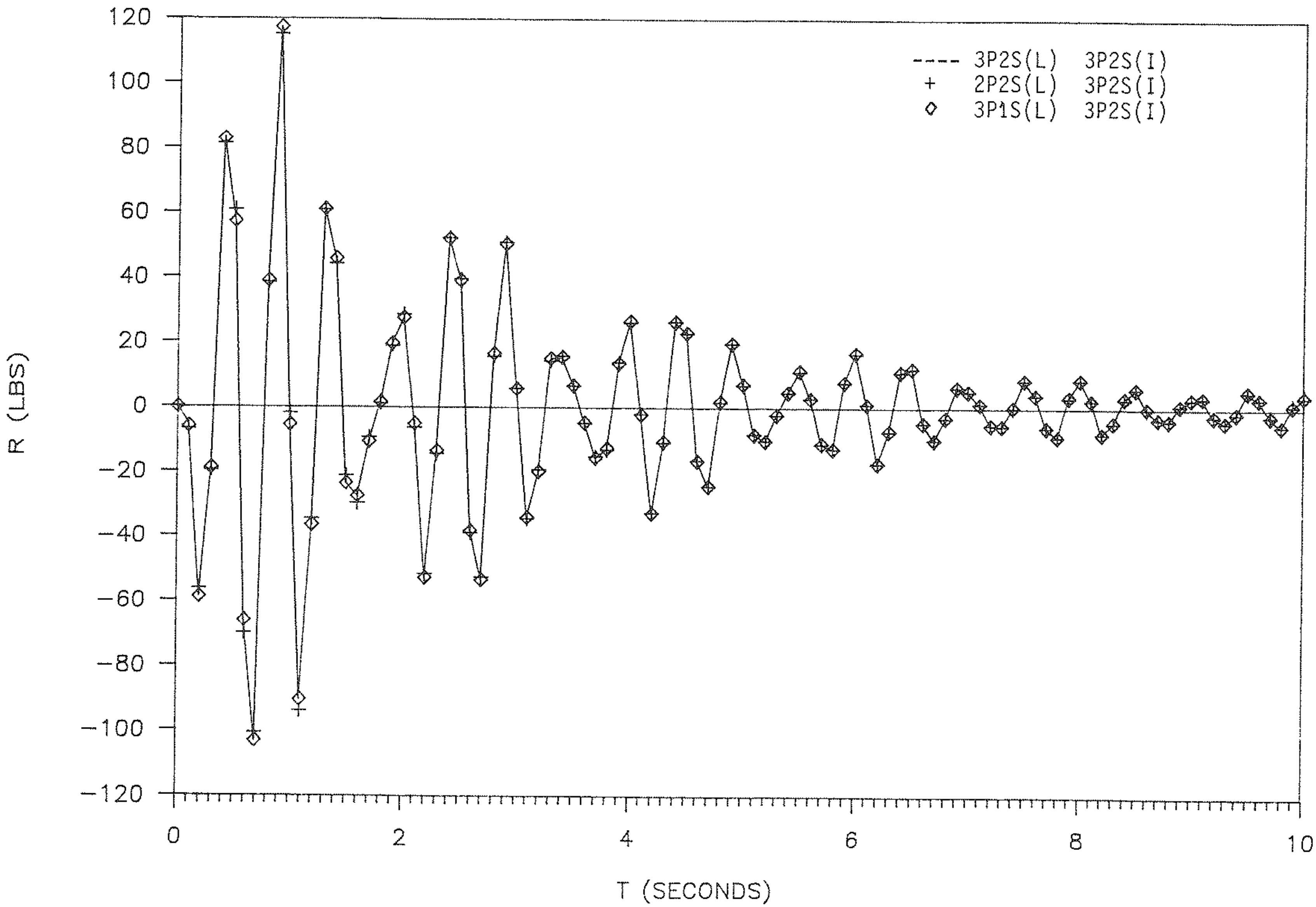

Figure 13 - Effect of mode condensation on the interaction force for primary-secondary substructure case (d) under high frequency sine excitation - full (I) representation.

