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Simplified Seismic Collapse Capacity-Based Evaluation and Design of Frame Buildings with and without Supplemental Damping Systems

by
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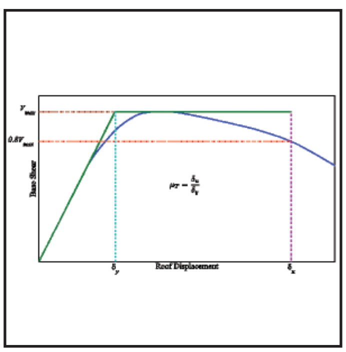

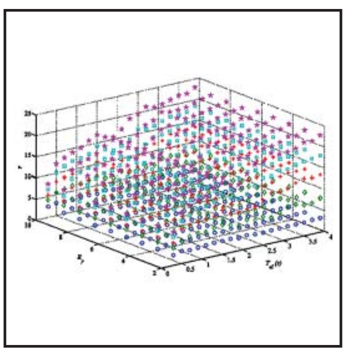

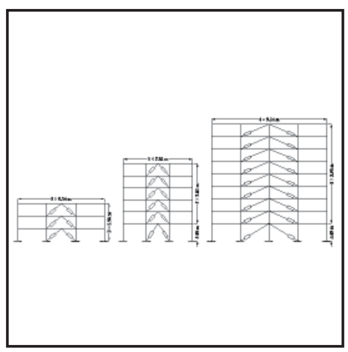

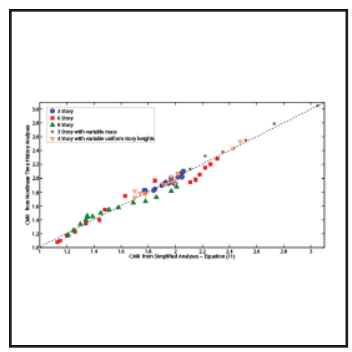

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PREFACE

MCEER is a national center of excellence dedicated to the discovery and development of new knowledge, tools and technologies that equip communities to become more disaster resilient in the face of earthquakes and other extreme events. MCEER accomplishes this through a system of multidisciplinary, multi-hazard research, education and outreach initiatives.

Headquartered at the University at Buffalo, State University of New York, MCEER was originally established by the National Science Foundation (NSF) in 1986, as the first National Center for Earthquake Engineering Research (NCEER). In 1998, it became known as the Multidisciplinary Center for Earthquake Engineering Research (MCEER), from which the current name, MCEER, evolved.

Comprising a consortium of researchers and industry partners from numerous disciplines and institutions throughout the United States, MCEER's mission has expanded from its original focus on earthquake engineering to one which addresses the technical and socioeconomic impacts of a variety of hazards, both natural and man-made, on critical infrastructure, facilities, and society.

MCEER investigators derive support from the State of New York, National Science Foundation, Federal Highway Administration, National Institute of Standards and Technology, Department of Homeland Security/Federal Emergency Management Agency, other state governments, academic institutions, foreign governments and private industry.

This report describes a simplified procedure for estimating the seismic sidesway collapse capacity of frame building structures. The procedure is then extended to quantify the seismic collapse capacity of buildings incorporating supplemental damping systems. The proposed procedure is based on a robust database of seismic peak displacement responses of viscously damped nonlinear single-degree-of-freedom systems for various seismic intensities and uses nonlinear static (pushover) analysis without the need for nonlinear time history dynamic analysis. The proposed procedure is assessed by comparing its collapse capacity predictions on 1,470 different building models with those obtained from incremental nonlinear dynamic analyses. A straightforward unifying collapse capacity based design procedure aimed at achieving a pre-determined probability of collapse under maximum considered earthquake event is also introduced for structures equipped with viscous dampers (linear and nonlinear) and hysteretic dampers. The proposed simplified procedure offers a simple, yet efficient, computational/analytical tool that is capable of predicting collapse capacities with acceptable accuracy for a wide variety of frame building structures incorporating several types of supplemental damping systems.

ABSTRACT

A simplified procedure is developed for estimating the seismic sidesway collapse capacity of frame building structures. The procedure is then extended to quantify the seismic collapse capacity of buildings incorporating supplemental damping systems. The proposed procedure is based on a robust database of seismic peak displacement responses of viscously damped nonlinear single-degree-of-freedom systems for various seismic intensities and uses nonlinear static (pushover) analysis without the need for nonlinear time history dynamic analysis. The proposed procedure is assessed by comparing its collapse capacity predictions on 1,470 different building models with those obtained from incremental nonlinear dynamic analyses. A straightforward unifying collapse capacity based design procedure aimed at achieving a pre-determined probability of collapse under maximum considered earthquake event is also introduced for structures equipped with viscous dampers (linear and nonlinear) and hysteretic dampers. The proposed simplified procedure offers a simple, yet efficient, computational/analytical tool that is capable of predicting collapse capacities with acceptable accuracy for a wide variety of frame building structures incorporating several types of supplemental damping systems.

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SECTION 1

INTRODUCTION

1.1 Objectives and scope of research

Quantification of structural seismic performance based on collapse capacity under extreme events provides a rational and powerful framework for evaluating existing buildings and designing new ones. Recently, ASCE7-10standard (2010) adopted the "anticipated reliability" concept as the design objective for seismic force-resisting systems (SFRS). Table C.1.3.1.b of ASCE7-10 (reprinted in Table 1-1) lists the required maximum probability of global collapse of seismic force-resisting systems under maximum considered earthquake (MCE) ground motions by various risk categories. Note that listed values are based on assumed fragility curves with a single logarithmic standard deviation value of 0.6.

Table 1-1 Anticipated reliability for newly designed SFRS per ASCE7-10

Risk Category	Maximum probabilty of global collapse under MCE ground motions
I and II	10%
III	6%
IV	3%

The Applied Technology Council (ATC), under the direction of the Federal Emergency Management Agency (FEMA), published the FEMA P695 document (FEMA, 2009), which describes a rational methodology for evaluating seismic design parameters based on pre-determined structural collapse safety margins under MCE ground motions. The new systems to be introduced as SFRS in ASCE 7-10 are required to satisfy prescribed minimum probabilities of collapse per FEMA P-695 (10% for average for each performance group and 20% for individual archetype buildings) under MCE ground motions. The FEMA P695 methodology requires numerous rigorous, and computationally demanding nonlinear time history dynamic analyses of several archetype building models, up to their collapse limit under an ensemble of prescribed ground motions.

On the other hand, buildings incorporating supplemental damping systems that are designed according to Chapter 18 of ASCE 7-10 are not required to meet any specific probability of collapse at MCE level. Although it is clear that incorporating supplemental damping systems raises the seismic performance of frame buildings, this improved performance is unknown in terms of probability of collapse under MCE ground motions. The simplified procedure developed in this report allows the evaluation and design of frame buildings with and without supplemental damping systems based on probability of collapse under MCE ground motions, thereby unifying the two types of building systems (i.e. conventional and equipped with supplemental damping systems). In addition, the procedure takes into account the various sources of uncertainty in the development of the reference fragility curves.

Simplified methods of analysis, for which equivalent single-degree-of-freedom (SDOF) systems are used as a proxy for estimating peak displacements of nonlinear fixed base and base isolated structural systems, have been widely used in seismic design(Ramirez *et al*., 2002a). For example, such procedures were adopted by the ASCE7-10standard (2010) and the NEHRP (2003) and AASHTO (2010) provisions. The simplified procedure proposed in this reports built upon the same concepts to estimate the sidesway collapse capacity of nonlinear building structures incorporating supplemental damping systems and to design new ones.

The proposed procedure requires only the application of a standard pushover analysis of the building under evaluation without the need of specialized software, thereby making the application of the FEMA P695 methodology less computationally demanding and more user-friendly. The proposed procedure is based on replacing a multi-degrees-of-freedom (MDOF) structural system with or without supplemental damping systems with a fictitious SDOF system characterized by a bilinear relationship between the lateral force and roof displacement obtained from pushover analysis. The collapse level spectral acceleration associated with the resulting nonlinear SDOF system is then quantified by utilizing the developed parameters from a large database of SDOF responses obtained from nonlinear time history dynamic analyses. The procedure is outlined with simple steps, and its effectiveness is demonstrated by comparing predicted collapse capacities with the values obtained from incremental dynamic analyses. Simplified design procedures are also developed for frame buildings incorporating linear and nonlinear viscous and hysteretic dampers based on achieving a pre-determined probability of collapse under MCE ground motions. The proposed simplified procedure offers a simple, yet efficient, computational/analytical tool that is capable of predicting collapse capacities with acceptable accuracy for a wide variety of frame building structures incorporate several types of supplemental damping systems.

1.2 Outline of the report

The report is organized into seven sections with the following contents:

Section 2 presents a literature review of the various seismic collapse analysis methods that have been developed. The types of collapse are described and the factors causing local or global dynamic instability in the structure are introduced. Previous studies on estimating the seismic capacity from nonlinear static (pushover) analyses are reviewed and the general methodology of FEMA P695 for the quantification of seismic collapse capacity of buildings is summarized. The development of the seismic design procedures for the buildings equipped with supplemental damping systems is also introduced in this section.

The proposed simplified procedure for quantifying collapse capacity of frame buildings is developed and assessed in Section 3. The procedure aims at estimating the collapse margin ratio (*CMR*) of frame building structures without supplemental damping systems, as defined by FEMA P695. The proposed procedure is based on the development of a robust database of seismic peak displacement responses of nonlinear SDOF systems for various seismic intensities and uses pushover analysis without the need for nonlinear time history dynamic analysis. Nonlinear response database of the SDOF oscillators are developed and the pertinent key parameters of the proposed simplified analysis are presented. The analysis is extended to MDOF systems and summarized in a step-by-step procedure. The proposed simplified procedure is then assessed by comparing its collapse capacity predictions on 72 different building structures with those obtained by incremental dynamic analyses.

Section 4 describes the application of the simplified procedure developed in Section 3 for buildings equipped with linear and nonlinear viscous dampers. For this purpose, the proposed procedure addressed in Section 3 is modified to account for higher critical viscous damping ratios. The theory behind the development of the damping ratios of SDOF system models is presented and extended to MDOF systems through modal analysis. Again, a step-by-step procedure is presented for quantification of frame buildings incorporating viscous dampers. The proposed procedure is then assessed by comparing its collapse capacity predictions on 1,190 different building models incorporating linear and nonlinear viscous dampers with those obtained from incremental nonlinear dynamic analyses. A straightforward collapse capacity based design procedure aimed at achieving a pre-determined probability of collapse under MCE event is also introduced for viscously damped structures without extreme soft story irregularities. The detailed calculation of the proposed design procedure is also presented for an example building equipped with linear and nonlinear viscous dampers.

Section 5 describes the application of the simplified procedure developed in Section 3 for buildings with hysteretic dampers. Characteristics of hysteretic dampers are reviewed and the parameters affecting the response of structures with hysteretic dampers are investigated. The proposed simplified procedure is assessed by comparing its collapse capacity predictions on 210 different building models equipped with hysteretic dampers with those obtained by incremental nonlinear dynamic analyses. A straightforward collapse capacity based design procedure aimed at achieving a pre-determined probability of collapse under MCE event is also introduced for hysteretically damped structures without extreme soft story irregularities. This section is concluded by a detailed design calculation for an example building with hysteretic dampers.

Section 6 includes detailed design calculations for a five-story office building equipped with viscous dampers (linear and nonlinear) or hysteretic dampers. The considered building is first designed through equivalent lateral force (ELF) procedure according to Section 18 of ASCE 7-10.The collapse safety of the building with and without each of the three considered damping systems is then quantified and the properties of the supplemental damping systems are re-designed for a pre-determined 2% probability of collapse under MCE ground motions. The actual collapse capacity of this re-designed building is also evaluated through nonlinear time history dynamic analysis.

The main findings and contributions from the study are summarized and areas of future research are suggested inspection 7.

SECTION 2
LITERATURE REVIEW

2.1 Seismic collapse definition and assessment

Collapse prevention has been accepted by building codes as a major objective of performance-based seismic design of buildings. Seismic collapse occurs when the structural system, locally or globally, loses the capacity to carry gravity loads in a strong ground motion. The imposed dynamic instability in a sidesway mode is usually caused by second order (P-Δ) effects, backbone deterioration in stiffness and strength of the components and cyclic deteriorations. As addressed by this report, collapse is referred to as global incremental sidesway instability of at least one story of the frame building structure. Such a phenomenon happens when the displacement of an individual story is large and second order effects overcome the story shear resistance. Large P-Δ effects can cause structural instability and collapse even without the presence of degrading components. A number of studies have focused on P-Δ effects on the seismic response of non-degrading structures with positive post-yield stiffness (Jennings and Husid, 1968; Sun et al., 1973; MacRae, 1994; Bernal, 1998; Takizawa and Jennings, 1980; Vian and Bruneau, 2001, 2003; Wei et al., 2011; Adam and Jäger, 2012a, 2012b). Also, research has been carried out in developing deteriorating nonlinear component models in order to reproduce hysteretic behavior obtained from experimental results (Takeda et al., 1970; Bouc, 1967; Wen, 1976, 1980; Otani, 1981; Sivaselvan and Reinhorn, 2000, 2006; Song and Pincheria, 2000; Ibarra et al. 2002; Lignos and Krawinkler, 2009).

Several studies focused on the assessment of the dynamic instability of 5% damped SDOF systems due to the combined action of P-Δ effects and degrading components (Takizawa and Jennings, 1980; Bernal, 1987; MacRae, 1994; Williamson, 2003; Ibarra and Krawinkler, 2011).Miranda and Akkar (2003) studied lateral strengths required to avoid dynamic instability of bilinear SDOF systems with negative post-yield stiffness. The results showed that the lateral strength required to avoid collapse increases as the post-yield descending branch of the force–deformation relationship becomes steeper. Adam et al. (2004) proposed a procedure that considers P-Δ effects in non-degrading multi-degrees-of-freedom (MDOF) structures through an equivalent SDOF system with properties defined based on the results from a pushover analysis.

2.2 Seismic collapse capacity estimation from pushover curve

Several studies have aimed at predicting the collapse capacity of buildings from nonlinear static (pushover) analysis. Vamvatsikos and Cornell (2006) developed the SPO2IDA software that converts a first-mode dominated static pushover curve into 16th-, 50th- and 84th-percentiles incremental dynamic analysis (IDA) curves. The tool is based on equating the elastic segment of the normalized pushover curve with that of the elastic fractile IDA region. The curve is then characterized by the post-yield hardening ratio, the end-of-hardening ductility, the slope of the post-capping branch, the residual strength and the ultimate ductility. The tool then uses the nonlinear time history dynamic analysis results of numerous SDOF oscillators with various quadri-linear backbone curves and pinching hysteretic behavior, all subjected to the same 30 ground motion records. Although the tool is developed originally for structures with 5% critical damping ratio, it is also applicable to other damping ratios. Han et al. (2010) used a tri-linear fit of a modal pushover analysis to estimate the collapse fragility curve considering the empirical equations of collapse strength ratio for strength-limited bilinear SDOF systems. Higher damping values are taken into account by a modification factor applied to the collapse strength ratio at 5% critical damping. Shafei et al. (2011) also introduced a pushover-based approach, which uses generic MDOF systems and best-fitted first-order regression models to predict the collapse capacity of buildings. The proposed procedure is different from the aforementioned studies in which an equivalent SDOF system was utilized to estimate the collapse capacity.

Considering the destabilizing gravity loads due to P-Δ effects during seismic events for various critical damping ratios, Adam and Jäger (2012) assessed the earthquake-induced sidesway collapse of inelastic non-deteriorating moment-resisting frame structures. The proposed collapse capacity spectrum methodology requires double nonlinear static pushover analyses with and without gravity loads to obtain global hardening and post-yielding stiffness ratios. Recently, Peruš et al. (2013) introduced an expandable web-based methodology for the prediction of approximate incremental dynamic analysis (IDA) curves of reinforced concrete structures with various damping ratios that requires seven parameters, five of which describe the idealized pushover curve.

De Luca et al. (2013) investigated an array of combinations of different yielding plateau levels and elastic secant values in search of the optimal bilinear fit to twelve generalized highly- nonlinear force-displacement backbone curves for the purpose of seismic response prediction. The initial stiffness was set at 10% or 60% of the nominal yield strength combined with two yield plateau levels set at 80% (L) or 100% (H) of the peak shear strength, respectively (see Figure 2-1). The results showed that the bilinear fit with the same elastic stiffness and peak shear strength of the original force-displacement backbone curve (10%H) can be an unbiased and robust alternative to the original curve, at least up to the point where the structure loses approximately 20% of its maximum strength.

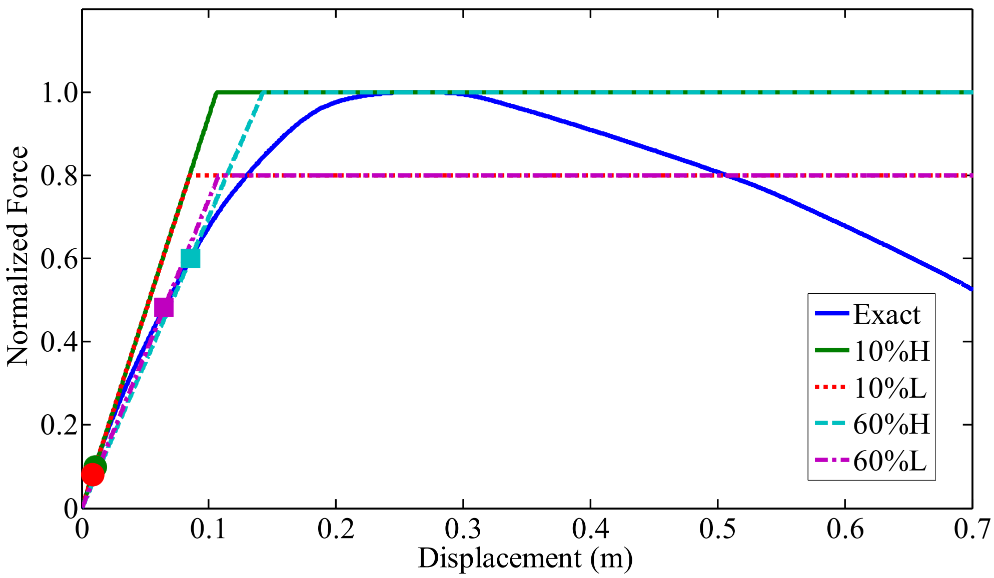


Figure 2-1 Generalized elastic-hardening-negative capacity curve and its corresponding fits (after De Luca et al. (2013))

2.3 Overview of the FEMA P695 methodology

The overall objective of the FEMA P695 (ATC-63) methodology is validating the seismic performance factors of a newly-proposed seismic force-resisting system as defined in ASCE 7-10 (i.e. the seismic response modification factor, R, the displacement amplification factor, C_d, and the overstrength factor, Ω_o) by quantifying its overall collapse capacity. The FEMA P695 assessment process is illustrated by a flowchart in Figure 2-2. The procedure includes gathering design provisions, component testing information and professional design experience. Several prototypical representatives of the current building stock are then designed following the design provisions. These building prototypes are called archetypes. The archetypes need to cover the expected range of building geometrical and structural parameters such as: building heights, seismic design categories and site classes, seismically effective weight tributary to lateral load resisting system, number of bays and bay width, story height, etc. Analytical models are then developed for the archetypes using modeling techniques that take into account deterioration parameters of the components.

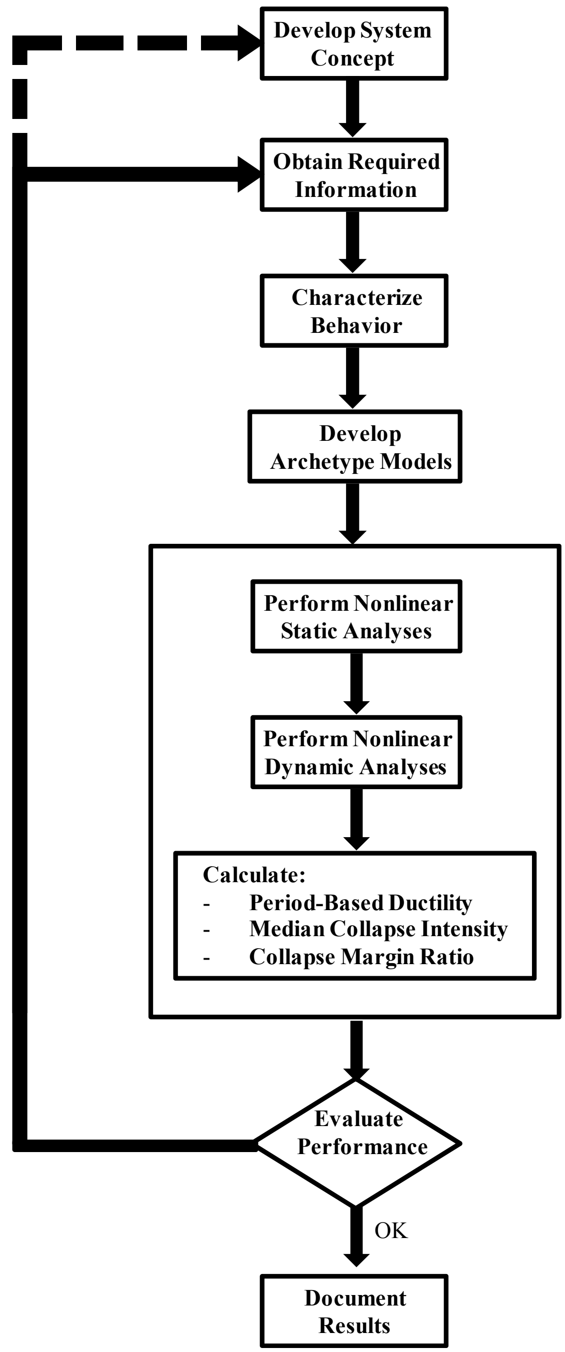


Figure 2-2 FEMA P695 procedure for quantitatively establishing and documenting seismic performance factors

Ibarra and Krawinkler (2005) proposed using IDAs (Vamvatsikos and Cornell, 2002) to determine the spectral acceleration of each ground motion that causes the structure to collapse. This approach requires nonlinear time history dynamic analyses with increasing values of spectral acceleration until the structure reaches the collapse point. Results of an example incremental dynamic analysis are illustrated in Figure 2-3, where each symbol corresponds to the results of a nonlinear time history dynamic analysis under a ground motion scaled to an intensity level. Collapse occurs by excessive lateral displacement of the SFRS as a result of dynamic instability, which is reflected by the flat portion (collapse capacity) of each IDA curve.

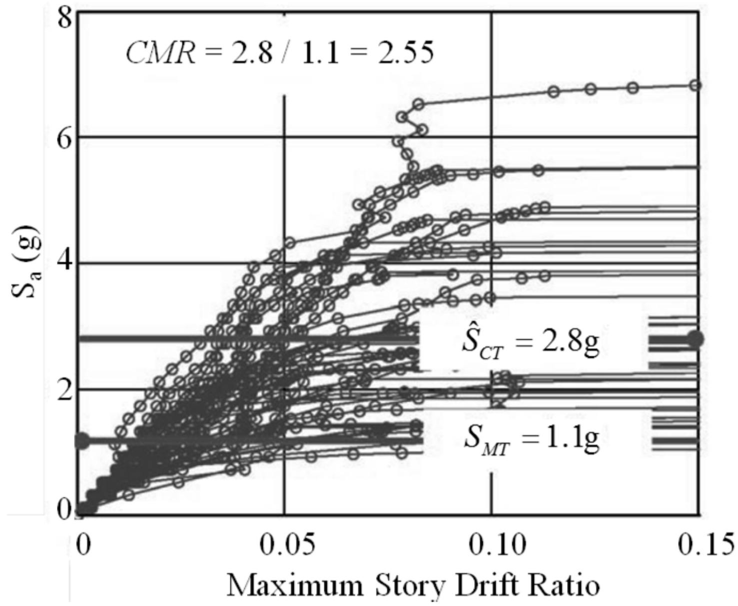


Figure 2-3 Example IDA plot (after FEMA (2009))

The probability of collapse at a given spectral acceleration is then quantified as the number of records causing collapse when divided by the total number of records. The collapse fragility curve shows the probability of collapse versus median ground motion spectral acceleration at the fundamental period of the building. The fragility curve is basically a cumulative distribution function of median collapse spectral accelerations from individual records. The collapse fragility curve is assumed to follow approximately a cumulative lognormal distribution (Vamvatsikos and Cornell, 2002; Bradley and Dhakal, 2008; Ghafory-Ashtiany et al., 2011). Figure 2-4a shows an example of a cumulative distribution plot obtained by fitting a lognormal distribution through the collapse data points from Figure 2-3.

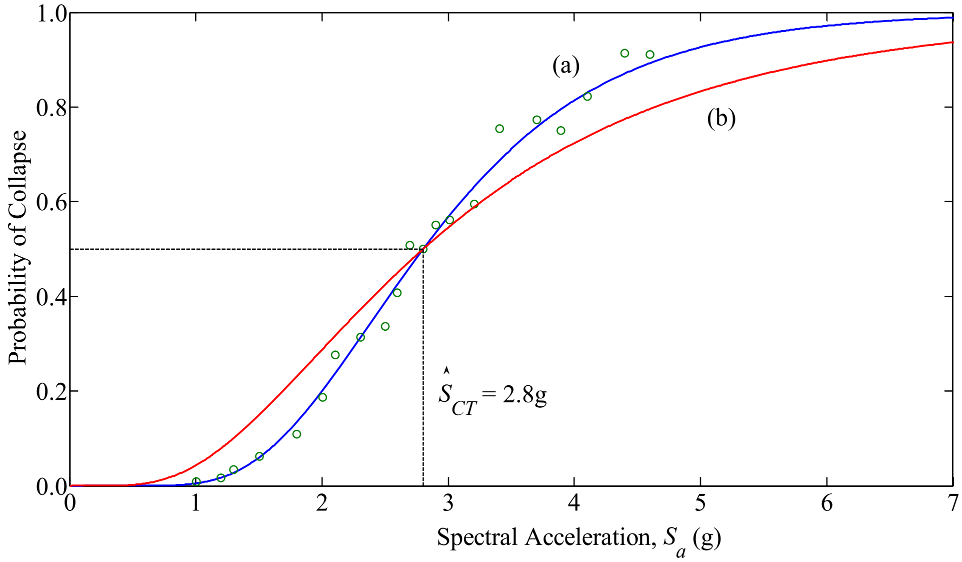


Figure 2-4 Example collapse fragility curve with a) ground motion record-to-record uncertainty b) total system collapse uncertainty

The cumulative lognormal fragility is determined by two parameters: 1) median collapse spectral acceleration at the fundamental period of the structure ($\hat{S}_{CT}$) that corresponds to a 50% probability of collapse, and 2) the standard deviation of the natural logarithm of the values (β_{RTR}) that shows the dispersion in the results due to record-to-record variability (uncertainty).To consider all other sources of uncertainty, the standard deviation value is increased to the total system collapse uncertainty (β_{TOT}). The total system collapse uncertainty includes record-to-record collapse uncertainty, design requirements-related collapse uncertainty, test data-related collapse uncertainty and modeling-related collapse uncertainty. Figure 2-4b illustrates the example collapse fragility curve with the same median collapse spectral acceleration but with an increased total system collapse uncertainty. As shown in the figure, additional uncertainty flattens the curve and causes an increase in the probability of collapse at the MCE spectral acceleration located in the low tail region of the curve.

The ratio between the median 5% damped collapse spectral acceleration at the fundamental period of the structure and the MCE spectral acceleration (S_{MT}) is defined as collapse margin ratio:

$$CMR = \frac{\hat{S}_{CT}}{S_{MT}} \tag{2.1}$$

The collapse capacity can be influenced by the frequency content and spectral shape of each record of the ground motion record set. FEMA P695 defines an adjusted collapse margin ratio (*ACMR*) as the product of the *CMR* and spectral shape factor (*SSF*) to account for the effect of the spectral shape. The spectral shape factor is basically dependent on the fundamental elastic period of the building under evaluation and the target ductility factor (μ_T).

The acceptance criteria includes exceedance of the average and individual value of the *ACMR* for each performance group from *ACMR* value corresponding to 10% and 20% probabilities of collapse under MCE ground motions, respectively. If either does not meet the required performance, the assumed seismic performance factors must be modified and the archetypes are redesigned. The procedure will be repeated until the SFRS with a proposed set of seismic performance factors provides adequate collapse safety. Further information about the detailed requirement of the methodology can be found in FEMA P695 (2009) and Haselton et al. (2011)

2.4 Seismic design of supplemental damping systems

A considerable amount of research has been conducted to develop innovative earthquake-resistant systems that improve seismic performance levels while keeping construction costs reasonable. Most of these systems are designed to dissipate the seismic energy introduced into a structure by supplemental damping mechanisms. Supplemental damping systems use special devices that are often referred to as mechanical dampers. This supplemental mechanical energy dissipation, activated through movement of the main structural system, reduces the overall dynamic response of the structure during a major earthquake. Furthermore, the main elements of the structure are protected by diverting the seismic energy to these mechanical devices that can be inspected or even replaced following an earthquake. Ideally, if all the seismic energy is absorbed by the mechanical dampers, the main structure will not sustain any damage. Further information about the supplemental damping systems and passive control of the structures can be found in Soong and Constantinou (1994), Housner et al. (1997), Soong and Spencer (2002), Christopoulos and Filiatrault (2006), Zahrai and Hamidia (2009a, 2009b) and Hamidia and Hosseini (2010).

Force-based seismic design methods are the basis of the seismic design provisions contained in most building codes worldwide and require certain levels of lateral stiffness, strength and ductility. In low-intensity earthquakes, sufficient lateral stiffness allows for protection against damage to architectural and nonstructural components. In moderate-intensity earthquakes, sufficient lateral strength limits important damage to the main structure. In major earthquakes, adequate lateral ductility of the structures needed to allow for large inelastic displacements but without collapse of the vertical load-carrying system. In most current design codes, the force-based seismic design procedure is applied to first-mode dominated structures through equivalent lateral force (ELF) methods. Design level spectral acceleration, seismic weight of the structure, importance factor and response modification factor are the ingredients of this method. For tall and irregular structures for which higher mode and torsional effects are important, the force-based seismic design procedure is applied through the response spectrum analysis (RSA) method. This method is based on a dynamic analysis procedure using modal superposition. However, the design base shear needs to be scaled to the design base shear obtained from ELF method. Thus, the main purpose of the RSA method is to obtain an improved distribution of the design lateral forces along the height of the structure.

