NATIONAL CENTER FOR EARTHQUAKE ENGINEERING RESEARCH
State University of New York at Buffalo

PARAMETRIC STUDIES OF FREQUENCY RESPONSE OF SECONDARY SYSTEMS UNDER GROUND-ACCELERATION EXCITATIONS

by

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ABSTRACT

Numerical investigation is carried out into the variation of frequency response to ground acceleration of a single-degree-of-freedom secondary system due to variations of system parameters. Selected system parameters include the mass ratio γ between the secondary and primary systems, the tuning parameter β indicating the proximity of the secondary-system frequency to a primary-system frequency, and damping of the primary system. The term "response" refers to the displacement of the secondary system relative to its support. As expected, tuning of the secondary mode to a given primary mode enhances the contributions to the total response from both modes; the highest frequency response occurs when the secondary mode is tuned to the first primary mode. Nevertheless, the first primary mode always contributes importantly even when it is not tuned to the secondary system, especially when the secondary system is heavy. When a primary mode is sufficiently detuned from the secondary system, its contribution is insensitive to the mass ratio. Increasing the damping in the primary system for the purpose of reducing the response is not very effective for heavier secondary systems. As unexpected result is found that detuning may give rise to higher response when damping is extremely low in the secondary system but is rather high in the primary system. No apparent discrepancy in the results is found whether the primary system is classically or nonclassically damped.

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INTRODUCTION

A secondary system is a lighter equipment or appendage, supported by a heavier primary system. It receives seismic or other ambient excitations transmitted indirectly through the primary system. Most analyses of secondary systems in the literature have placed much emphasis on the changes in the natural frequencies and mode shapes due to the interaction between primary-secondary systems. The objective was to find approximate solutions by use of purtubation or iterative schemes, see for example, references [1-9] with the implied presumptions that an exact solution for the composite system, while possible, is too costly and hence not feasible. Three types of system parameters were found to be important: (1) the mass ratio γ which is the ratio between a representive secondary-system mass and a representive primary-system mass, (2) the tuning parameter $\beta = 2(\omega_p-\omega_s)/(\omega_p+\omega_s)$ where ω_p and ω_s are the natural frequencies of a primary-system mode and a secondary-system mode, respectively, which are close enough to cause significant changes in these frequencies in the composite system, and (3) the damping parameter $\zeta_a = (\zeta_p+\zeta_s)/2$ which is the average of the modal-damping coefficients of a pair of primary and secodary modes with close natural frequencies, referred to in (2).

In this paper, emphasis is placed directly on the response characteristics of the secondary-system, instead of eigenvalues and eigenvectors. In particular, we study the magnitude of the frequency response function as the key physical parameters are varied. For this purpose, the input ground motion is a unit sinusoidal ground acceleration in the horizontal direction. The primary system is chosen to be a four-story building of shear-wall type construction shown in figure 1, with the total mass of each story lumped at the floor level, and with a secondary system attached to the third floor. The exact solution for such a simple composite system can be obtained easily, and the intended parametric study can be carried out exactly. The varying physical parameters selected for the present study are γ, β and ζ_0 where γ and β have been defined previously, and ζ_0 is the would-be modal damping of the primary system if it were a single story biulding. We choose ζ_0 rather than ζ_a because damping in a primary system is usually more difficult to predict than that of a secondary system; therefore, it is subject to greater statistical uncertainty. Both classically damped and non-classically damped primary structures are considered.

FORMULATION

The equation of motion for the composite system shown in figure 1 is given by

$$[\tilde{M}]\{\ddot{X}\} + [\tilde{C}]\{\dot{X}\} + [\tilde{K}]\{X\} = \{\psi\} \tag{1}$$

where

$$\{X\} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_S \end{Bmatrix} \tag{2}$$

$$[\tilde{M}] = \begin{bmatrix} m_1 & & & & \\ & m_2 & & 0 & \\ & & m_3 & & \\ & 0 & & m_4 & \\ & & & & m_S \end{bmatrix}$$

$$[\tilde{C}] = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 & 0 \\ -c_2 & c_2 + c_3 & -c_3 & 0 & 0 \\ 0 & -c_3 & c_3 + c_4 + c_S & -c_4 & -c_S \\ 0 & 0 & -c_4 & c_4 & 0 \\ 0 & 0 & -c_S & 0 & c_S \end{bmatrix}$$

$$[\tilde{K}] = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 + k_S & -k_4 & -k_S \\ 0 & 0 & -k_4 & k_4 & 0 \\ 0 & 0 & -k_S & 0 & k_S \end{bmatrix} \tag{3}$$

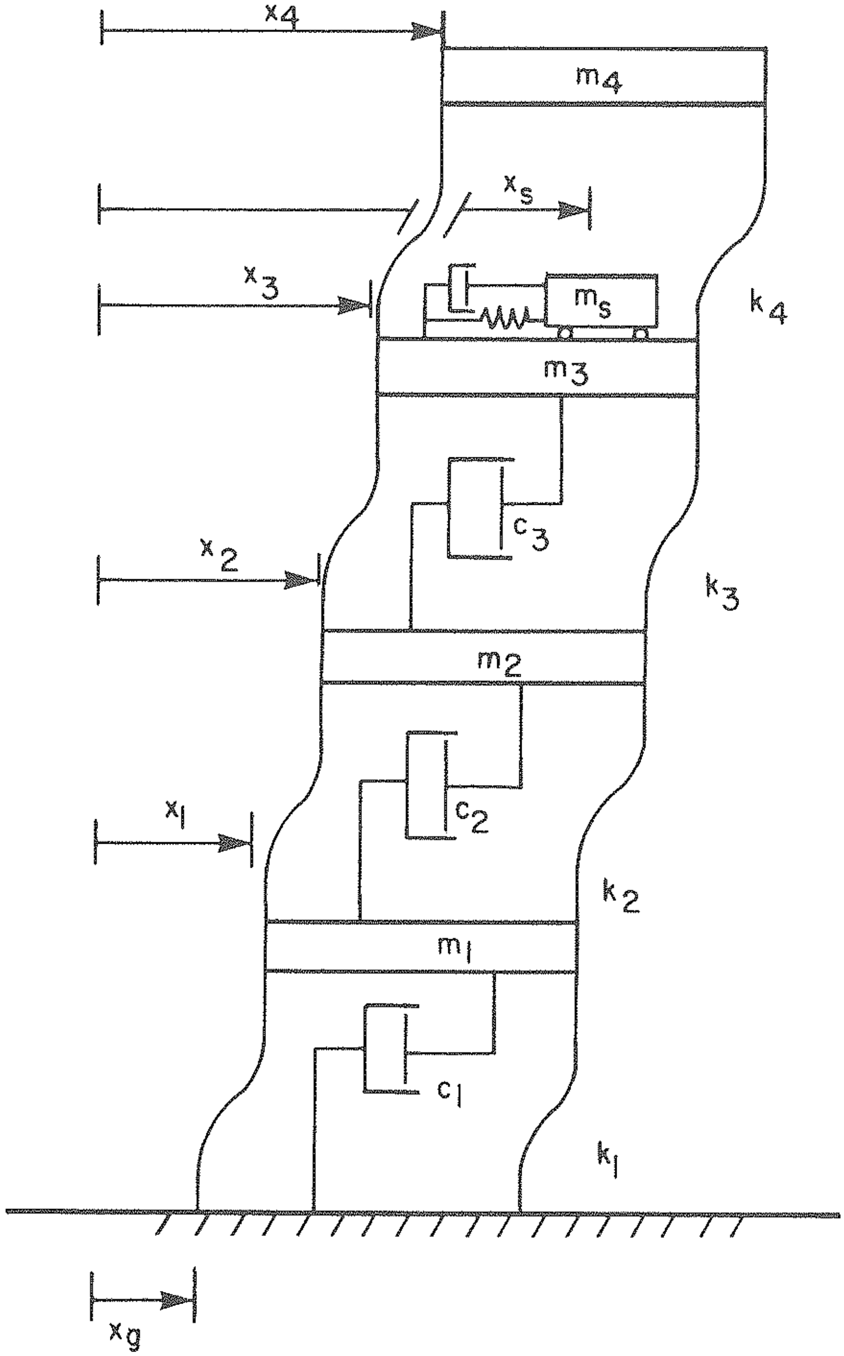

FIGURE 1 Model of a Four-Story Primary Structure Supporting a Secondary System on the Third Floor. Internal Damping Mechanism in the Fourth Story Omitted to Avoid Crowding.

$$\{\psi\} = \begin{Bmatrix} k_1 x_g + c_1 \dot{x}_g \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

in which the subscript j (=1, 2, 3, or 4) refers to the jth story or jth floor, the subscript s refers to the secondary-system and the subscript g refers to the ground. Introduce the following transformation

$$\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_s \end{Bmatrix} = \begin{Bmatrix} x_1 - x_g \\ x_2 - x_g \\ x_3 - x_g \\ x_4 - x_g \\ x_s - x_3 \end{Bmatrix} \tag{4}$$

The governing equation (1) is changed to

$$[M]\{\ddot{Y}\} + [C]\{\dot{Y}\} + [K]\{Y\} = -[M]\{1\}x_g \tag{5}$$

where {1} is a 5x1 column matrix with every element equal to one, and where

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 \\ 0 & 0 & m_s & 0 & m_s \end{bmatrix}$$

$$[C] = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 & 0 \\ -c_2 & c_2 + c_3 & -c_3 & 0 & 0 \\ 0 & -c_3 & c_3 + c_4 & -c_4 & -c_s \\ 0 & 0 & -c_4 & c_4 & 0 \\ 0 & 0 & 0 & 0 & c_s \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 & -k_S \\ 0 & 0 & -k_4 & k_4 & 0 \\ 0 & 0 & 0 & 0 & k_S \end{bmatrix} \qquad (6)$$

The displacement y_s of the secondary system relative to the supporting floor is considered to be a more relevant response variable for assessing the safety of the secondary system.

To determine the frequency response functions for various response variables y's due to the ground acceleration input. Let $y_j = \bar{y}_j\, e^{i\omega t}$ and $x_g = e^{i\omega t}$. We obtain

$$[\bar{Y}] = [Z]^{-1}[M]\{1\} = -[A][M]\{1\} \qquad (7)$$

in which [Z] and [A] are known as matrix of impedances and matrix of admittances, respectively, and

$$[Z] = \left[-\omega^2[M] + i\omega[C] + [K]\right] \qquad (8)$$

Then the frequency response function for y_S may be computed from

$$H_S(\omega) = \bar{y}_S = -\sum_0^4 a_{5j}\, m_j - a_{55}\, m_S \qquad (9)$$

where a_{ij} is the (i,j) element of the admittance matrix.