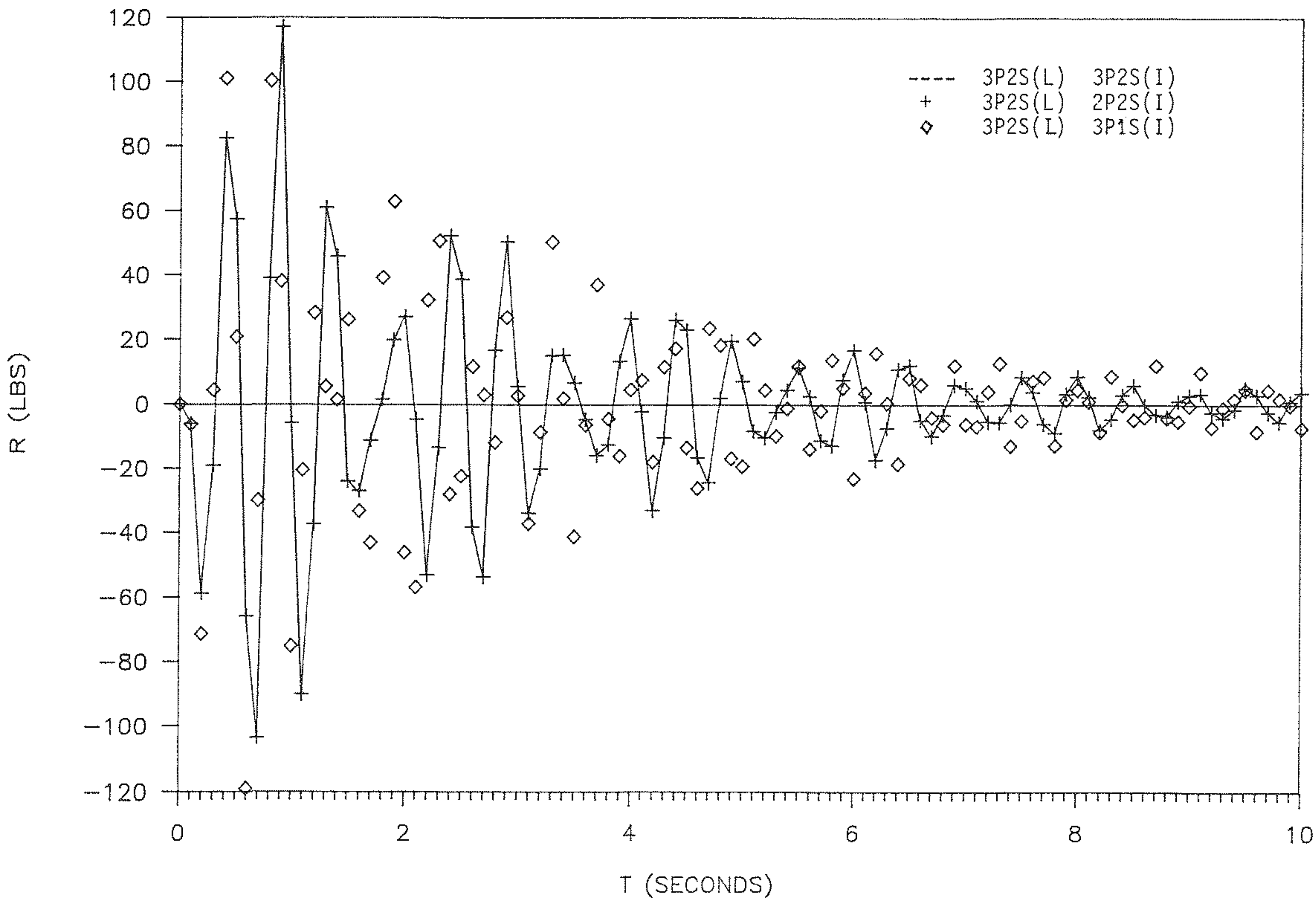

Figure 14 - Effect of mode condensation on the interaction force for primary-secondary substructure case (d) under high frequency sine excitation - full (L) representation.