Ramirez et al. (2001, 2002a, 2002b) developed an effective method to simplify the design procedures and reduce computational efforts for frame building with supplemental damping systems. This method assumes that a viscously damped structure with equivalent linear stiffness of the SFRS as a proxy to the damped structure characterized with nonlinear behavior of the SFRS. The total effective damping is introduced as the summation of the structural inherent damping, viscous damping of the supplemental damping system, and hysteretic damping from post-yield hysteretic behavior of the structure. The spectral acceleration, reduced by a function of the total effective damping, is applied to compute the maximum responses, velocities, and accelerations of the damping system and the SFRS. The procedure takes only two modes into account: the fundamental and residual modes. FEMA 368/369 (2001) adopted the simplified method and incorporated it into an appendix to Chapter 13. In NEHRP (2003), this appendix became Chapter 15 with editorial revisions. ASCE 7-10 (2010) adopted the entire Chapter 15 of FEMA 450 and formed Chapter 18 for the seismic design of structures with damping systems. Chapter 6 of this report includes a complete design example using the ASCE7-10 methodology. Also noteworthy, the current research is built upon the concept of the research conducted by Ramirez et al. (2001, 2002a, 2002b) and Whittaker et al. (2003).

Seismic response modification factors (R) are key components of force-based seismic design approaches, and are difficult to justify for various structural systems. Variations in R values result in significant alterations in the design base shear. The issue is more dominant for structures expected to have larger ductility demands during major earthquakes. In addition, the force-based methods are essentially developed to meet a single performance level (life safety).

Filiatrault et al. (2001) proposed a simple procedure to directly determine the damping constant of each linear viscous damper in the building given the equivalent viscous damping ratio in a particular mode of vibration. The proposed procedure uses the fictitious elastic springs and that are incorporated in the structure at the location of the viscous dampers in order to obtain a modified fundamental period associated with the desired level of equivalent viscous damping ratio. The fictitious spring constants are distributed according to the interstory lateral stiffness of the unbraced structure thus the damping matrix would be proportional to the global stiffness matrix of the structure. A simple relationship is then used to link the required spring constants of the fictitious springs to the required damping constants of the corresponding viscous dampers. The procedure can be also used to determine the nonlinear viscous damping constants based on equal energy dissipation with the linear viscous dampers.

Design of SFRS with hysteretic dampers is generally aimed at obtaining stiffness of the connecting braces and the slip load of the damping device. Baktash and Marsh (1987) proposed to choose a slip load such that the maximum amount of energy is dissipated when the yield moment is reached at some locations in the frame. This approach suggests a slip load independent of the type and intensity of seismic excitation and only based on structural property. Filiatrault and Cherry (1988) proposed a procedure for the design of the buildings with hysteretic damping systems. They proposed the incorporation of the connecting braces to achieve a fundamental period of the braced structure less than or equal to 40 percent of the unbraced structure. The total slip load of the dampers can then be determined by a design slip load spectrum that takes into account the frequency content of the ground motion and the dynamic properties of the structure with and without the added bracing members. Ciampi et al. (1995) considered the design of hysteretic bracing systems using an approach based on inelastic response spectrum in terms of ductility demands obtained by averaging the results of nonlinear time history dynamic analyses over an ensemble of five artificial accelerograms. Dowdell and Cherry (1996) proposed a design procedure for SDOF systems equipped with friction dampers taking advantage of transfer functions concept. The procedure is based on minimizing the steady-state root-mean-square (RMS) interstory drift response of a structure exited by a stationary random process. Fu and Cherry (2000) proposed a simplified design procedure for SDOF friction damped steel frames based on the determination of an equivalent seismic response modification factor (R). They adopted a tri-linear model to represent the hysteretic properties of the friction-damped structure to account for the slipping of the friction damper and yielding of the structure. The optimal planar positioning of the friction dampers in asymmetric elastic single-story structures has been investigated by De LaLlera et al. (2005). A simple equation determining the optimum location of the dampers was derived assuming that the empirical center of balance needs to be at equal distance from all edges of the building. Design of such systems can also be performed using the Chapter 18 of ASCE7-10. For this purpose, the equivalent damping ratio must be related to the slip load and the associated stiffness of the bracing members as addressed by Whittaker et al. (2003).

The concept of performance-based earthquake engineering relies primarily on the definition of multiple performance objectives for different intensity levels of the seismic input. The capacity spectrum analysis method is a graphical representation of the equivalent secant linearization of a nonlinear system. Seismic demand spectra are plotted in a spectral acceleration-displacement format for different values of the equivalent viscous damping and are compared to the pushover capacity curve of the structure. The intersection of the capacity curve with the seismic demand spectra represents possible maximum responses of the system with respect to displacements and accelerations. Recently, Guo and Christopoulos (2013) developed a direct performance-based design procedure for the low- to medium-rise frame structures using supplemental dampers. The key aspect of the procedure is the ability to directly assess the performance of different damping scenarios that satisfy a given set of performance targets using graphical tools called performance spectra (P-Spectra). With the help of the proposed procedure, performance of any selected damping solution can be assessed for multiple hazard levels.

Displacement of the structural system is the center of direct displacement-based seismic design methods. The structure is modeled as an equivalent SDOF system with equivalent elastic lateral stiffness and total damping equivalent viscous damping properties representative of the global behavior of the structure at a pre-determined target displacement. The total viscous damping of the structure is obtained by adding inherent damping to the equivalent viscous damping provided by the hysteretic response of the structure at the target displacement. Recently, Lago (2011) proposed a direct displacement-based design procedure using the spectral displacement spectra for design of structures with supplemental damping systems.

The capacity spectrum method and direct displacement-based design methods require an estimation of the equivalent viscous damping ratios as well as knowledge of the damping coefficients for the reduction of the spectral acceleration and spectral displacement where they are not well established for some type of structural systems. To obtain this information, system level testing is required in parallel with the development of the specialized structural analysis methods.

Nonlinear response history dynamic analysis is prescribed for very tall and/or high irregular important structures. The ground motion input must be represented by an ensemble of acceleration time-histories that are compatible with the seismic hazard level at specific site. A nonlinear model of the structure needs to be generated. The corresponding model must include a realistic plasticity model of the components. The response is then estimated as the peak response between selected ground motion and the mean of the response parameters. The scaling and also selection methods of the ground motions can affect the results of the analysis. Additional information about various seismic design methods for structures equipped with various types of supplemental damping systems can be found in (Christopoulos and Filiatrault, 2006).

In addition to inherent shortcomings of the previous methods, none of them aim at achieving a pre-determined probability of collapse. The ASCE7-10 standard has recently introduced new "risk consistent" seismic design maps for a collapse risk of 1% in 50 years (Luco et al., 2007). This represents an assumed collapse probability of 10% under MCE intensity. Although this is a significant step forward, the approach does not account for various sources of uncertainty in the development of the fragility curves and uses a limited database of fragility functions regardless of the structural systems. In addition, different probabilities of collapse cannot be explicitly addressed by the proposed design maps.

The general objective of the FEMA P695 methodology is to predict the median collapse capacity of the newly-proposed structural system. However, it provides a rational framework for quantification of the seismic sidesway collapse capacity of the structures. The approach is adopted in this report for development of a unifying design procedure aiming at achieving a pre-determined probability of collapse under MCE event for building incorporating both conventional seismic force-resisting systems and supplemental damping devices of various types.

SECTION 3

SIMPLIFIED SEISMIC COLLAPSE ANALYSIS OF FRAME BUILDINGS

3.1 Introduction

This section presents the development and assessment of the simplified procedure for estimating the seismic sidesway collapse margin ratio (*CMR*) of frame building structures. The proposed procedure is based on the development of a robust database of seismic peak displacement responses of nonlinear single-degree-of-freedom systems for various seismic intensities and uses nonlinear static (pushover) analysis without the need for nonlinear time history dynamic analysis. The proposed simplified procedure is assessed by comparing its collapse capacity predictions on 72 different frame building structures with those obtained by incremental dynamic analyses.

3.2 Nonlinear response database of single-degree-of-freedom oscillators

Nonlinear time history dynamic analyses are first performed on a wide range of SDOF oscillators with a bilinear force-displacement relationship characterized by a linear segment with initial stiffness (K_{el}), followed by a post-yield perfectly plastic segment with constant yield force (F_y) without hardening or cyclic deterioration. Figure 3-1 illustrates the force-displacement relationship of the analyzed systems and identifies the pertinent key parameters. The dynamic response of each SDOF system is obtained in terms of its elastic period of vibration (T_{el}) and its yield reduction factor (R_y), determined as follows:

$$T_{el} = 2\pi\sqrt{\frac{m}{K_{el}}} \tag{3.1}$$

$$R_y = \frac{S_{MT}}{A_y} \tag{3.2}$$

where m is the mass of the oscillator (normalized herein to1kg), S_{MT} is the MCE level elastic spectral acceleration at a period T_{el} and critical damping ratio of 5% and A_y represents the yield pseudo-acceleration, defined as:

$$A_y = \frac{F_y}{m} \tag{3.3}$$

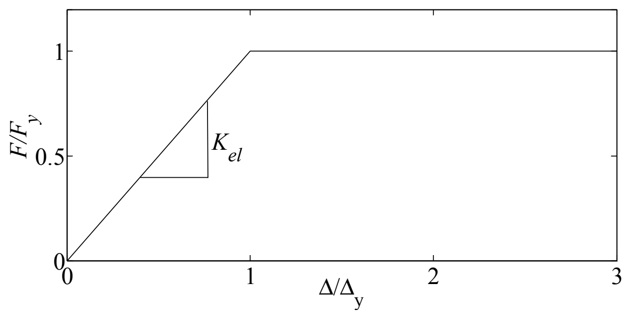


Figure 3-1 Force–displacement characteristics of bilinear SDOF system with target ductility of 3

Elastic periods (T_{el}) ranging from 0.1 s to 4.0 s with 0.1 s increments were selected for the analyses thereby encompassing the fundamental periods of vibration of a wide range of building structures. Furthermore, a range of R_y values between 1 and 10 with an increment of 0.5 was considered. All the nonlinear SDOF systems considered in this study were assumed to have an inherent equivalent viscous damping ratio of 5% of critical to be consistent with common seismic design assumptions.

All SDOF systems described above were subjected to the set of 44 far-field ground motions adopted by the FEMA P695 methodology at various intensities. Table 3-1 lists the earthquake events and corresponding stations. The selection of the historical records for the FEMA P695 far-field ensemble was based on a number of criteria related to source magnitude, source type, site condition, site-source, amplitudes, number of records per event, and strong-motion instrument capability (Haselton, 2006). The set of ground motions used in this study was first normalized and scaled to the MCE level spectral acceleration at a period T_{el} (S_{MT}) consistent with a seismic design category (SDC) D_{max} per ASCE7-10and according to the FEMA P695 methodology. Normalization by peak ground velocity is a simple way to remove unwarranted variability between records due to inherent differences in event magnitude, distance to source, source type and site conditions, while still maintaining the inherent aleatory (i.e., record-to-record variability) (FEMA, 2009). Figure 3-2 shows normalized far-field record set spectra and median of the responses. Figure 3-3 shows the 5% damped MCE elastic response spectrum of ASCE 7-10 for SDC D_{max} and the median spectrum of the record set anchored to it at period of one second. The MCE response spectrum for SDC Dmax is also described by the equation shown in Figure 3-3.

Table 3-1 Summary of earthquake events and recording station data for the far-field record set

ID No.	Earthquake			Name of Recording	1-Sec Spec. Acc. (g)		Median of PGV	Normalization	PGA max	PGV max
	M	Year	Name	Station	Comp. 1	Comp.2	(in/sec)	Factor	(g)	(in/sec)
1	6.7	1994	Northridge	BeverlyHills-Mulhol	1.02	0.94	22.5	0.65	0.34	16.4
2	6.7	1994	Northridge	CanyonCountry-WLC	0.38	0.63	17.6	0.83	0.4	15.2
3	7.1	1999	Duzce,	TurkeyBolu	0.72	1.16	23.3	0.63	0.52	15.6
4	7.1	1999	Hector Mine	Hector	0.35	0.37	13.4	1.09	0.37	18.4
5	6.5	1979	Imperial Valley	Delta	0.26	0.48	11.2	1.31	0.46	17.2
6	6.5	1979	Imperial Valley	ElCentroArray#11	0.24	0.23	14.4	1.01	0.39	17.2
7	6.9	1995	Kobe, Japan	Nishi-Akashi	0.31	0.29	14.2	1.03	0.53	15.6
8	6.9	1995	Kobe, Japan	Shin-Osaka	0.33	0.23	13.3	1.1	0.26	16.8
9	7.5	1999	Kocaeli, Turkey	Duzce	0.43	0.61	21.3	0.69	0.25	16.4
10	7.5	1999	Kocaeli, Turkey	TurkeyArcelik	0.11	0.11	10.8	1.36	0.3	21.6
11	7.3	1992	Landers	YermoFireStation	0.5	0.33	14.8	0.99	0.24	20.4
12	7.3	1992	Landers	Coolwater	0.2	0.36	12.8	1.15	0.48	19.6
13	6.9	1989	Loma Prieta	Capitola	0.46	0.28	13.5	1.09	0.58	15.2
14	6.9	1989	Loma Prieta	GilroyArray#3	0.27	0.38	16.7	0.88	0.49	15.6
15	7.4	1990	Manjil, Iran	Abbar	0.35	0.54	18.6	0.79	0.4	17.2
16	6.5	1987	Superstition Hills	ElCentroImp.Co.	0.31	0.25	16.9	0.87	0.31	16
17	6.5	1987	Superstition Hills	PoeRoad(temp)	0.33	0.34	12.5	1.17	0.53	16.8
18	7	1992	Cape Mendocino	RioDellOverpass	0.54	0.39	17.9	0.82	0.45	14.4
19	7.6	1999	Chi-Chi, Taiwan	CHY101	0.49	0.95	35.7	0.41	0.18	18.8
20	7.6	1999	Chi-Chi, Taiwan	TCU045	0.3	0.43	15.3	0.96	0.49	15.2
21	6.6	1971	San Fernando	LA-HollywoodStor	0.25	0.15	7.0	2.1	0.44	16
22	6.5	1976	Friuli, Italy	Tolmezzo	0.25	0.3	10.2	1.44	0.5	17.6

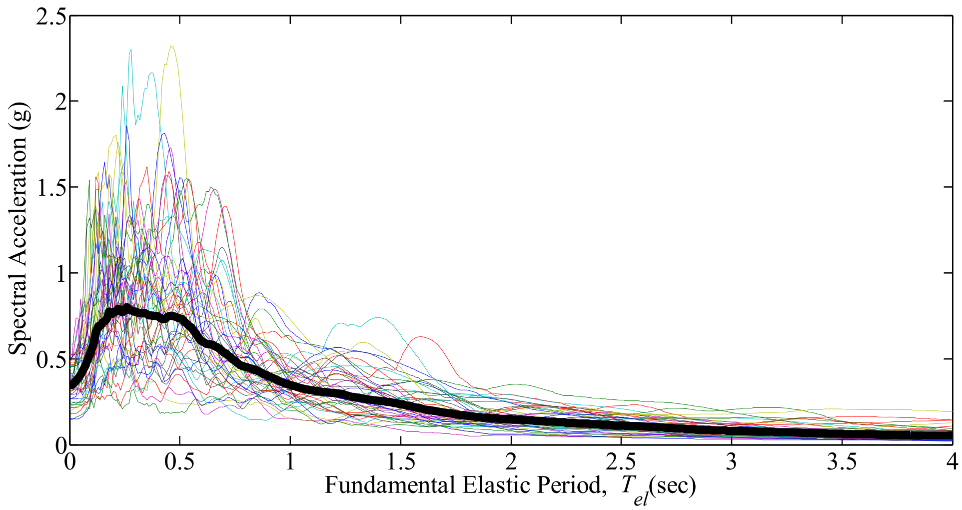


Figure 3-2 Normalized far-field record set spectra and median of the responses

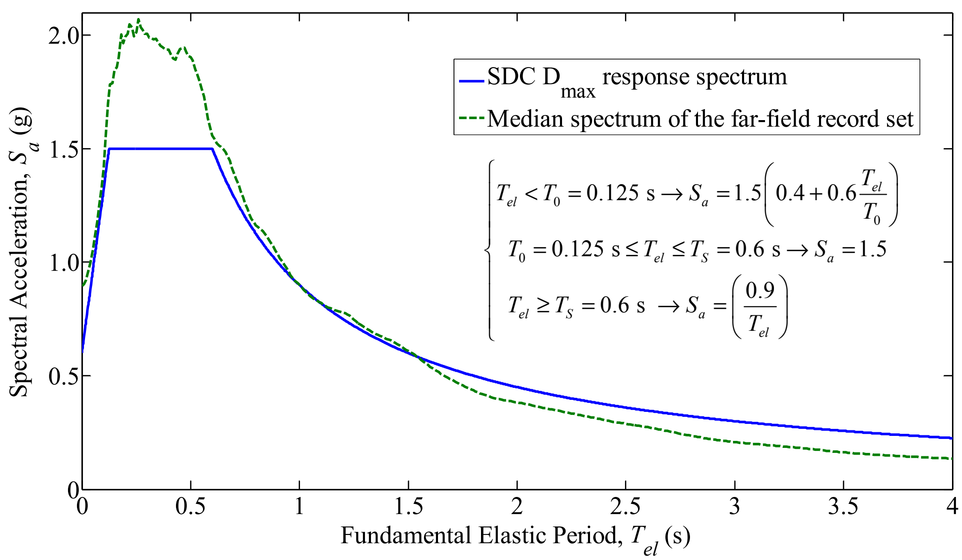


Figure 3-3 SDC D$_{max}$ response spectral acceleration

The amplitudes of the ground motions were then expressed for each SDOF system by an intensity factor (I) defined as:

$$I = \frac{S_a}{S_{MT}} \tag{3.4}$$

where S_a is the median 5% damped elastic spectral acceleration at a period T_{el}. Nonlinear time history dynamic analyses of each SDOF system were performed for intensity factors ranging from 0.1 to 150 at increments of 0.1. The intensity factor of 1.0 represents the scaled records at the MCE level of SDC D$_{max}$. Thus, a total of 40(T_{el}) × 19(R_y) × 1500(I) × 44(records) >50 million analyses were performed using the University at Buffalo Center for Computational Research (CCR) clusters, taking advantage of high performance computing concepts to generate a robust database of peak displacement seismic responses of nonlinear SDOF systems. Although SDC D$_{max}$ is used to define the MCE spectral accelerations, the results of this study can be extended to other seismic design categories by appropriately scaling the denominator of Equation 3.4.

For each considered SDOF system with known T_{el} and R_y, a family of fragility curves, giving the probability of exceeding a certain target peak displacement (Δ_T), can be constructed. Accordingly, the probability of exceedance at a given intensity factor is obtained by counting the number of records that cause the system to exceed Δ_T and dividing this number by the total number of records (44). The target peak displacement can also be expressed in terms of a target ductility ratio (μ_T) as follows:

$$\mu_T = \frac{\Delta_T}{\Delta_y} \tag{3.5}$$

where Δ_y is the yield displacement. For each SDOF oscillator under consideration, the intensity at which half of the records cause the system to exceed the considered target ductility ratio is defined as the median exceedance intensity (I_{med}). As shown in Figure 3-4 and Figure 3-5, significant scatter is observed for the median exceedance intensity in terms of various target ductility ratios, elastic periods and yield reduction factors. In general, I_{med} increases with a reduction of R_y and an increase of T_{el}.

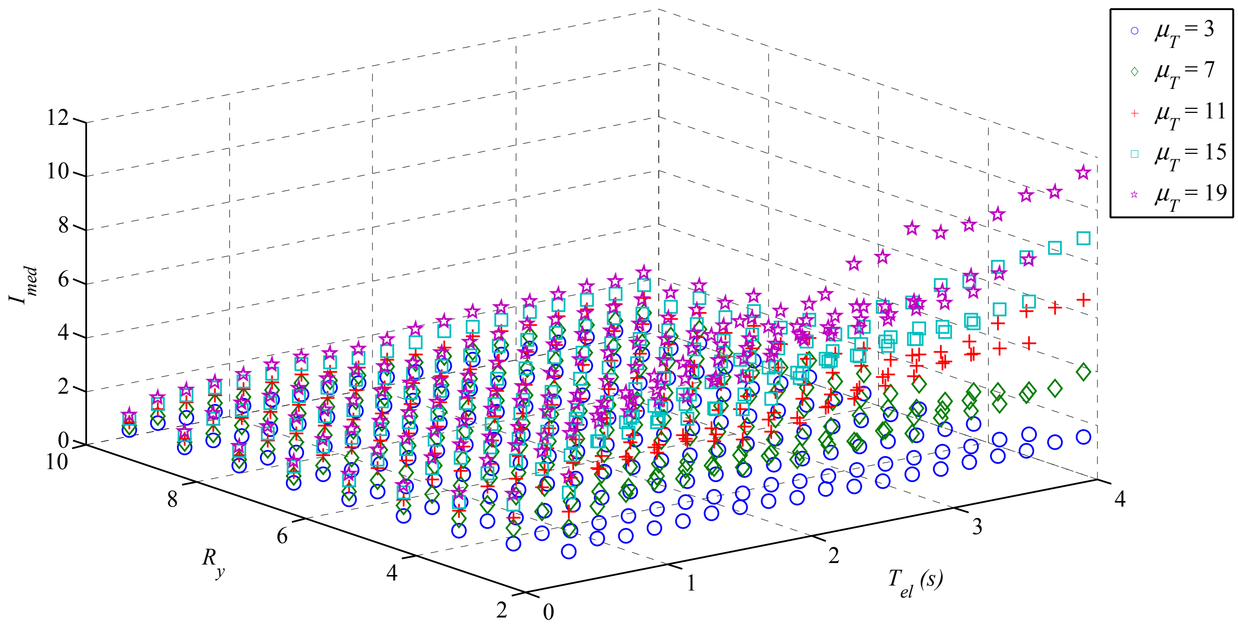


Figure 3-4 Median exceedance intensity (I_{med}) for selected studied systems with target ductility ratios from 1 to 19

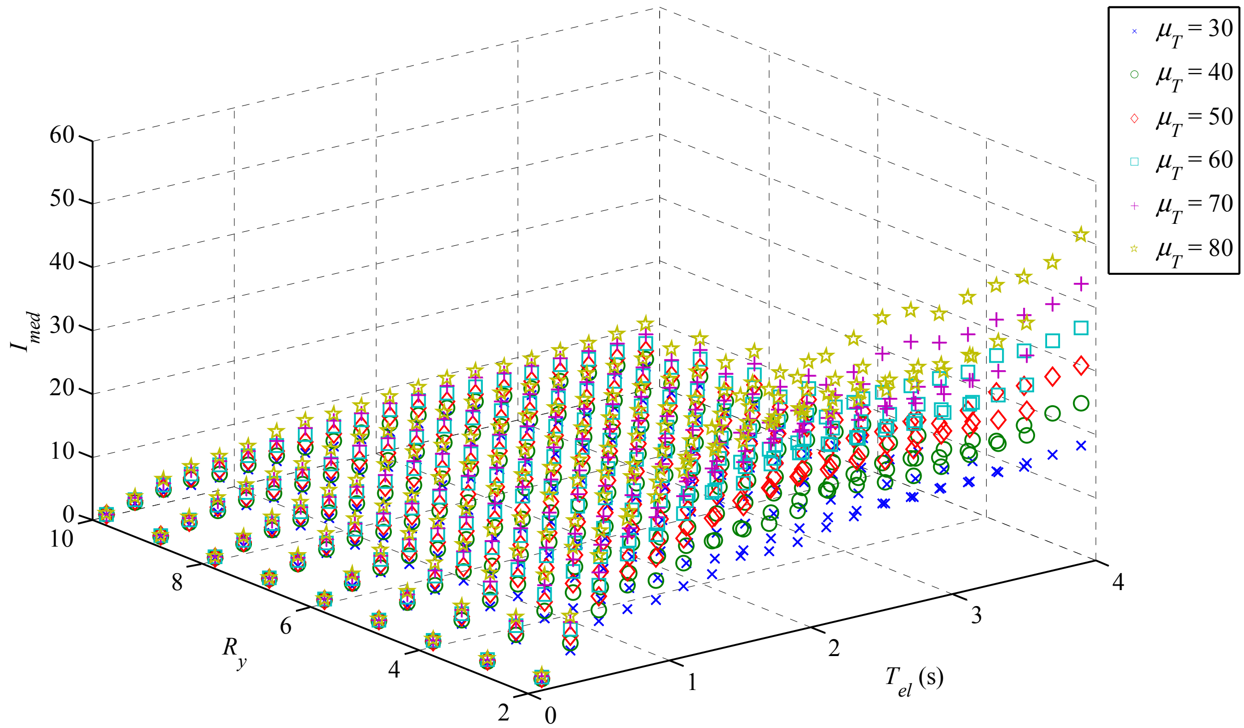


Figure 3-5 Median exceedance intensity (I_{med}) for selected studied systems with target ductility ratios from 30 to 80

3.3 Simplified analysis of SDOF systems

For each SDOF system considered, a simplified analysis procedure is developed and evaluated to predict the associated I_{med} for a given target ductility.

The simplified procedure is based on a reduction factor (r) defined as the ratio of the spectral acceleration at which half of the records cause the bilinear inelastic SDOF system to exceed the target ductility (S_{aMed}= $I_{med}\times$ S_{MT} or median exceedance spectral acceleration), to the yield pseudo-acceleration, as shown in Figure 3-6.

$$r = \frac{S_{aMed}}{A_y} = I_{med} R_y \qquad (3.6)$$

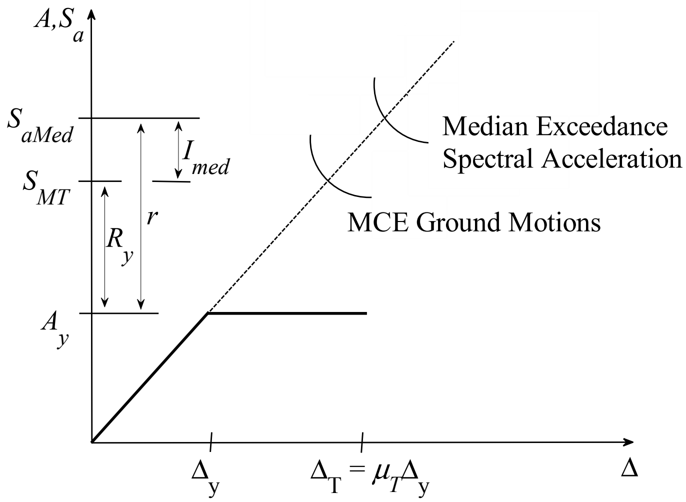


Figure 3-6 Schematic illustration of seismic performance factors as defined by the simplified analysis

Figure 3-7 and Figure 3-8 show the variation of the *r*-factor with the target ductility ratio (μ_T), elastic period (T_{el}) and yield reduction factor (R_y). Although the *r*-factor varies with the change in elastic period, it is not highly dependent on R_y. This could be attributed to the "equal-displacement rule" (Blume et al., 1961), which is known to be generally satisfied for far-field ground motions. Therefore, the average values taken across R_y are considered as characteristic values of the reduction factors, and are listed in Table 3-2 to Table 3-5 for different target ductility ratios and elastic periods.