We digress to note that if the primary system is composed of N identically constructed stories, namely, $m_j = m_p$, $c_j = c_p$ and $k_j = k_p$ for $j = 1,\ldots,N$, then there exists an efficient way to determine the eigen properties for the primary system alone. A structure so constructed is called a periodic structure. In the present case, the jth natural frequency of a periodic shear-wall type primary-system is given by

$$\omega_j = 2\,\omega_o \sin\frac{\theta_j}{2}\,, \qquad j = 1, \ldots, N \tag{10}$$

where $\omega_o = \sqrt{k_p/m_p}$ and $\theta_j = (\pi/2)(2j-1)/(2N+1)$. The damping ratio of the jth mode also can be easily calculated as

$$\zeta_j = 2\,\zeta_o \sin\frac{\theta_j}{2} \tag{11}$$

where $\zeta_o = c_p/(2m_p\omega_o)$.

The numerical results to be presented in the following pertains to a periodic primary structure, unless noted otherwise.

SENSITIVITY WITH RESPECT TO MASS RATIO VARIATION

Mass ratio is an important parameter because it has a controlling effect on the interaction between the primary-secondary systems, and it has been used in early investigations as the major perturbation parameter. See, for example, references 4 and 5. To study the changes in the frequency response function H_s of the secodary system as the mass ratio γ is varied, the parameters of the secondary system itself are kept unchanged. Thus different γ values refer to different primary systems. Specifically, we selected for our calculation the following parameter values

$m_s = 3.456 \times 10^4$ kg, $\omega_s = 5$ rad/sec, $\zeta_s = 0.02$

$m_p = m_s/\gamma$, $0.001 \leqslant \gamma \leqslant 0.1$, $k_p = 7.313 \times 10^7$ N/m, $\zeta_o = 0.05$.

The secondary system becomes tuned to the first, second, third and fourth modes of the primary system when γ takes the values of 0.0980, 0.0119, 0.0053 and 0.0033, respectively.

Figure 2 is a 3-dimentional plot of $|H_s|$ as a function of mass ratio γ and frequency ω. The high ridge on the left represents the contribution from the first mode of the primary-system. As γ increases, the first modal frequency of the primary system also increases and the ridge moves toward right to higher frequencies. The high ridge on the right represents the contribution from the secondary mode. The heights of both left and right ridges increase when tuning between the secondary mode and the first primary mode is approached. It is well-known that tuning gives rise to a repelling effect between two frequencies; thus two ridges do not coalesce into one, but bend away from each other at the tuning γ value which, in the present case, is near $\gamma = 0.1$. It will be shown later that this frequency repelling effect at tuning is not strong enough to be noticeable if the secondary system is light (a small γ value), or if tuning occurs at a higher primary mode.

Contribution from a higher primary mode also increases when tuning occurs between this mode and the secondary system; however, the magnitude is much smaller to be noticeable in figure 2. It will be discussed later with the help of other figures.

The behavior of $|H_s|$ when γ increases beyond a tuning value is illustrated in

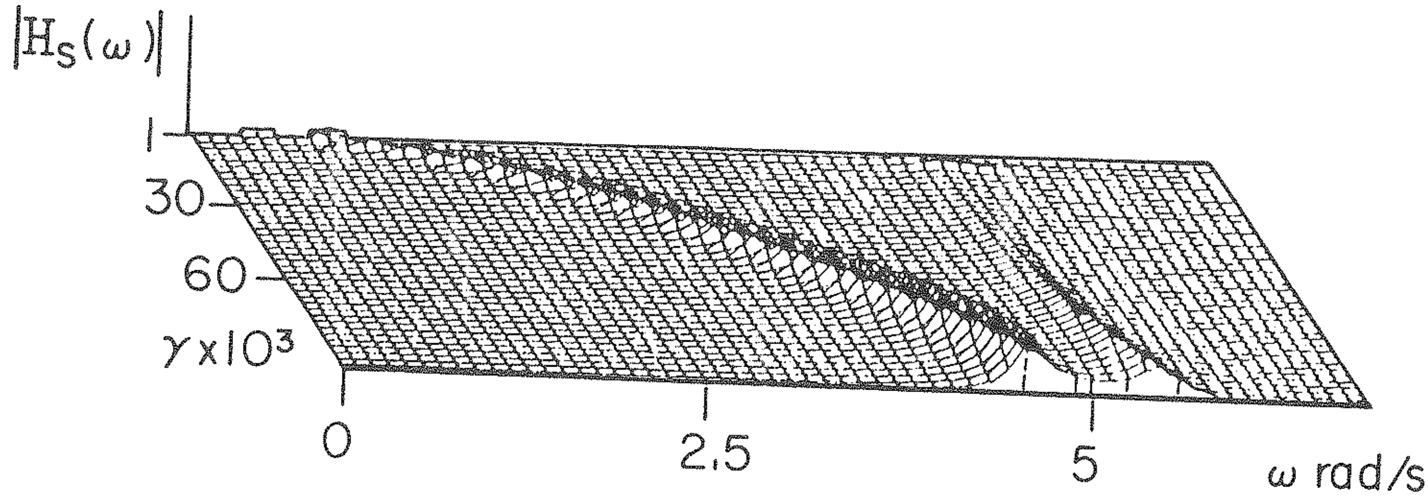

FIGURE 2 Magnitude of Frequency Response of a Secondary System as Function of Mass Ratio and Frequency.
$\omega_s = 5$ rad/s, $\zeta_s = 0.02$, $\zeta_0 = 0.05$.

figure 3, which is obtained using a higher k_p value of 1.46×10^8 N/m. For this higher k_p value, tuning between the first primary mode and the secondary mode takes place at $\gamma = 0.0491$. It can be seen that $|H_s|$ behaves similarly to what has been shown in figure 2 in the range of $\gamma < 0.0491$. In the range $\gamma >$ 0.0491, however, the left ridge representing the contribution from the first primary mode remains near $\omega = \omega_s$. In contrast, the right ridge representing the contribution from the secondary mode, which begins originally at ω_s, turns further to the right to the higher frequencies. The heights of both ridges are reduced as γ increases beyond the tuning value.

It is of interest to compare the effects of tuning when the secondary system is tuned to different primary modes. For this purpose, we choose a fixed mass ratio, but vary the stiffness of the primary system so that the secondary system is tuned to the first, or second, or third, or fourth mode of the primary system, one at a time. The results obtained for a rather light secondary system with $\gamma = 0.001$ are shown in figures 4 through 7. It is seen that tuning to a higher primary mode does enhance the contribution from that mode, but contribution from the first primary mode remains quite important in all cases. Contribution from the first mode becomes even more important if the secondary system is heavier, as shown in figures 8 through 11 for $\gamma =$ 0.1, whether or not the secondary system is tuned to the first mode of the primary system. As expected, the highest response occurs when the secondary system is tuned to the first mode of the primary system, in which case the frequency repelling effect is again evident for a heavy secondary system (see figure 8).

The result shown in figures 4 through 11 were obtained when the secondary system is tuned to one of the primary modes. When tuning does not take place, called detuned case in the sequel, response of the secondary system is expected to be lower. This is confirmed in fiqures 12 and 13 for the case of light and heavy secondary systems, respectively. The dotted line in fiqure 12 represents the magnitude of frequency response of the secondary system when it is de-tuned from the first primary mode obtained by reducing the value for k_p. The two peaks are associated with the contributions from the first primary mode and the secondary mode, respectively, and the response magnitude is seen to be much lower than that of the tuned case, shown in

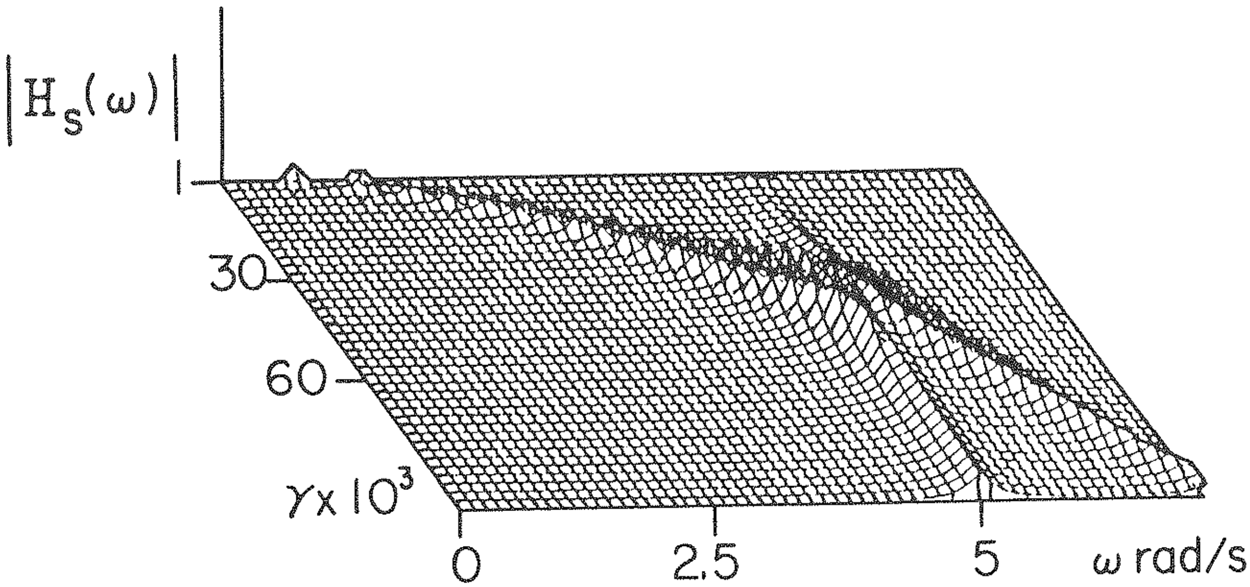

FIGURE 3 Magnitude of Frequency Response of a Secondary System as Function of Mass Ratio and Frequency. $\omega_s = 10$ rad/s, $\zeta_s = 0.02$, $\zeta_o = 0.05$.