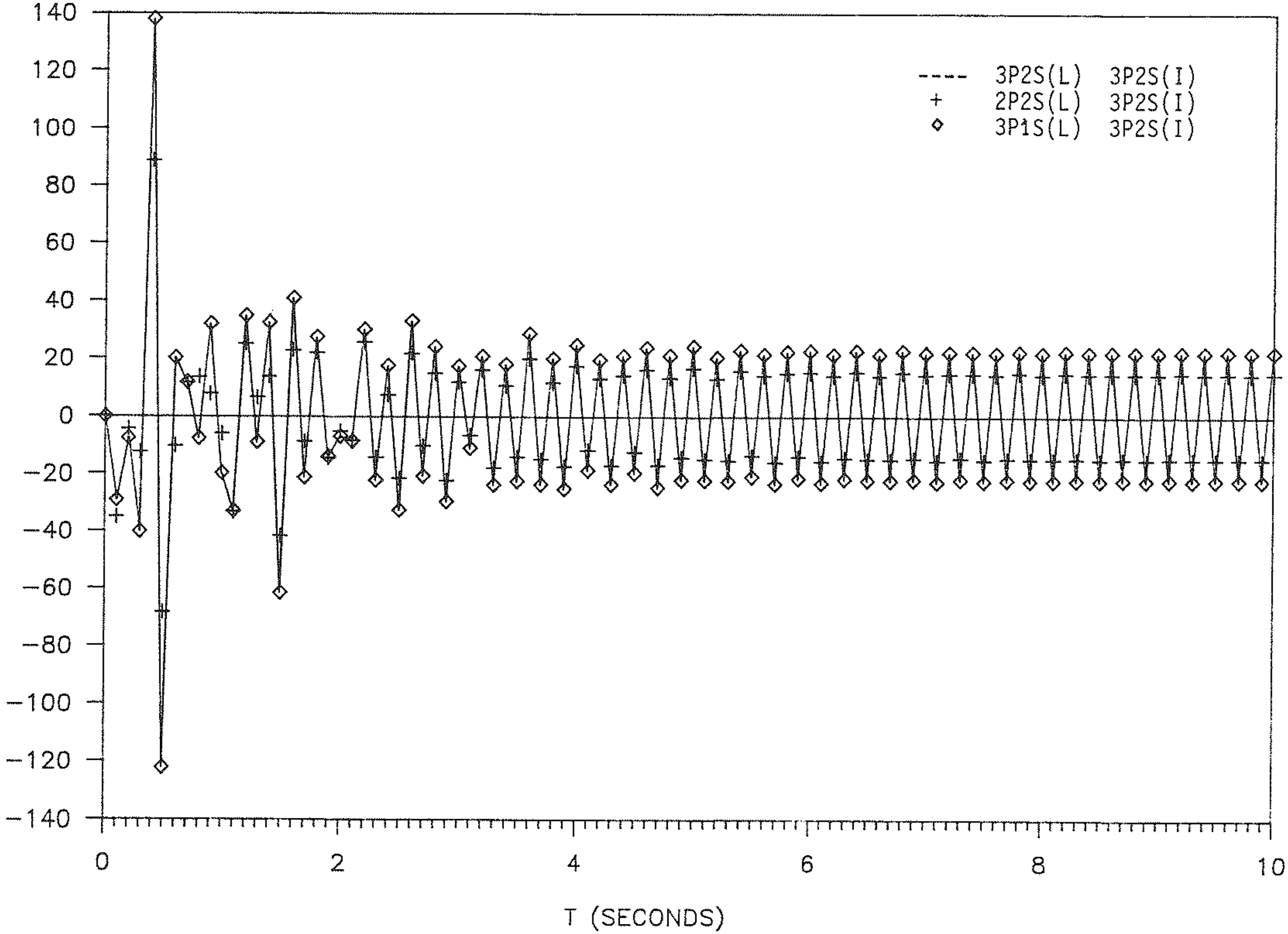

Figure 15 - Effect of mode condensation on the interaction force for primary-secondary substructure case (e) under high frequency sine excitation.