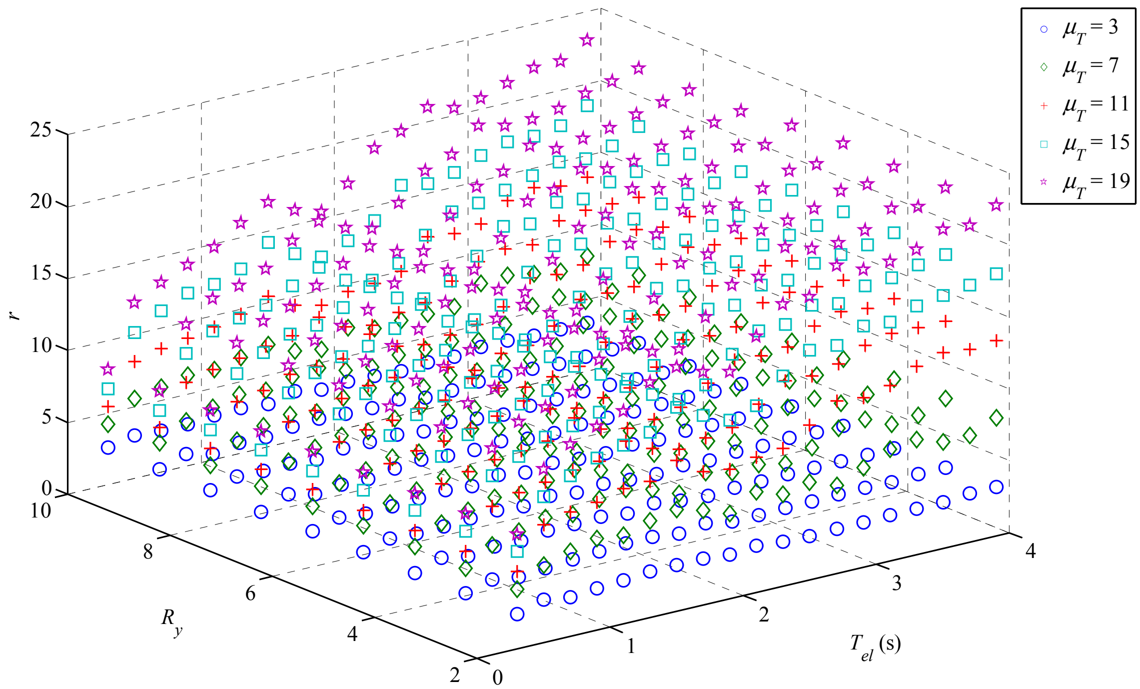


Figure 3-7 **Reduction factor (*r*) for selected studied systems with target ductility ratios from 1 to 19**

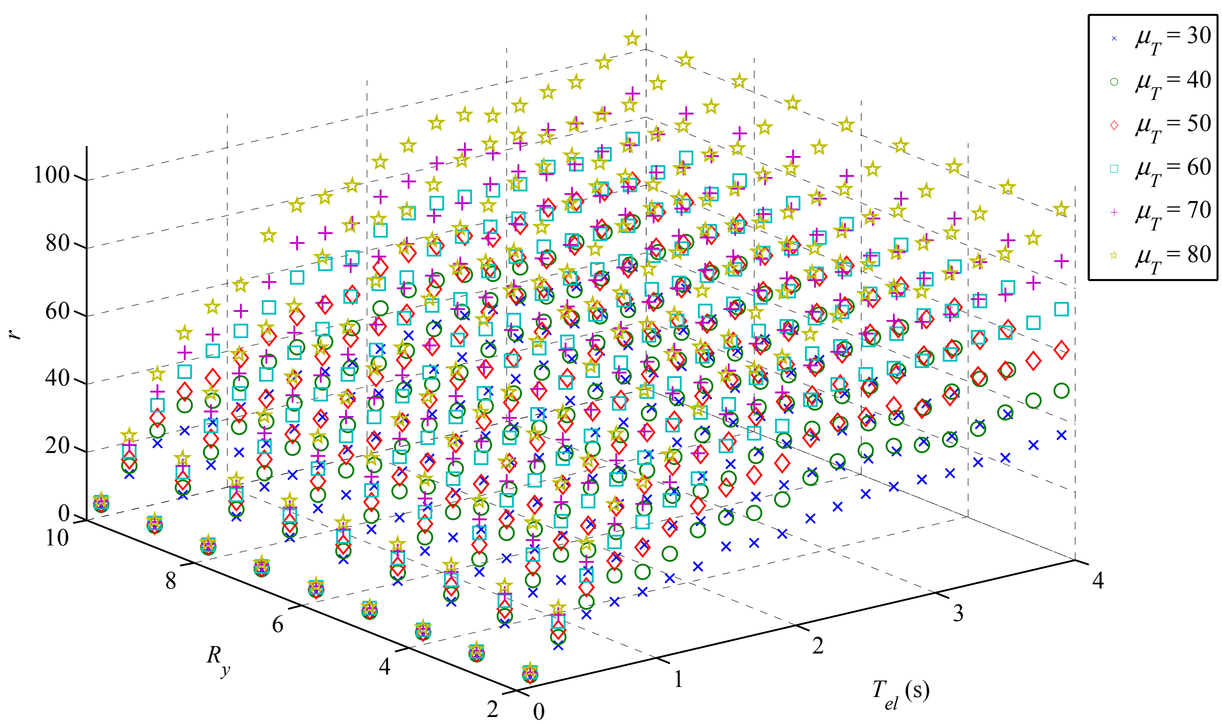


Figure 3-8 Reduction factor (*r*) for selected studied systems with target ductility ratios from 30 to 80

Table 3-2 Average reduction factors (*r*) for systems with μ_T=1 to 20

Fundamental Period (T_{el})	μ_T = 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	0.96	1.41	1.53	1.64	1.76	1.82	1.85	1.92	2	2.06	2.1	2.16	2.32	2.39	2.44	2.48	2.53	2.57	2.61	2.69
0.20	0.94	1.7	2.09	2.53	2.84	3.04	3.25	3.55	3.73	3.96	4.08	4.27	4.33	4.62	4.81	5.07	5.21	5.34	5.5	5.66
0.30	0.94	1.90	2.46	3.02	3.31	3.68	4.28	4.64	4.95	5.19	5.39	5.73	6.08	6.43	6.73	7.07	7.35	7.69	8.00	8.25
0.40	0.97	1.92	2.50	3.22	3.55	4.18	4.55	5.11	5.54	5.94	6.32	7.01	7.29	7.69	8.17	8.54	8.89	9.38	9.77	10.14
0.50	0.99	2.15	3.08	3.61	4.10	4.68	5.43	6.42	6.92	7.54	8.15	8.68	9.03	9.68	10.20	10.65	11.18	11.61	12.14	12.58
0.60	0.99	2.09	2.88	3.67	4.44	5.28	5.70	6.16	6.63	7.15	7.73	8.40	8.75	9.43	10.03	10.69	11.20	11.71	12.54	13.13
0.70	1.06	2.05	2.79	3.77	4.60	5.32	5.92	6.54	7.25	8.11	8.61	9.33	9.97	10.57	11.22	11.78	12.28	12.70	13.43	13.86
0.80	1.00	2.03	2.85	3.50	4.38	5.27	6.10	6.76	7.38	8.19	8.85	9.53	10.13	10.75	11.14	11.61	12.03	12.62	13.33	13.84
0.90	1.05	2.07	2.95	3.80	4.62	5.24	6.36	7.18	7.76	8.42	8.98	9.72	10.50	11.04	11.92	12.34	13.07	13.62	14.09	14.66
1.00	1.02	2.06	3.06	3.64	4.56	5.53	6.28	7.11	7.77	8.44	9.07	9.71	10.28	10.89	11.43	12.22	12.84	13.42	13.95	14.58
1.10	1.00	2.02	3.00	3.81	4.61	5.38	6.21	7.10	7.93	8.73	9.33	10.29	10.96	11.66	12.32	12.93	13.61	14.22	14.77	15.31
1.20	1.02	2.11	3.17	4.00	4.93	5.72	6.60	7.43	8.14	8.98	9.57	10.28	11.13	11.83	12.70	13.44	14.16	15.00	15.73	16.52
1.30	1.04	2.17	3.19	4.08	5.02	5.96	6.76	7.48	8.21	8.89	9.56	10.52	11.53	12.10	12.93	13.94	14.66	15.47	16.31	17.30
1.40	1.02	2.30	3.17	4.15	5.09	6.09	6.97	7.68	8.63	9.55	10.51	11.20	11.89	12.62	13.75	14.63	15.74	16.41	17.41	17.79
1.50	1.04	2.29	3.21	4.19	5.27	6.09	7.03	8.19	9.19	9.94	10.48	11.20	12.17	13.33	14.18	14.80	15.42	16.35	16.96	17.50
1.60	1.07	2.22	3.34	4.26	4.92	5.68	6.41	7.69	8.72	9.34	10.05	10.59	11.41	12.64	13.18	13.71	14.38	15.26	17.06	17.58
1.70	1.06	2.11	3.30	4.03	4.77	5.49	6.51	7.22	8.06	8.51	9.37	10.14	11.32	12.24	12.97	13.61	14.66	15.28	16.11	16.96
1.80	1.04	2.15	3.21	3.89	4.56	5.31	6.16	6.74	7.65	8.36	9.07	9.79	10.80	11.53	12.28	13.06	13.99	14.68	16.76	17.53
1.90	1.01	2.14	3.12	3.96	4.71	5.39	5.97	6.65	7.59	8.29	8.97	9.74	10.94	11.76	12.35	12.98	13.52	14.80	15.73	17.23
2.00	1.02	2.15	3.26	4.22	4.89	5.44	6.14	7.05	8.00	8.45	9.39	10.22	11.07	11.68	12.41	13.37	14.64	16.11	16.97	18.08
2.10	0.98	2.21	3.33	4.20	4.87	5.51	6.79	7.34	7.98	8.61	9.36	10.01	10.83	11.79	12.96	14.13	15.39	16.94	17.53	18.41
2.20	1.00	2.27	3.19	4.02	4.81	5.99	6.66	7.20	7.84	8.60	9.59	10.46	11.27	12.08	13.42	14.18	15.44	16.79	18.06	19.04
2.30	0.96	2.23	3.09	3.98	4.88	5.74	6.47	7.13	8.02	8.77	9.57	10.45	11.25	13.13	14.44	15.84	16.85	17.96	19.37	20.52
2.40	0.95	2.15	2.99	3.89	4.78	5.58	6.44	7.17	8.17	9.02	9.76	10.69	12.38	14.10	15.38	16.54	17.89	18.84	19.56	20.39
2.50	0.95	2.16	3.05	3.92	4.87	5.77	6.66	7.45	8.13	8.87	10.61	12.36	13.46	15.02	15.93	16.82	17.74	18.60	19.75	21.06
2.60	0.98	2.21	3.15	4.06	4.92	5.78	6.51	7.52	8.32	10.05	12.10	12.87	13.99	14.93	16.10	17.09	18.21	19.17	20.43	22.15
2.70	1.00	2.24	3.17	4.06	4.82	5.63	6.34	7.63	9.05	11.18	11.97	12.98	14.03	14.88	15.85	17.11	18.02	20.26	21.17	22.63
2.80	1.02	2.25	3.13	3.99	4.72	5.48	6.24	8.11	9.99	10.91	11.95	12.93	13.74	14.68	15.74	17.92	19.04	20.22	21.41	22.35
2.90	1.04	2.26	3.15	3.92	4.71	5.54	6.86	8.50	9.99	10.84	11.89	12.68	13.57	14.79	16.56	17.72	18.87	19.74	20.61	21.90
3.00	1.05	2.31	3.20	3.92	4.79	5.86	7.18	8.92	9.93	10.76	11.91	12.72	14.12	15.53	16.73	17.60	18.53	19.80	20.61	21.59
3.10	1.06	2.33	3.25	4.05	4.78	6.16	7.79	8.95	9.85	10.86	12.11	13.60	14.80	15.84	16.62	18.06	18.98	19.83	20.73	21.69
3.20	1.07	2.38	3.28	4.06	5.18	6.38	7.90	8.93	9.76	10.84	12.23	14.09	15.41	16.35	17.27	18.25	19.24	20.00	20.77	21.70
3.30	1.07	2.35	3.26	4.19	5.26	6.72	7.90	8.68	9.59	11.06	13.12	14.61	15.65	16.45	17.30	18.15	19.21	20.00	21.17	21.92
3.40	1.07	2.28	3.25	4.20	5.24	6.79	7.59	8.49	9.63	10.89	13.56	14.67	15.50	16.45	17.40	18.22	18.99	20.09	21.58	22.15
3.50	1.06	2.27	3.26	4.12	5.20	6.57	7.30	8.32	9.66	11.95	13.60	14.44	15.42	16.44	17.43	18.16	19.16	20.38	21.98	22.65
3.60	1.06	2.29	3.31	4.11	5.18	6.30	7.32	8.63	9.74	12.14	13.49	14.41	15.38	16.58	17.48	18.52	19.49	21.03	21.96	22.87
3.70	1.06	2.30	3.32	4.18	5.30	6.29	7.36	8.69	10.36	12.23	13.46	14.42	15.59	16.81	17.78	19.07	19.88	21.06	21.99	22.84
3.80	1.08	2.27	3.34	4.27	5.28	6.51	7.72	8.61	10.58	12.14	13.39	14.51	16.11	17.09	18.10	18.98	19.84	21.15	22.10	23.23
3.90	1.08	2.23	3.30	4.42	5.59	6.62	7.96	8.65	10.72	12.10	13.54	14.84	16.22	17.31	18.23	19.04	19.91	20.96	22.76	23.96
4.00	1.08	2.23	3.34	4.47	5.74	6.80	8.06	8.88	11.10	12.09	13.65	15.27	16.49	17.42	18.34	19.17	19.96	21.80	23.54	24.70

Table 3-3 Average reduction factors (r) for systems with μT=21 to 40

Fundamental Period (T_{el})	μ_T 21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
0.10	2.77	2.82	2.86	2.88	2.98	3.01	3.03	3.07	3.12	3.17	3.21	3.27	3.31	3.37	3.41	3.45	3.57	3.57	3.6	3.6
0.20	5.78	6	6.35	6.47	6.6	6.79	6.97	7.1	7.1	7.1	7.39	7.48	8.03	8.06	8.29	8.44	8.6	8.71	8.98	9.15
0.30	8.43	8.67	9.13	9.36	9.58	9.76	10.10	10.23	10.42	10.73	10.93	11.16	11.43	11.67	11.99	12.11	12.26	12.47	12.72	12.99
0.40	10.58	10.87	11.23	11.70	12.13	12.32	12.72	13.04	13.31	13.63	13.86	14.23	14.60	15.00	15.53	15.85	16.28	16.59	16.79	17.47
0.50	13.51	13.86	14.48	15.06	15.53	16.07	16.50	16.98	17.45	17.62	18.05	18.51	18.93	19.47	19.80	20.08	20.41	20.78	21.14	21.47
0.60	13.60	13.97	14.75	15.44	15.60	15.88	16.23	16.75	17.18	17.68	18.25	18.93	19.00	19.49	19.74	20.08	20.43	20.85	21.26	21.66
0.70	14.29	14.72	15.13	15.58	16.13	16.55	17.16	17.96	19.00	19.49	19.96	20.52	21.26	21.80	22.48	22.81	23.26	23.91	24.48	26.90
0.80	14.54	14.98	16.13	16.72	17.21	17.77	18.33	19.00	19.80	20.39	21.05	21.32	21.80	22.48	22.82	23.50	24.04	24.62	24.89	25.56
0.90	15.25	16.01	16.53	17.06	17.56	17.99	18.53	19.00	19.68	20.24	20.89	21.68	22.31	23.35	23.96	24.65	24.99	25.48	25.87	26.41
1.00	15.31	15.83	16.47	17.38	17.96	18.59	19.16	19.86	20.49	21.23	21.80	22.55	23.60	24.52	24.95	25.56	26.26	26.78	27.44	27.95
1.10	16.17	16.85	17.56	18.33	19.10	19.80	20.40	21.11	21.88	22.53	23.74	24.48	25.15	26.03	26.53	27.02	27.77	28.45	29.19	29.82
1.20	17.55	18.32	19.14	19.59	20.17	21.60	22.48	23.23	23.81	24.48	25.15	25.81	26.59	27.28	28.03	28.70	29.45	31.58	32.38	32.94
1.30	17.88	19.00	19.68	20.30	21.04	21.79	22.18	22.81	23.41	24.19	24.94	25.91	29.28	30.03	30.83	31.50	32.25	32.83	33.70	34.45
1.40	18.44	19.38	19.96	20.70	21.88	22.55	23.35	24.10	26.03	26.90	27.95	28.83	29.95	31.50	31.95	32.83	33.50	34.17	35.05	35.50
1.50	18.93	20.25	21.13	21.80	22.48	23.80	24.59	25.55	26.59	27.95	28.83	29.92	31.00	31.78	32.59	33.25	34.12	34.89	35.74	36.55
1.60	18.43	19.20	20.93	22.08	23.04	24.10	24.88	26.03	26.90	27.46	28.39	29.39	30.14	30.94	31.58	32.68	33.69	34.86	35.58	36.49
1.70	19.00	20.05	21.00	22.05	22.93	23.60	24.67	25.46	26.38	27.25	28.26	29.09	29.90	30.63	31.50	32.63	34.00	34.38	35.13	36.01
1.80	18.80	19.80	20.79	21.42	22.18	22.98	23.87	24.87	26.29	27.33	27.95	28.76	29.58	30.54	31.27	32.63	33.28	34.07	35.25	36.30
1.90	18.42	19.40	20.14	20.85	21.88	23.35	24.48	25.55	26.43	27.95	28.83	29.58	30.45	31.50	32.06	33.13	34.58	35.39	36.34	37.85
2.00	18.93	19.68	21.11	22.93	23.80	24.48	25.35	26.51	27.95	28.90	29.95	30.70	31.78	32.68	34.38	35.31	36.49	37.73	38.72	39.84
2.10	20.25	21.88	22.67	23.43	24.40	25.15	26.17	27.35	28.96	30.45	31.20	32.45	33.50	34.68	35.62	36.94	38.80	40.35	41.40	42.35
2.20	20.07	22.05	23.23	24.48	26.23	26.90	28.03	29.01	30.03	30.94	32.25	33.21	34.56	36.17	36.98	39.05	40.60	41.48	42.84	44.08
2.30	21.42	22.05	24.00	25.35	26.25	26.99	27.96	29.15	30.54	31.98	34.17	35.05	35.50	36.53	38.93	40.53	41.48	42.84	44.08	45.29
2.40	21.66	23.35	23.91	24.98	26.23	27.74	29.00	30.90	32.63	33.50	34.38	35.92	36.98	38.23	39.85	41.09	42.41	43.65	44.95	46.08
2.50	22.67	23.80	24.99	26.13	27.35	29.15	30.03	31.50	32.90	33.81	34.86	35.92	37.35	39.05	40.60	41.85	43.20	44.28	45.51	46.81
2.60	23.44	24.69	25.84	26.90	28.03	28.90	30.90	32.25	33.13	34.09	35.09	36.49	38.80	39.92	41.40	42.58	43.70	44.95	46.08	46.95
2.70	23.80	24.59	25.65	27.65	28.70	29.58	30.70	31.95	32.94	33.90	35.39	37.05	38.41	39.72	40.96	42.16	43.11	44.15	45.11	46.15
2.80	23.35	24.64	25.63	26.59	27.80	29.06	30.22	31.50	32.58	33.75	35.35	36.68	37.85	38.80	39.79	40.80	41.93	42.90	43.83	44.87
2.90	23.00	23.97	25.15	26.48	27.95	28.90	30.03	30.90	32.08	34.17	35.28	36.22	37.17	38.06	39.02	40.05	41.02	42.16	43.20	44.21
3.00	22.69	24.10	25.35	26.90	28.23	29.28	30.03	31.01	32.90	34.10	35.04	35.81	36.73	37.81	38.80	39.61	40.78	41.80	42.86	43.88
3.10	22.56	23.80	25.35	26.90	28.23	29.78	30.83	32.07	33.33	34.26	35.28	36.31	37.31	38.19	39.10	40.15	41.04	42.00	43.14	44.14
3.20	22.59	23.81	25.15	26.48	28.03	29.95	31.16	32.58	33.67	34.71	35.92	36.98	37.85	38.80	39.73	40.78	41.74	42.70	43.80	44.95
3.30	22.56	23.66	24.90	26.40	28.29	29.84	31.20	32.76	34.17	35.25	36.30	37.25	38.00	39.05	39.92	41.04	42.30	43.40	44.48	45.29
3.40	22.76	23.77	25.15	26.90	28.03	29.30	31.03	32.63	34.38	35.25	36.30	37.05	37.92	39.25	40.19	41.34	42.15	43.11	44.08	45.70
3.50	23.35	24.25	25.43	26.48	27.69	29.15	30.76	32.38	34.00	35.05	36.19	37.33	38.41	39.29	40.35	41.28	42.15	43.51	45.70	47.83
3.60	23.91	24.81	25.84	26.79	28.10	29.63	31.06	32.74	34.45	35.92	36.98	37.85	38.80	39.85	40.60	41.78	43.60	46.08	48.20	49.38
3.70	24.00	25.35	26.48	27.65	28.90	30.29	31.95	33.13	34.58	36.38	37.25	38.30	39.48	40.35	41.96	44.15	46.28	47.33	48.50	49.63
3.80	24.55	26.03	27.29	28.42	29.42	30.83	32.25	33.13	34.86	36.98	37.92	38.80	40.09	41.85	44.08	45.40	46.58	47.83	48.88	49.83
3.90	25.35	26.99	28.40	29.58	30.26	31.50	32.46	33.50	35.13	37.25	38.60	39.96	41.85	43.47	44.95	46.08	47.03	48.50	49.55	50.50
4.00	26.48	27.65	28.70	29.78	31.06	32.31	33.28	34.24	36.30	38.30	39.79	41.85	43.03	44.28	45.40	46.75	47.83	48.95	50.43	51.55

Table 3-4 Average reduction factors (r) for systems with μ_T=41 to 60

Fundamental Period (T_{el})	μ_T 41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
0.10	3.6	3.61	3.66	3.7	3.78	3.82	3.85	3.88	3.92	3.95	4.01	4.07	4.07	4.1	4.15	4.18	4.21	4.31	4.32	4.4
0.20	9.25	9.44	9.65	9.84	9.9	9.92	10.07	10.11	10.21	10.41	10.52	10.58	10.76	10.86	11.02	11.1	11.21	11.32	11.39	11.49
0.30	13.21	13.54	13.64	13.92	14.08	14.36	14.62	14.69	14.96	15.01	15.27	15.51	15.70	15.92	16.13	16.50	16.81	17.06	17.10	17.28
0.40	17.75	18.05	18.40	18.53	18.93	18.93	19.26	19.42	19.74	19.96	20.40	20.93	21.20	21.67	21.87	22.01	22.34	22.72	22.99	23.28
0.50	21.86	22.48	22.78	23.08	23.40	23.75	24.07	24.55	24.94	25.48	26.03	26.39	26.75	27.10	27.49	27.80	28.25	28.42	28.76	29.10
0.60	22.08	22.48	23.60	24.10	24.48	24.88	25.55	25.69	26.33	26.65	28.70	28.90	29.28	29.58	30.03	30.32	30.90	31.50	31.78	32.38
0.70	27.35	27.95	28.03	28.23	28.70	29.10	29.58	30.06	30.63	31.10	31.81	32.51	32.99	33.54	33.98	34.64	35.17	35.77	36.41	36.92
0.80	26.08	26.65	27.27	27.83	28.45	29.06	29.61	30.24	30.83	31.50	31.78	32.38	32.74	33.28	33.81	35.05	35.92	37.05	38.00	38.60
0.90	26.84	27.44	28.16	28.92	29.68	30.51	31.10	31.95	32.63	33.50	34.17	35.05	35.92	36.38	37.25	38.00	38.97	39.47	39.92	40.73
1.00	28.59	29.43	30.10	30.83	31.50	32.10	32.64	33.09	33.65	34.17	35.05	35.50	36.38	37.05	37.25	37.73	39.17	39.92	40.73	41.48
1.10	30.35	30.86	31.23	31.84	32.31	34.17	35.05	35.92	36.38	37.25	38.00	39.05	39.85	40.73	41.48	42.53	43.40	44.08	44.53	45.03
1.20	33.61	34.39	35.05	35.67	36.49	37.25	37.93	38.80	39.73	40.53	41.40	42.15	43.03	43.60	44.28	45.03	45.70	46.28	46.95	47.63
1.30	35.50	36.38	37.05	37.73	38.30	38.97	39.56	40.44	41.06	41.78	42.64	43.40	44.08	44.95	45.82	46.51	47.31	48.23	49.03	49.86
1.40	36.17	36.98	37.81	38.51	39.43	40.24	41.09	42.00	42.96	43.86	44.99	45.88	46.48	47.29	48.07	48.76	49.46	50.13	51.13	52.14
1.50	37.45	38.41	39.47	40.53	41.40	42.15	42.90	43.67	44.55	45.44	46.19	47.13	48.05	49.83	52.05	53.30	54.43	55.60	56.65	57.53
1.60	37.45	38.31	39.19	40.05	40.88	41.68	42.78	44.15	45.33	46.28	47.63	48.88	49.83	51.18	52.43	53.18	54.05	54.93	55.98	56.85
1.70	36.73	37.81	39.04	39.96	40.80	42.15	43.40	44.35	45.70	46.95	48.20	49.38	50.30	51.18	51.86	54.05	56.18	57.22	58.40	59.15
1.80	37.31	38.13	39.05	40.53	41.48	42.73	44.21	45.85	47.23	48.31	49.06	50.30	51.18	52.14	53.10	53.98	55.12	56.09	57.15	58.11
1.90	38.97	40.04	41.04	42.27	43.33	44.38	45.29	46.39	47.63	48.58	49.63	50.50	51.63	52.50	53.55	54.73	55.69	57.13	58.40	59.53
2.00	40.92	41.77	42.70	43.59	44.64	45.70	46.88	47.83	49.46	51.63	52.63	53.98	54.85	55.98	57.10	58.40	59.15	60.01	60.89	61.89
2.10	43.40	44.28	45.51	47.03	48.15	49.43	50.43	51.55	52.74	53.98	55.10	56.18	57.34	58.15	59.30	60.40	61.95	63.20	64.63	65.50
2.20	45.01	46.31	47.48	48.95	50.02	51.24	52.54	53.64	54.91	56.01	57.09	58.40	59.45	60.33	61.28	62.20	63.08	63.95	65.00	65.88
2.30	46.58	47.97	49.21	50.43	51.52	52.59	53.52	54.79	55.75	56.65	57.53	58.49	59.45	60.33	61.36	62.20	63.08	64.25	65.88	67.80
2.40	47.35	48.56	49.79	50.86	51.86	52.80	53.68	54.72	55.60	56.48	57.53	58.49	59.30	60.40	62.12	63.95	65.20	66.63	67.68	68.55
2.50	47.97	48.99	50.02	50.90	51.80	52.76	53.64	54.60	55.60	56.54	57.98	59.53	61.08	62.40	63.75	64.63	65.88	66.75	67.88	68.75
2.60	48.03	49.06	50.02	50.90	51.86	52.80	53.82	55.30	56.85	57.98	59.45	60.56	61.79	62.70	64.08	65.25	66.55	67.68	68.86	70.10
2.70	47.15	48.03	49.06	50.21	51.39	52.76	54.22	55.33	56.65	57.75	58.89	59.93	61.08	62.20	63.40	64.44	65.59	66.55	67.85	68.95
2.80	46.08	47.33	48.50	49.38	50.50	51.55	52.61	53.72	55.09	56.24	57.15	58.28	59.30	60.56	61.56	62.66	63.66	64.97	66.75	69.23
2.90	45.33	46.39	47.63	48.50	49.55	50.50	51.55	52.61	53.72	54.94	56.05	57.15	58.28	59.35	61.28	63.45	65.50	66.63	67.80	68.75
3.00	45.04	46.08	47.14	48.19	49.06	50.30	51.18	52.43	53.50	54.66	56.29	58.40	60.33	61.45	62.40	63.88	65.00	65.95	67.00	68.47
3.10	45.28	46.37	47.42	48.83	50.02	51.30	52.50	54.24	55.69	57.05	58.15	59.45	60.83	62.04	63.30	64.25	65.50	66.85	67.85	68.95
3.20	46.28	47.63	48.58	49.75	51.33	53.30	54.85	56.05	57.53	58.78	60.20	61.45	62.40	63.95	65.20	66.55	67.80	69.23	70.30	71.43
3.30	46.25	47.63	49.75	52.05	53.98	55.30	56.65	57.98	59.45	60.40	61.95	63.08	64.33	65.50	66.55	67.68	68.75	70.10	70.97	72.10
3.40	47.83	49.83	52.43	53.98	55.10	56.05	57.53	58.40	59.53	60.65	61.65	63.00	63.95	65.08	66.25	67.43	68.55	69.50	70.68	72.03
3.50	50.30	51.38	52.50	53.55	54.85	55.97	56.85	57.98	59.45	60.40	61.45	62.40	63.75	65.00	65.88	67.00	68.18	69.43	70.47	71.43
3.60	50.50	51.55	52.63	53.98	54.93	56.05	57.22	58.40	59.53	60.65	61.65	63.00	63.95	65.08	66.55	67.43	68.55	69.80	70.97	72.10
3.70	50.88	52.05	53.18	54.18	55.30	56.65	57.73	58.85	60.20	61.08	62.33	63.45	64.63	65.88	67.00	67.88	69.35	70.30	71.43	72.78
3.80	51.30	52.43	53.38	54.73	55.97	56.93	57.98	59.45	60.40	61.53	63.00	63.95	65.08	66.55	67.50	68.55	69.80	70.97	72.22	73.35
3.90	51.75	53.10	54.18	55.30	56.65	57.73	58.85	60.20	61.45	62.40	63.88	65.00	65.95	67.43	68.55	69.50	70.97	72.28	73.46	74.82
4.00	52.63	53.98	55.10	56.18	57.53	58.85	60.20	61.28	62.40	63.75	65.00	66.21	67.72	68.85	70.25	71.54	72.97	74.23	75.78	77.33

Table 3-5 Average reduction factors (*r*) for systems with μ_T=61 to 80

Fundamental Period (T_{el})	μ_T 61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
0.10	4.4	4.46	4.48	4.48	4.48	4.5	4.53	4.55	4.66	4.68	4.71	4.76	4.8	4.82	4.86	4.88	4.89	4.94	4.96	4.99
0.20	11.62	11.77	11.96	12.07	12.12	12.23	12.45	12.58	12.58	12.64	12.83	12.92	12.98	13.15	13.21	13.21	13.45	13.45	13.64	13.88
0.30	17.43	17.71	17.82	17.99	18.21	18.46	18.56	18.93	19.13	19.49	19.74	19.93	20.11	20.30	20.36	20.64	20.73	21.04	21.09	21.47
0.40	23.63	23.88	24.10	24.48	24.79	25.15	25.60	26.03	26.48	26.90	27.35	27.95	28.03	28.70	28.90	29.58	29.78	29.95	30.03	30.45
0.50	29.39	29.66	30.06	30.42	31.50	31.70	32.06	32.38	32.94	33.13	33.80	35.05	36.30	37.05	37.25	37.73	37.85	37.93	38.30	38.60
0.60	32.71	33.16	33.80	34.24	34.72	35.34	35.92	36.45	36.83	37.73	38.26	38.96	39.43	40.06	40.68	41.08	41.70	42.30	42.88	43.40
0.70	37.52	38.13	38.60	39.20	39.59	40.25	40.86	41.43	42.00	42.55	42.97	43.51	44.08	44.61	45.21	45.91	46.66	47.06	47.63	48.03
0.80	39.05	39.47	39.92	40.60	40.98	41.48	42.15	42.64	43.18	43.74	44.48	45.03	46.08	46.58	47.03	47.63	48.20	48.88	49.38	49.83
0.90	41.40	41.85	42.60	43.40	44.08	44.53	45.40	45.70	46.08	46.08	46.39	46.96	47.35	47.83	50.88	51.63	52.43	53.10	53.98	54.43
1.00	42.28	43.03	43.60	44.35	45.33	46.08	46.75	47.63	48.08	48.95	49.63	50.50	51.18	52.05	52.50	53.30	53.98	54.43	54.85	55.30
1.10	45.70	46.28	46.88	47.15	47.83	48.50	48.95	49.63	50.30	50.90	51.54	52.20	53.01	53.70	54.54	55.41	56.28	57.15	58.05	58.65
1.20	48.19	48.99	49.76	50.50	51.39	52.31	53.28	54.11	54.94	55.75	56.41	57.15	57.93	58.70	59.30	59.93	60.66	61.22	61.91	62.73
1.30	50.50	51.38	52.14	52.83	53.50	54.31	54.88	55.54	56.39	57.61	59.11	60.50	61.76	63.08	64.63	65.95	67.50	68.75	70.30	71.43
1.40	53.10	53.98	55.60	57.53	59.45	61.07	61.95	63.00	63.88	64.75	65.88	66.75	67.88	68.75	69.80	70.98	72.03	72.97	74.03	74.97
1.50	58.40	59.45	60.20	61.07	62.13	63.00	63.95	65.00	65.88	66.75	67.88	68.75	69.80	70.98	72.03	72.97	74.97	76.53	77.33	78.25
1.60	57.60	58.65	59.53	60.40	62.20	64.75	66.55	67.43	68.18	68.75	69.80	70.55	71.35	72.10	72.90	73.65	74.52	75.30	76.20	77.16
1.70	60.20	61.07	61.95	63.00	63.88	64.75	65.68	66.25	67.00	67.88	68.73	69.72	70.59	71.54	72.59	73.65	74.90	75.85	77.01	78.45
1.80	59.11	60.03	61.16	62.13	63.08	64.04	64.87	66.15	67.43	68.55	69.80	70.98	72.10	73.35	74.52	75.65	76.90	78.08	79.33	80.45
1.90	60.40	61.07	62.04	63.00	63.79	64.78	65.68	66.75	67.88	69.43	70.98	72.30	74.03	74.97	75.78	76.53	77.57	78.38	79.13	80.00
2.00	63.00	64.12	65.20	66.63	67.80	68.75	70.30	71.43	72.50	73.80	74.97	76.45	77.33	78.25	79.13	80.00	81.05	81.93	82.80	83.93
2.10	66.25	67.00	67.88	68.75	69.91	71.06	72.47	73.65	74.52	75.57	76.45	77.33	78.25	79.13	80.00	81.05	82.00	82.88	83.93	84.80
2.20	66.63	67.95	69.43	71.22	72.10	72.97	73.85	74.70	75.57	76.45	77.33	78.38	79.33	80.22	81.39	82.30	83.55	84.68	85.93	87.10
2.30	69.23	70.10	70.97	72.03	72.90	73.65	74.70	75.57	76.53	77.57	78.71	79.75	80.96	82.00	83.25	84.68	85.93	87.10	88.35	89.90
2.40	69.50	70.55	71.43	72.47	73.35	74.52	75.57	76.75	77.89	79.13	80.38	81.80	82.88	84.13	85.55	86.80	88.15	89.40	90.35	91.23
2.50	70.10	71.06	72.36	73.28	74.61	75.78	77.01	78.45	79.70	81.13	82.20	83.25	84.60	85.55	86.80	87.88	89.30	90.55	91.43	92.73
2.60	71.35	72.30	73.35	74.70	75.78	76.90	77.78	79.13	80.38	82.00	83.93	85.48	86.42	87.48	88.43	89.48	90.65	91.83	92.95	93.90
2.70	69.91	71.09	72.78	74.52	76.45	78.38	79.58	80.45	81.93	82.80	83.93	84.88	85.93	87.10	88.15	89.23	90.28	91.23	92.58	93.63
2.80	71.43	72.90	74.03	74.90	76.03	76.90	78.25	79.33	80.38	81.33	82.38	83.55	84.60	85.55	86.60	87.68	88.73	89.90	90.95	91.90
2.90	70.10	71.35	72.30	73.65	74.90	75.78	76.90	78.25	79.33	80.45	81.33	82.68	83.93	84.80	85.93	86.80	88.15	89.03	90.15	91.03
3.00	69.43	70.55	71.85	72.90	74.03	74.98	76.45	77.57	78.45	79.58	80.88	82.00	83.05	84.00	85.48	86.35	87.48	88.60	89.90	90.95
3.10	70.25	71.46	72.59	73.78	74.90	76.14	77.33	78.45	79.58	80.88	82.00	83.13	84.43	85.55	86.80	88.15	89.10	90.35	91.70	92.58
3.20	72.78	73.85	74.98	76.45	77.57	78.45	79.58	80.88	81.93	82.88	84.00	85.48	86.35	87.48	88.43	89.48	90.95	91.90	93.03	93.90
3.30	73.35	74.23	75.57	76.53	77.78	78.83	80.00	81.13	82.20	83.25	84.60	85.55	86.80	87.68	89.03	90.15	91.23	92.28	93.63	94.58
3.40	72.97	74.10	75.10	76.45	77.57	78.53	79.70	81.05	82.00	83.13	84.43	85.48	86.60	87.68	89.03	89.90	91.03	92.28	93.45	94.50
3.50	72.90	73.85	74.90	76.03	77.33	78.45	79.33	80.45	81.93	82.88	83.93	85.05	86.35	87.48	88.43	89.59	90.95	92.20	93.54	94.70
3.60	72.97	74.23	75.57	76.53	77.78	78.83	80.00	81.13	82.20	83.55	84.60	85.68	86.91	88.21	89.55	90.95	92.28	93.82	95.25	96.58
3.70	73.85	74.98	76.03	77.33	78.45	79.58	80.88	82.00	83.13	84.24	85.48	86.87	88.21	89.62	91.23	92.95	94.50	95.82	97.38	98.93
3.80	74.52	75.78	77.01	78.16	79.53	80.71	81.93	83.30	84.60	85.93	87.19	88.69	90.20	91.61	93.14	94.70	96.50	98.05	99.38	100.93
3.90	76.05	77.41	78.76	80.07	81.33	82.88	84.43	85.68	87.10	88.43	89.90	91.34	92.78	94.29	95.85	97.49	99.01	100.73	102.48	104.00
4.00	78.53	80.00	81.33	82.88	84.43	85.93	87.40	88.78	90.33	92.00	93.29	94.70	96.50	98.05	99.68	101.30	102.85	104.48	106.20	109.28