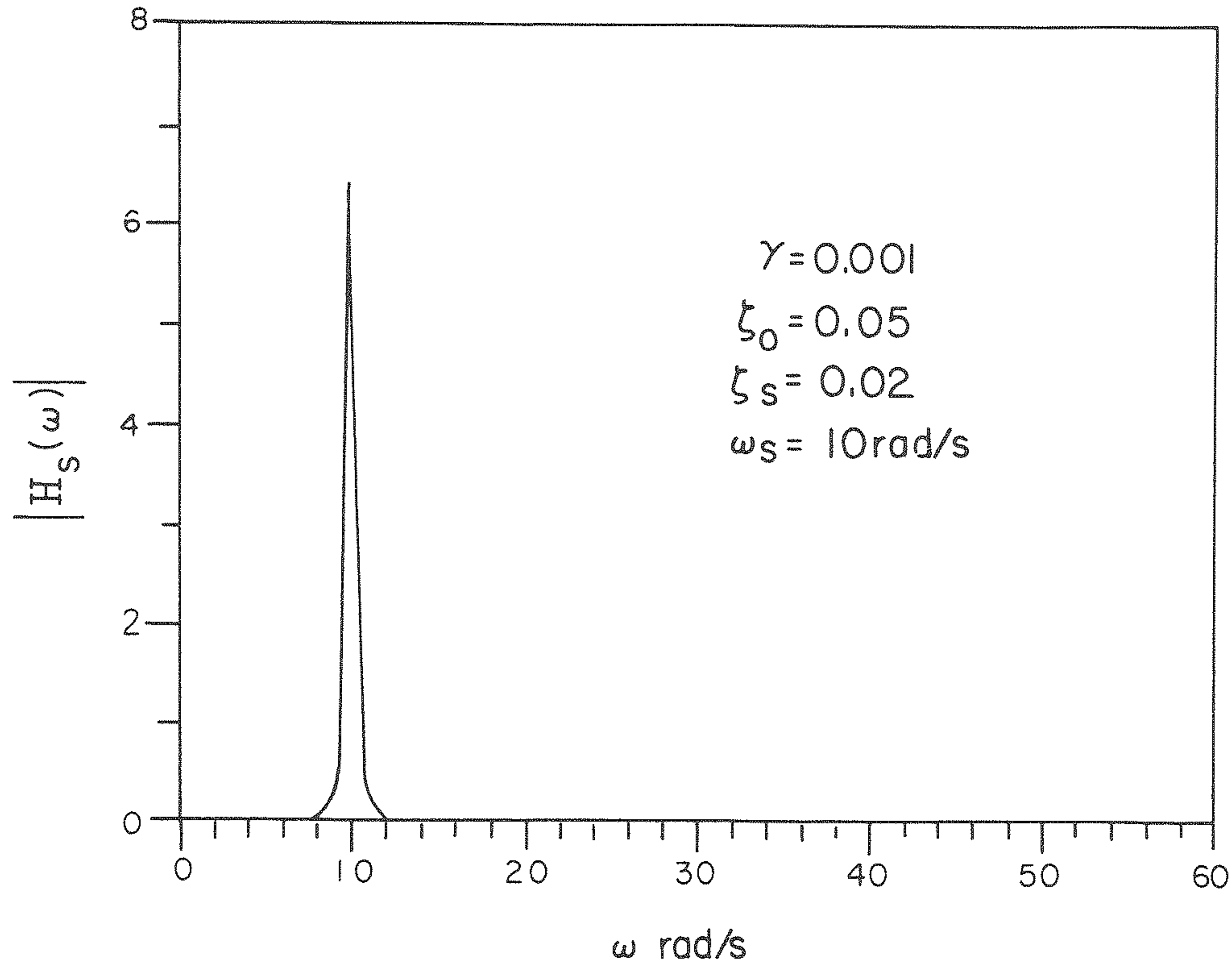

FIGURE 4 Magnitude of Frequency Response of a Light Secondary System Tuned to the First Mode of the Primary System.

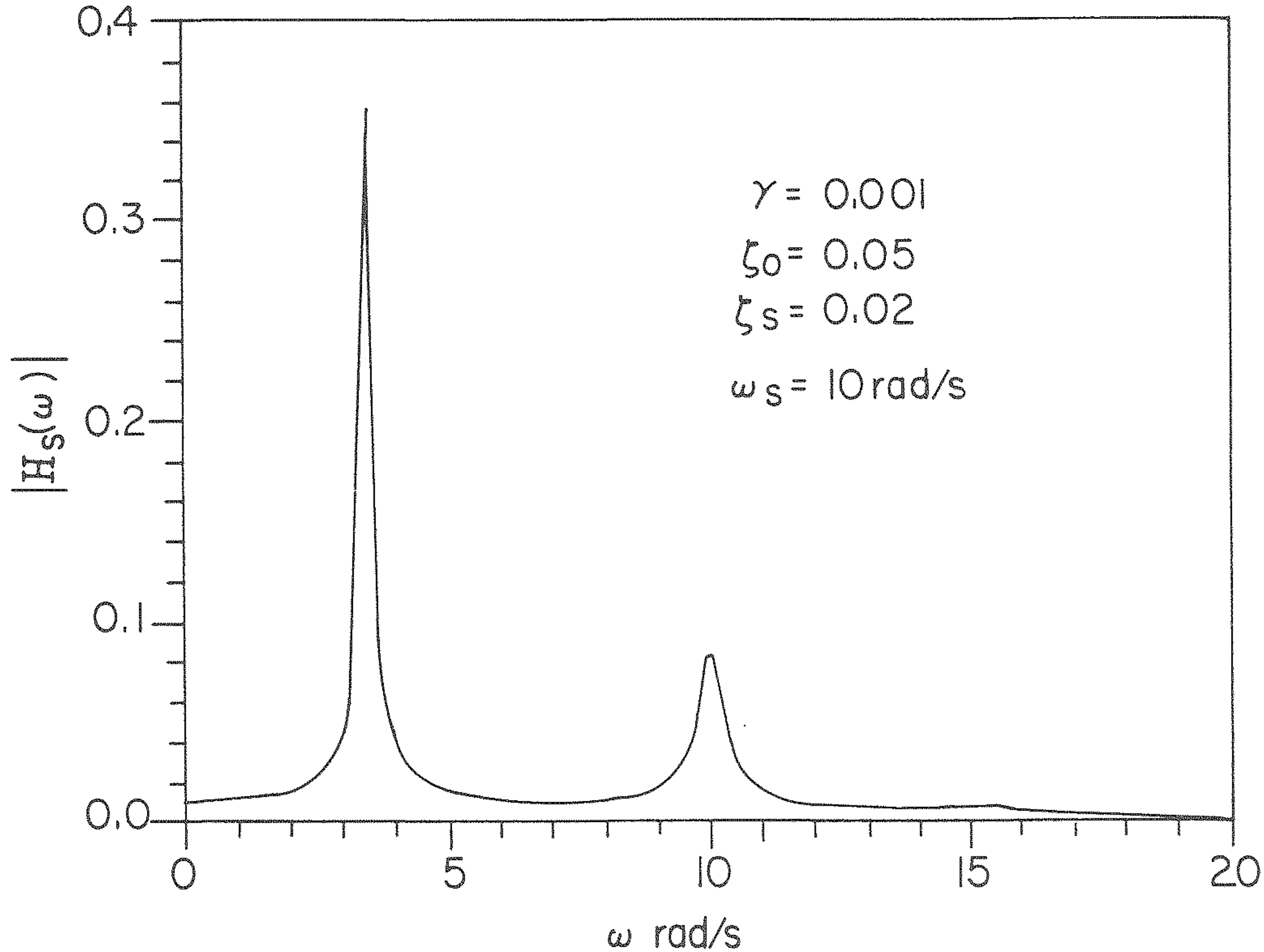

FIGURE 5 Magnitude of Frequency Response of a Light Secondary System Tuned to the Second Mode of the Primary System.

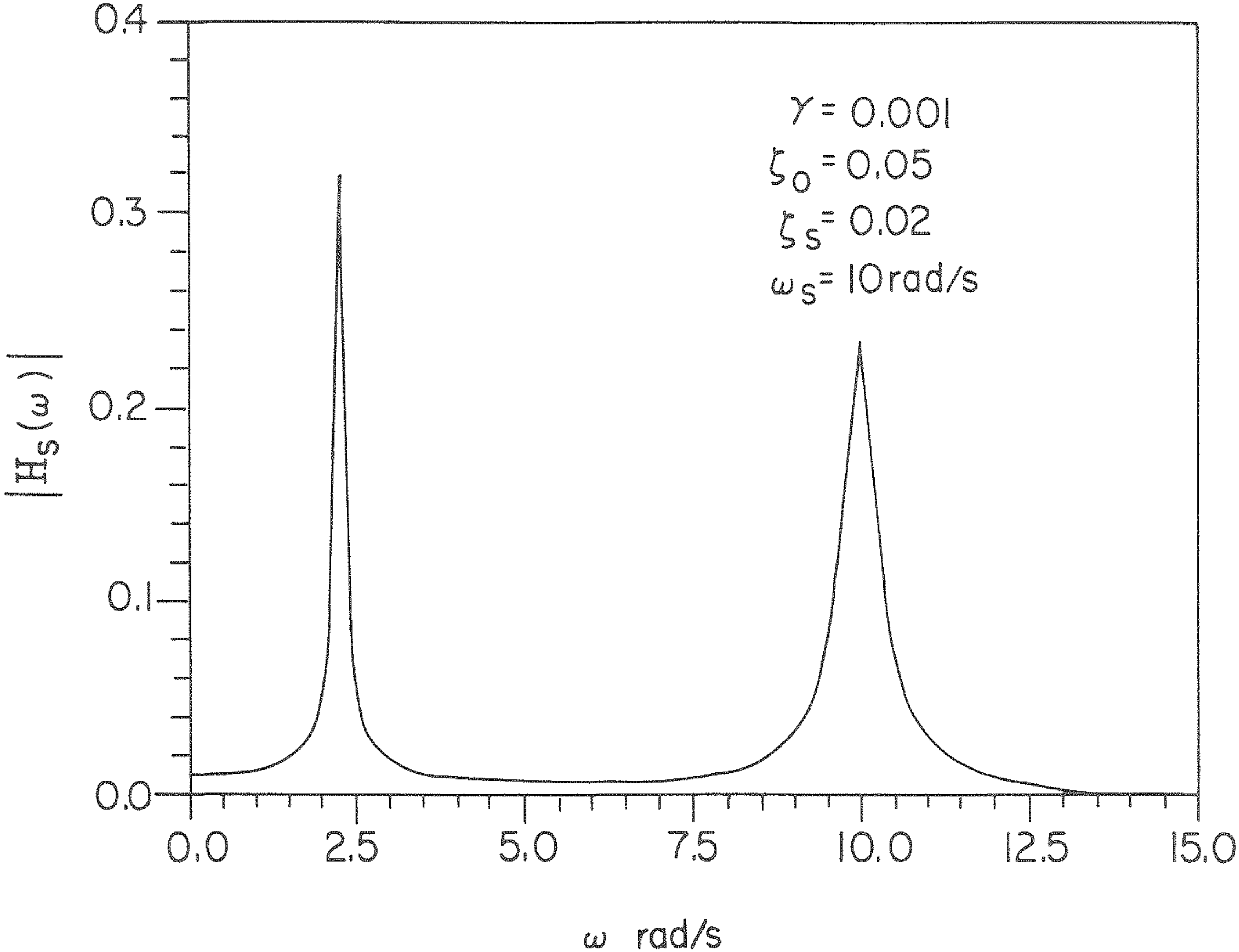

FIGURE 6 Magnitude of Frequency of a Light Secondary System Tuned to the Third Mode of the Primary System.