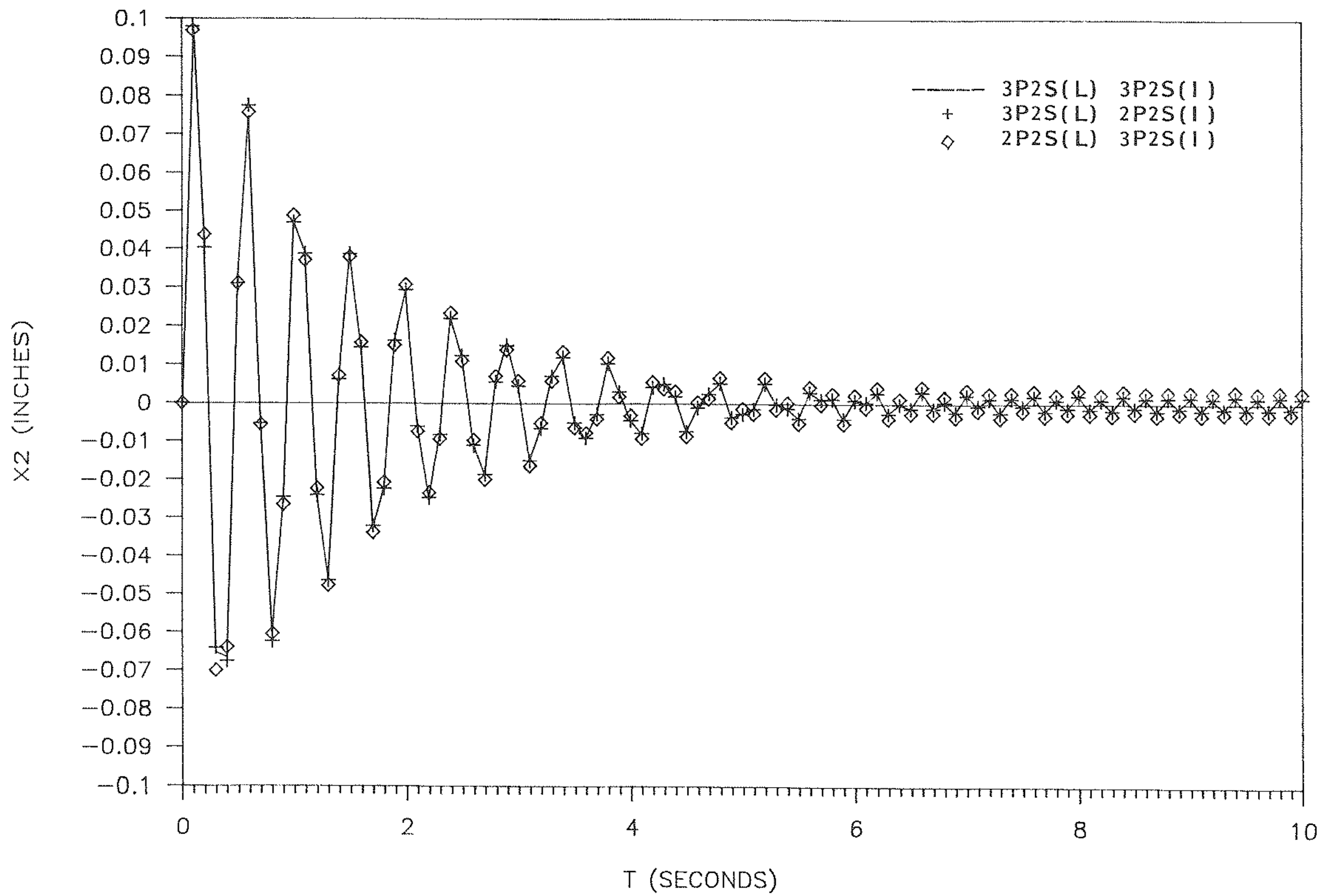

Figure 16 - Effect of mode condensation on the displacement at dof 2 for primary-secondary substructure case (f) under high frequency sine excitation.