3.4 Extension of the analysis to multi-degrees-of-freedom systems

The main idea of the proposed simplified collapse analysis procedure is to simulate the response of multi-degrees-of-freedom (MDOF) frame building systems using nonlinear static (pushover) analysis. For this purpose and consistent with the FEMA P695 methodology, the building is subjected to a monotonic lateral load pattern that is proportional to the first elastic mode shape until a loss of 20% of the base shear capacity is observed. The target ductility factor (μ_T) is then obtained from the bilinear curve fitted to the base shear-roof displacement pushover curve according to the FEMA P695 methodology (see Figure 3-9). The proposed bilinear fitting curve is easy to obtain since only the maximum strength of the building and slope of the elastic segment of the curve are required. The adequacy of using a simple bilinear fitting pushover curve was also suggested by De Luca et al (2013). For evaluating the proposed procedure, the fundamental period of vibration of the building derived from an eigenvalue analysis is considered as the elastic period of the corresponding SDOF system. The static deformed shape of the building at the ultimate displacement (δ_u), which corresponds to the roof displacement at the point of 20% strength loss, is defined as the "inelastic mode shape" (ϕ_I). A simplifying assumption is made herein by considering the structure near collapse to vibrate according to its inelastic mode shape pattern. Therefore, an inelastic mode participation factor (Γ_I) can be defined as:

$$\Gamma_I = \frac{\phi_I^T \mathbf{M}\{1\}}{\phi_I^T \mathbf{M}\phi_I} \tag{3.7}$$

where {1} is the unity vector and **M** is the global mass matrix. The yield pseudo-acceleration for a MDOF system can be obtained as follows:

$$A_y = \frac{4\pi^2 \delta_y}{T_{el}^2 \Gamma_I \phi_{I,r}} = \frac{4\pi^2 \delta_u}{\mu_T T_{el}^2 \Gamma_I \phi_{I,r}} \tag{3.8}$$

where $\phi_{I,r}$ is the roof component of the inelastic mode shape and δ_u is the ultimate roof displacement, as defined in Figure 3-9.

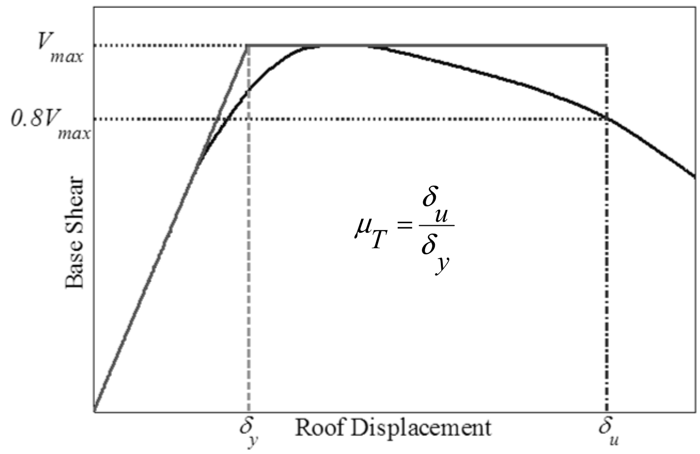


Figure 3-9 Idealized nonlinear static pushover curve (after FEMA (2009))

The reduction factor (r) can be obtained from Table 3-2 to Table 3-5and is defined for MDOF systems as the ratio of the median value of the 5-percent damped spectral acceleration at the fundamental period (T_{el}) of the building ($\hat{S}_{CT}$) causing the roof of the building to reach δ_u to the yield pseudo-acceleration (A_y). Thus, the reduction factor is defined as:

$$r = \frac{\hat{S}_{CT}}{A_y} \tag{3.9}$$

The FEMA P695 methodology also defines $\hat{S}_{CT}$ as the product of the *CMR* and the MCE level, 5-percent damped, spectral acceleration at the fundamental period of the building (T_{el}), for the assumed seismic design category (S_{MT}):

$$\hat{S}_{CT} = CMR \times S_{MT} \tag{3.10}$$

Using the MCE spectral shape of ASCE7-10 (see Figure 3-3) and substituting Equation 3.8 and Equation 3.9 into Equation3.10, the *CMR* can be expressed as:

$$\begin{cases} T_{el} < T_S \rightarrow CMR = \dfrac{4\pi^2 \delta_u r}{S_{MS} T_{el}^{\;2} \mu_T \Gamma_I \phi_{I,r}} \\ T_{el} \geq T_S \rightarrow CMR = \dfrac{4\pi^2 \delta_u r}{S_{M1} T_{el} \mu_T \Gamma_I \phi_{I,r}} \end{cases} \tag{3.11}$$

where S_{M1} is the 5-percent damped MCE spectral acceleration at a period of one second and S_{MS} is the 5-percent damped MCE spectral acceleration at short periods, both for the assumed seismic design category. In addition, T_s is the short-period transition of the building in Equation 3.11, as defined in ASCE7-10 (see

Figure 3-3). Figure 3-10 illustrates the schematic relationship between the defined parameters of the proposed procedure.

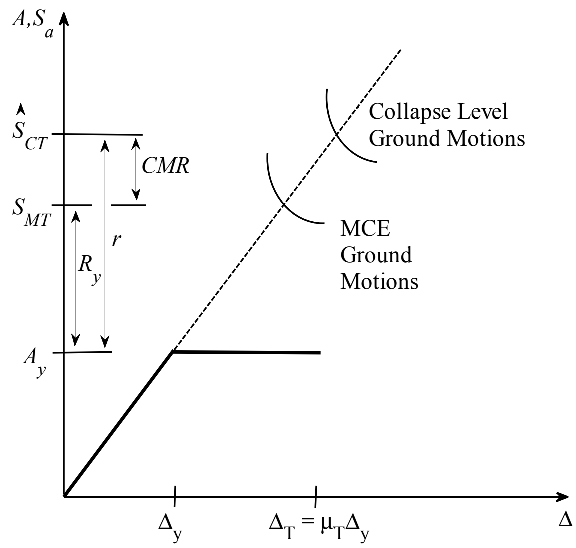


Figure 3-10 Schematic illustration of seismic performance factors as defined by the proposed procedure

3.5 Discussion of fundamental period determination

The fundamental period used within the FEMA P695 methodology for collapse evaluation of conventional building systems is defined by the following approximate equation:

$$T = C_u T_a = C_u C_t h_n^x \geq 0.25 \text{ s} \tag{3.12}$$

where h_n is the building height, the values of the coefficient, C_u, are given in Table 12.8-1 of ASCE7-10, and values of period parameters C_t and x are given in Table 12.8-2 of ASCE7-10. The value of the coefficient, C_u, ranges from 1.4 in high seismic regions to 1.7 in low seismic regions, and the product, $C_u T_a$, approximates the average value of building period.

The fundamental period determination affects the collapse margin ratio by change in S_{MT} as shown in Equation 2.1. The philosophy behind the introduction of the approximate period is to avoid large values of fundamental period that results in smaller values of collapse level spectral acceleration and accordingly larger *CMR* values. Thus, the approximate period formulation aims at providing conservative *CMR* values. However, application of the proposed simplified methodology results in unconservative *CMR* if the fundamental period is decreased simply because of the usage of displacement-based analysis approach where yield pseudo-acceleration is derived from pseudo-spectral displacement using Equation 3.8.

This issue is also addressed in the commentary of Chapter 18 of ASCE7-10 where all the equations for design of buildings with supplemental damping systems are based on fundamental period of the SFRS obtained from eigenvalue analysis. FEMA P695 methodology also does not present any specific guideline for evaluation of fundamental period of SFRS with damping systems. Thus, the fundamental period obtained from eigenvalue analysis is used in this report for the evaluation and design of buildings with supplemental damping systems.

In the case of conventional building systems where the approximate fundamental period is less than the fundamental period obtained from eigenvalue analysis, the user can modify the *CMR* obtained by equation 3.11 by replacing $S_{MT}(T_{el})$ with $S_{MT}(T)$ or use the following equation:

$$\begin{cases} T < T_S \rightarrow CMR = \dfrac{4\pi^2 \delta_u r}{S_{MS} {T_{el}}^2 \mu_T \Gamma_I \phi_{I,r}} \\ T \geq T_S \rightarrow CMR = \dfrac{4\pi^2 T \delta_u r}{S_{M1} T_{el}^2 \mu_T \Gamma_I \phi_{I,r}} \end{cases} \tag{3.13}$$

If the approximate fundamental period is larger than the fundamental period obtained from eigenvalue analysis, Equation 3.11 should be used which results in conservative results.

3.6 Step by step procedure

The proposed simplified collapse assessment procedure described above can be summarized by the following steps:

1. Develop a numerical model of the building structure under evaluation which incorporates monotonic nonlinear behavior and degradation characteristics of the structural components.
2. Obtain the elastic fundamental period of vibration (T_{el}) and the corresponding first mode shape using eigenvalue analysis of the elastic building structure.
3. Perform a nonlinear static (pushover) analysis of the building structure with the lateral force distribution that is proportional to its first mode shape until the point of 20% strength loss.
4. Fit a bilinear curve to the base shear-roof displacement pushover curve according to the FEMA P695 methodology (see Figure 3-9) and obtain the ultimate roof displacement (δ_u) and target ductility ratio (μ_T).

5. Construct the inelastic mode shape (ϕ_I) from pushover data, which represents the relative displacement of each story when the ultimate roof displacement (δ_u) is reached.
6. Calculate the inelastic mode participation factor (Γ_I) using Equation 3.7.
7. Extract the reduction factor (r) by linear interpolation from Table 3-2 to Table 3-5 based on the given elastic fundamental period of vibration (T_{el}) and target ductility (μ_T).
8. Calculate the *CMR* fromEquation3.11 or Equation 3.13 as addressed in Section 3.5.

3.7 Assessment of proposed procedure

In order to assess the validity and robustness of the proposed procedure, the collapse capacities of three steel frame building models (three-, six- and nine-story) that were designed for the SAC Steel Project (ATC, 1994) are evaluated. The buildings are idealized as two dimensional structures. The structural sections of the building models meet post-Northridge seismic design requirements for Los Angeles, California. These buildings were chosen because they also served as benchmark structures for the SAC studies and, thus, provide a direct means for comparing the findings of the proposed procedure with those available in the literature. Figure 3-11 shows the geometry and dimensions of the considered frame buildings and Table 3-6 to Table 3-8 list sections and seismic masses of the buildings. Detailed information about the considered buildings can be found in Hall (1995), Gupta and Krawinkler (1999) and FEMA (2005).

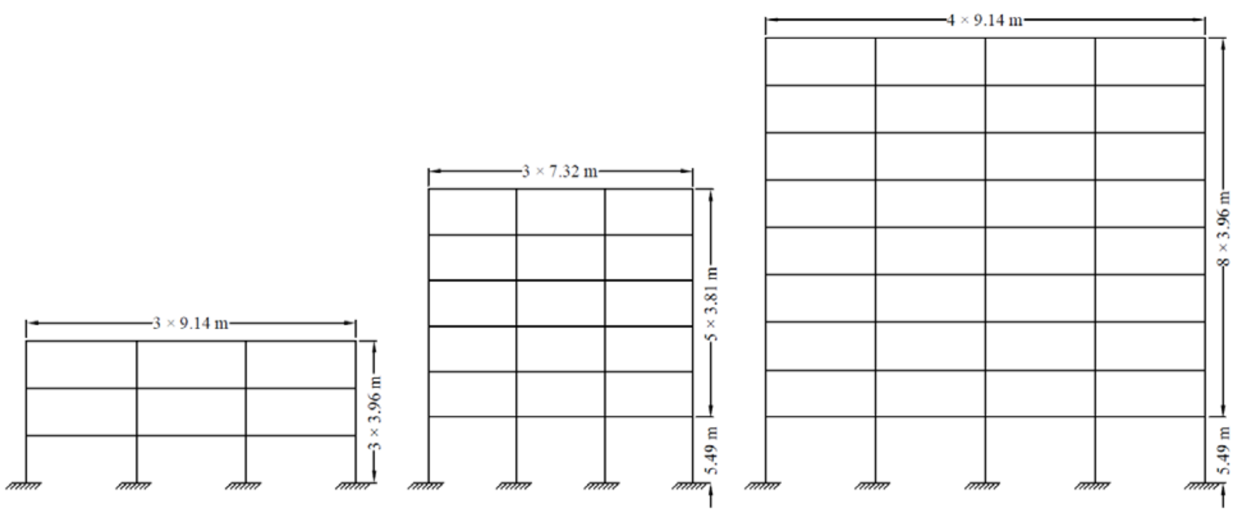


Figure 3-11 Elevation view of the selected steel frame buildings

Table 3-6 **Sections and floor seismic masses of three-story building**

Floor Number	Column		Girder	Seismic Mass (kN.s^2/m)
	Exterior	Interior		
1	W14×257	W14×311	W30×116	478.15
2	W14×257	W14×311	W30×116	478.15
3	W14×257	W14×311	W24×62	517.3

Table 3-7 **Sections and floor seismic masses of six-story building**

Floor Number	Column		Girder	Seismic Mass (kN.s^2/m)
	Exterior	Interior		
1	W14×193	W30×173	W30×99	265.2
2	W14×193	W30×173	W30×99	256.6
3	W14×159	W27×146	W27×94	256.6
4	W14×159	W27×146	W27×94	256.6
5	W14×109	W24×104	W24×76	256.6
6	W14×109	W24×104	W24×76	185.3

Table 3-8 **Sections and floor seismic masses of nine-story building**

Floor Number	Column		Girder	Seismic Mass (kN.s^2/m)
	Exterior	Interior		
1	W14×370	W14×500	W36×150	503.8
2	W14×370	W14×500, W14×455	W36×150	495.2
3	W14×370	W14×455	W33×141	495.2
4	W14×370, W14×283	W14×455, W14×370	W33×141	495.2
5	W14×283	W14×370	W33×141	495.2
6	W14×283, W14×257	W14×370, W14×283	W33×130	495.2
7	W14×257	W14×283	W27×102	495.2
8	W14×257, W14×233	W14×283, W14×257	W27×94	495.2
9	W14×233	W14×257	W24×62	533.4

The Open Systems for Earthquake Engineering Simulation (OpenSees Version 2.3.0, 2013) software was used to carry out the structural analyses. To simulate P-Δ effects, a leaning column, made of axially rigid beams pinned at both ends, was placed parallel to the frame. This leaning column was loaded with a vertical gravity load at each floor level and was constrained to the floor translational displacements. Rigid-end offsets were specified at the end of the frame members to account for the actual size of the members at the beam-column joints. The panel zones of the beam-column connections were assumed to be stiff and strong enough to avoid any panel shear deformation and yielding under seismic loading. This assumption represents the most critical condition for the inelastic curvature demand on the welded beam-to-column joints, as all the hysteretic energy must be dissipated only through plastic hinging in the beams and the columns. Inherent Rayleigh damping of 5% of critical was assigned based on the first and third elastic modes of vibration of the structureas suggested by Haselton et al. (2011).

Inelastic materials response was modeled by concentrated plasticity through nonlinear fiber hinges at both ends of the framing members. The hysteretic behavior of the fiber hinges was governed by a modified version of the Ibarra–Krawinkler (IK) deterioration model (Lignos and Krawinkler, 2011). The phenomenological IK model (see Figure 3-12) is based on a backbone curve that defines a boundary for the behavior of a structural component and establishes strength and deformation bounds, as well as a set of rules that define the basic characteristics of the hysteretic behavior between these bounds. Cyclic deterioration is quantified using a deterioration parameter that is based on a reference energy dissipation capacity of the component. Parameters of this model were determined by multivariate regression analysis of experimental results from over 300 steel specimens.

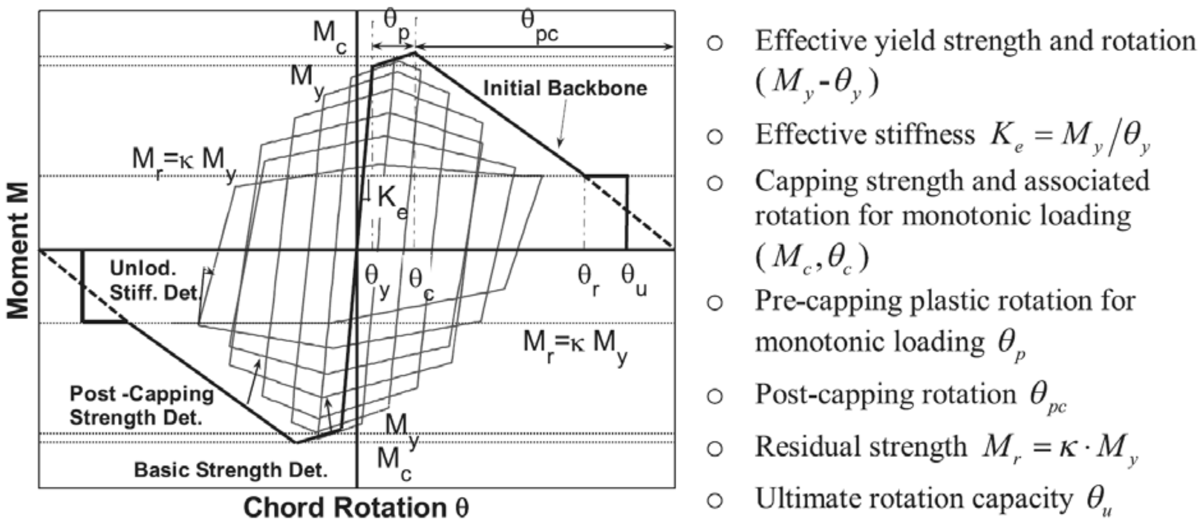


Figure 3-12 Modified IK deterioration model (from Lignos et al. (2011))

The following regression equations were used to obtain pre-capping plastic rotation (θ_p):

$$\begin{cases} \bullet \text{ For } d < 21 \text{ in}: \\ \theta_p = 0.0865 \cdot \left(\dfrac{h}{t_w}\right)^{-0.365} \cdot \left(\dfrac{b_f}{2 \cdot t_f}\right)^{-0.140} \cdot \left(\dfrac{L}{d}\right)^{0.340} \cdot \left(\dfrac{c_{unit}^1 \cdot d}{533}\right)^{-0.721} \cdot \left(\dfrac{c_{unit}^2 \cdot F_y}{355}\right)^{-0.230} \\ \bullet \text{ For } d \geq 21 \text{ in}: \\ \theta_p = 0.318 \cdot \left(\dfrac{h}{t_w}\right)^{-0.550} \cdot \left(\dfrac{b_f}{2 \cdot t_f}\right)^{-0.345} \cdot \left(\dfrac{L_b}{r_y}\right)^{-0.023} \cdot \left(\dfrac{L}{d}\right)^{0.090} \cdot \left(\dfrac{c_{unit}^1 \cdot d}{533}\right)^{-0.330} \cdot \left(\dfrac{c_{unit}^2 \cdot F_y}{355}\right)^{-0.130} \end{cases} \tag{3.14}$$

where h is depth of the web, t_w is thickness of the web, b_f is width of the flange, t_f is thickness of the flange, d is depth of the section, F_y is the yield strength of the section, L is the distance from plastic hinge location to point of inflection, L_b is the distance from connection face to the nearest lateral brace and r_y is the radius of gyration about the y-axis of the beam. Also, c^1_{unit} and c^2_{unit} are coefficients for unit conversion. They both are 1.0 if millimeters and megapascals are used, and c^1_{unit}=25.4 and c^2_{unit}=6.895 if d is in inches and F_y is in ksi. In addition it was assumed that L_b/r_y=50 and M_c/M_y=1.10 as suggested by Lignos and Krawinkler (2011).

The following regression equations were used to obtain post-capping rotation (θ_{pc}):

$$\begin{cases} \bullet \text{ For } d < 21 \text{ in}: \\ \theta_{pc} = 5.63 \cdot \left(\dfrac{h}{t_w}\right)^{-0.565} \cdot \left(\dfrac{b_f}{2 \cdot t_f}\right)^{-0.800} \cdot \left(\dfrac{c^1_{unit} \cdot d}{533}\right)^{-0.280} \cdot \left(\dfrac{c^2_{unit} \cdot F_y}{355}\right)^{-0.430} \\ \bullet \text{ For } d \geq 21 \text{ in}: \\ \theta_{pc} = 7.50 \cdot \left(\dfrac{h}{t_w}\right)^{-0.610} \cdot \left(\dfrac{b_f}{2 \cdot t_f}\right)^{-0.710} \cdot \left(\dfrac{L_b}{r_y}\right)^{-0.110} \cdot \left(\dfrac{c^1_{unit} \cdot d}{533}\right)^{-0.161} \cdot \left(\dfrac{c^2_{unit} \cdot F_y}{355}\right)^{-0.320} \end{cases} \tag{3.15}$$

The rate of cyclic deterioration was also defined by reference cumulative plastic rotation (Λ) parameter determined as follows:

$$\begin{cases} \bullet \text{ For } d < 21 \text{ in}: \\ \Lambda = \dfrac{E_t}{M_y} = 495 \cdot \left(\dfrac{h}{t_w}\right)^{-1.34} \cdot \left(\dfrac{b_f}{2 \cdot t_f}\right)^{-0.595} \cdot \left(\dfrac{c^2_{unit} \cdot F_y}{355}\right)^{-0.360} \\ \bullet \text{ For } d \geq 21 \text{ in}: \\ \Lambda = \dfrac{E_t}{M_y} = 536 \cdot \left(\dfrac{h}{t_w}\right)^{-1.26} \cdot \left(\dfrac{b_f}{2 \cdot t_f}\right)^{-0.525} \cdot \left(\dfrac{L_b}{r_y}\right)^{-0.130} \cdot \left(\dfrac{c^2_{unit} \cdot F_y}{355}\right)^{-0.291} \end{cases} \tag{3.16}$$

The IK deterioration model does not account for the effect of a variable axial force on the bending deterioration parameters. The use of fiber elements, however, results in reductions of the bending strength of the frame elements due to the variable axial-moment interactions. This approach results in smooth backbone curves for flexural members, similar to that observed in experiments, but it increases the computational overhead. Force-based *beam with hinges* elements (OpenSees, 2013) incorporating the modified Gauss–Radau plastic hinge integration method developed by Scott and Fenves (2006) were used. The implementation divides the element into two hinges at the ends with one integration point per hinge, and a linear-elastic region in the central portion of the element with two integration points. There are advantages to this formulation over the standard force-based beam-column elements as addressed by Scott and Fenves (2006) and Scott and Ryan (2013). This formulation ensures a unique relationship between the resulting moment-rotation hardening ratio and the parameters defining the plastic hinge length and the moment-curvature hardening ratio (Scott and Ryan, 2013).The fiber hinge sections were composed of the *Bilin* material (OpenSees, 2013) that simulates the modified IK deterioration model. Plastic hinge lengths were set equal to the depth of the sections as suggested in Section C8.7 of AISC 358-10 (AISC, 2010).The hinges were then calibrated individually with the multi-linear deteriorating concentrated hysteresis model.

To assess the performance of the proposed procedure for a wide range of fundamental periods of the benchmark buildings, the first story height (h_1) was incrementally varied while all the other stories height (h_{typ}) remained unchanged. A ratio of h_1/h_{typ} from 1 to 2.4 at increments of 0.1 was considered and led to a total of 45 different building structures to be evaluated with elastic periods ranging from 0.94 s to 2.96 s. Note that h_1/h_{typ} is approximately 1.4 for the original six- and nine-story buildings and 1.0 for the three-story building. To assess the performance of the proposed procedure for short period buildings, the floor masses of the original three-story building were reduced from 10% to 90% of their original values by an increment of 10%. The elastic periods of these nine supplemental buildings varied ranging from 0.30 s to 0.89 s. To avoid a bias of the results towards first floor sidesway collapse, the heights of the three-story building were also increased uniformly by a factor (h_{unif}/h_{typ}) ranging from 0.5 to 2.0 of their initial height at increments of 0.1. This led to a total of 16 more building models with elastic periods ranging from 0.41 s to 2.29 s. Further information can be found in Hamidia et al. (2014a, 2014b, 2014c, 2014d) and Hamidia (2014).

The computation of the collapse capacity for each considered building is based on the set of 44 records of the P695 far-field ground motions as required by the FEMA P695 methodology. Accordingly, as defined in FEMA P695, the *CMR* is the ratio of the median 5%-damped collapse spectral acceleration across the ground motion ensemble ($\hat{S}_{CT}$) to the corresponding 5%-damped MCE spectral acceleration (S_{MT}), both values are taken at the fundamental period of the building obtained through eigenvalue analysis. In this study, it is assumed that the collapse level ground motion, $\hat{S}_{CT}$, occurs when the structure experiences maximum inter-story drift ratios for 22 out of the 44 records exceeding the larger inter-story drift ratio associated with the ultimate roof displacement (δ_u) calculated from the pushover analysis.

The results of the application of the proposed simplified collapse evaluation procedure to the ensemble of frame building structures considered above are summarized in Table 3-9 to Table 3-11 for the buildings with varying first story heights, floor masses and uniform story heights respectively. Figure 3-13 compares the *CMR* values derived from the proposed simplified procedure with those obtained from nonlinear time history dynamic analyses. The predicted *CMR* values based on the proposed simplified procedure are in very good agreement with those obtained from nonlinear time history dynamic analyses, but with much reduced computational demand. The maximum average absolute difference between the two approaches is only 4.5%, with a maximum standard deviation of 5% for the buildings with varying first story height (Table 3-9). The corresponding values for the buildings with varying floor masses (Table 3-10) are 1.7% and 2.0% respectively, and for the building with varying uniform story heights (Table 3-11) both values are 2.0%. These results are acceptable considering that the proposed simplified procedure alleviates the need for nonlinear time history dynamic analyses. Figure 3-14 compares the differences in *CMR* values predicted by the proposed simplified procedure with those obtained from nonlinear time history dynamic analyses as a function of the elastic period of the 72 building structures considered. Clearly, for the entire period range considered (0.30 s to 2.96 s), there is no discernible divergence in trend, which is indicative of the robustness of the proposed procedure.