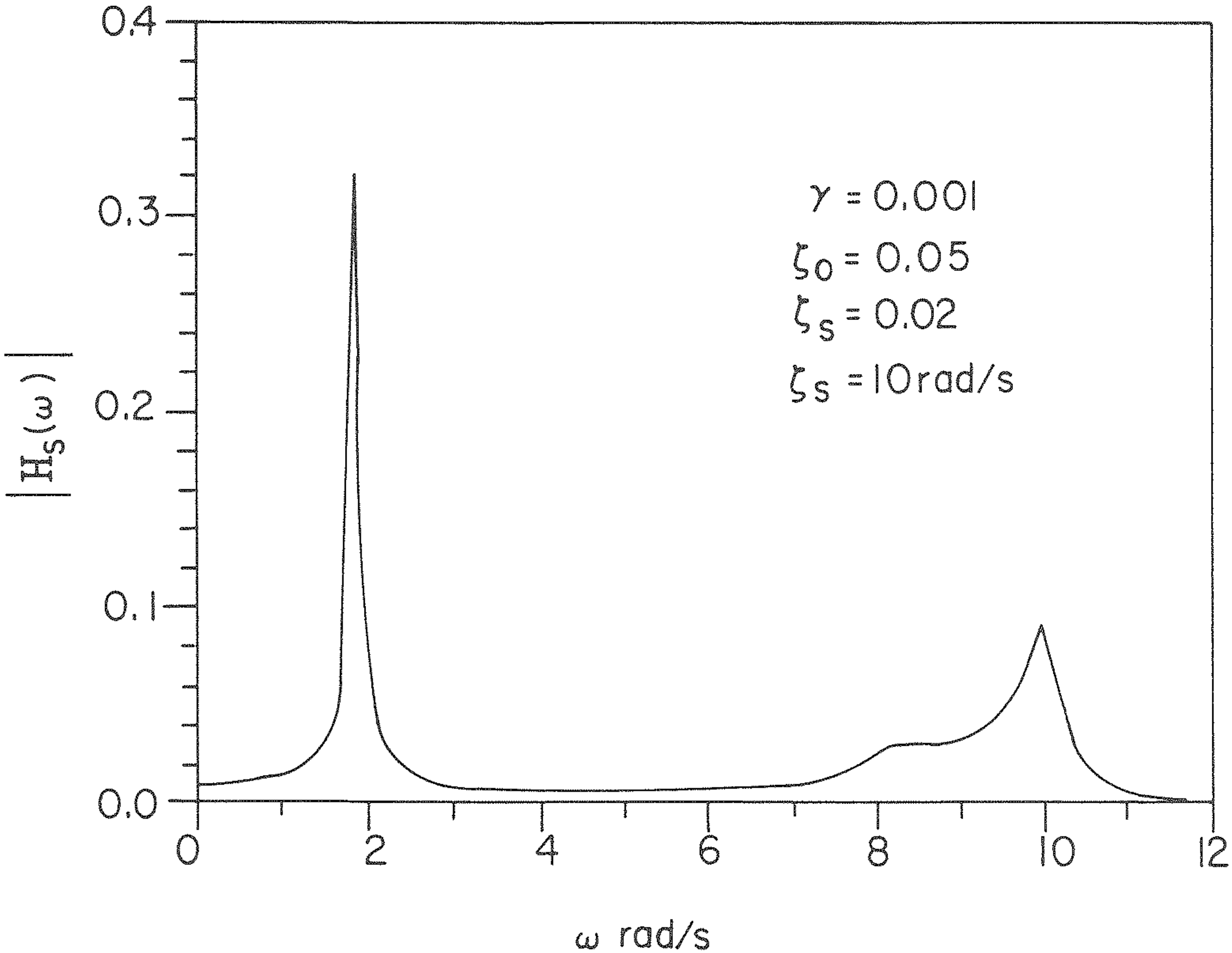

FIGURE 7 Magnitude of Frequency Response of a Light Secondary System Tuned to the Fourth Mode of the Primary System.

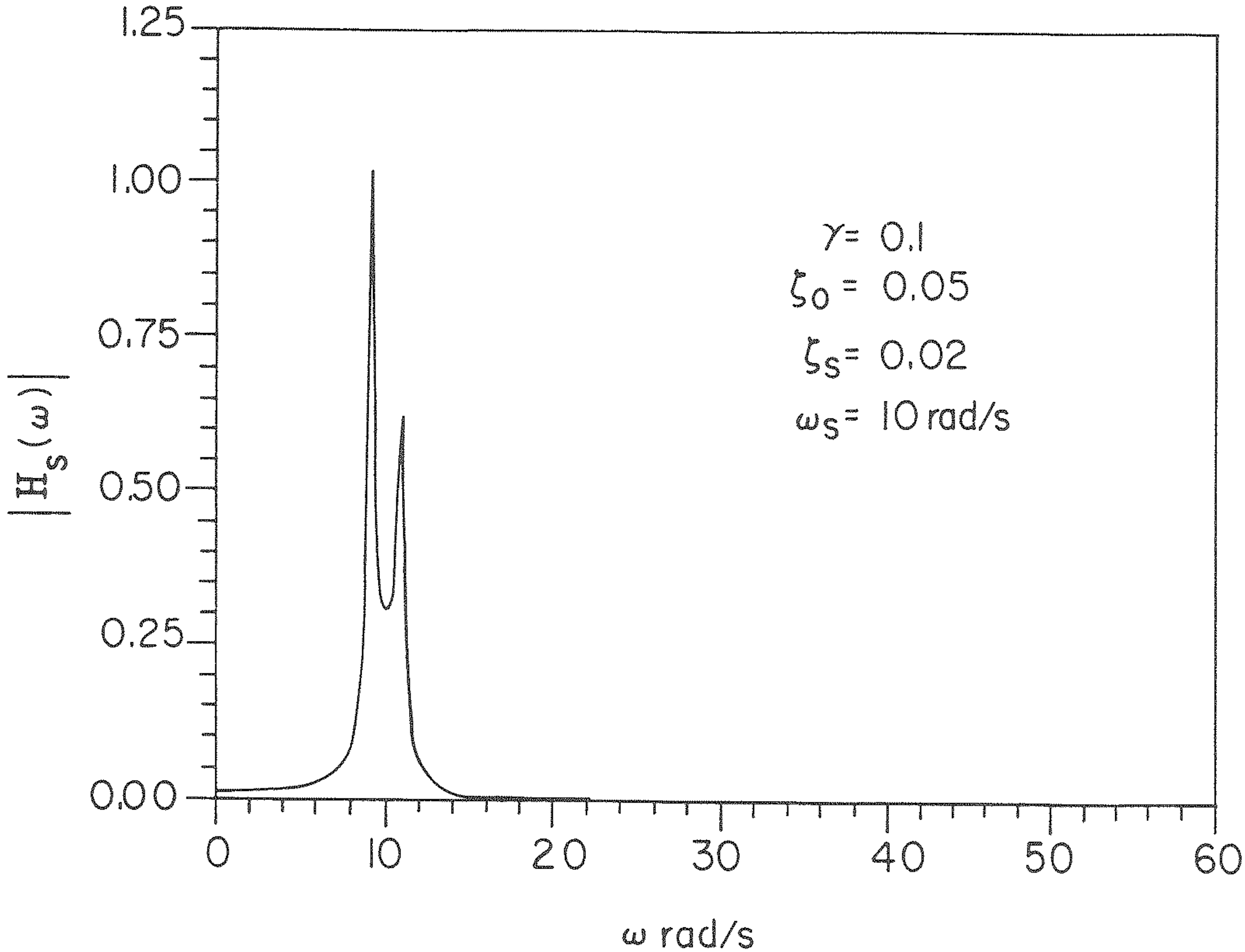

FIGURE 8 Magnitude of Frequency Response of a Heavy Secondary System Tuned to the First Mode of the Primary System.

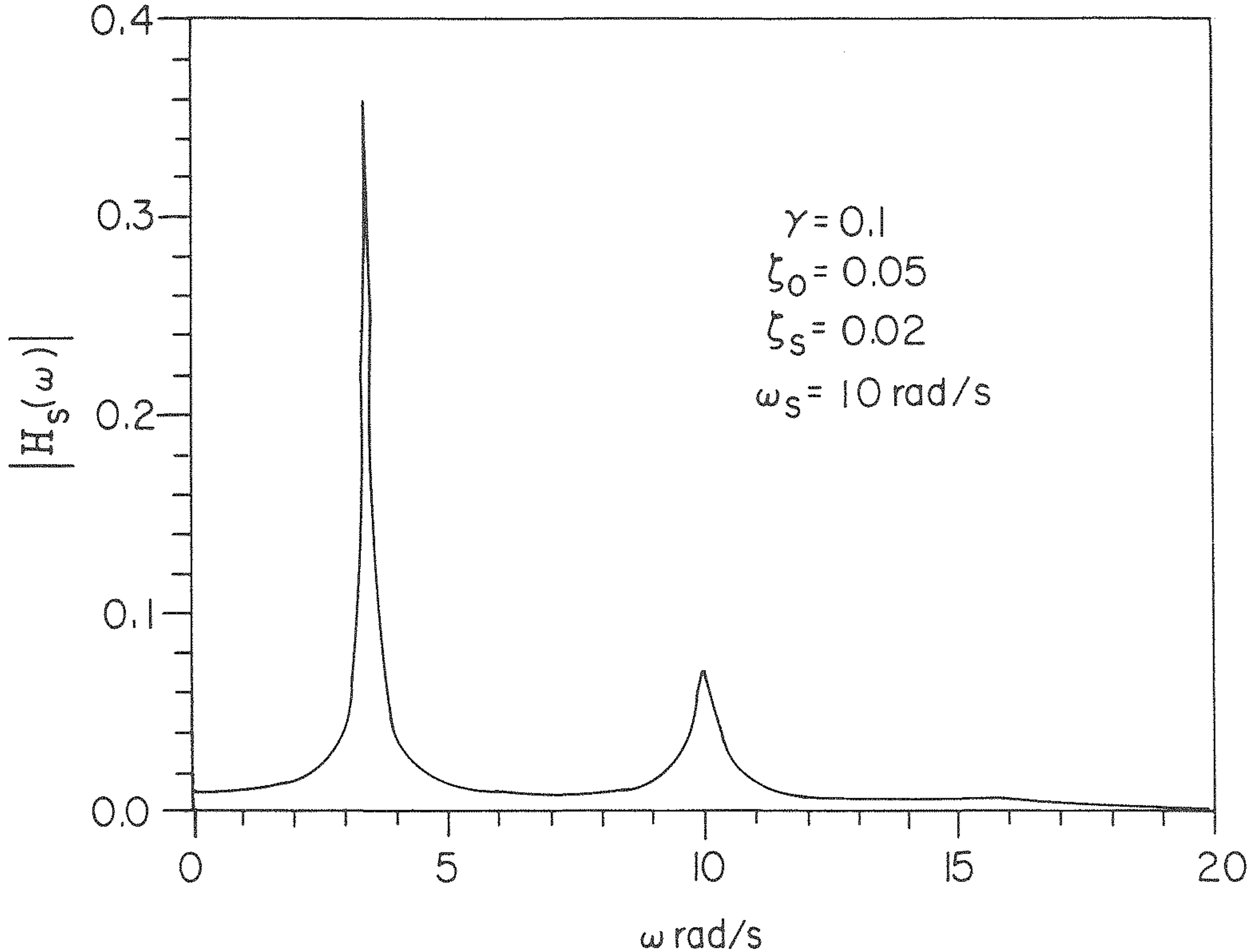

FIGURE 9 Magnitude of Frequency Response of a Heavy Secondary System Tuned to the Second Mode of the Primary System.