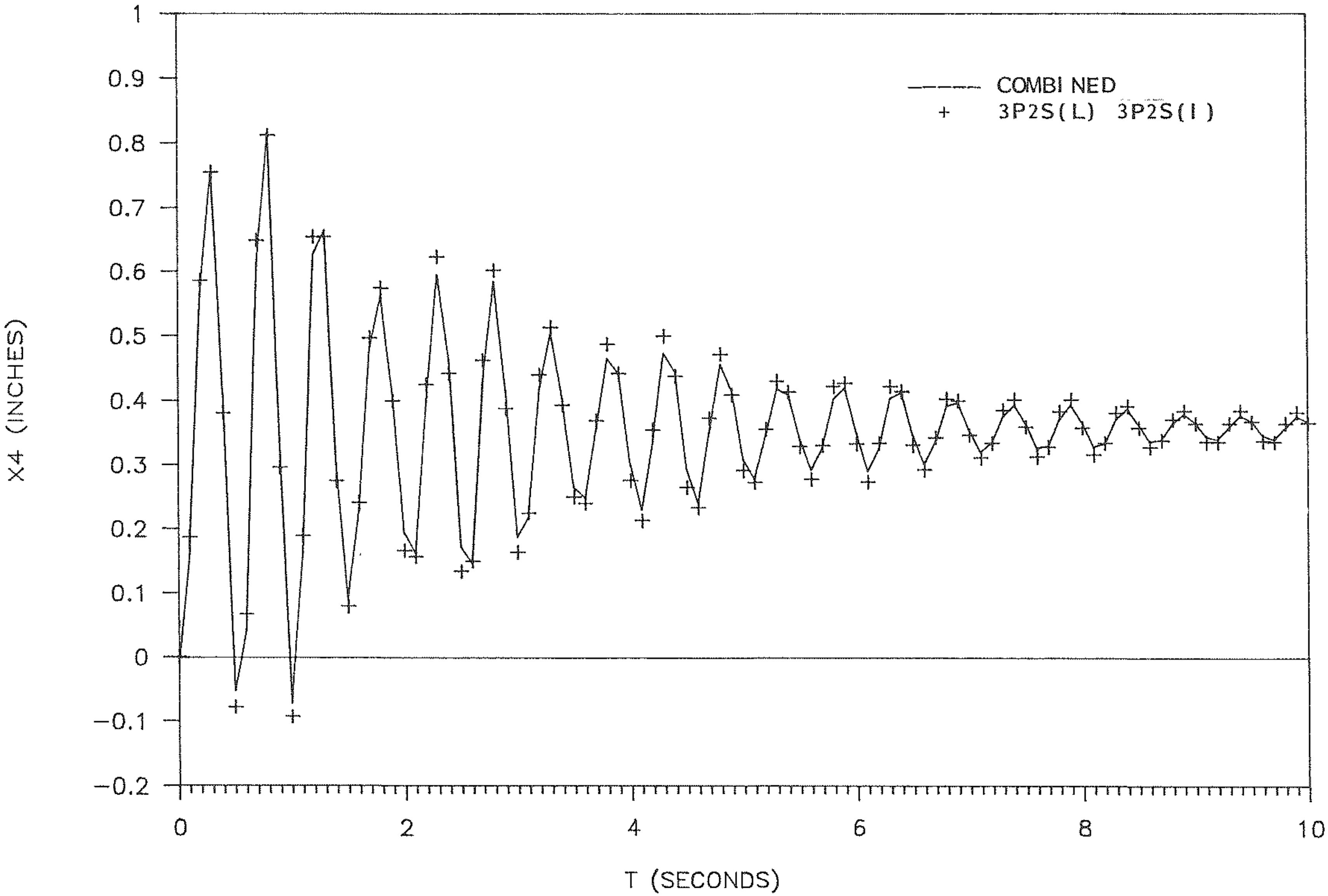

Figure 17 - Comparison between base solution and solution obtained using all the modes of the primary-secondary substructure case (e) under suddenly applied and maintained load.