Table 3-9 Simplified analysis results and comparison with CMR obtained from time history analyses for buildings with varying first story heights

Type	h_1/h_{typ}	T_{el} (sec)	δ_y (cm)	δ_u (cm)	Maximum Interstory Drift Ratio	μ_T	r	$\Gamma_1\phi_{1,r}$	*CMR* fom Equation 11	*CMR* from Nonlinear Time History	Differences
3 Story	1.0	0.94	8.96	59.28	5.39%	6.61	5.95	1.28	2.00	1.93	+3%
	1.1	0.99	9.32	61.58	5.30%	6.60	5.98	1.27	1.99	1.93	+3%
	1.2	1.04	9.78	64.11	5.34%	6.56	5.91	1.27	1.95	1.93	+1%
	1.3	1.11	10.28	66.77	5.38%	6.50	5.83	1.26	1.92	1.93	0%
	1.4	1.17	10.90	69.50	5.42%	6.38	5.94	1.26	1.97	2.00	-2%
	1.5	1.25	11.59	72.37	5.44%	6.24	6.04	1.25	2.02	2.02	0%
	1.6	1.32	12.37	75.42	5.48%	6.10	6.06	1.24	2.05	2.02	+1%
	1.7	1.40	13.21	78.57	5.56%	5.95	6.04	1.23	2.06	2.10	-2%
	1.8	1.49	14.11	81.91	5.73%	5.81	5.93	1.22	2.05	2.08	-1%
	1.9	1.58	15.10	85.35	5.89%	5.65	5.49	1.22	1.93	1.95	-1%
	2.0	1.68	16.13	88.93	6.05%	5.51	5.17	1.21	1.84	1.83	+1%
	2.1	1.78	17.25	92.67	6.22%	5.37	4.88	1.20	1.77	1.83	-3%
	2.2	1.88	18.44	96.57	6.39%	5.24	4.84	1.19	1.78	1.83	-2%
	2.3	1.98	19.68	101.14	6.61%	5.14	4.94	1.19	1.85	1.85	0%
	2.4	2.09	20.97	108.36	7.11%	5.17	4.97	1.17	1.90	1.90	0%
								Average of Absolute Differences:			1.3%
								STDEV of Differences:			2%
6 Story	1.0	1.16	13.52	76.84	5.66%	5.68	5.33	1.20	2.31	2.28	+2%
	1.1	1.19	13.50	74.70	5.58%	5.53	5.31	1.19	2.26	2.20	+3%
	1.2	1.23	13.46	73.20	5.57%	5.44	5.32	1.18	2.22	2.15	+3%
	1.3	1.26	13.54	72.80	5.61%	5.38	5.31	1.17	2.18	2.05	+7%
	1.4	1.30	13.53	72.59	5.70%	5.36	5.36	1.16	2.15	1.98	+9%
	1.5	1.35	13.60	72.17	5.99%	5.31	5.35	1.15	2.11	1.95	+8%
	1.6	1.40	13.68	63.90	6.26%	4.67	4.78	1.13	1.85	1.97	-6%
	1.7	1.45	13.78	56.54	5.82%	4.10	4.27	1.12	1.63	1.75	-7%
	1.8	1.51	13.83	52.52	5.49%	3.80	4.00	1.11	1.48	1.55	-4%
	1.9	1.57	13.83	52.52	5.20%	3.80	4.04	1.11	1.44	1.40	+3%
	2.0	1.63	14.00	49.74	5.08%	3.55	3.80	1.09	1.34	1.35	0%
	2.1	1.70	14.12	49.29	4.93%	3.49	3.66	1.08	1.26	1.23	+3%
	2.2	1.77	14.29	49.32	4.82%	3.45	3.55	1.07	1.20	1.17	+2%
	2.3	1.84	14.52	49.51	4.72%	3.41	3.48	1.07	1.15	1.10	+5%
	2.4	1.92	14.77	50.02	4.64%	3.39	3.48	1.06	1.13	1.08	+5%
								Average of Absolute Differences:			4.5%
								STDEV of Differences:			5%
9 Story	1.0	1.97	21.97	112.01	5.76%	5.10	4.90	1.22	2.01	1.88	+7%
	1.1	2.01	22.18	108.21	5.30%	4.88	4.81	1.21	1.97	1.82	+8%
	1.2	2.04	22.42	100.50	5.10%	4.48	4.54	1.20	1.86	1.73	+7%
	1.3	2.08	22.66	94.96	4.85%	4.19	4.33	1.18	1.78	1.67	+7%
	1.4	2.13	22.91	90.70	4.87%	3.96	4.11	1.17	1.69	1.65	+2%
	1.5	2.18	23.04	88.01	4.86%	3.82	3.90	1.16	1.58	1.58	0%
	1.6	2.24	23.20	86.46	4.83%	3.73	3.76	1.16	1.51	1.55	-3%
	1.7	2.30	23.33	85.63	4.79%	3.67	3.67	1.15	1.45	1.50	-3%
	1.8	2.38	23.53	85.31	4.73%	3.63	3.57	1.14	1.39	1.45	-4%
	1.9	2.46	23.77	85.40	4.80%	3.59	3.54	1.13	1.35	1.46	-7%
	2.0	2.54	24.05	85.67	4.97%	3.56	3.58	1.13	1.35	1.44	-6%
	2.1	2.63	24.41	85.80	5.10%	3.51	3.62	1.12	1.34	1.40	-4%
	2.2	2.73	24.82	85.93	5.22%	3.46	3.56	1.11	1.30	1.34	-3%
	2.3	2.84	25.26	85.75	5.32%	3.39	3.46	1.10	1.25	1.25	0%
	2.4	2.96	25.76	85.34	5.37%	3.31	3.41	1.10	1.21	1.18	+3%
								Average of Absolute Differences:			4.3%
								STDEV of Differences:			5%

Table 3-10 Simplified analysis results and comparison with *CMR* obtained from time history analyses for three-story buildings with varying floor masses

Type	Percentage of Total Mass	T_{el} (sec)	δ_y (cm)	δ_u (cm)	Maximum Interstory Drift Ratio	μ_T	r	$\Gamma_I \phi_{I,r}$	*CMR* fom Equation 11	*CMR* from Nonlinear Time History	Differences
3 Story	10	0.30	9.07	62.69	5.74%	6.91	4.23	1.28	9.00	8.90	1%
	20	0.42	9.06	62.29	5.70%	6.88	4.67	1.28	5.12	5.25	-2%
	30	0.51	9.05	61.88	5.66%	6.84	5.34	1.28	3.93	4.03	-2%
	40	0.59	9.03	61.49	5.62%	6.81	5.59	1.28	3.05	3.05	0%
	50	0.66	9.02	61.11	5.58%	6.77	5.71	1.28	2.73	2.79	-2%
	60	0.72	9.01	60.73	5.54%	6.74	5.79	1.28	2.52	2.55	-1%
	70	0.78	9.00	60.36	5.50%	6.71	5.83	1.28	2.35	2.38	-1%
	80	0.84	8.97	60.01	5.47%	6.69	5.90	1.28	2.22	2.32	-4%
	90	0.89	8.96	59.65	5.43%	6.66	5.95	1.28	2.11	2.13	-1%
								Average of Absolute Differences:			1.7%
								STDEV of Differences:			2%

Table 3-11 Simplified analysis results and comparison with *CMR* obtained from time history analyses for three-story buildings with varying uniform story heights

Type	h_{unif}/h_{typ}	T_{el} (sec)	δ_y (cm)	δ_u (cm)	Maximum Interstory Drift Ratio	μ_T	r	$\Gamma_I \phi_{I,r}$	*CMR* fom Equation 11	*CMR* from Nonlinear Time History	Differences
3 Story	0.5	0.41	3.50	31.03	5.40%	8.86	5.62	1.27	2.48	2.53	-2%
	0.6	0.50	4.45	36.85	5.39%	8.28	6.56	1.27	2.43	2.43	0%
	0.7	0.61	5.47	42.57	5.39%	7.79	6.10	1.28	1.93	1.92	+1%
	0.8	0.71	6.56	48.22	5.39%	7.35	6.15	1.28	1.99	1.97	+1%
	0.9	0.82	7.73	53.80	5.39%	6.96	6.12	1.28	2.02	2.07	-3%
	1.0	0.94	8.96	59.28	5.39%	6.61	5.95	1.28	2.00	1.93	+3%
	1.1	1.05	10.28	64.76	5.40%	6.30	5.69	1.28	1.95	1.95	0%
	1.2	1.18	11.66	70.17	5.41%	6.02	5.67	1.28	1.97	2.02	-3%
	1.3	1.30	13.12	75.54	5.42%	5.76	5.73	1.28	2.02	2.04	-1%
	1.4	1.43	14.64	80.88	5.44%	5.53	5.64	1.28	2.02	2.07	-3%
	1.5	1.57	16.25	86.19	5.46%	5.30	5.26	1.28	1.91	1.92	-1%
	1.6	1.70	17.89	91.50	5.48%	5.11	4.85	1.28	1.78	1.79	0%
	1.7	1.84	19.64	96.78	5.51%	4.93	4.57	1.28	1.70	1.73	-2%
	1.8	1.99	21.41	102.10	5.55%	4.77	4.72	1.28	1.78	1.77	0%
	1.9	2.14	23.25	107.49	5.60%	4.62	4.57	1.28	1.74	1.79	-3%
	2.0	2.29	25.14	112.88	5.65%	4.49	4.42	1.28	1.70	1.82	-7%
								Average of Absolute Differences:			2%
								STDEV of Differences:			2%

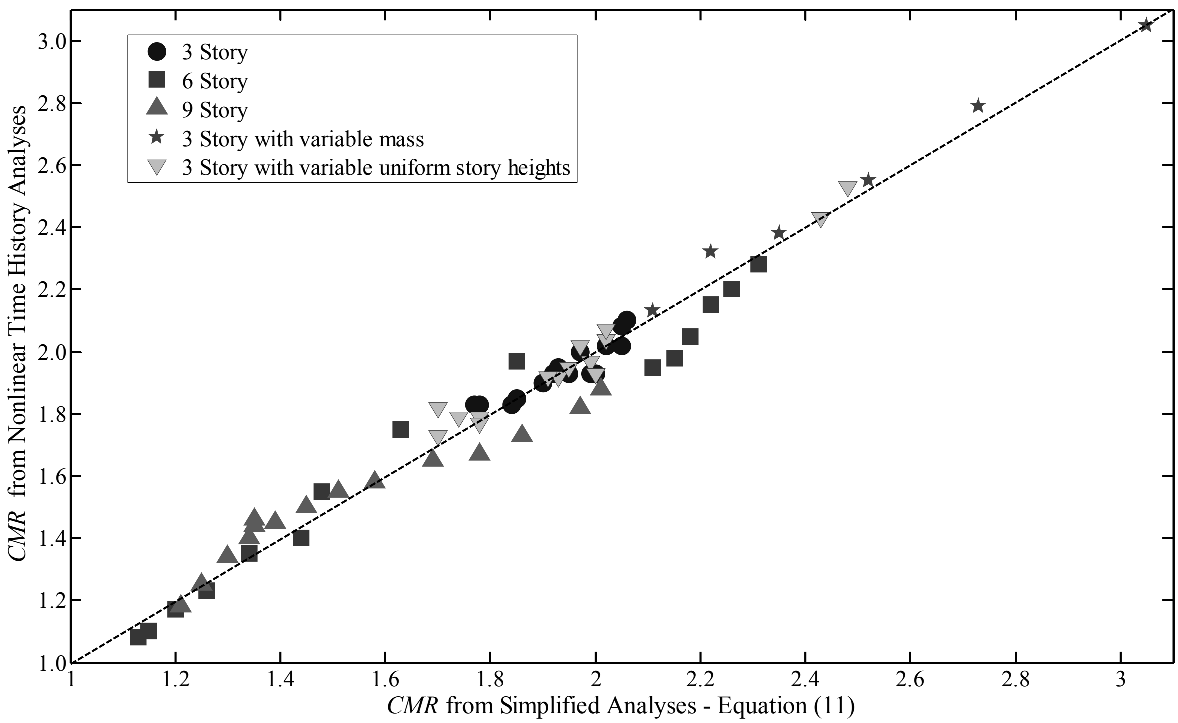


Figure 3-13 Comparison of collapse margin ratios (*CMR*) obtained from simplified analyses and nonlinear time history dynamic analyses

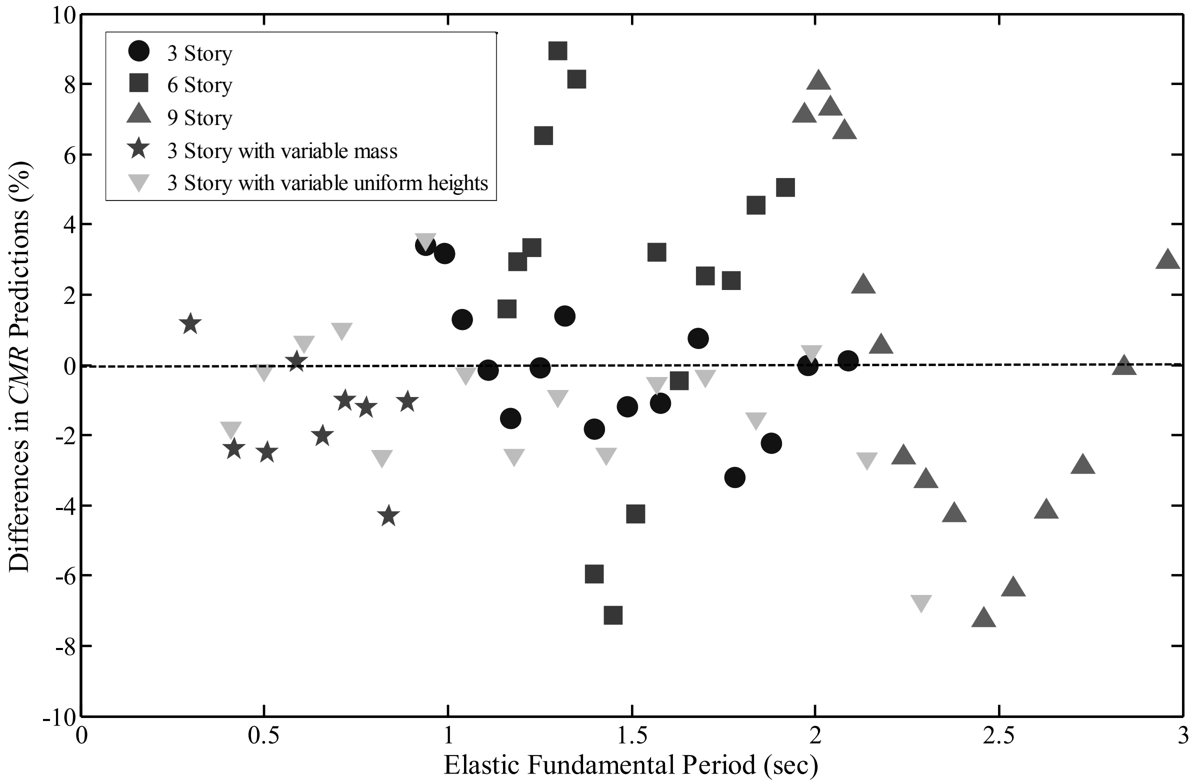


Figure 3-14 Error in estimating collapse margin ratio (*CMR*) for various fundamental periods

3.8 Detailed comparison with example application from FEMA P695

Chapter 9 of FEMA P695 illustrates the application of its methodology to several archetype buildings. Detailed calculations are presented for a four-story reinforced concrete building with a special moment resisting frame system (archetype ID 1010). The first story of this building is 4.57 m high, while the height for the other three stories is 3.96 m, for a total building height of 16.45m. In this example, FEMA P695 uses the full IDA approach to compute the collapse capacity of the building. The fundamental period of the building is approximated at 1.03 s by the empirical formula of ASCE7 larger than the value obtained from eigenvalue analysis. The lateral load pattern used for pushover analysis is also consistent with ASCE7. Figure 3-15 shows the results of the pushover analysis.

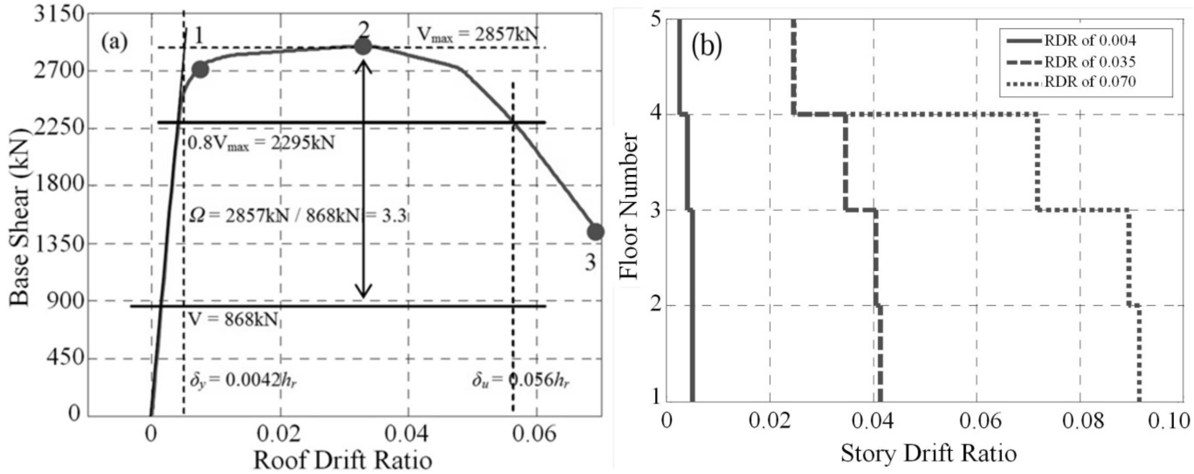


Figure 3-15 Pushover results for FEMA P695 example concrete building (from (FEMA, 2009))

The target ductility ratio μ_T= 13.2 and at the point of ultimate roof displacement, the building drift ratio is 0.056, which results in a target displacement of 0.056×16.45 = 0.92 m at the roof level. By linear interpolation in Table 3-2, r = 10.61. Although the pushover data is not given for the point of ultimate roof displacement (0.92 m), a linear interpolation of the pushover curve presented in Figure 3-15 for a roof drift ratio of 0.056 results in approximately 7%, 7%, 5.5% and 2.5% inter story drift ratios from the first story to the roof, respectively, at the point of ultimate roof displacement. Therefore, the inelastic mode shape can be approximated as:

$$\phi_I = \begin{Bmatrix} 3.02 \\ 2.68 \\ 1.96 \\ 1.05 \end{Bmatrix} \tag{3.17}$$

From Equation 3.7 and considering a uniform distribution of the seismic mass along the building height $\Gamma_I \phi_{I,r}$ = 1.23. Given the spectral acceleration at a period of 1.0 s (S_{M1}=0.9g for SDC D_{max}) the *CMR* predicted by the proposed simplified seismic collapse analysis procedure is then obtained from Equation 3.11 as:

$$CMR = \frac{4\pi^2 \delta_u r}{S_{M1} T_{el} \mu_T \Gamma_I \phi_{I,r}} = \frac{4\pi^2 \times 0.92 \times 10.61}{0.9 \times 9.81 \times 1.03 \times 13.2 \times 1.23} = 2.61 \tag{3.18}$$

The *CMR* obtained from nonlinear time history analysis is 2.48 as given in FEMA P695, and this represents a difference of 5% with that predicted by Equation 3.11, which is quite acceptable.

The median intensity measured for the roof displacement capacity of δ_u=0.92 m is estimated at 2.60 by the SPO2IDA software (Vamvatsikos and Cornell, 2006), as shown in Figure 3-16. This value correlates well with the *CMR* obtained from Equation3.15. The ultimate collapse capacity is also 2.87 as appeared on the flat-line of the 50%-fractile IDA, close to *CMR* obtained by FEMA P695.

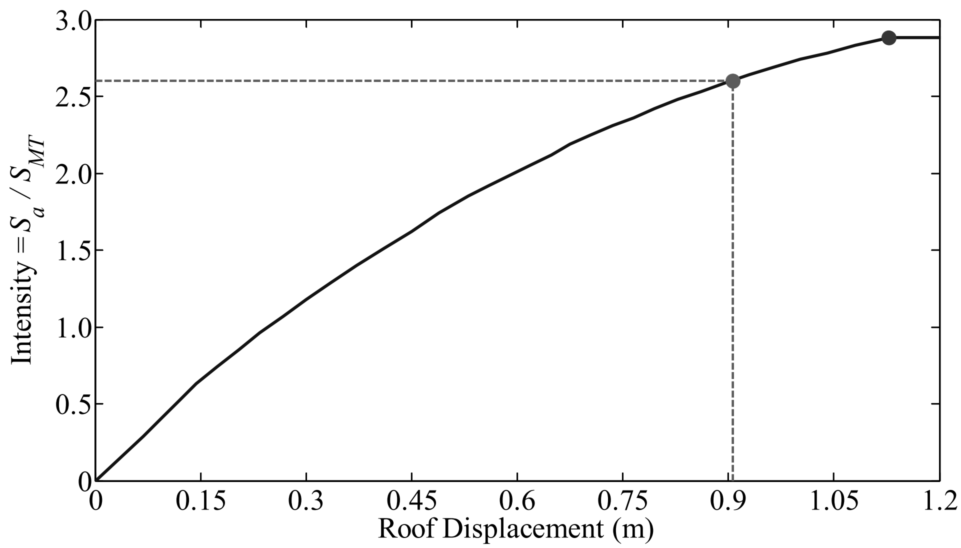


Figure 3-16 Estimated 50% IDA curve estimated by SPO2IDA software

3.9 Detailed comparison with example application from NIST steel frame

The National Institute of Standard and Technology (NIST) report on the evaluation of the FEMA P695 methodology (NIST, 2010) also presents detailed calculation for a 20-story special steel moment resisting frame (archetype ID: 6RSA). Figure 3-17 shows the pushover results for various roof drift ratios. The target ductility ratio μ_T = 1.9 and the fundamental period is approximated at 3.37 sec by the empirical formula of ASCE7. The ultimate roof drift ratio at 20% strength loss is 1.4%, which results in a target displacement of 1.13 m at the roof level. Again, by linear interpolation in Table 3-2, *r* = 2.16. Considering, the interpolation of the data provided in Figure 3-17 for roof drift ratios equal to 1.25% and 1.5%, the corresponding inelastic mode shape $\Gamma_I \phi_{I,r}$ = 1.3 from Equation3.7. The *CMR* is obtained from Equation 3.11 as:

$$CMR = \frac{4\pi^2 \delta_u r}{S_{M1} T_{el} \mu_T \Gamma_I \phi_{I,r}} = \frac{4\pi^2 \times 1.13 \times 2.16}{0.9 \times 9.81 \times 3.37 \times 1.9 \times 1.3} = 1.31 \quad (3.19)$$

The collapse margin ratio from nonlinear time history dynamic analysis is reported as 1.21 (NIST, 2010), which represents an 8% deviation from the predictions by Equation3.11, which is quite acceptable considering the simplicity of the proposed procedure.

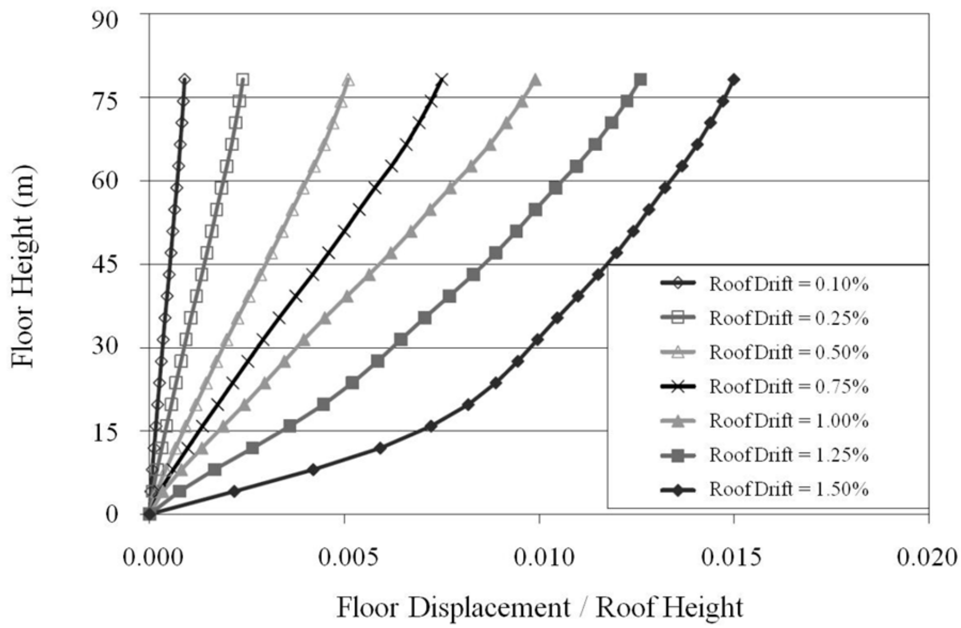


Figure 3-17 Pushover results for example steel building (from (NIST, 2010))

3.10 Limitations of the proposed procedure

The estimation of the seismic response parameters through a SDOF model in combination with nonlinear static (pushover) analysis has some inherent limitations (Krawinkler and Seneviratna, 1998; Fajfar, 2000). For example, the effect of cyclic deterioration in strength and stiffness of structural components is not considered. The proposed procedure is aimed at estimating the *CMR* due to sidesway seismic collapse of buildings dominated by a single mode of deformation and is unable to predict collapse modes dominated by forces and overturning moments along the height of a frame building. Prediction of complete sidesway collapse fragility curves is also out of the scope of the proposed procedure. In addition, the proposed procedure has been assessed for steel frame buildings only and its application to other structural systems, such as reinforced concrete structures, is subject to further investigations especially on the approximation of the elastic period. Furthermore, the proposed procedure is strictly valid for the P695 far-field ground motions ensemble adopted in this study.

3.11 Summary

The simplified procedure to evaluate the seismic collapse capacity of building structures presented in this section is based on the development of a robust database of nonlinear seismic responses of single-degree-of-freedom systems for various seismic intensities and uses nonlinear static (pushover) analysis without the need for nonlinear time history dynamic analyses. The procedure was assessed using a total of 72 different frame building structures with a wide range of fundamental periods (0.30 s to 3.37 s). The collapse capacities predicted by the proposed simplified procedure are in very good agreement with those obtained by nonlinear time history dynamic analyses. The average absolute difference between the two approaches was only 4.5%, with a maximum standard deviation of 5% for long period buildings (elastic periods between 0.90 s to 2.96 s). The corresponding values for short period buildings (elastic periods between 0.3 s to 0.89 s) were 1.7% and 2.0%, respectively, while both values for buildings with varying uniform heights were 2%. Given its inherent simplicity combined with its consistency with current methodologies used in seismic design practice, the proposed procedure represents a useful tool for practicing engineers and researchers interested in evaluating the safety of structures against sidesway collapse with minimal computational overhead. At the same time, it is important to note that the proposed procedure is only an approximate approach for the quick estimation of the collapse capacity of frame buildings that cannot replace more rigorous nonlinear time history dynamic analyses.

SECTION 4

COLLAPSE CAPACITY BASED EVALUATION AND DESIGN OF FRAME BUILDINGS WITH VISCOUS DAMPERS

4.1 Introduction

The simplified procedure developed in Section 3.4 is modified in this section for estimating the seismic sidesway collapse capacity of frame building structures incorporating linear and nonlinear viscous dampers. The proposed procedure is based on a robust database of seismic peak displacement responses of viscously damped nonlinear SDOF systems for various seismic intensities and uses nonlinear static (pushover) analysis without the need for nonlinear time history dynamic analysis. The proposed procedure is assessed by comparing its collapse capacity predictions on 1,190 different building models with those obtained from incremental nonlinear dynamic analyses. A straightforward collapse capacity based design procedure aimed at achieving a pre-determined probability of collapse under MCE event is also introduced for frame building structures incorporating linear and nonlinear viscous dampers without extreme soft story irregularities. Several equations derived in section 3 are repeated in this section in order to facilitate the derivation of the procedure and make the section self-contained.

4.2 Characteristics of nonlinear viscous dampers

4.2.1 Single-degree-of-freedom systems

The force developed by a nonlinear viscous damper (F_D) is expressed by:

$$F_D = C_{NL}\,\mathrm{sgn}(\dot{u}_D)\left|\dot{u}_D\right|^{\alpha} \tag{4.1}$$

Where C_{NL} is the nonlinear viscous damping constant, $\dot{u}_D$ is the relative velocity between the two ends of the damper, sgn() stands for signum function and α is a predetermined velocity exponent in the range of 0.2 to 1.0 for seismic application. The advantage of nonlinear viscous dampers over linear ones is the reduction of damper forces at high velocities to avoid overloading the dampers or the system to which they are connected (Christopoulos and Filiatrault, 2006). When α= 1.0, the device acts as a linear viscous damper and Equation 4.1 is reduced to:

$$F_D = C_L \dot{u}_D \tag{4.2}$$

where C_L stands for linear viscous damping constant. Consider a nonlinear viscous damper subjected to a harmonic relative displacement time-history between its ends given by:

$$u(t) = u_0 \sin \Omega t \tag{4.3}$$

Where u_0 is the displacement amplitude and Ω stands for the excitation frequency. Substituting Equation4.3 into Equation 4.1 yields:

$$F_D = C_{NL} \operatorname{sgn}(\cos \Omega t) \left| \Omega X_0 \cos \Omega t \right|^{\alpha} \tag{4.4}$$

The energy dissipated by the nonlinear viscous damper for each cycle (W_D) is the area under the force-displacement relationship:

$$W_D = \int F_D du = \int_0^{\frac{2\pi}{\Omega}} F_D \dot{u} dt = \int_0^{\frac{2\pi}{\Omega}} C_{NL} \left| \dot{u} \right|^{1+\alpha} dt \tag{4.5}$$

which yields:

$$W_D = \lambda C_{NL} \Omega^{\alpha} u_0^{1+\alpha} \tag{4.6}$$

The constant λ is equal to:

$$\lambda = \frac{2^{2+\alpha} \Gamma^2 (1 + \alpha/2)}{\Gamma(2+\alpha)} \tag{4.7}$$

where Γ is the Euler Gamma function. Table 4-1 lists values of λ for various values of the exponent α. For a linear damper with $\alpha = 1.0$, $\lambda = \pi$ andEquation4.6reduces to:

$$W_D = \pi C_L \Omega u_0^2 \tag{4.8}$$

Table 4-1 Values of λ for various values of exponent α

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
λ	3.88	3.77	3.67	3.58	3.50	3.42	3.34	3.27	3.20	3.14

For a linear elastic SDOF system, the strain energy at a displacement equal to u_0 can be defined as:

$$W_S = \frac{1}{2} m \omega^2 u_0^2 \tag{4.9}$$

where ω is the natural frequency of the SDOF system and m is the mass of the oscillator. The supplemental damping ratio (ξ) of the SDOF system can be defined using the concept of equivalent viscous damping ratio (Chopra, 2011):

$$\xi = \frac{W_D}{4\pi W_S} \tag{4.10}$$

Substituting Equations 4.6 and 4.9 into Equation 4.10 yields an expression for the equivalent viscous damping ratio provided by a nonlinear viscous damper:

$$\xi = \frac{\lambda}{\pi} \frac{1}{2m\omega} \frac{C_{NL}}{(\Omega u_0)^{1-\alpha}} \tag{4.11}$$

For seismic application, the excitation frequency (Ω) is taken equal to natural frequency of the SDOF system (ω). (Lin and Chopra, 2002). For the case of a linear viscous damper (α = 1.0), Equation 4.11 becomes:

$$\xi = \frac{C_L}{2m\omega} \tag{4.12}$$

4.2.2 Multi-degrees-of-freedom systems

Assuming that a MDOF system undergoes a harmonic vibration such that:

$$\{u\} = D_{roof}\,\phi_1 \sin\left(\frac{2\pi t}{T_{el}}\right) \tag{4.13}$$

where D_{roof} is the amplitude of roof displacement, T_{el} is the undamped elastic fundamental period of vibration, ϕ_1 is the first undamped mode shape (normalized to have a unit component at the roof level). The energy dissipated by the damping system per cycle of motion in the first mode can be expressed by:

$$W_D = \sum_{j=1}^{N_d} \left(\frac{2\pi}{T_{el}}\right)^{\alpha_j} C_j \lambda_j \left(D_{roof} f_j (\phi_{1j} - \phi_{1(j-1)}\right)^{1+\alpha_j} \tag{4.14}$$

where C_j is the sum of all the damping constants for all viscous dampers (linear or nonlinear) at floor level j, f_j is a displacement magnification factor that depends on the geometrical arrangement of the dampers at floor level j ($f_j = cos\theta_j$ for a device j in diagonal configuration at an angle of inclination θ_j, see (Christopoulos and Filiatrault, 2006)), ϕ_{1j} is the first mode shape ordinate at floor level j, N_F is the number of floors, N_d stands for the number of dampers and λ_j is given by Equation 4.7 as a function of α_j, which is the velocity exponent of the device j (See Table 4-1).