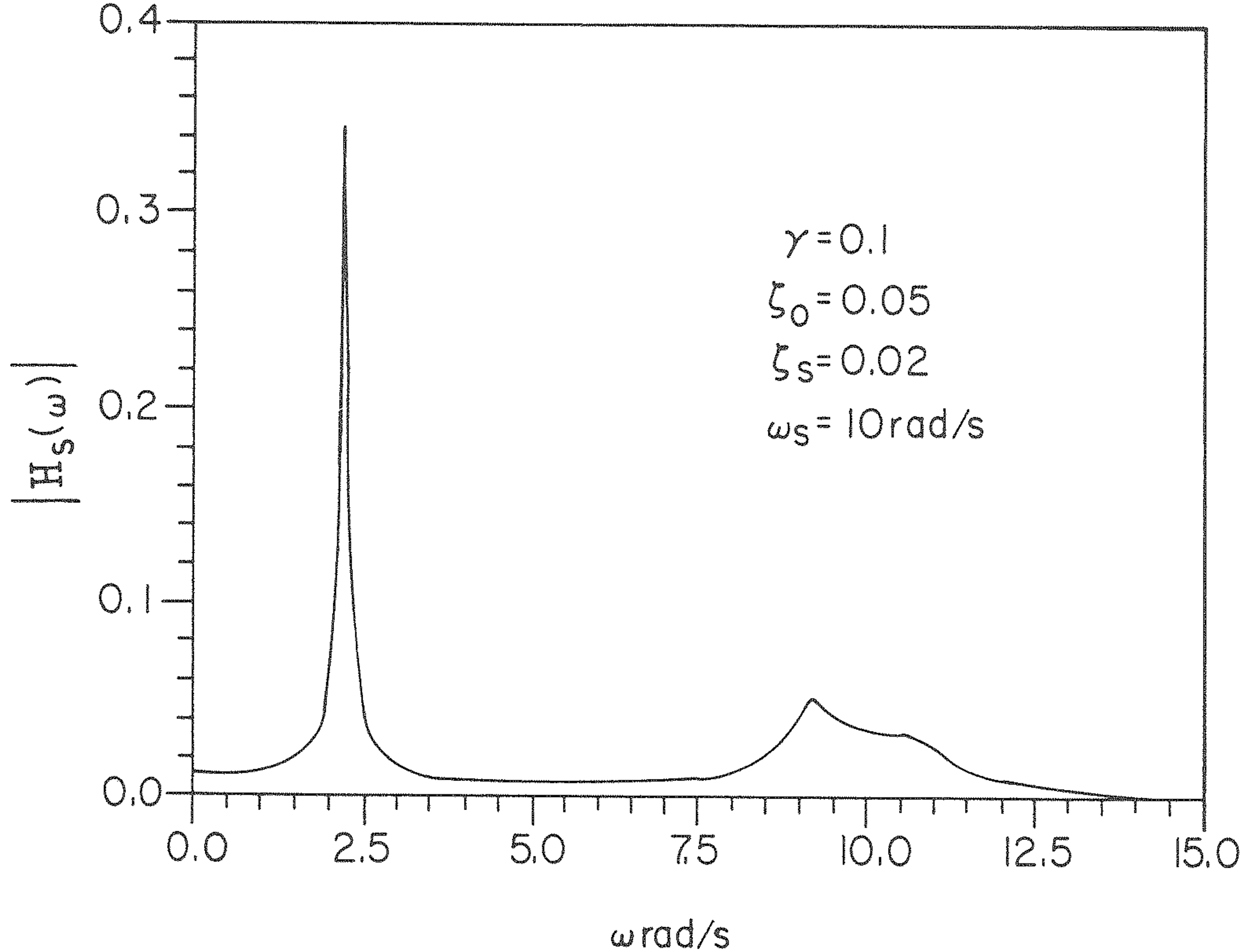

FIGURE 10 Magnitude of Frequency Response of a Heavy Secondary System Tuned to the Third Mode of the Primary System.

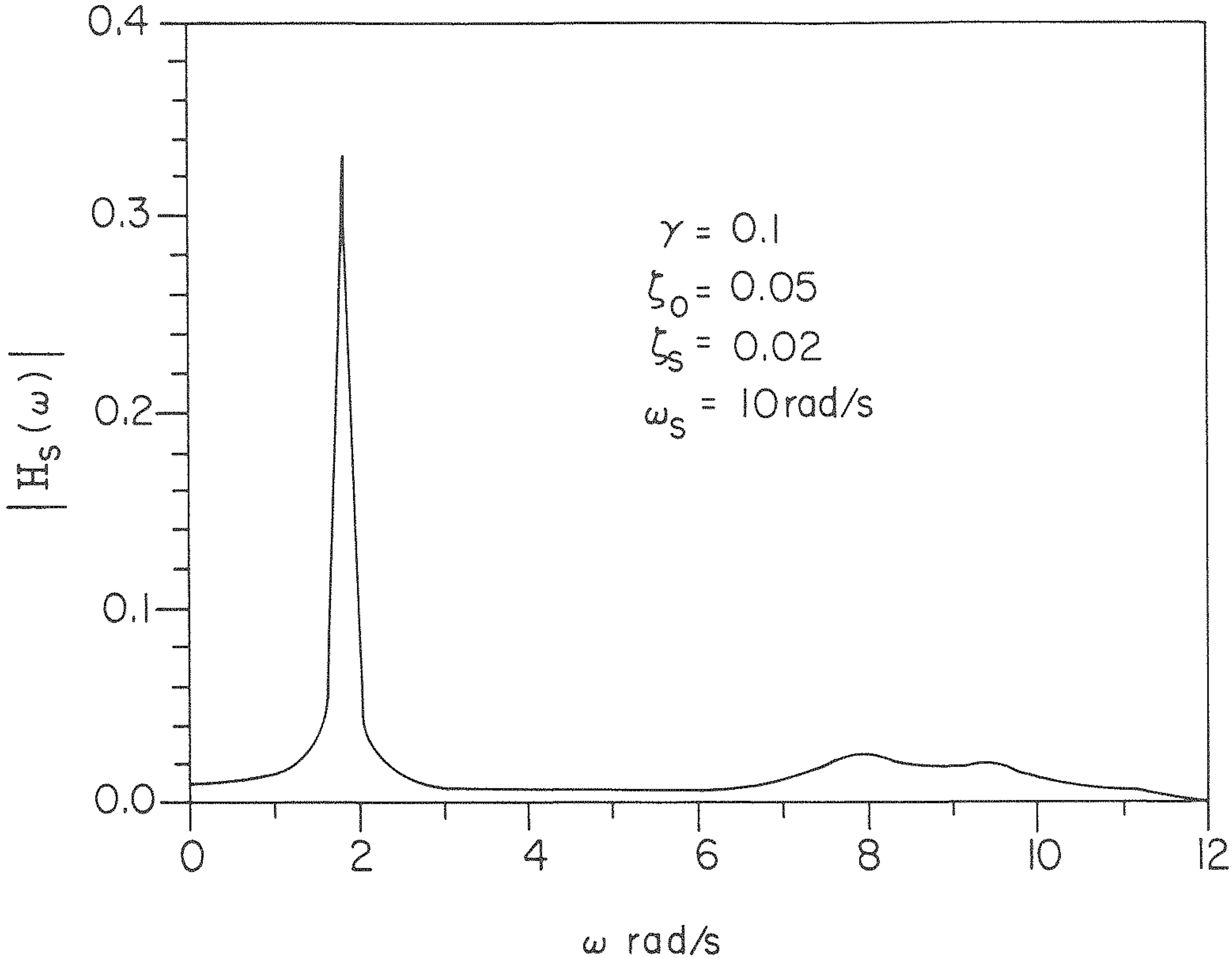

FIGURE 11 Magnitude of Frequency Response of a Heavy Secondary System Tuned to the Fourth Mode of the Primary System.

solid line for comparison. As commented earlier (refer to figure 4), the frequency repelling effect at tuning is too weak to be noticeable in this case of light secondary system.

Figures 12 and 13 were obtained when the primary system is classically damped. A preliminary investigation has been carried out into the effect of non-classical damping in the primary system by increasing the damping in the second story by three folds. The results of this investigation are shown in figures 14 amd 15. It is seen by comparing these with figures 12 and 13 that very little change takes place by such modifications in the primary system.

In certain applications, we might be interested in the maximum value of the frequency response $|H_s(\omega)|_{max}$ as a function of the mass ratio γ and the tuning parameter $\beta_1 = 2(\omega_{p1}-\omega_s)/(\omega_{p1}+\omega_s)$ where ω_{p1} is the natural frequency of the first primary mode. We focus our attention on the first primary mode since tuning to this mode gives rise to the highest response. Behavior of $|H(\omega)|_{max}$ in the case of tuning to a different primary mode should be similar.

As seen in figure 16, tuning has a greater effect for lighter secondary systems, say $\gamma < 0.01$, and it becomes less and less important as γ increases. Furthermore, for $|\beta| > 0.2$, $|H(\omega)|_{max}$ remains nearly constant or all γ values indicating that if the primary and secondary systems are very much not in tuned, then the relative weight of the secondary system is immaterial.

In the foregoing dicussion concerning the sensitivity of secondary-system response with respect to the variations of mass and tuning parameters, the secondary system itself is treated as being fixed. In usual engineering practice, however, a primary system either already exists or its design has been selected. One must then choose a secondary system to meet a number of requirements, among which is the safety of such a secondary system. Figure 17 is prepared with the needs of a design engineer in mind. In the forefront of this figure, the left ridge represents the contribution from the secondary mode, and the right ridge the contribution from a neighboring primary mode. The two modes are not in tune at higher γ values. As γ value decreases, the natural frequency of the secondary system increases, and the contributions from both the primary and secondary modes are enhanced when tuning is