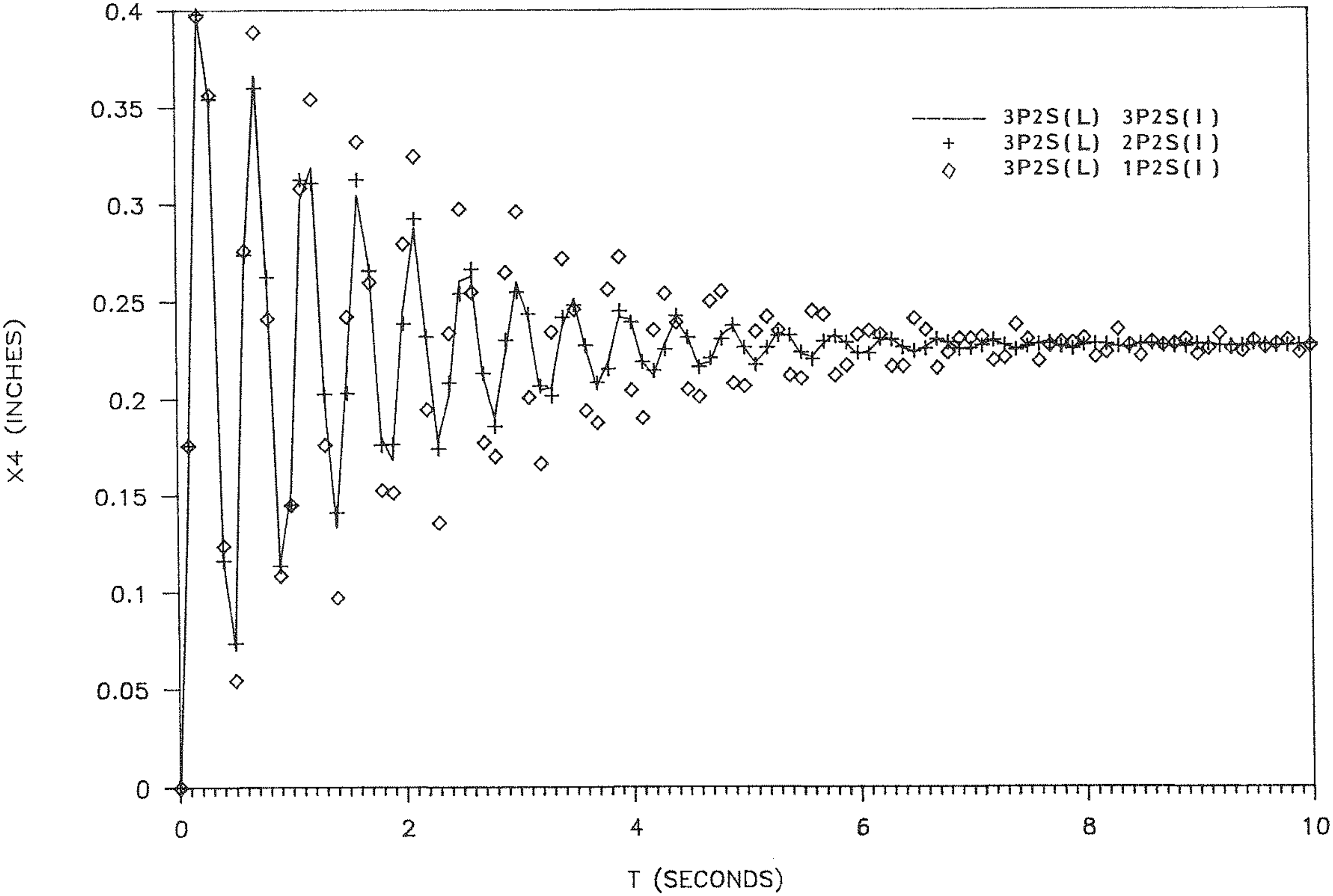

Figure 18 - Effect of mode condensation on the displacement at dof 4 for primary-secondary substructure case (d) under suddenly applied and maintained load using all modes for the applied loads.