The maximum elastic strain energy of a MDOF system, also equals to its maximum kinetic energy, can be expressed by:

$$W_S = \frac{1}{2} \sum_{j=1}^{N_F} m_j \dot{u}_{j0}^2 = \frac{2\pi^2}{T_{el}^2} \sum_{j=1}^{N_F} m_j D_{roof}^2 \phi_{1j}^2 \tag{4.15}$$

where m_j is the lumped mass at floor level j. Substituting Equations 4.14 and 4.15 into Equation 4.10, the equivalent first modal viscous damping ratio due to the incorporation of nonlinear viscous dampers can be estimated as:

$$\xi_1 = \frac{\sum_{j=1}^{N_d} (2\pi)^{\alpha_j} T_{el}^{2-\alpha_j} \lambda_j C_j f_j^{1+\alpha_j} D_{roof}^{\alpha_j -1} (\phi_{1j} - \phi_{1(j-1)})^{1+\alpha_j}}{8\pi^3 \sum_{j=1}^{N_F} m_j \phi_{j1}^2} \tag{4.16}$$

Considering linear viscous dampers case, Equation 4.16 reduces to:

$$\xi_1 = \frac{T_{el} \sum_{j=1}^{N_d} C_j f_j^{\,2} (\phi_{1j} - \phi_{1(j-1)})^2}{4\pi \sum_{j=1}^{N_F} m_j \phi_{1j}^{\,2}} \tag{4.17}$$

4.3 Nonlinear response database of single-degree-of-freedom oscillators

Nonlinear time history dynamic analyses are first performed on a wide range of SDOF oscillators with a bilinear force-displacement relationship characterized by a linear segment with initial stiffness (K_{el}), followed by a post-yield perfectly plastic segment with constant yield force (F_y) without hardening or cyclic deterioration. Supplemental damping is provided by a nonlinear velocity-dependent dashpot element with viscous damping constant (C_{NL}) and velocity exponent (α). Figure 4-1 illustrates the force-displacement relationship of the analyzed systems and identifies the pertinent key parameters. The dynamic response of each SDOF system is obtained in terms of its elastic period of vibration (T_{el}), yield reduction factor (R_y), equivalent viscous damping ratio (ξ) and velocity exponent (α) determined as follows:

$$T_{el} = 2\pi \sqrt{\frac{m}{K_{el}}} \tag{4.18}$$

$$R_y = \frac{S_{MT}}{A_y} \tag{4.19}$$

$$A_y = \frac{F_y}{m} \tag{4.20}$$

$$\xi = \frac{\lambda C_{NL}}{2\pi m} \Delta_y^{\alpha-1} \left(\frac{2\pi}{T_{el}}\right)^{\alpha-2} \tag{4.21}$$

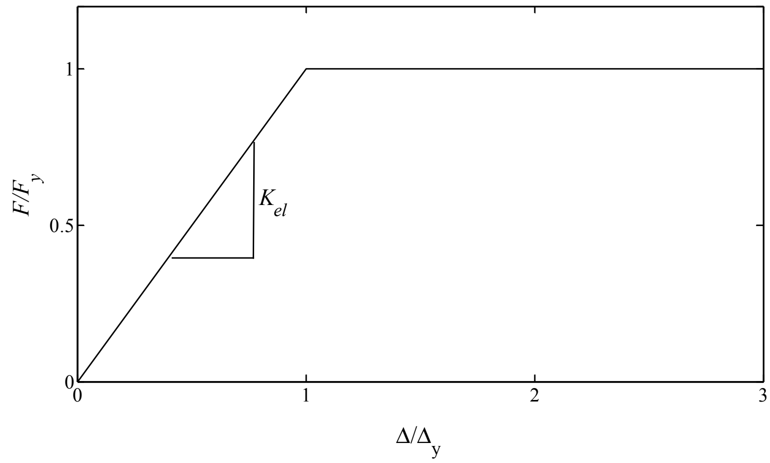


Figure 4-1 Force–displacement characteristics of bilinear SDOF system with target ductility of 3

where m is the mass of the oscillator (normalized herein to 1kg), S_{MT} is the MCE level elastic spectral acceleration at a period T_{el} and critical damping ratio of 5% and A_y represents the yield pseudo-acceleration. Note that C_{NL} in Equation 4.21 is obtained from Equation 4.11 by considering the natural frequency of the SDOF system as the excitation frequency and yield displacement (Δ_y) as the displacement amplitude. Elastic periods (T_{el}) ranging from 0.1 s to 4.0 s with 0.1 s increments were selected for the analyses thereby encompassing the fundamental periods of vibration of a wide range of building structures. Also, a range of R_y values between 1 and 10 with an increment of 0.5 was considered. The nonlinear SDOF systems considered in this study were assumed to have added equivalent viscous damping ratio (ξ) provided by the nonlinear dashpot element from 5% to 35% of critical at 5% increments and velocity exponent (α) ranging from 0.2 to 1.0 at 0.2 increments. In addition, 5% inherent viscous damping was considered for all the studied systems.

All SDOF systems described above were subjected to the set of 44 far-field ground motions adopted by the FEMA P695 methodology at various intensities. The set of ground motions used in this study was first normalized and scaled to the MCE level spectral acceleration at a period T_{el} (S_{MT}) consistent with a Seismic Design Category (SDC) D_{max} per ASCE7-10and according to the FEMA P695 methodology. The amplitudes of the ground motions were then expressed for each SDOF system by an intensity factor (I) defined as:

$$I = \frac{S_a}{S_{MT}} \tag{4.22}$$

where S_a is the 5% damped elastic spectral acceleration at a period T_{el}. Nonlinear time history dynamic analyses of each SDOF system were performed for intensity factors ranging from 0.1 to 35 at increments of 0.1. The intensity factor of 1.0 represents the scaled records at the MCE level for SDC D_{max}. Thus, a total of 40(T_{el}) × 19(R_y) × 350(I) × 44(records) × 7(ξ)× 5(α)>400 million analyses were performed using the University at Buffalo clusters, taking advantage of high performance computing concept to generate a robust database of peak displacement seismic responses of nonlinear SDOF systems for various seismic intensities. Although the SDC D_{max} is used herein to define the MCE spectral accelerations, the results of this study can be extended to other seismic design categories by appropriately scaling the denominator of Equation 4.22.

For each considered SDOF system with known T_{el}, R_y, ξ and α, a family of fragility curves giving the probability of exceeding a certain target peak displacement (Δ_T), can be constructed. Accordingly, the probability of exceedance at a given intensity factor is obtained by counting the number of records that cause the system to exceed Δ_T and dividing this number by the total number of records (44). The target peak displacement can also be expressed in terms of a target ductility ratio (μ_T) as follows:

$$\mu_T = \frac{\Delta_T}{\Delta_y} \tag{4.23}$$

For each SDOF oscillator analyzed, the intensity at which half of the records cause the system to exceed the considered target ductility ratio is defined as the median exceedance intensity (I_{med}). As shown in Figure 4-2 to Figure 4-9, significant scatter is observed for the median exceedance intensity in terms of various target ductility ratios, elastic periods, yield reduction factors, equivalent viscous damping ratios and velocity exponents. In general, I_{med} increases with a reduction of R_y and an increase of T_{el}, μ_T, ξ and α.

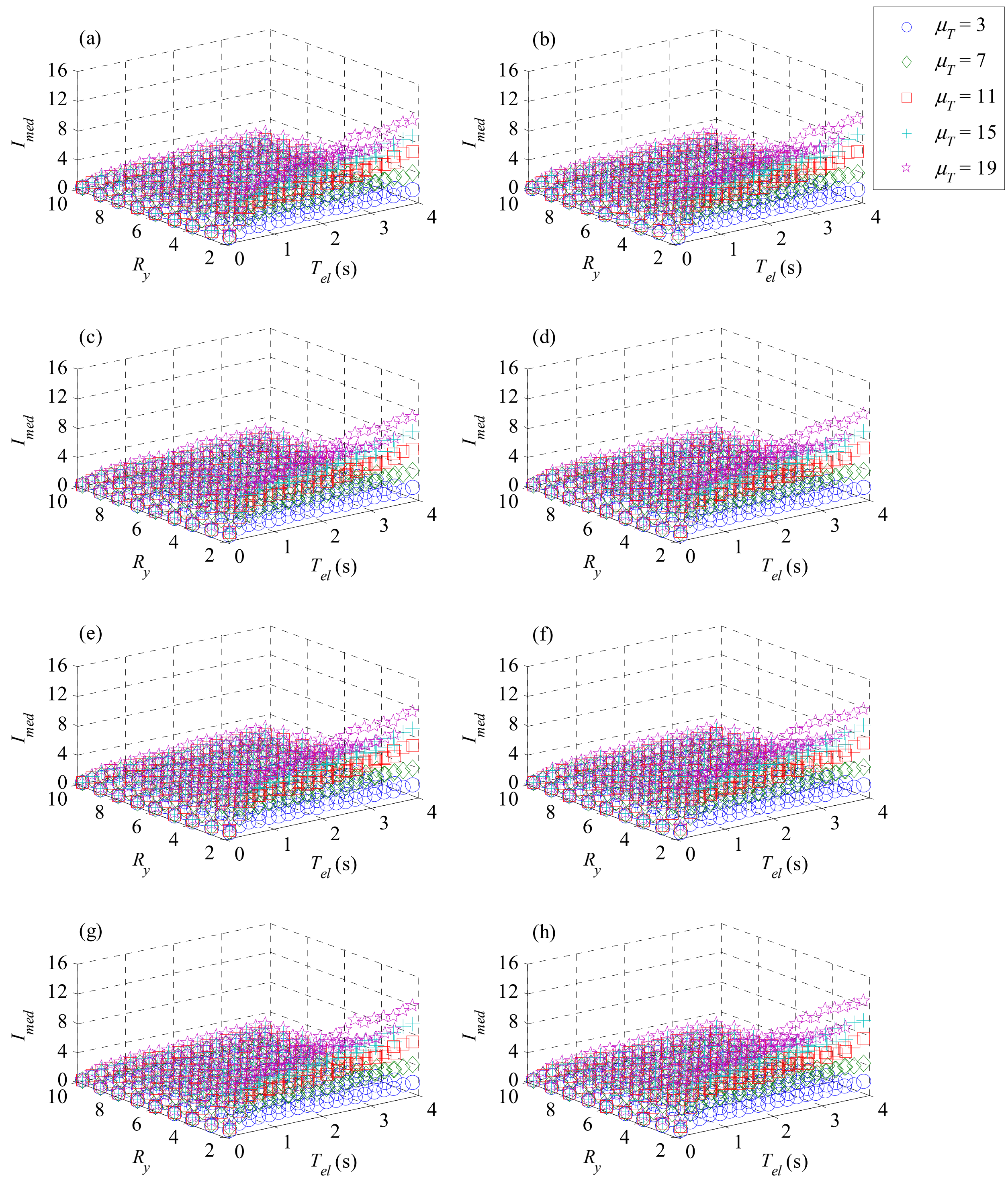


Figure 4-2 Median exceedance intensity (I_{med}) for selected studied systems with nonlinear viscous dampers a) α=0.2, ξ=5% b) α=0.2, ξ=10% c) α=0.4, ξ=5% d) α=0.4, ξ=10% e)α=0.6, ξ=5% f)α=0.6, ξ=10% g)α=0.8, ξ=5% h)α=0.8, ξ=10%

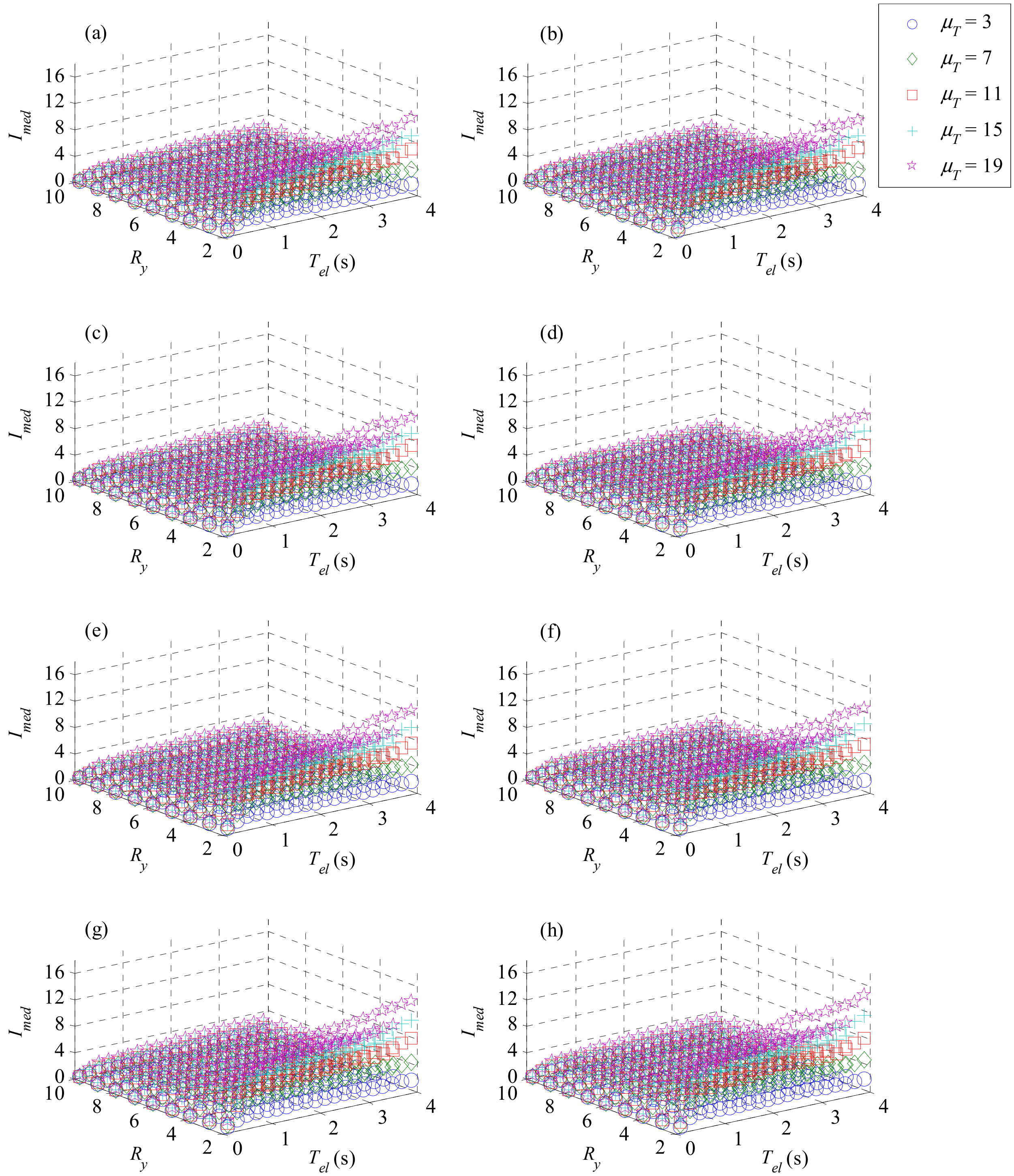


Figure 4-3 Median exceedance intensity (I_{med}) for selected studied systems with nonlinear viscous dampers a) α=0.2, ξ=15% b) α=0.2, ξ=20% c) α=0.4, ξ=15% d) α=0.4, ξ=20% e)α=0.6, ξ=15% f)α=0.6, ξ=20% g)α=0.8, ξ=15% h)α=0.8, ξ=20%

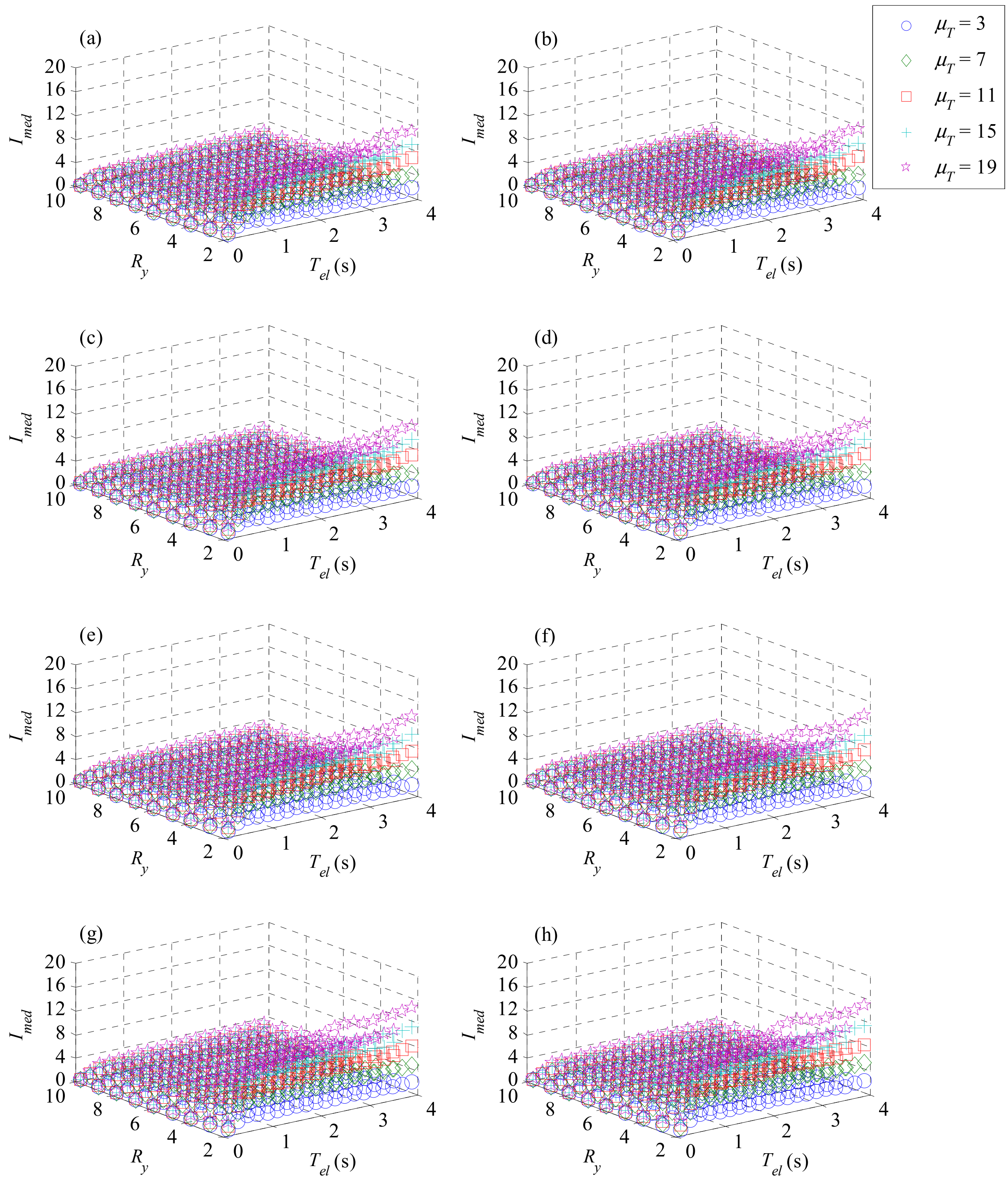


Figure 4-4 Median exceedance intensity (I_{med}) for selected studied systems with nonlinear viscous dampers a) α=0.2, ξ=25% b) α=0.2, ξ=30% c) α=0.4, ξ=25% d) α=0.4, ξ=30% e)α=0.6, ξ=25% f)α=0.6, ξ=30% g)α=0.8, ξ=25% h)α=0.8, ξ=30%

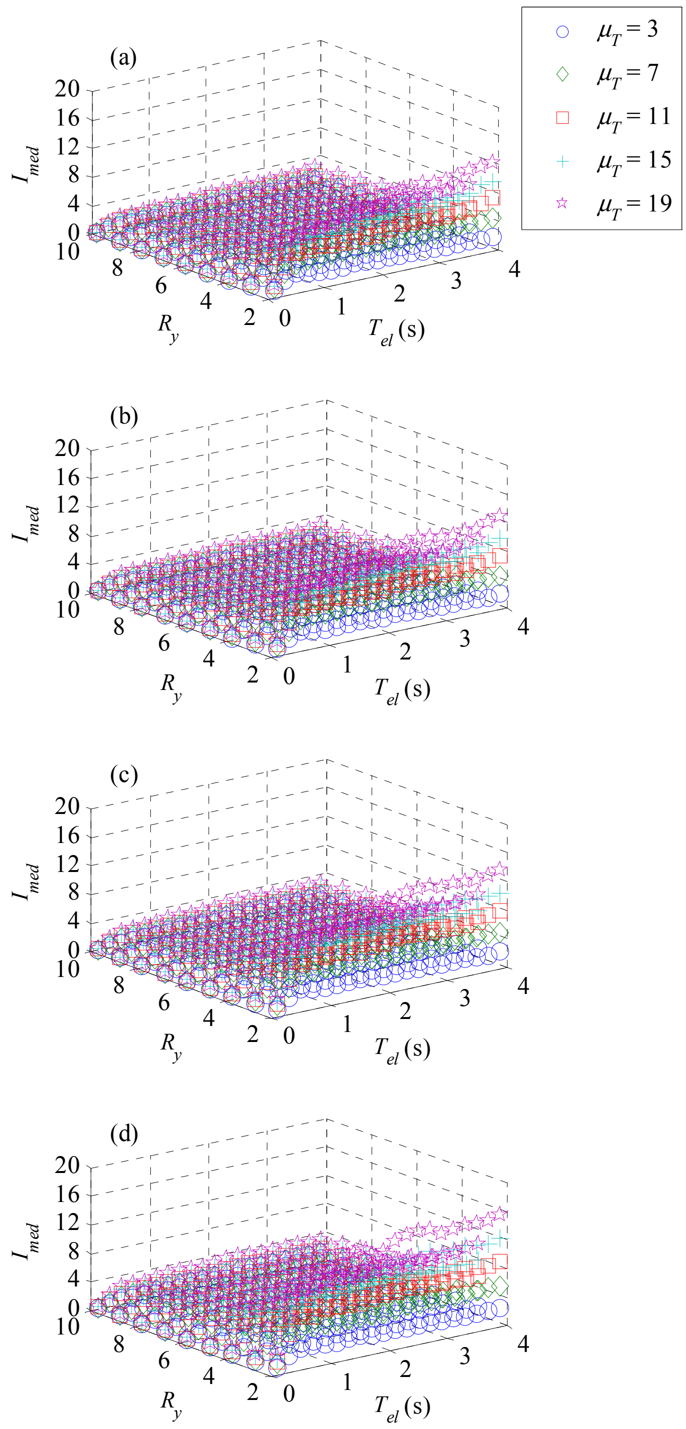


Figure 4-5 Median exceedance intensity (I_{med}) for selected studied systems with nonlinear viscous dampers a) α=0.2, ξ=35% b) α=0.4, ξ=35% c) α=0.6, ξ=35% d) α=0.8, ξ=35%

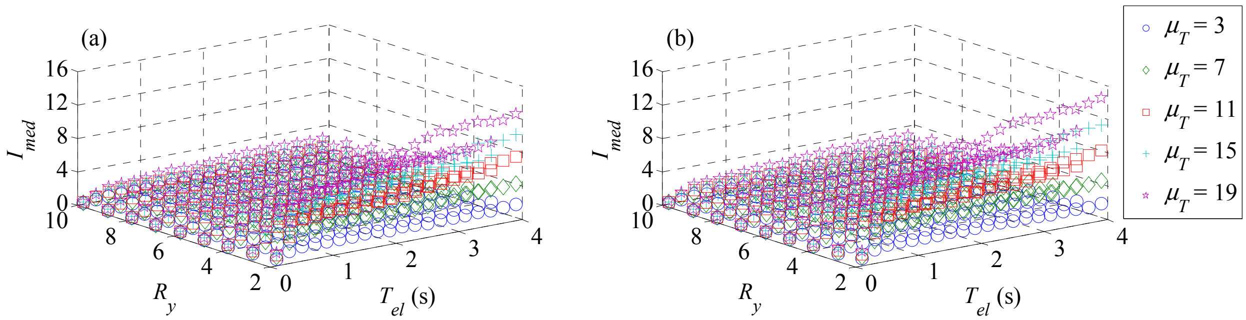


Figure 4-6 Median exceedance intensity (I_{med}) for selected studied systems with linear viscous dampers (α= 1.0) a) ξ=5% b)ξ=10%

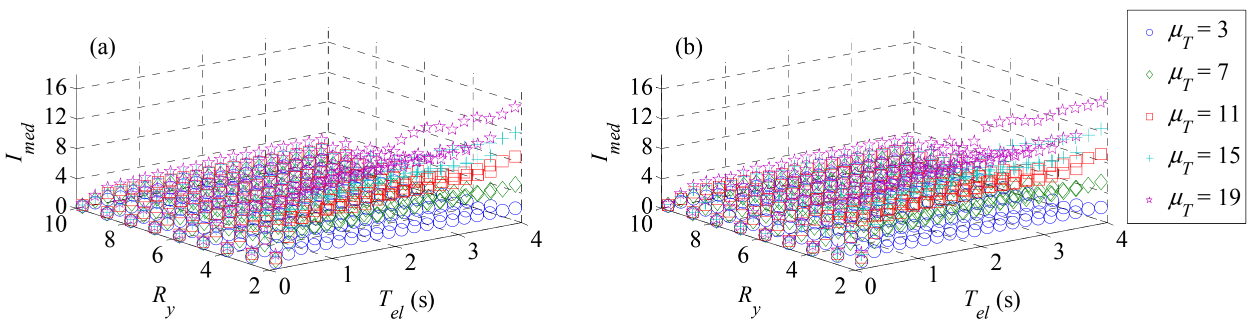


Figure 4-7 Median exceedance intensity (I_{med}) for selected studied systems with linear viscous dampers a) ξ=15% b)ξ=20%

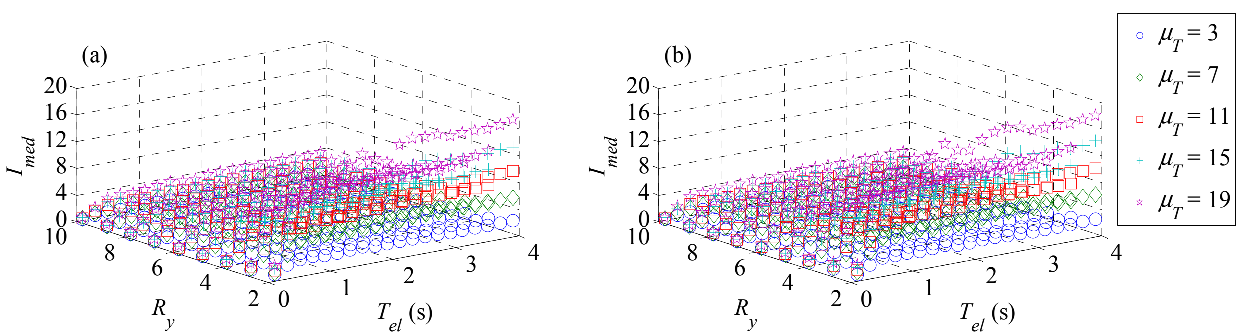


Figure 4-8 Median exceedance intensity (I_{med}) for selected studied systems with linear viscous dampers a) ξ=25% b)ξ=30%

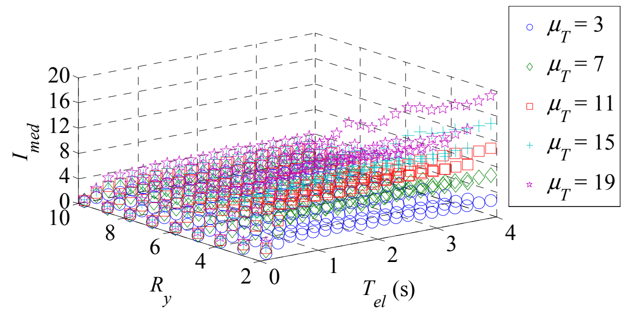


Figure 4-9 Median exceedance intensity (I_{med}) for selected studied systems with linear viscous dampers ξ = 35%

4.4 Simplified Analysis of SDOF Systems

For each SDOF system considered in this study, a simplified analysis procedure is developed and evaluated to predict the associated I_{med} for a given target ductility. The simplified procedure is based on a reduction factor (r) defined as the ratio of the spectral acceleration at which half of the records cause the bilinear inelastic SDOF system to exceed the target ductility ($S_{aMed} = I_{med} \times S_{MT}$ or median exceedance spectral acceleration), to the yield pseudo-acceleration, as shown in Figure 4-10.

$$r = \frac{S_{aMed}}{A_y} = I_{med} R_y \tag{4.24}$$

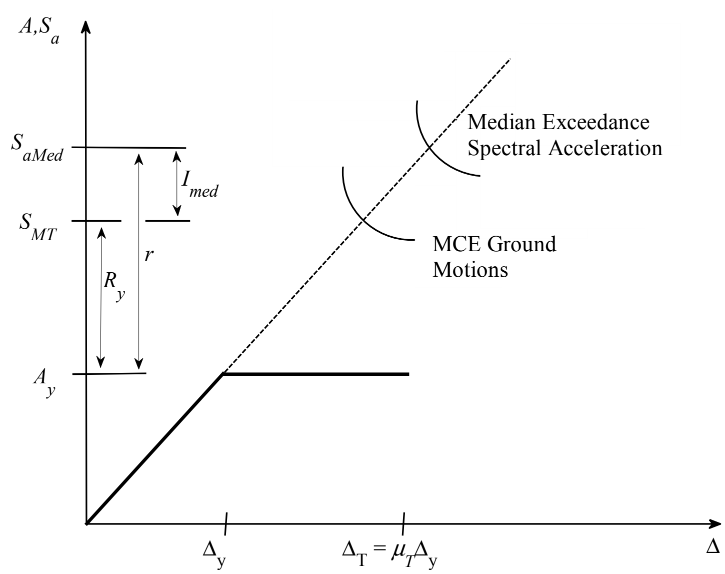


Figure 4-10 Schematic illustration of seismic performance factors as defined by the simplified analysis

Figures 4-11 to 4-18 show the variation of the *r*-factor with the target ductility ratio (μ_T), elastic period (T_{el}) and yield reduction factor (R_y) for selected critical damping ratios (ξ) and velocity exponents (α). In general, the *r*-factor increases with an increase of μ_T, ξ and α.