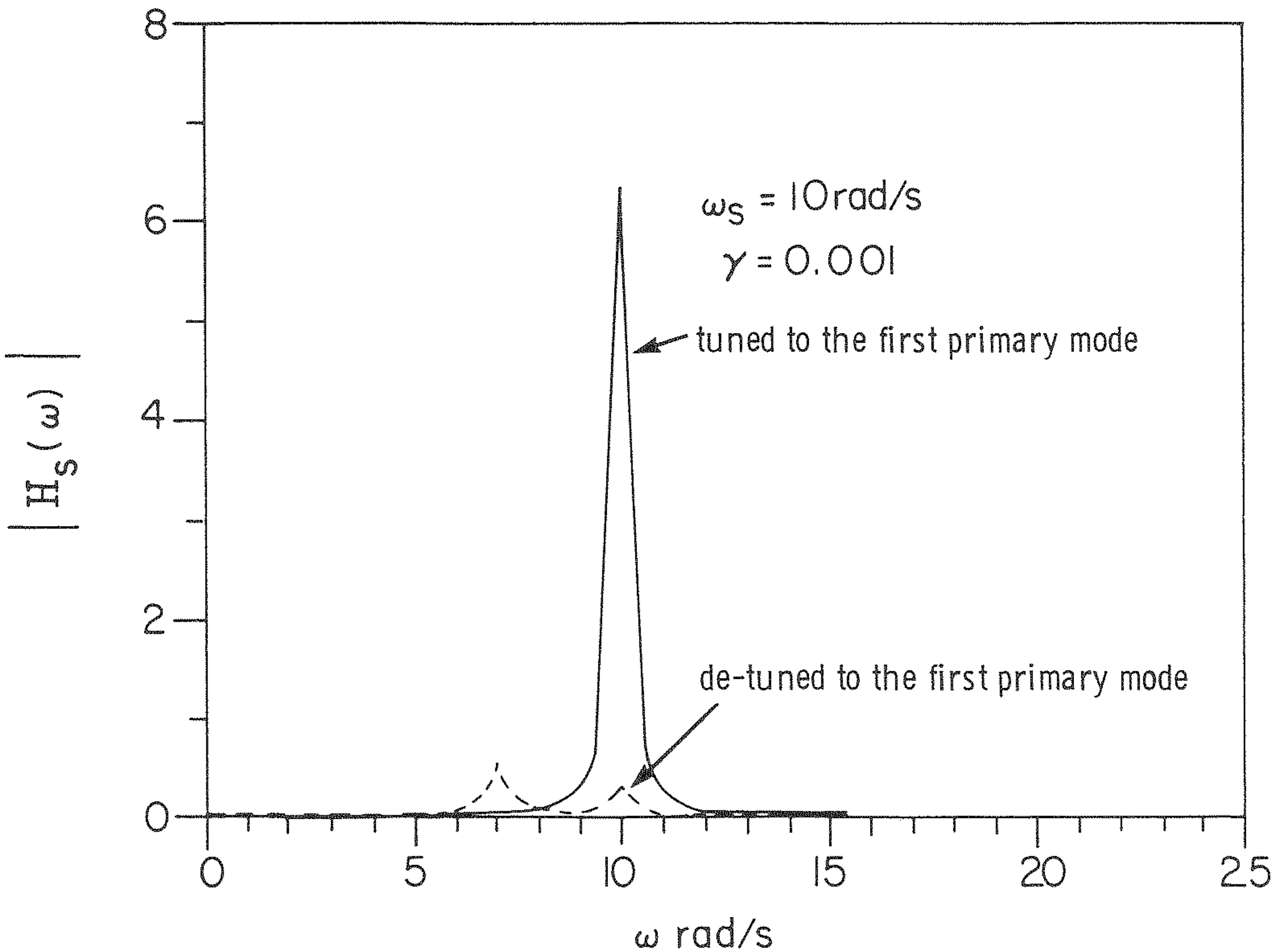

FIGURE 12 Comparison between Tuned and Detuned Responses for Light Secondary System.

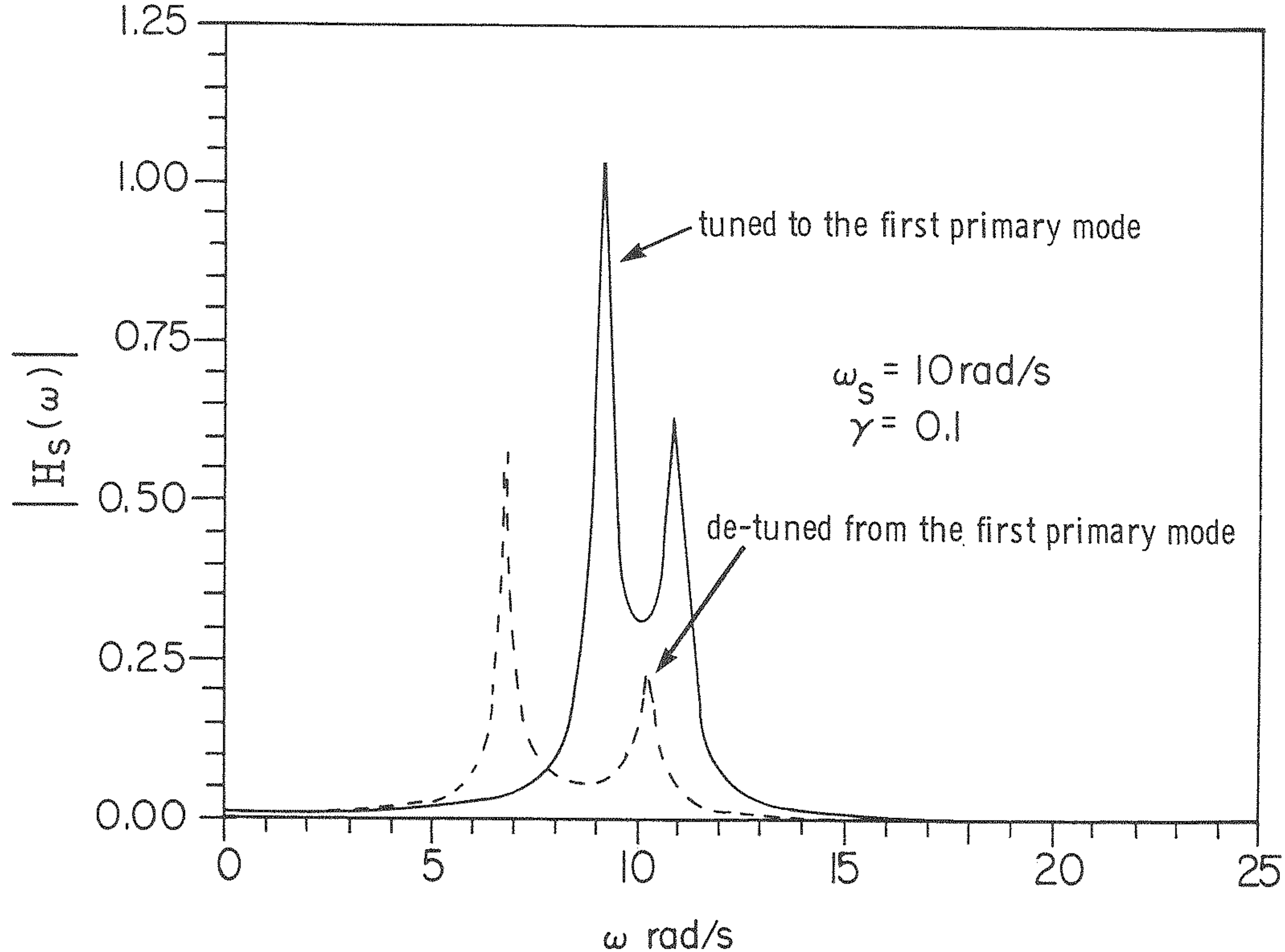

FIGURE 13 Comparison between Tuned and Detuned Responses for Heavy Secondary System.

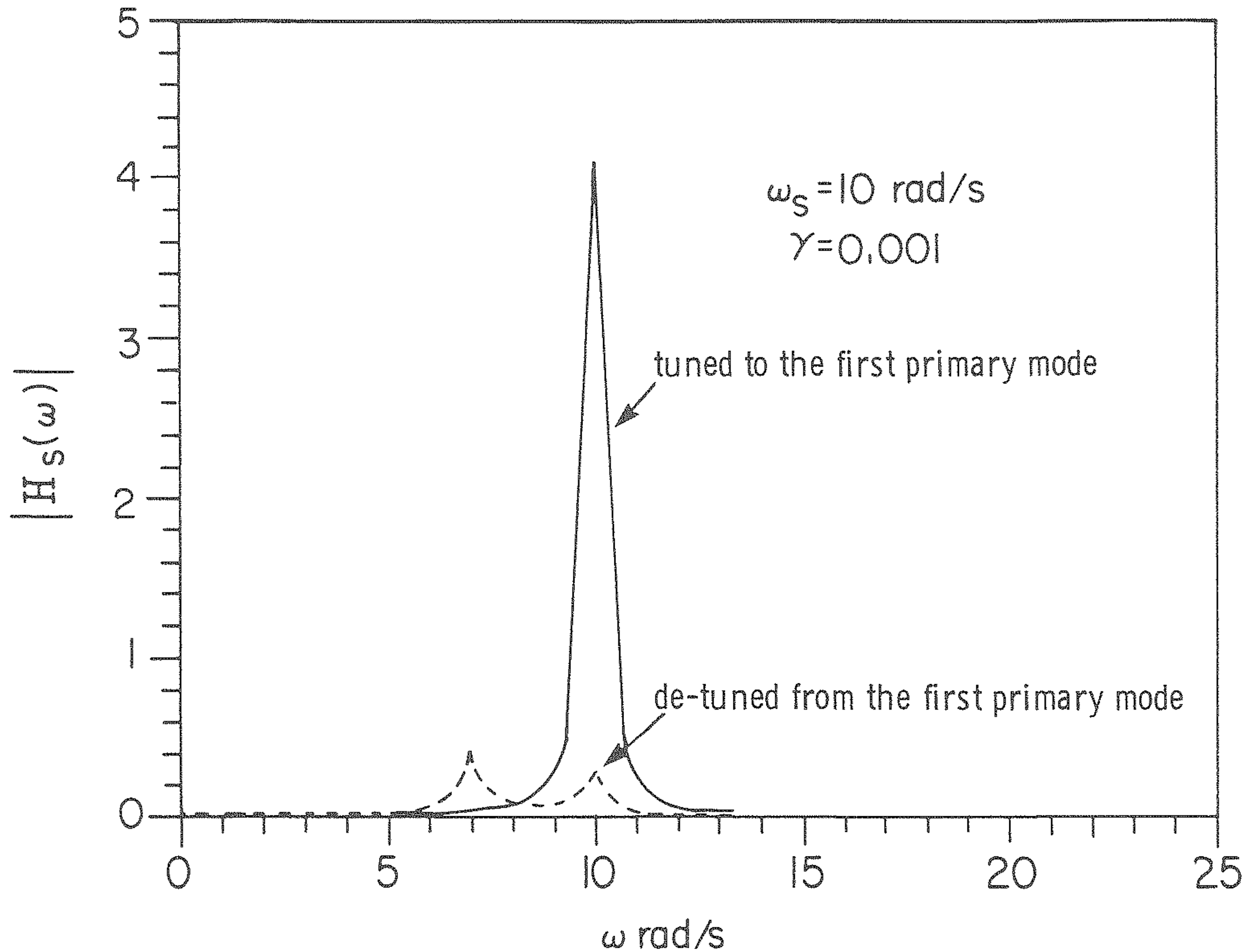

FIGURE 14 Comparison of Tuned and Detuned Responses of a Light Secondary System When Primary System is Nonclassically Damped.