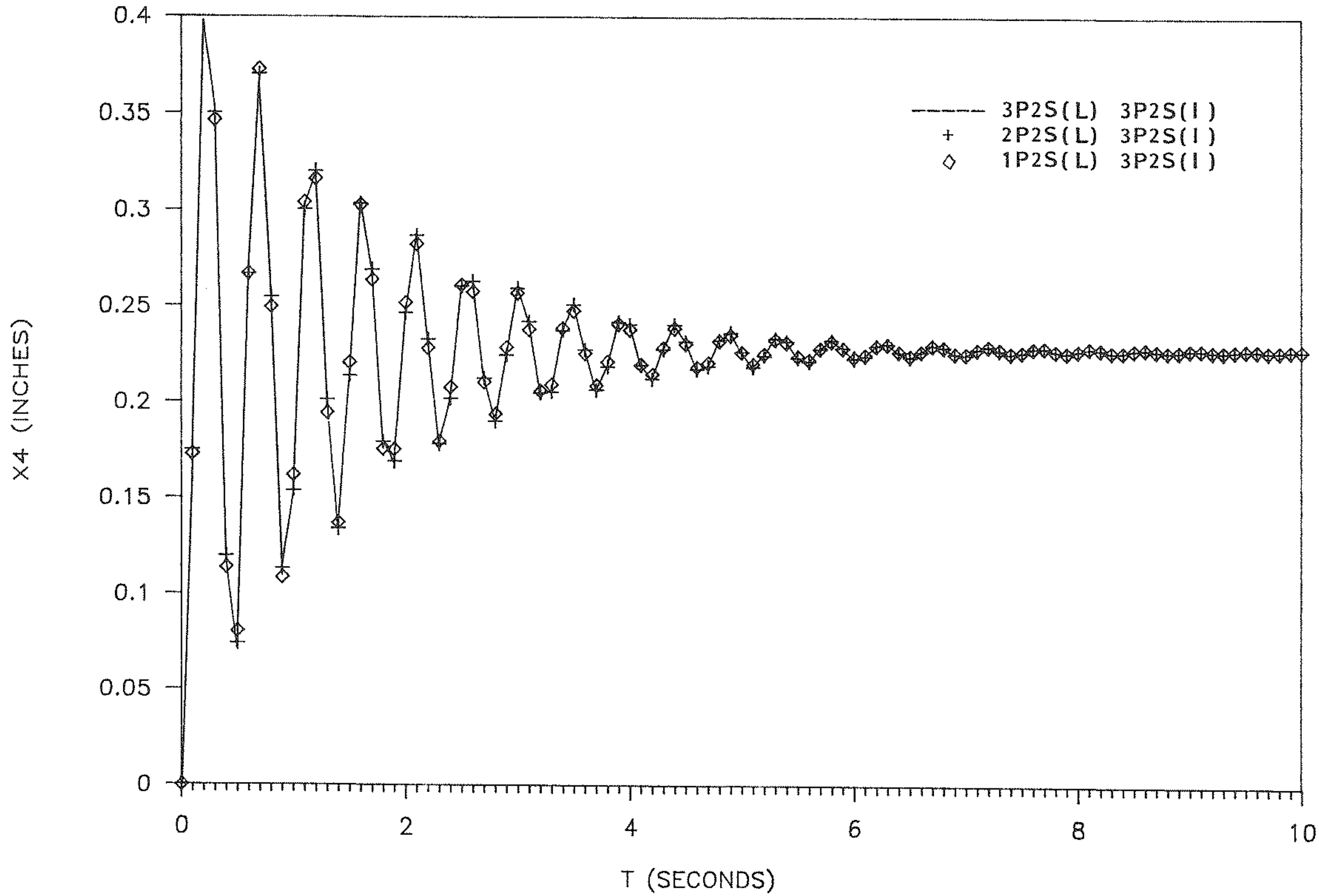

Figure 19 - Effect of mode condensation on the displacement of dof 4 for primary-secondary substructure case (d) under suddenly applied and maintained load using all modes for the interaction force.