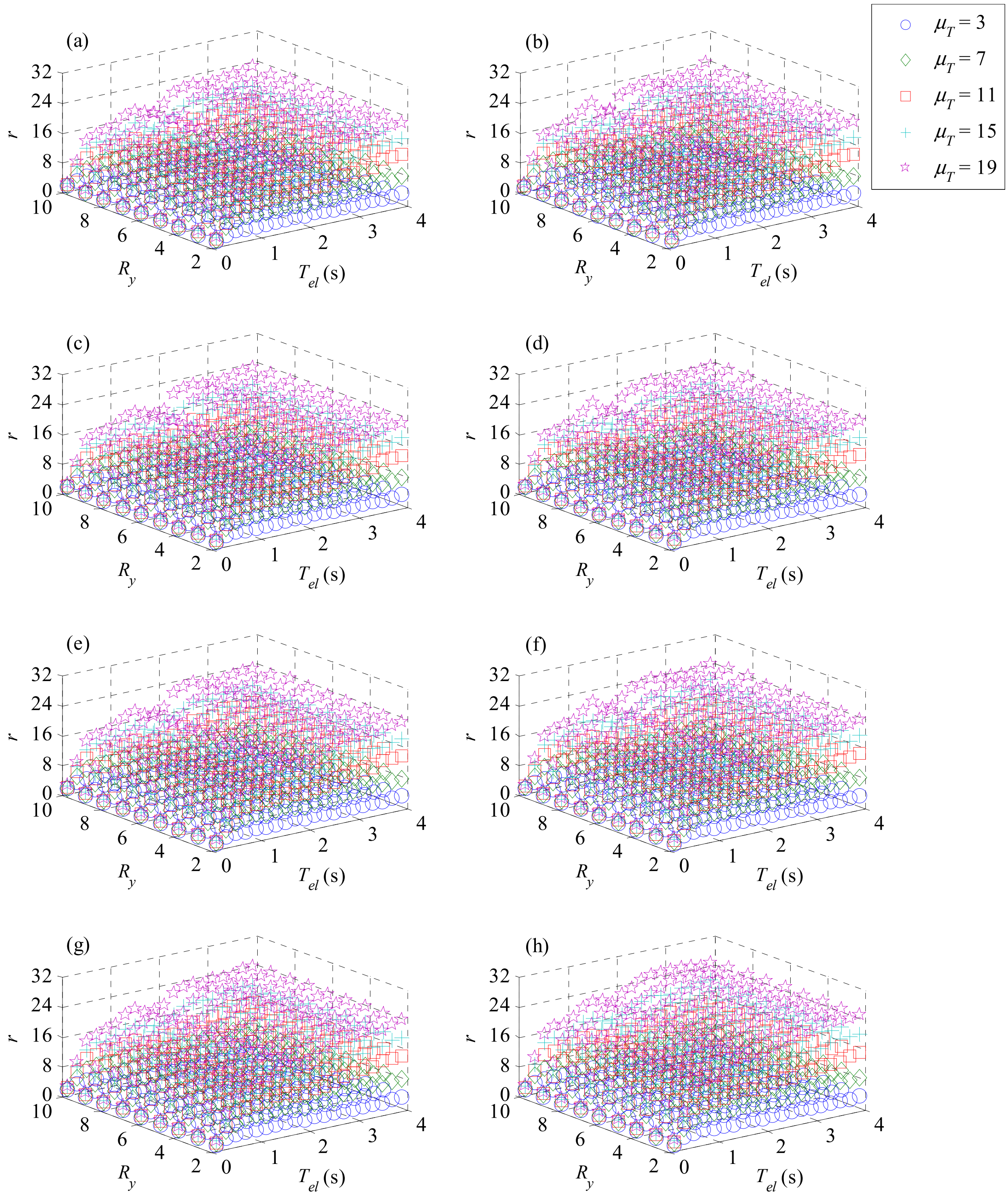


Figure 4-11 Reduction factor (*r*) for selected studied systems with nonlinear viscous dampers a) *α*=0.2, *ξ*=5% b) *α*=0.2, *ξ*=10% c) *α*=0.4, *ξ*=5% d) *α*=0.4, *ξ*=10% e)*α*=0.6, *ξ*=5% f)*α*=0.6, *ξ*=10% g)*α*=0.8, *ξ*=5% h)*α*=0.8, *ξ*=10%

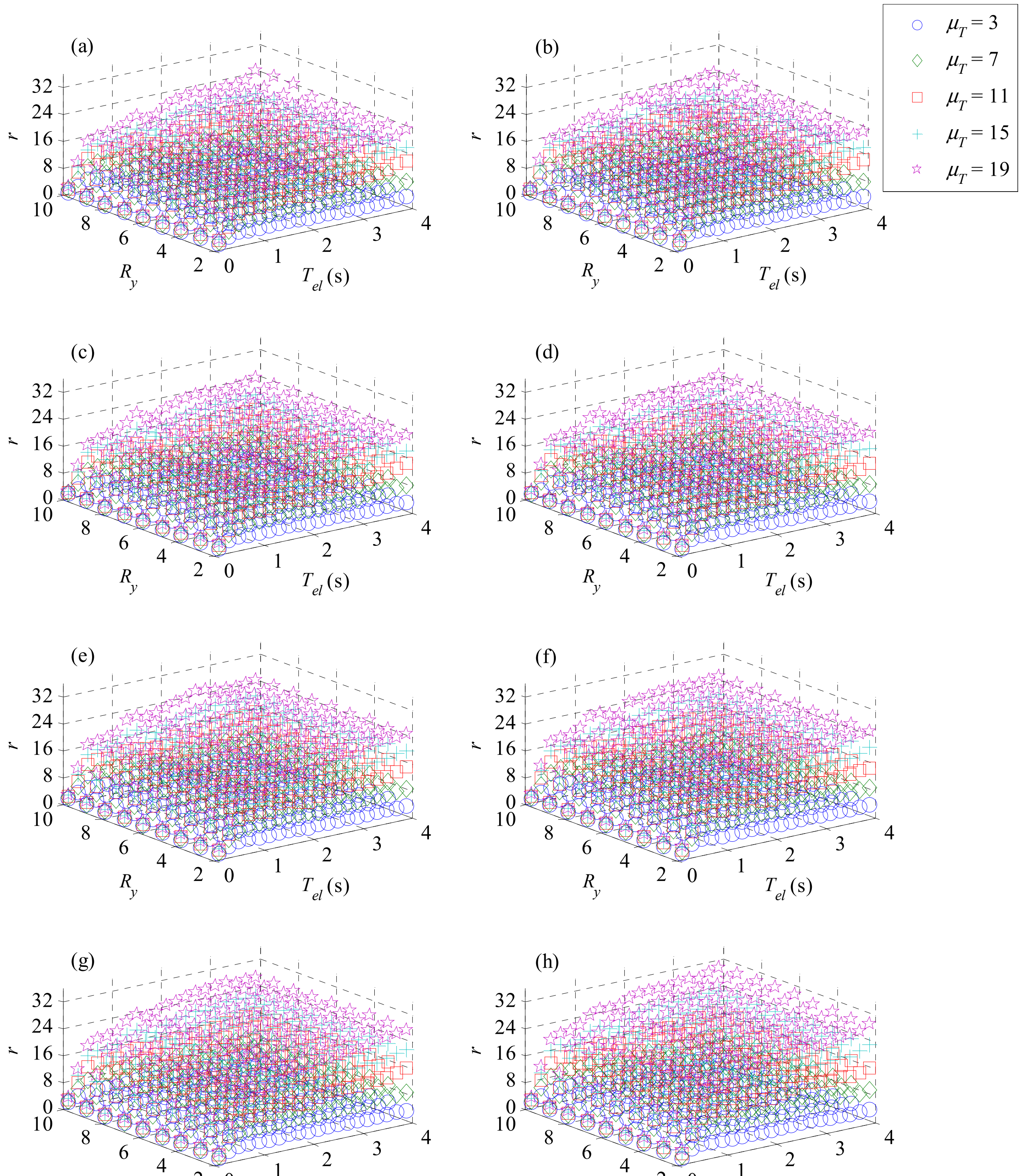


Figure 4-12 Reduction factor (r) for selected studied systems with nonlinear viscous dampers a) α=0.2, ξ=15% b) α=0.2, ξ=20% c) α=0.4, ξ=15% d) α=0.4, ξ=20% e)α=0.6, ξ=15% f)α=0.6, ξ=20% g)α=0.8, ξ=15% h)α=0.8, ξ=20%

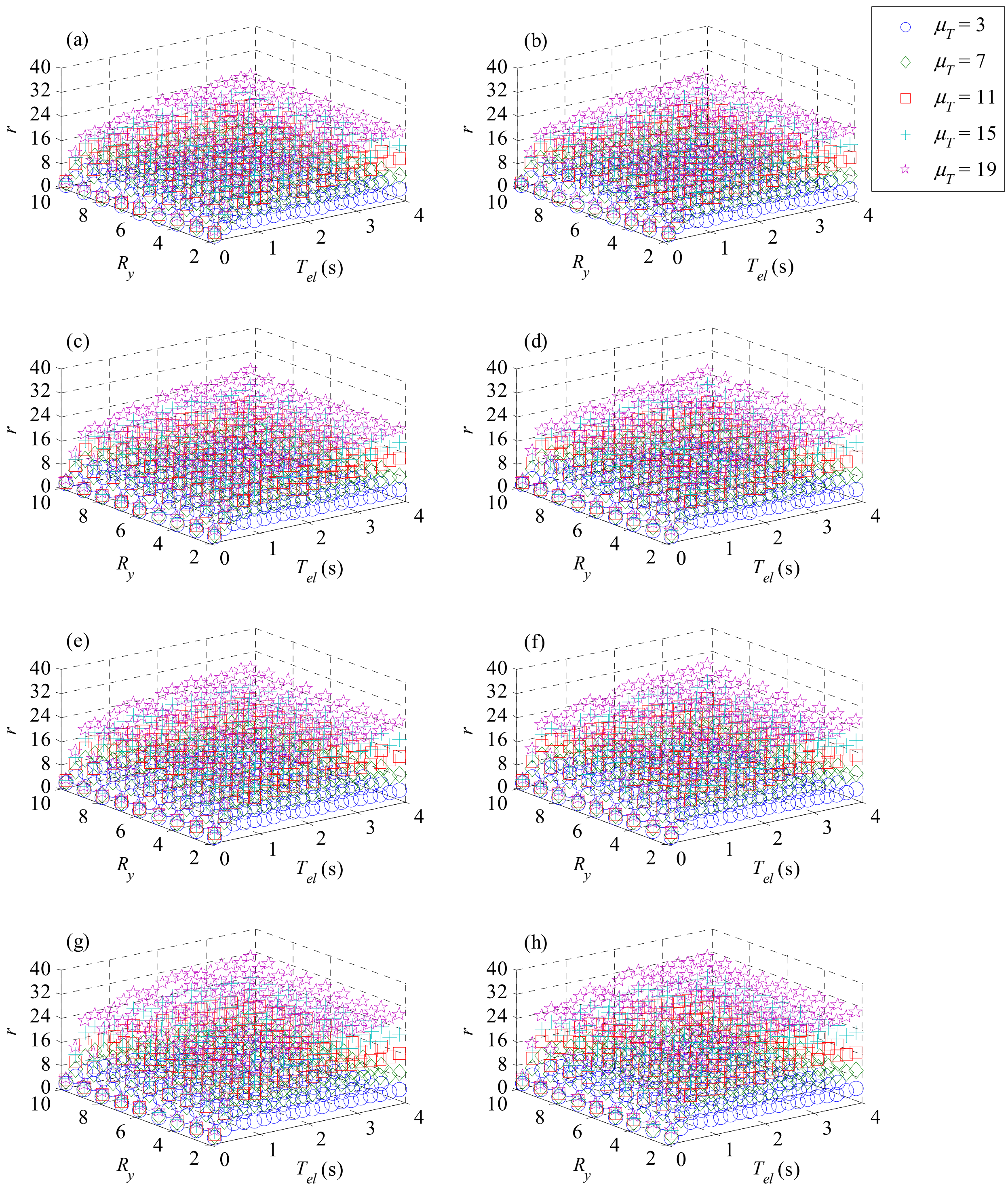


Figure 4-13 Reduction factor (r) for selected studied systems with nonlinear viscous dampers a) α=0.2, ξ=25% b) α=0.2, ξ=30% c) α=0.4, ξ=25% d) α=0.4, ξ=30% e) α=0.6, ξ=25% f) α=0.6, ξ=30% g) α=0.8, ξ=25% h) α=0.8, ξ=30%

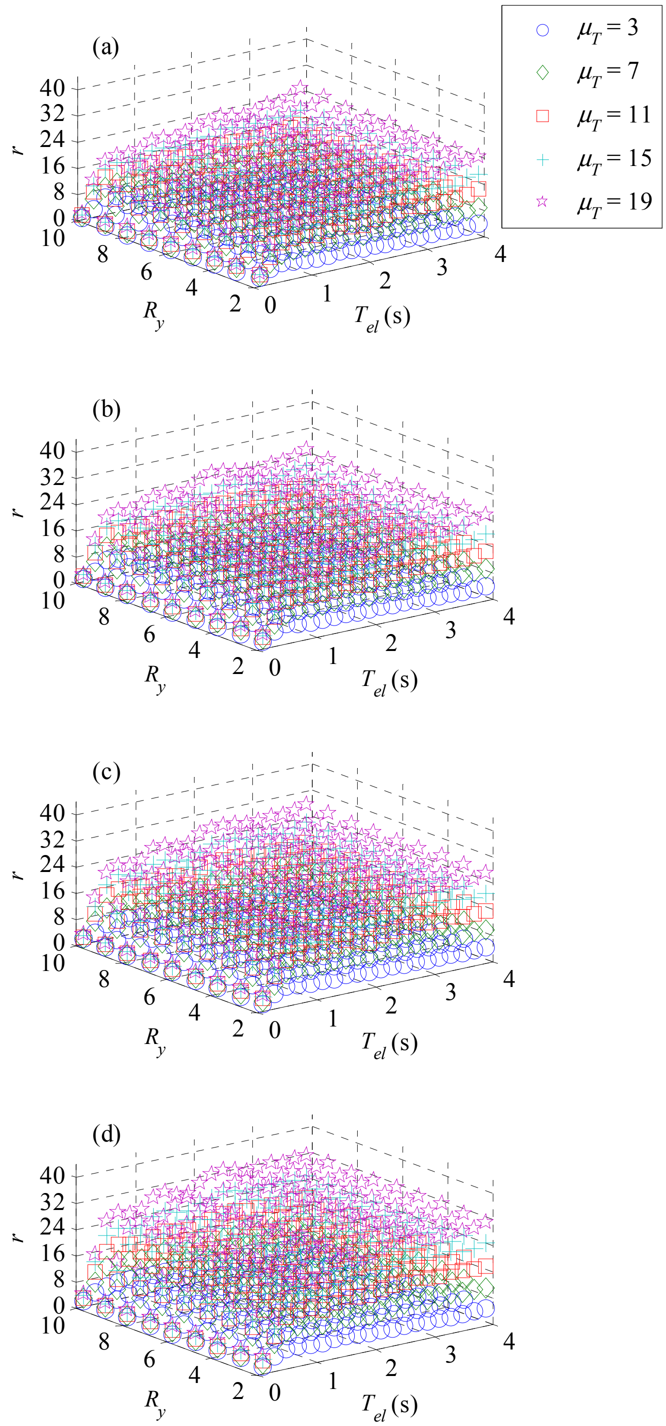


Figure 4-14 Reduction factor (*r*) for selected studied systems with nonlinear viscous dampers a) *α*=0.2, *ξ*=35% b) *α*=0.4, *ξ*=35% c) *α*=0.6, *ξ*=35% d) *α*=0.8, *ξ*=35%

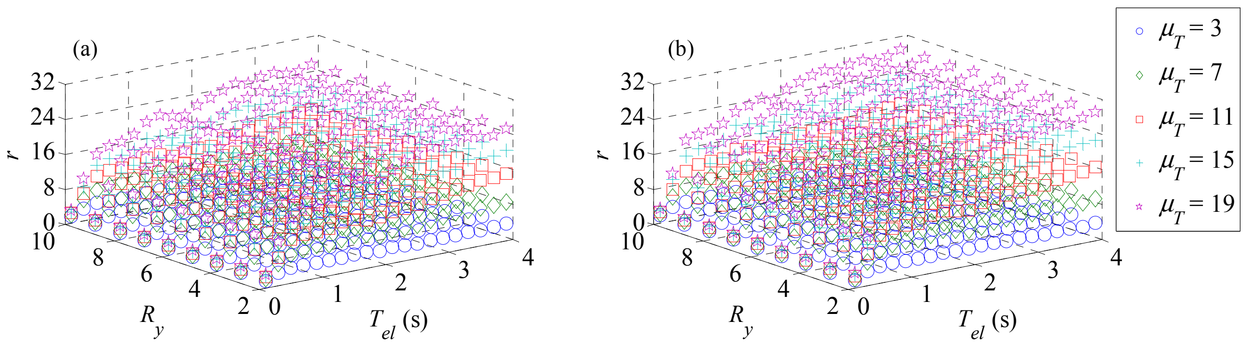


Figure 4-15 Reduction factor (*r*) for selected studied systems with linear viscous dampers (α=1.0) a) *ξ*=5% b) *ξ*=10%

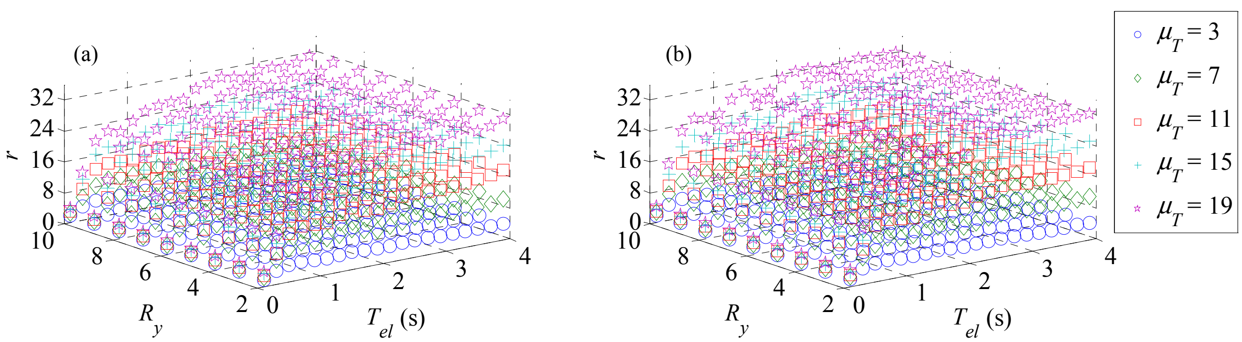


Figure 4-16 Reduction factor (*r*) for selected studied systems with linear viscous dampers (α=1.0) a) *ξ*=15% b) *ξ*=20%

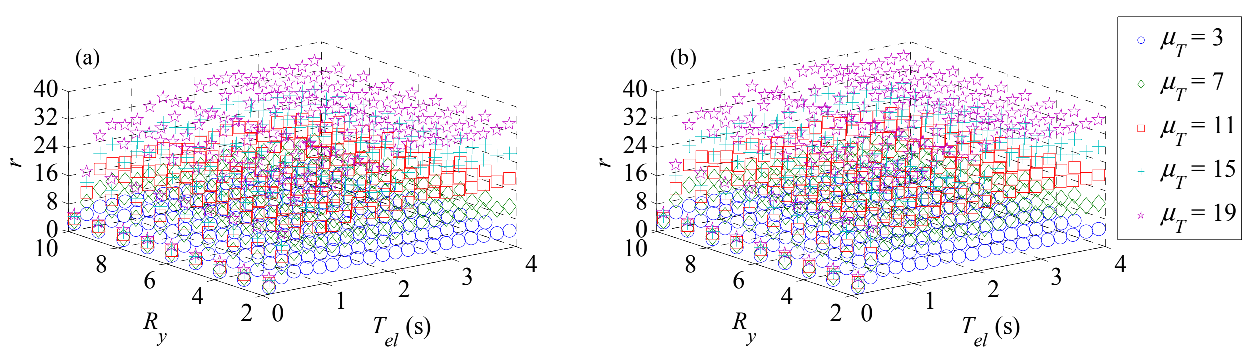


Figure 4-17 Reduction factor (*r*) for selected studied systems with linear viscous dampers (α=1.0) a) *ξ*=25% b) *ξ*=30%

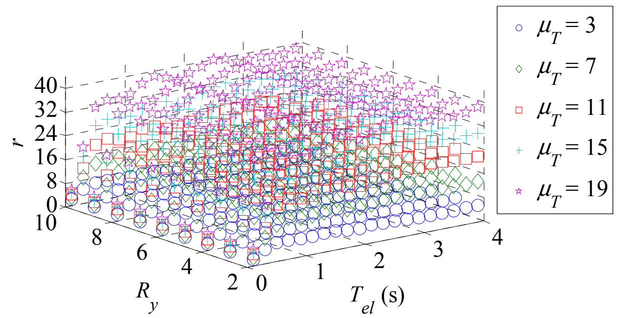


Figure 4-18 Reduction factor (*r*) for selected studied systems with linear viscous dampers (α=1.0) *ξ*=35%

Although the *r*-factor varies with the change in elastic period, it is not highly dependent on R_y. Therefore, the averages taken across R_y are considered as appropriate characteristic values for the reduction factors. This variable reduction leads to a database of 40(T_{el}) × 20(μ_T) × 7(ξ) × 5(α)= 28000 different *r*-factors. The values of all the r-factors are listed in Tables 4-2 to 4-36. In order to avoid interpolation between large numbers of values, empirical equations were determined by symbolic regression analysis (Schmidt and Lipson, 2009) of the database of *r*-factors for short, intermediate and long-period systems. Eureqa software (Schmidt and Lipson, 2013) was used for this purpose, taking advantage of genetic programming (GP). In addition, the analysis was performed for systems with nonlinear and linear viscous dampers separately to reduce the error in estimating the reduction factors. The equations obtained for nonlinear viscous dampers (α < 1.0) are as follows:

$$\begin{cases} \bullet \text{ For } T_{el} \le 1\text{s}: \\ r = 0.5 + 4.93\xi + 3.7T_{el}\mu_T + 1.55\,\mu_T\,\xi\tan(\alpha)\sqrt{T_{el}} - 0.061\mu_T - 0.0096\mu_T^2 - 2.82T_{el}\mu_T\sqrt{T_{el}} \\ \bullet \text{ For } 1\text{s} < T_{el} \le 3\text{s}: \\ r = 2.36 + 4.33\xi + 0.53\mu_T + 0.2T_{el}\mu_T + 2.73^{\alpha}\mu_T\xi\alpha - T_{el} - 4.3\xi\alpha + 0.052\mu_T\sin(5.68T_{el}) \\ \bullet \text{ For } T_{el} \ge 3\text{s}: \\ r = 3.69\xi + 2.15\mu_T + 2.6\mu_T\xi\alpha^2 - 0.43 - \mu_T\sin(0.51T_{el}) - 0.51T_{el}\xi\alpha(1 + \mu_T\xi) \end{cases} \tag{4.25}$$

and the following equations are obtained for linear viscous dampers (α = 1.0).

$$
\begin{cases}
\bullet \text{ For } T_{el} \le 1\text{s}: \\
r = 1.91 T_{el}\mu_T + 7.57 T_{el}\mu_T \xi + \cos(T_{el}) - 0.0088\mu_T^2 - 4.76 T_{el}^2 \mu_T \xi - 2 T_{el}^3 \mu_T \cos(T_{el}) \\
\bullet \text{ For } 1\text{s} < T_{el} \le 3\text{s}: \\
r = 0.55 + 0.79\mu_T + 2.79\mu_T \xi + 0.0023 T_{el}^2 \mu_T^2 + 0.065\mu_T \sin(5.67 T_{el}) \\
\bullet \text{ For } 3\text{s} < T_{el} \le 4\text{s}: \\
r = \dfrac{0.87}{\mu_T} + 1.09 T_{el}\mu_T + \mu_T \sin(T_{el}) + 7.78\mu_T \dfrac{\xi}{T_{el}} - 0.99 - 2.19\mu_T
\end{cases}
\tag{4.26}
$$

As shown in Figure 4-19 and 4-20, for selected systems with ξ=15% and 25%, the regression analysis equations yield acceptable predictions of the reduction factors (r) in comparison with those obtained from nonlinear time history dynamic analyses.

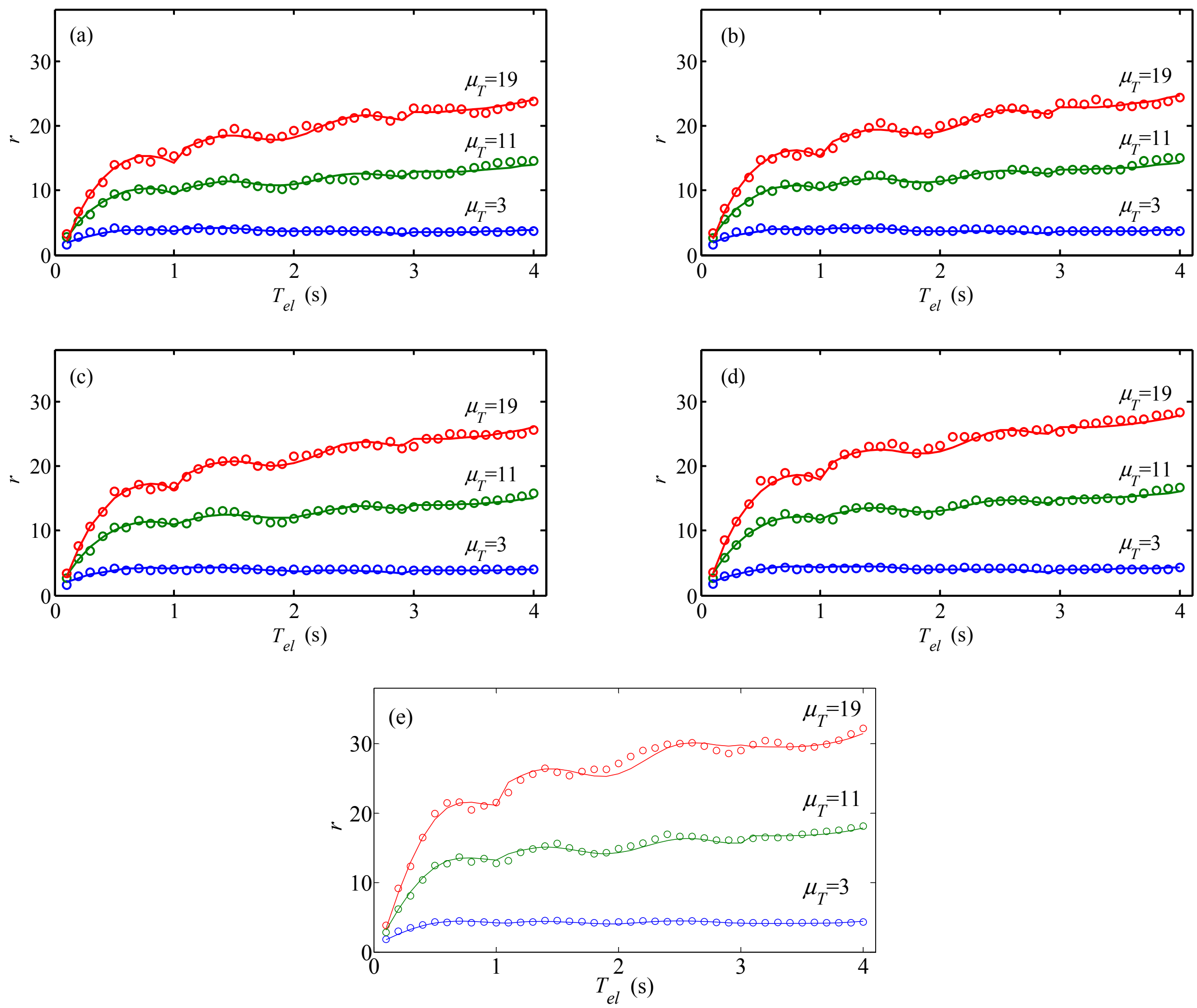


Figure 4-19 Comparison of *r*-factors obtained from regression analysis equations (solid lines) and nonlinear time history dynamic analyses (symbols) for selected studied systems with *ξ*=15%a) *α*=0.2 b) *α*=0.4 c) *α*=0.6 d) *α*=0.8 e) *α*=1.0