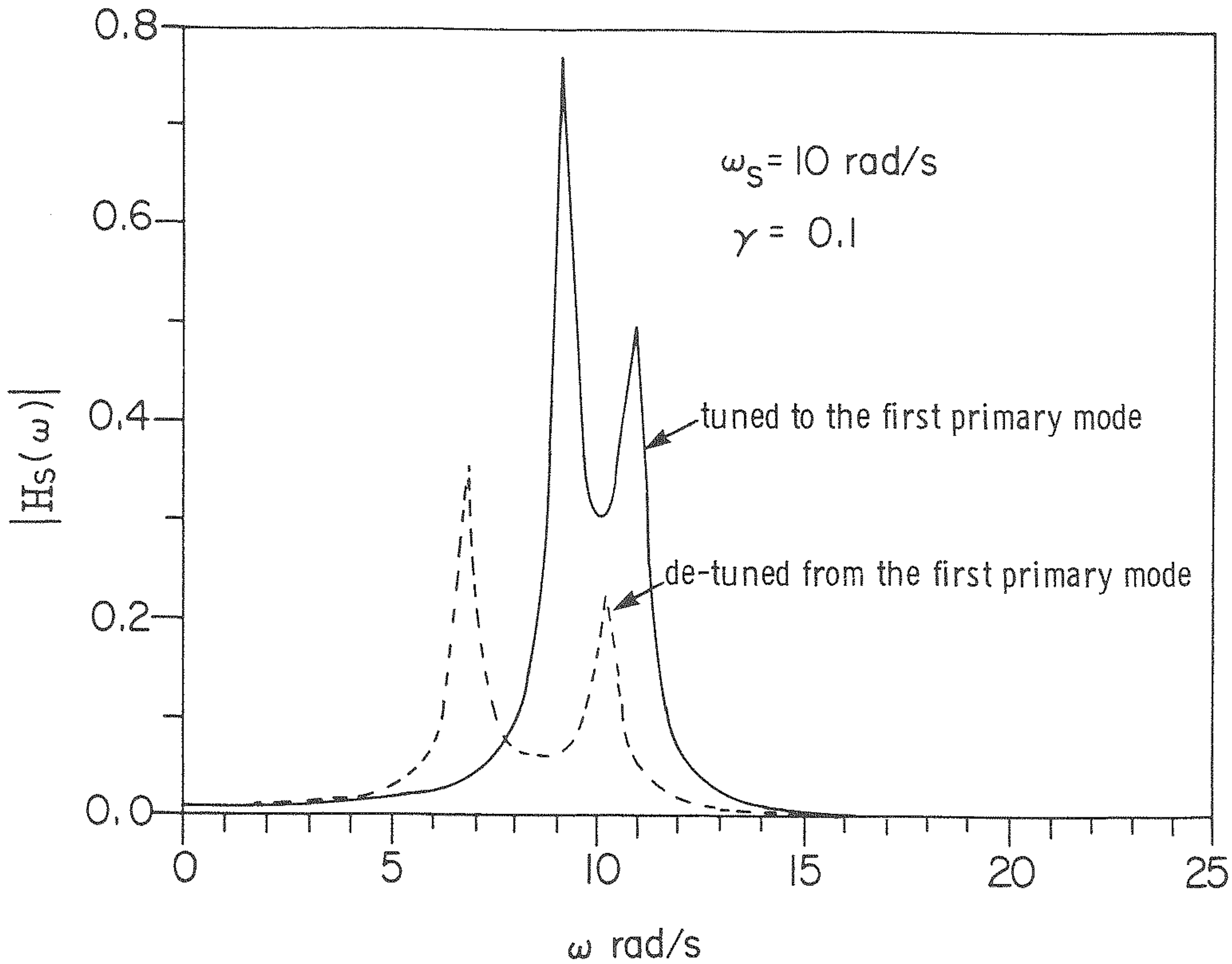

FIGURE 15 Comparison of Tuned and Detuned Responses of a Heavy Secondary System When Primary System is Nonclassically Damped.

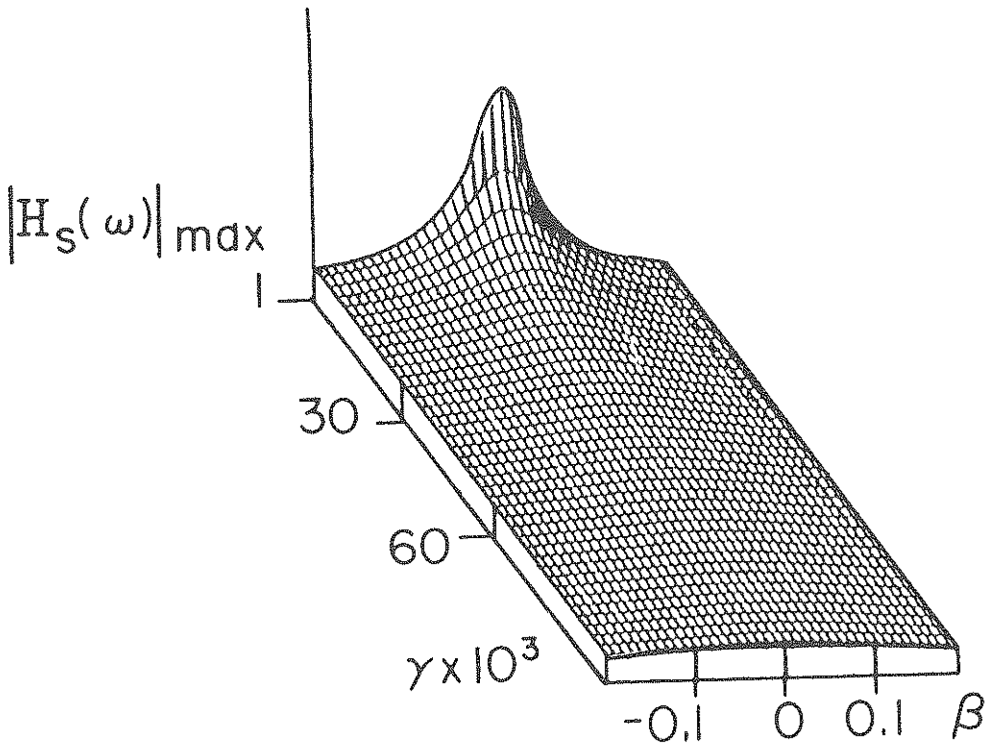

FIGURE 16 Peak Frequency Response Magnitude of a Secondary System as Function of Mass Ratio and Tuning Parameter.

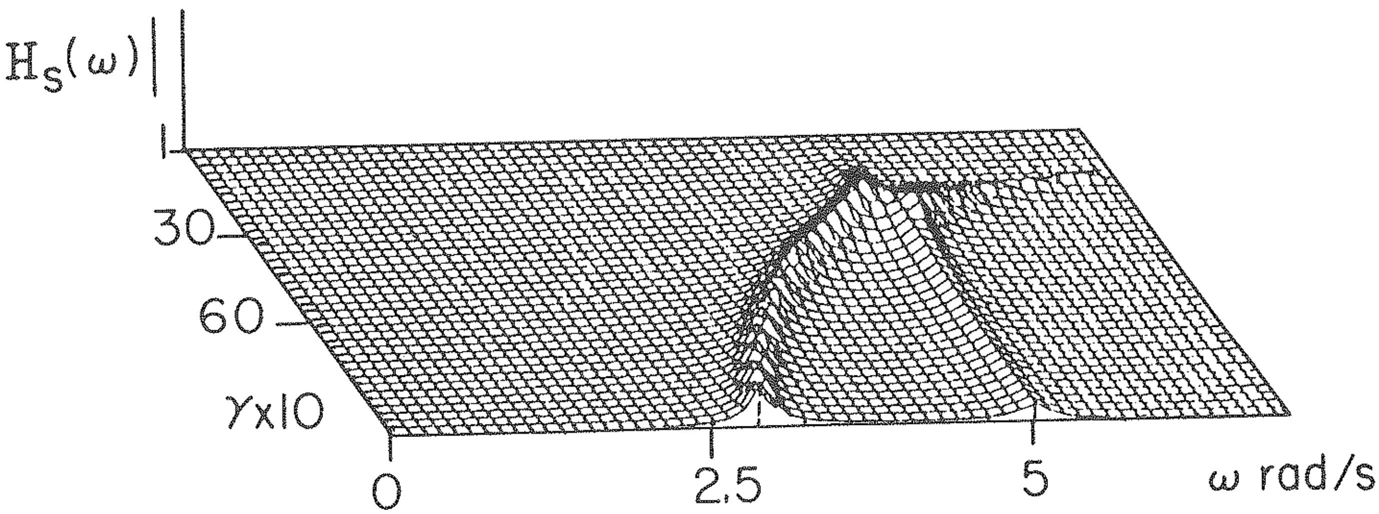

FIGURE 17 Variation of Frequency Response Magnitude of a Secondary System with Respect to Mass Ratio and Damping, Given a Primary System. ω_p=5rad/s

approached. Finally, as the natural frequency of the secondary system increases further and exceeds the primary system natural frequency, the ridge representing the contribution from the secondary system proceeds further to the right to seek tuning with a higher primary mode.

SENSITIVITY WITH RESPECT TO PRIMARY-SYSTEM DAMPING VARIATION

Because of its greater statistical uncertainty, damping in the primary system has been chosen also as a key parameter in the present investigation. Intuition suggests that response of the secondary system can be reduced by increasing the damping in either the primary or the secondary system. Therefore, we are mainly interested in how effective is this reduction effect under various conditions. There may also be some exceptional cases which are contrary to intuition.

Figure 18 is a 3-dimentional plot of the maximum frequency response magnitude $|H(\omega)|_{max}$ of a secondary system versus mass ratio γ and the damping parameter ζ_o of the primary system, computed at the tuning condition $\beta = 0$. As expected, $|H(\omega)|_{max}$ is monotonically decreasing with increasing ζ_o; however, the rate of decreasing is smaller at larger mass ratios, and the rate is nearly zero for $\gamma \geqq 0.1$.

A plot of $|H(\omega)|_{max}$ versus the tuning parameter β and the primary-system damping ζ_o is shown in figure 19 for a given mass ratio $\gamma = 0.01$. The value of $|H(\omega)|_{max}$ is seen to decrease with increasing ζ_o at similar rates for almost all β values.

In the course of our investigation, unexpected results were found in the case of a heavy secondary system ($\gamma = 0.1$) with extremely light damping ($\zeta_s = 0.005$). While primary-system damping reduces the maximum frequency response of the secondary system in both tuned and de-tuned cases, the reduction quickly loses effectiveness in the de-tuned case, as shown in figure 20. It is then possible for the response of the de-tuned case to be higher than that of the tuned case when the primary damping is increased beyond certain value.