and again indicate that better accuracy is obtained if the reduced representation in the primary substructure is used in conjunction with the applied loads (L) rather than with the interaction force (I).

4.4 Discussion of Results

Based on the results presented in the previous two sections, the following points are summarized:

(i) The magnitude of the interaction force is proportional to the mass ratio of the secondary to the primary substructure. The frequency content of the interaction force depends on the frequency content of the applied loads as well as on the tuning effect between secondary and primary substructures.

(ii) The response of a secondary substurcture is very sensitive to the magnitude and frequency content of the interaction force.

(iii) The effect of the interaction force on the response of the primary substructure is negligible for low mass ratios of secondary to primary substructure and considerable if these ratios are high.

(iv) Substructuring without modal condensation and substructuring with modal condensation give identical results as those obtained from solution of the entire structure provided in the latter case complete orthongonal Ritz vector sets are used, i.e., all the modes are included.

(v) If any of the modes of the secondary substructure is tuned to a mode of the primary substrcture, then this tuned mode should be included in the modal representation of the secondary substructure.

(vi) In general, reduced orthogonal Ritz vector sets can be safely employed for either primary or secondary substructure and for either the applied load or the interaction force case if the excitation frequencies are much lower than the first natural frequency of the entire structure.

(vii) It does not make much difference if the reduction in the modal representation is done in conjunction with the secondary substructure or in conjunction with the primary substructure.

(viii) For a given substructure, it is preferable to reduce the orthogonal Ritz vector set representation for the applied loading case than to reduce the orthogonal Ritz vector representation for the interaction force case.

SECTION 5

CONCLUSIONS

A substructuring approach in the time domain for the analysis of structures composed of a primary substructure to which secondary substructures are attached is presented in this work. This substructuring approach allows for a separate analysis of the uncoupled primary and secondary substuctures, with the coupling effect accounted for through the interaction forces. Uncoupling also nesessitates definition of two loading states, one being the applied loads and the other being the interaction forces. The response of the primary and secondary substructures is then determined under each of these two loading states, with the reponse due to the interaction forces acting as a correction to the response due to the applied loads. Modal condensation is introduced by employing incomplete orthogonal Ritz vector sets for both primary and secondary substructures.

Direct numerical integration is used for implementing this substructuring approach. The complication that arises is that the interaction forces are not known for a time step past the current one and therefore a predictor - corrector method is introduced to remedy that. For the case of modal condensation, numerical integration is applied at the modal equation level but computation of the interaction forces is done in the physical coordinate level.

The numerical results obtained indicate that the substructuring approach yields results indentical to those obtained by solving the entire structural system in one try. If modal condensation is employed, then good quality results can be obtained for a small number of orthogonal Ritz vectors. Modal condensation is, however, sensitive to the forcing frequencies of the applied loads and the tuning effect between primary and secondary substructure. The advantage gained by using substructuring is obviously that one works with smaller structural systems.

In general, the substructuring approach presented here is applicable to cases where there are more than one secondary substructure and there are multiple points of attachment between primary and secondary substructures. Also, the secondary substructure does not need to be light compared to the primary substructure and the applied loads may have arbitrary spatial and temporal variations.

At present, the substructuring methodology is applicable to linearized systems under deterministic loads. Current work is aimed at relaxing both assumptions, i.e., to further the methodology so that it will be possible to treat random vibrations of hysteretic systems.

SECTION 6

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