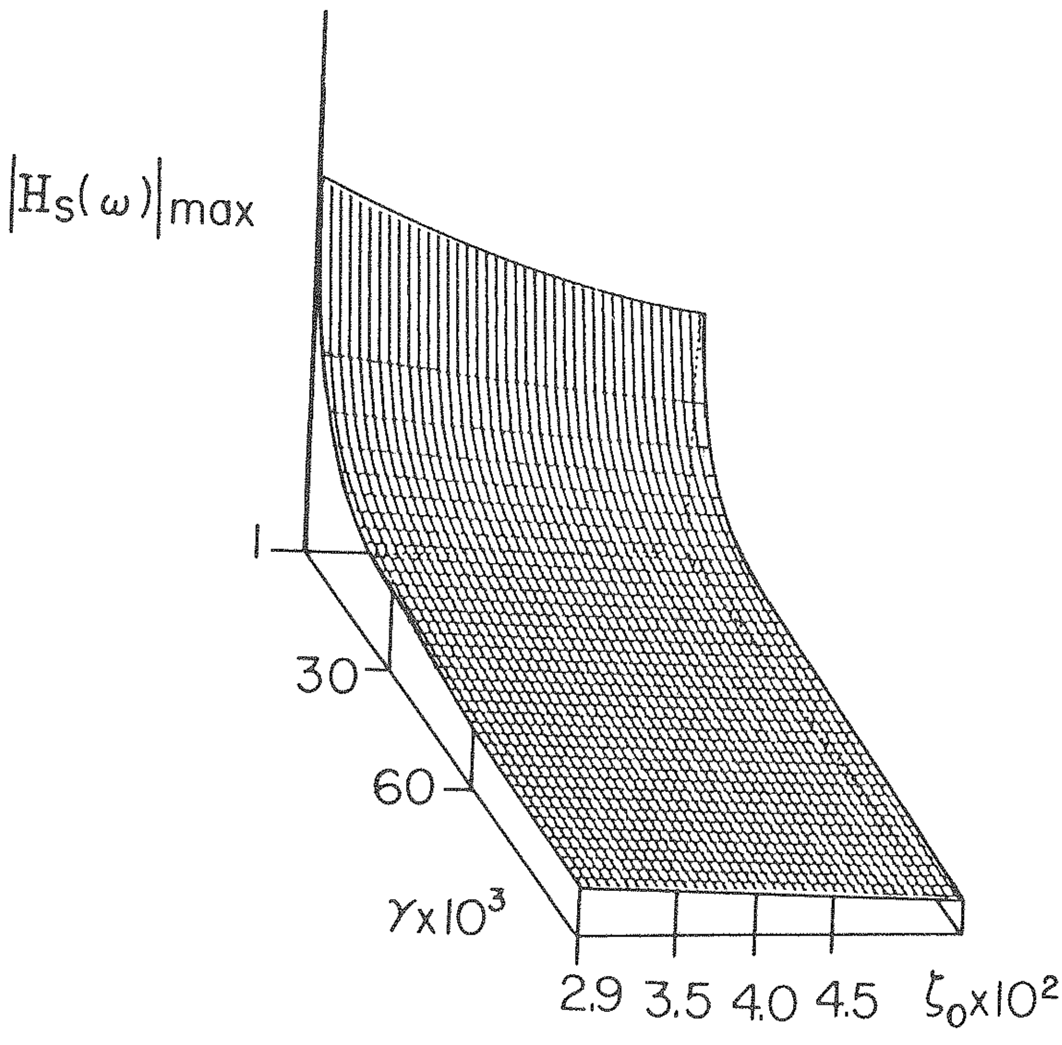

FIGURE 18 Maximum Frequency Response Magnitude of a Secondary System as Function of Mass Ratio and Primary System Damping. $\beta = 0$.

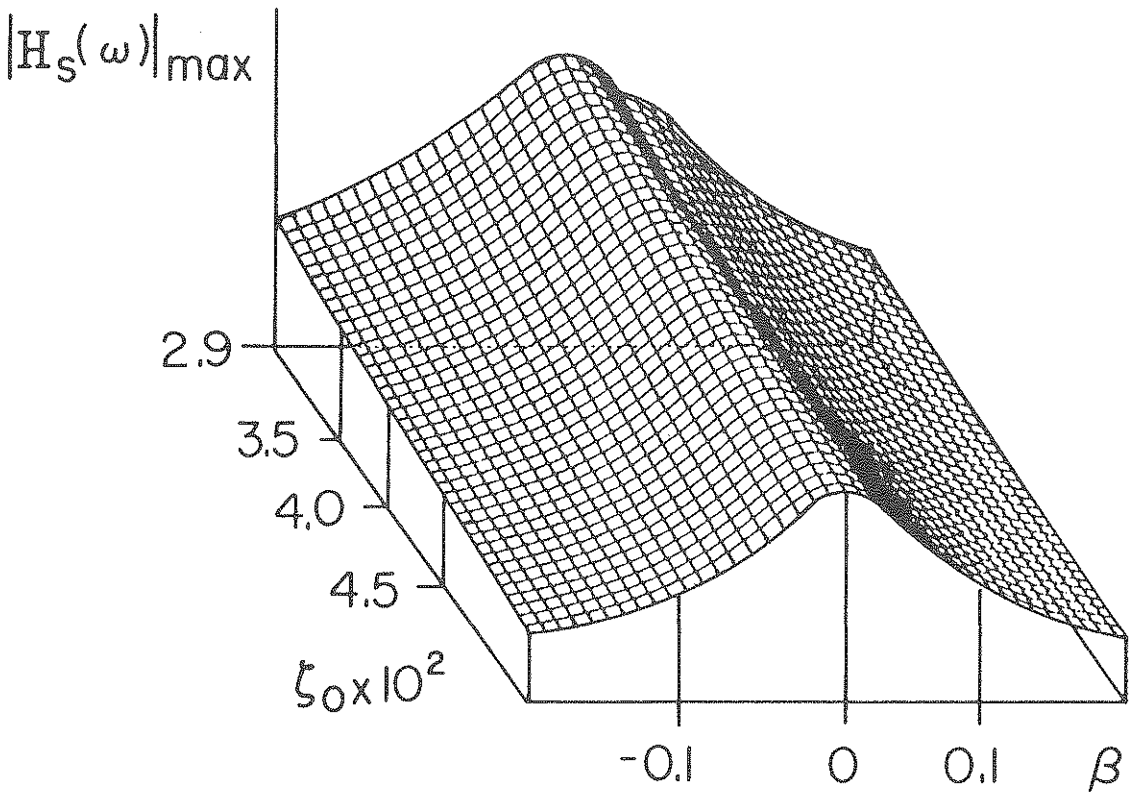

FIGURE 19 Maximum Frequency Response Magnitude of a Secondary System as Function of Tuning Parameter and Primary System Damping. $\gamma = 0.01$.

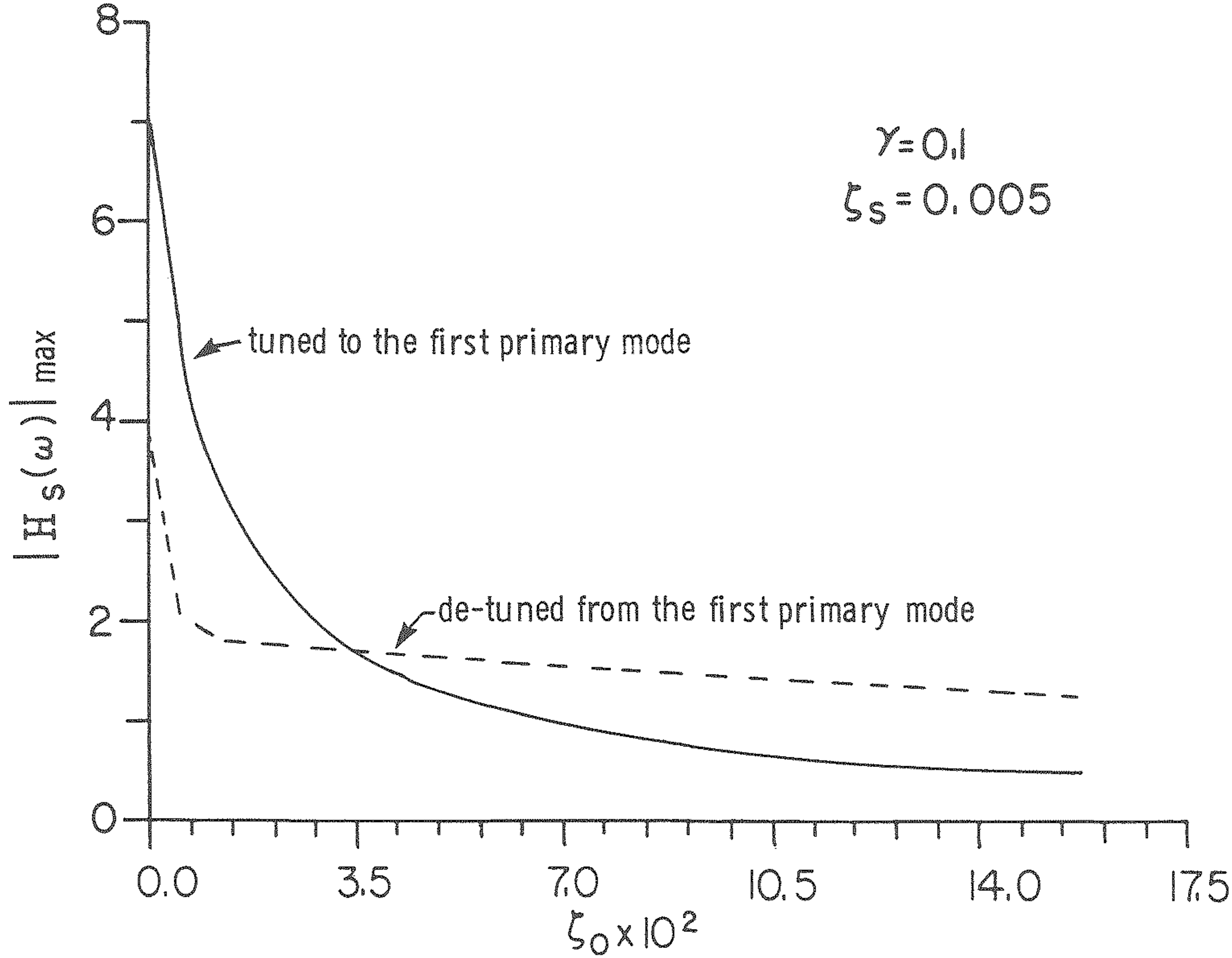

FIGURE 20 Maximum frequency response magnitude of a lightly damped heavy secondary system vs. damping of primary system.

CONCLUSIONS

The following conclusions can be drawn from the present numerical study of frequency response of a single-degree-of-freedom secondary system due to horizontal ground acceleration, which is transmitted through a 4-story primary structure of shear-wall type construction. The term "response" refers herein to the displacement of the secondary system relative to the supporting floor.

(1) The total response is contributed by the primary mode and the secondary mode. When tuning is approached, contributions from both the secondary mode and the tuned primary mode are enhanced.

(2) The highest response occurs when the secondary system is tuned to the first primary mode.

(3) Although tuning to a higher primary mode enhances contribution from that mode, contribution from the de-tuned first primary mode remains important, especially when the secondary system is heavy.

(4) When a primary mode is sufficiently de-tuned from the secondary mode, its contribution toward the total response is insensitive to the relative weight of the secondary system.

(5) Increasing the damping in the primary system for the purpose of reducing the response is not very effective for heavy secondary systems.

(6) Variation of the tuning parameter has no apparent effect on the effectiveness in which the response is reduced by increasing the primary system damping.

(7) For a heavy secondary system with an extremely light damping, response in a de-tuned case may be higher than that of the tuned case when damping in the primary system is increased beyond a certain level.

(8) Essentially same results are obtained for classically damped and nonclassically damped primary structures.

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