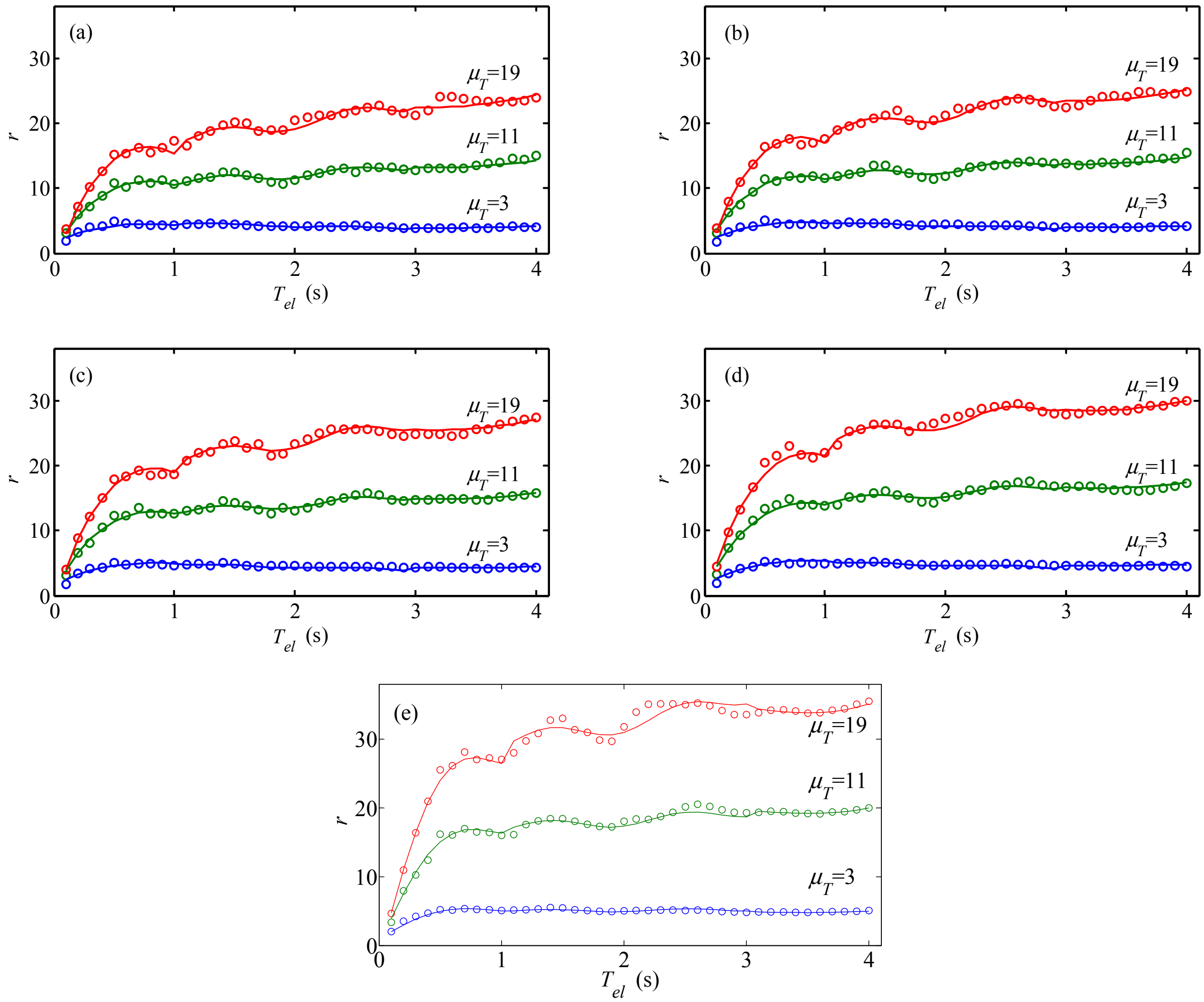


Figure 4-20 Comparison of *r*-factors obtained from regression analysis equations (solid lines) and nonlinear time history dynamic analyses (symbols) for selected studied systems with *ξ*=25%

a) *α*=0.2 b) *α*=0.4 c) *α*=0.6 d) *α*=0.8 e) *α*=1.0

Table 4-2 Average reduction factors (r) for systems with α=0.2 and ξ=5%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	0.60	1.25	1.42	1.50	1.59	1.75	1.79	1.86	1.93	2.05	2.10	2.33	2.42	2.53	2.56	2.64	2.75	2.81	2.81	2.92
0.20	0.92	1.79	2.29	2.80	3.08	3.31	3.56	3.88	4.08	4.19	4.42	4.63	4.75	4.92	5.17	5.33	5.50	5.88	6.25	6.50
0.30	1.06	2.11	2.82	3.31	3.60	4.00	4.75	5.00	5.25	5.50	5.75	6.25	6.50	6.75	7.08	7.38	7.81	8.25	8.44	8.75
0.40	1.16	2.21	2.90	3.56	4.06	4.50	5.00	5.50	6.00	6.38	6.88	7.50	7.75	8.25	8.75	9.08	9.42	9.75	10.25	10.75
0.50	1.25	2.44	3.33	4.08	4.56	5.25	6.00	6.50	7.50	7.75	8.50	8.92	9.50	10.00	10.75	11.25	11.83	12.25	12.75	13.25
0.60	1.22	2.33	3.05	3.88	4.67	5.50	6.30	6.63	7.25	8.00	8.33	8.92	9.25	10.25	10.50	11.19	11.69	12.25	12.88	13.58
0.70	1.31	2.29	3.08	4.15	4.92	5.75	6.42	7.00	7.88	8.75	9.42	9.88	10.38	11.00	11.80	12.25	12.67	13.25	13.75	14.75
0.80	1.20	2.33	3.04	3.81	4.75	5.75	6.42	7.13	7.75	8.38	9.00	10.17	10.63	11.00	11.44	12.13	12.63	13.25	13.50	14.17
0.90	1.21	2.22	3.32	4.08	4.85	5.75	6.50	7.50	8.25	8.88	9.50	10.00	10.63	11.33	12.00	13.00	13.75	14.13	14.63	15.25
1.00	1.20	2.15	3.13	4.13	4.83	5.75	6.63	7.19	7.88	8.56	9.40	10.08	10.75	11.13	11.69	12.25	13.50	13.88	14.38	14.88
1.10	1.20	2.31	3.34	4.00	4.92	5.75	6.58	7.38	8.19	9.00	9.75	10.50	11.13	11.88	12.63	13.38	14.13	14.75	15.50	16.00
1.20	1.15	2.41	3.50	4.31	5.25	6.00	7.00	7.75	8.75	9.50	10.38	10.94	11.75	12.50	13.25	14.00	14.75	15.38	16.25	17.13
1.30	1.08	2.38	3.39	4.20	5.25	6.17	7.13	7.83	8.75	9.50	9.92	11.00	12.00	12.83	13.50	14.50	15.13	16.00	17.00	18.00
1.40	1.19	2.58	3.42	4.33	5.13	6.15	7.25	8.33	9.13	10.00	11.13	11.75	12.38	13.25	14.25	15.25	16.25	17.00	17.75	19.25
1.50	1.13	2.54	3.33	4.44	5.50	6.50	7.50	8.75	9.63	10.33	11.00	11.50	12.38	13.00	15.00	15.50	16.25	17.00	18.25	18.75
1.60	1.13	2.50	3.50	4.50	5.33	6.06	7.13	8.00	9.13	10.00	10.63	11.13	12.00	13.25	14.00	14.75	15.25	15.75	17.50	18.00
1.70	1.14	2.25	3.42	4.25	4.91	5.63	6.83	7.75	8.63	9.13	9.67	10.25	11.13	12.50	13.25	13.88	15.50	16.25	17.00	17.88
1.80	1.03	2.19	3.25	4.14	4.88	5.69	6.56	7.00	8.00	8.58	9.50	10.13	11.50	12.25	13.13	14.13	14.63	15.38	16.13	18.00
1.90	1.00	2.21	3.21	4.19	4.88	5.69	6.17	6.88	8.00	8.75	9.50	10.00	11.25	12.50	13.00	13.75	14.25	17.00	18.00	18.00
2.00	1.08	2.33	3.42	4.50	5.17	5.69	6.25	7.08	8.13	9.13	10.00	10.63	12.00	12.63	13.13	15.00	16.00	17.00	17.75	18.00
2.10	1.07	2.25	3.50	4.50	5.17	5.67	6.50	8.00	8.50	9.25	10.08	11.00	11.75	12.75	14.25	15.50	16.25	17.00	18.00	19.38
2.20	0.91	2.30	3.33	4.31	5.00	6.25	7.00	7.75	8.25	9.25	10.25	11.25	12.25	13.25	14.25	15.13	16.38	17.83	19.25	20.00
2.30	0.97	2.38	3.20	4.15	5.25	6.19	6.88	7.50	8.50	9.38	10.17	11.00	12.08	13.38	14.63	16.25	17.38	18.50	19.50	20.00
2.40	0.89	2.29	3.25	4.31	5.25	6.00	6.75	7.75	8.58	9.58	10.42	11.50	12.92	14.75	16.25	17.00	17.88	19.00	20.50	21.25
2.50	0.92	2.18	3.20	4.33	5.25	6.06	6.88	8.06	8.92	9.75	10.83	12.75	14.25	15.50	16.50	17.50	18.25	19.13	20.00	21.38
2.60	0.93	2.33	3.31	4.33	5.25	6.25	7.25	8.25	9.00	10.75	12.25	13.25	14.38	15.13	16.50	17.50	18.50	19.38	21.00	22.00
2.70	0.93	2.38	3.25	4.25	5.08	6.00	7.00	8.38	9.50	11.25	12.00	13.00	14.50	15.25	16.25	17.38	18.50	20.50	21.50	23.00
2.80	0.95	2.40	3.19	4.19	5.00	5.83	7.25	8.50	10.00	10.88	12.25	13.17	14.25	15.25	16.25	18.00	19.25	21.00	22.00	22.00
2.90	0.95	2.38	3.25	4.14	4.90	5.75	7.75	8.83	10.00	11.00	12.50	13.13	14.08	15.25	16.13	18.38	19.25	20.25	21.25	22.00
3.00	0.97	2.32	3.22	4.06	4.88	5.92	7.50	9.00	10.00	11.00	12.13	13.13	14.33	15.50	17.25	18.00	19.00	20.00	21.00	22.00
3.10	1.00	2.44	3.21	4.08	5.13	6.38	8.00	9.10	10.13	11.08	12.00	13.25	14.75	16.50	17.50	18.25	19.00	20.25	21.25	22.13
3.20	0.89	2.39	3.38	4.13	5.13	6.50	8.00	9.13	10.25	11.00	11.88	14.50	16.00	17.00	17.75	18.75	19.75	20.50	21.38	22.13
3.30	0.86	2.39	3.31	4.19	5.38	7.00	8.00	9.13	10.13	10.75	12.25	14.88	15.63	17.00	17.75	18.75	19.75	20.50	21.33	22.38
3.40	0.86	2.25	3.30	4.25	5.33	6.92	7.75	8.88	9.69	10.50	13.75	15.00	16.00	17.00	17.75	18.50	19.50	20.38	21.38	22.25
3.50	0.84	2.19	3.25	4.25	5.38	6.75	7.75	8.75	10.00	11.50	13.75	15.06	15.88	16.75	17.50	18.38	19.17	20.25	22.00	22.00
3.60	0.88	2.25	3.38	4.25	5.50	6.56	7.63	9.00	10.00	12.75	14.00	15.00	16.00	17.00	17.75	18.58	19.38	21.50	22.00	22.00
3.70	0.88	2.25	3.38	4.50	5.50	6.50	7.67	9.00	11.25	12.50	14.08	15.25	16.25	17.00	18.00	19.00	19.75	21.25	22.25	23.25
3.80	0.94	2.25	3.50	4.50	5.63	6.83	7.83	9.00	11.50	12.75	14.00	15.25	16.25	17.50	18.50	19.25	20.25	21.25	22.13	23.00
3.90	0.81	2.21	3.25	4.63	5.75	7.08	8.25	10.25	11.50	12.50	14.00	15.50	16.50	17.50	18.25	19.25	20.25	21.75	22.50	23.63
4.00	0.85	2.14	3.25	4.50	5.88	7.25	8.38	10.25	11.50	12.38	14.25	15.75	16.63	17.50	18.50	19.25	20.25	21.25	23.25	24.75

Table 4-3 Average reduction factors (r) for systems with α=0.2 and ξ=10%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	0.65	1.23	1.45	1.58	1.81	1.86	1.93	1.98	2.08	2.21	2.40	2.46	2.58	2.70	2.82	2.88	2.91	2.96	3.10	3.13
0.20	1.29	2.08	2.63	3.10	3.44	3.63	3.95	4.17	4.50	4.63	4.88	5.00	5.25	5.38	5.75	5.75	6.00	6.13	6.42	6.67
0.30	1.36	2.30	3.06	3.63	3.92	4.38	5.00	5.25	5.50	5.75	6.00	6.25	6.63	7.25	7.50	7.75	8.13	8.58	9.00	9.25
0.40	1.43	2.44	3.33	3.92	4.40	4.88	5.38	6.00	6.50	7.00	7.42	7.88	8.38	8.83	9.25	9.60	9.92	10.33	10.70	11.00
0.50	1.64	2.94	3.69	4.50	5.00	5.50	6.38	6.92	7.50	8.25	8.75	9.50	9.94	10.50	11.13	11.81	12.25	12.75	13.38	14.00
0.60	1.50	2.63	3.38	4.17	5.00	6.00	6.50	7.13	7.75	8.13	8.75	9.38	9.92	10.50	11.13	11.81	12.38	12.75	13.42	14.00
0.70	1.55	2.65	3.44	4.50	5.33	6.00	7.00	7.38	8.25	9.25	9.75	10.42	11.00	11.63	12.35	12.88	13.33	13.75	14.17	14.63
0.80	1.46	2.59	3.38	4.13	5.00	6.00	6.75	7.45	8.17	8.88	9.88	10.58	11.00	11.50	11.88	12.25	12.88	13.58	14.25	14.75
0.90	1.45	2.50	3.47	4.25	5.08	5.88	6.92	7.75	8.50	9.25	9.88	10.58	11.13	11.63	12.13	13.25	14.00	14.50	15.25	15.75
1.00	1.39	2.38	3.55	4.38	5.17	6.31	7.08	7.75	8.38	9.00	9.83	10.67	11.25	11.88	12.50	13.00	13.50	14.00	14.75	15.42
1.10	1.42	2.56	3.60	4.44	5.31	6.15	7.00	7.69	8.70	9.50	10.00	10.50	11.17	11.88	13.25	13.83	14.25	14.75	15.50	16.33
1.20	1.55	2.68	3.75	4.56	5.50	6.50	7.50	8.25	8.92	10.00	10.50	11.50	12.25	12.92	13.75	14.50	15.25	15.88	16.75	17.75
1.30	1.50	2.63	3.55	4.50	5.56	6.63	7.44	8.31	9.25	10.25	10.75	11.42	12.42	13.25	14.00	14.83	15.50	16.50	17.25	18.33
1.40	1.52	2.82	3.55	4.50	5.67	6.50	7.63	8.75	9.58	10.25	11.38	12.25	12.88	13.50	14.50	15.50	16.50	17.25	18.25	19.50
1.50	1.41	2.80	3.75	4.67	5.83	6.75	7.50	8.50	9.63	10.67	11.42	12.08	12.63	13.50	15.25	16.25	17.00	18.25	18.50	19.13
1.60	1.41	2.70	3.75	4.58	5.60	6.38	7.50	8.50	9.38	10.13	10.85	11.50	12.13	14.00	14.75	15.25	15.88	17.00	18.00	19.25
1.70	1.35	2.63	3.68	4.55	5.31	6.08	7.00	7.88	9.08	9.63	10.25	10.75	11.63	13.25	13.83	14.42	16.00	17.00	18.00	18.00
1.80	1.29	2.46	3.46	4.33	5.10	5.92	6.75	7.75	8.50	9.08	9.75	10.50	11.63	12.75	13.38	14.75	15.50	16.50	17.25	18.25
1.90	1.30	2.44	3.44	4.40	5.25	6.00	6.67	7.25	8.13	9.00	9.88	10.75	12.00	13.00	14.00	14.75	15.50	16.25	18.00	18.00
2.00	1.23	2.43	3.43	4.75	5.50	6.00	6.65	7.25	8.50	9.50	10.25	11.50	12.58	13.25	14.25	15.00	17.00	17.75	18.50	19.25
2.10	1.21	2.48	3.63	5.00	5.50	6.00	6.69	8.00	9.00	9.75	11.00	12.00	12.75	13.75	15.50	16.25	17.00	17.75	19.00	20.00
2.20	1.18	2.54	3.50	4.67	5.38	5.95	7.25	8.13	9.00	10.08	11.25	12.13	13.25	14.00	15.25	16.13	17.25	18.25	19.00	19.88
2.30	1.21	2.60	3.67	4.58	5.50	6.38	7.33	7.92	9.00	10.00	11.00	11.88	12.88	13.92	15.25	16.38	17.50	18.75	19.75	20.75
2.40	1.16	2.43	3.67	4.45	5.58	6.50	7.17	8.13	9.00	9.75	10.75	12.00	13.25	14.75	16.25	17.25	18.25	19.00	19.88	21.75
2.50	1.13	2.50	3.55	4.50	5.50	6.38	7.13	8.13	9.13	10.00	11.00	12.50	14.25	16.00	16.75	17.50	18.25	19.25	20.50	21.75
2.60	1.18	2.54	3.50	4.50	5.42	6.38	7.17	8.50	9.75	10.50	12.00	13.75	14.75	16.00	17.00	17.75	19.00	20.00	21.25	22.00
2.70	1.19	2.60	3.50	4.60	5.38	6.25	7.00	8.58	9.38	11.25	12.25	13.38	14.38	15.50	16.75	17.63	18.67	20.00	21.75	22.00
2.80	1.20	2.50	3.44	4.38	5.13	6.00	7.50	8.38	10.13	11.00	12.25	13.25	14.50	15.50	16.38	17.25	18.50	20.50	21.25	23.50
2.90	1.19	2.50	3.38	4.17	5.00	6.00	7.50	9.00	10.00	11.50	12.25	13.42	14.38	15.33	16.08	17.50	19.50	21.25	22.50	23.50
3.00	1.19	2.63	3.50	4.19	5.13	6.25	7.75	9.00	10.50	11.25	12.25	13.38	14.25	15.08	17.00	18.50	19.63	20.75	21.75	23.00
3.10	1.20	2.63	3.44	4.17	5.25	6.63	8.00	9.25	10.13	11.50	12.00	13.50	14.38	15.75	17.75	19.00	20.00	20.75	22.00	23.00
3.20	1.18	2.55	3.44	4.17	5.75	6.67	8.00	9.25	9.92	11.25	12.00	13.75	15.00	17.00	18.25	19.25	20.25	21.25	22.13	23.00
3.30	1.20	2.57	3.50	4.21	5.75	6.75	8.06	9.00	10.25	11.25	12.44	14.50	15.75	17.25	18.00	19.25	20.25	21.25	22.13	23.00
3.40	1.11	2.55	3.50	4.58	5.88	7.00	8.13	9.25	10.25	11.25	13.00	14.75	16.25	17.00	18.25	19.33	20.00	21.00	21.88	22.75
3.50	1.08	2.50	3.42	4.56	5.75	7.05	8.17	9.25	10.13	11.25	13.75	14.88	16.00	16.88	17.88	18.75	19.88	21.00	21.67	22.63
3.60	1.09	2.63	3.44	4.42	5.75	7.00	8.30	9.13	10.25	11.25	14.13	15.13	16.00	16.75	17.75	18.63	19.88	20.75	21.88	23.00
3.70	1.06	2.58	3.45	4.75	5.88	7.08	8.33	9.13	10.25	12.50	14.25	15.13	16.25	17.00	17.88	19.00	19.75	21.75	23.00	23.50
3.80	1.11	2.45	3.50	4.75	6.10	7.25	8.25	9.13	10.63	13.00	14.25	15.25	16.50	17.25	18.25	19.13	19.88	21.75	22.75	23.75
3.90	1.11	2.38	3.50	4.88	6.19	7.25	8.25	9.75	11.75	13.00	14.25	15.25	16.50	17.75	18.75	19.50	20.50	21.75	22.75	23.88
4.00	1.08	2.31	3.75	5.00	6.17	7.38	8.50	9.75	11.75	12.88	14.33	15.50	17.00	18.00	19.00	20.00	20.75	22.00	23.00	24.00

Table 4-4 Average reduction factors (r) for systems with α=0.2 and ξ=15%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	0.88	1.33	1.56	1.75	1.92	1.98	2.10	2.22	2.38	2.50	2.75	2.81	2.83	2.88	2.95	3.14	3.14	3.17	3.21	3.40
0.20	1.45	2.36	2.85	3.30	3.67	4.00	4.38	4.63	4.83	5.00	5.25	5.50	5.50	5.81	6.00	6.17	6.38	6.50	6.75	6.88
0.30	1.57	2.63	3.50	3.88	4.25	4.75	5.25	5.67	5.75	6.00	6.25	6.63	6.88	7.25	8.00	8.25	8.50	9.00	9.38	9.63
0.40	1.75	2.81	3.56	4.15	4.67	5.25	5.75	6.25	6.75	7.50	8.00	8.38	8.75	9.25	9.80	10.13	10.50	10.91	11.30	11.75
0.50	2.00	3.30	4.15	4.75	5.50	5.75	6.50	7.50	8.30	8.92	9.50	10.00	10.58	11.25	11.69	12.50	13.00	13.42	14.00	15.00
0.60	1.72	3.00	3.80	4.44	5.20	6.25	7.00	7.58	8.13	8.50	9.13	9.88	10.42	11.13	11.67	12.50	13.00	13.63	14.00	14.63
0.70	1.75	2.95	3.92	4.81	5.50	6.25	7.25	7.75	8.75	9.75	10.25	10.92	11.44	12.13	12.81	13.50	14.13	14.63	14.92	15.42
0.80	1.60	2.82	3.68	4.46	5.42	6.25	7.25	7.92	8.58	9.31	10.00	11.00	11.50	12.00	12.38	13.00	13.25	13.88	14.42	15.13
0.90	1.64	2.81	3.81	4.50	5.44	6.21	7.25	7.88	8.81	9.63	10.25	10.88	11.50	12.08	12.63	13.25	13.88	15.08	15.88	16.75
1.00	1.56	2.67	3.81	4.50	5.50	6.25	7.40	8.08	8.75	9.33	10.00	10.75	11.50	12.25	13.25	13.75	14.25	14.75	15.25	15.69
1.10	1.63	2.75	3.92	4.75	5.75	6.50	7.25	8.10	9.07	9.69	10.50	11.00	11.75	12.50	13.25	14.00	14.63	15.25	16.00	16.44
1.20	1.65	2.93	4.08	4.75	5.75	6.75	7.63	8.60	9.38	10.00	10.75	11.44	12.50	13.42	14.38	15.08	15.75	16.50	17.25	18.13
1.30	1.63	2.90	3.90	4.69	5.75	6.88	7.85	8.63	9.50	10.25	11.25	12.25	13.00	13.75	14.38	15.00	15.88	16.88	17.75	18.75
1.40	1.68	2.96	3.88	4.90	6.13	6.94	8.06	9.00	9.88	10.63	11.50	12.38	13.25	13.83	14.50	15.75	16.75	17.75	18.75	19.38
1.50	1.65	3.05	4.07	4.95	5.92	6.90	8.25	9.00	9.88	11.00	11.83	12.42	13.00	13.69	15.00	16.50	17.50	18.50	19.50	20.50
1.60	1.54	3.05	3.93	4.88	5.88	6.67	7.63	8.75	10.00	10.50	11.13	11.92	12.50	14.00	15.00	16.25	16.75	18.00	18.75	19.25
1.70	1.53	2.85	3.75	4.75	5.50	6.38	7.50	8.38	9.50	10.00	10.63	11.25	12.25	13.50	14.50	15.00	16.00	17.00	18.25	19.50
1.80	1.50	2.71	3.75	4.56	5.38	6.13	7.00	8.50	9.00	9.75	10.33	10.94	12.00	13.00	13.88	15.00	16.25	17.00	18.00	18.75
1.90	1.44	2.63	3.60	4.50	5.25	6.00	7.25	8.00	8.50	9.19	10.17	11.17	12.38	13.25	14.00	15.50	16.25	17.25	18.25	19.38
2.00	1.40	2.60	3.60	4.75	5.50	6.25	7.00	7.75	8.63	9.75	10.75	12.00	12.75	14.00	15.50	16.25	17.50	18.50	19.25	20.50
2.10	1.41	2.58	3.68	4.75	5.63	6.38	7.06	8.00	9.25	10.13	11.50	12.50	13.75	14.75	15.75	17.25	17.75	18.75	20.00	20.75
2.20	1.40	2.57	3.75	4.88	5.83	6.38	7.17	8.50	9.50	10.63	12.00	13.00	14.00	14.75	16.25	17.00	18.08	18.88	19.63	20.50
2.30	1.38	2.55	3.88	4.75	5.58	6.50	7.63	8.75	9.75	10.88	11.75	13.00	13.75	14.88	15.88	17.50	18.25	19.13	20.00	21.00
2.40	1.35	2.55	3.75	4.75	5.67	6.63	7.75	8.63	9.75	10.75	11.75	12.38	13.88	14.75	16.25	17.50	18.50	19.75	20.75	21.75
2.50	1.31	2.63	3.71	4.70	5.75	6.50	7.63	8.75	9.38	10.38	11.50	13.13	14.25	15.75	17.00	17.75	18.50	19.38	21.25	22.00
2.60	1.34	2.71	3.71	4.75	5.67	6.63	7.50	8.50	9.42	10.63	12.25	13.75	15.50	16.25	17.00	18.00	19.00	20.50	22.00	23.00
2.70	1.33	2.75	3.67	4.65	5.56	6.63	7.50	8.50	9.50	11.00	12.50	14.00	15.00	16.00	16.88	18.25	19.25	20.38	21.50	22.38
2.80	1.33	2.78	3.75	4.45	5.38	6.50	7.50	8.42	10.00	11.25	12.38	13.38	14.38	15.63	16.63	17.50	18.75	19.75	20.67	22.00
2.90	1.33	2.81	3.56	4.38	5.33	6.38	7.38	9.00	10.25	11.50	12.38	13.25	14.50	15.38	16.25	17.33	18.13	20.25	21.50	23.75
3.00	1.33	2.70	3.58	4.42	5.38	6.75	8.00	9.13	10.75	11.75	12.50	13.50	14.38	15.33	16.17	17.13	19.75	21.50	22.75	23.75
3.10	1.33	2.70	3.58	4.50	5.50	7.00	8.25	9.50	10.75	11.75	12.50	13.63	14.38	15.25	16.75	19.25	20.38	21.50	22.50	24.00
3.20	1.36	2.75	3.50	4.50	5.67	7.00	8.25	9.25	10.75	11.50	12.50	13.63	14.75	16.00	18.00	19.50	20.50	21.25	22.50	24.00
3.30	1.34	2.67	3.60	4.50	5.81	7.13	8.25	9.25	10.50	11.50	12.63	14.00	15.13	17.00	18.50	19.50	20.50	21.75	22.75	23.75
3.40	1.32	2.75	3.63	4.50	6.08	7.00	8.25	9.38	10.50	11.25	13.00	14.50	15.75	17.50	18.50	19.38	20.50	21.50	22.50	23.50
3.50	1.29	2.69	3.63	4.75	6.00	7.25	8.38	9.25	10.25	11.38	13.50	14.75	16.25	17.00	18.25	19.13	20.13	21.00	22.00	23.00
3.60	1.23	2.69	3.63	4.88	6.13	7.25	8.33	9.25	10.25	12.00	13.75	15.25	16.25	17.25	18.25	19.50	20.25	21.00	22.00	23.00
3.70	1.25	2.75	3.50	5.00	6.50	7.38	8.38	9.25	10.75	12.75	14.25	15.17	16.25	17.13	18.00	19.25	20.25	21.50	22.50	23.25
3.80	1.25	2.63	3.63	5.13	6.38	7.58	8.50	9.50	11.25	12.50	14.42	15.33	16.38	17.13	18.00	19.25	20.50	21.75	23.00	23.75
3.90	1.34	2.50	3.63	5.25	6.38	7.63	8.50	10.25	11.50	13.25	14.50	15.63	16.50	17.50	18.50	19.50	21.25	22.50	23.50	24.00
4.00	1.29	2.50	3.75	5.58	6.42	7.63	8.50	10.50	11.50	13.25	14.50	15.75	17.00	18.00	18.88	19.75	21.75	22.75	23.75	24.75

Table 4-5 Average reduction factors (r) for systems with α=0.2 and ξ=20%

Fundamental	μ_T																			
Period (T_{el})	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	0.88	1.42	1.75	1.96	2.08	2.29	2.45	2.55	2.65	2.81	2.82	2.92	3.05	3.13	3.13	3.25	3.35	3.40	3.44	3.50
0.20	1.67	2.58	3.07	3.61	3.92	4.25	4.67	5.00	5.29	5.50	5.63	5.83	5.95	6.13	6.33	6.38	6.63	6.75	6.94	7.17
0.30	1.75	2.88	3.75	4.25	4.50	5.00	5.42	5.92	6.17	6.38	6.67	6.92	7.50	7.75	8.00	8.50	9.00	9.50	9.75	10.13
0.40	2.00	3.08	3.75	4.42	5.08	5.63	6.25	6.75	7.25	7.88	8.50	9.00	9.50	9.88	10.42	10.75	11.10	11.56	11.93	12.38
0.50	2.21	3.79	4.56	5.17	5.75	6.25	7.00	8.00	8.75	9.50	10.13	10.67	11.17	11.67	12.44	13.00	13.63	14.13	14.75	15.38
0.60	1.97	3.34	4.11	4.75	5.45	6.50	7.33	8.00	8.56	9.13	9.67	10.38	11.00	11.67	12.17	13.00	13.63	14.25	14.75	15.25
0.70	2.00	3.25	4.17	5.13	6.00	6.58	7.50	8.38	9.25	10.25	10.75	11.50	11.94	12.69	13.25	13.83	14.50	15.25	15.75	16.50
0.80	1.75	3.08	4.00	4.75	5.67	6.75	7.75	8.38	9.00	9.75	10.40	11.00	12.00	12.50	13.08	13.50	13.88	14.42	14.88	15.44
0.90	1.79	3.11	4.00	4.85	5.75	6.63	7.83	8.42	9.25	10.25	10.75	11.33	11.75	12.38	12.94	13.50	14.38	15.00	15.75	16.75
1.00	1.72	3.00	4.05	4.75	5.50	6.58	7.58	8.50	9.08	9.67	10.25	11.00	11.67	12.50	13.38	14.25	14.75	15.25	15.63	16.13
1.10	1.72	3.07	4.13	5.00	6.00	6.88	7.50	8.38	9.25	10.05	10.75	11.38	11.88	12.63	13.17	13.88	14.50	15.25	15.95	16.75
1.20	1.79	3.13	4.19	5.13	6.00	6.94	7.92	9.00	9.81	10.42	11.13	11.75	12.50	13.75	14.75	15.42	16.08	16.75	17.50	18.50
1.30	1.83	3.17	4.31	5.13	6.15	7.33	8.13	9.00	9.83	10.58	11.50	12.25	13.50	14.25	14.88	15.75	16.42	17.25	18.25	19.50
1.40	1.81	3.17	4.25	5.25	6.50	7.33	8.25	9.13	10.50	11.00	11.88	12.88	13.63	14.38	15.00	16.00	16.75	18.00	19.13	20.13
1.50	1.84	3.31	4.35	5.31	6.33	7.30	8.19	9.75	10.25	11.25	12.13	13.00	13.58	14.25	15.00	16.25	17.75	18.75	19.88	20.88
1.60	1.78	3.25	4.13	5.08	6.00	7.00	8.00	8.92	10.25	11.00	11.60	12.38	12.94	14.00	15.00	16.50	17.75	18.50	19.25	20.00
1.70	1.70	3.04	4.04	5.00	5.75	6.69	7.63	9.00	9.75	10.50	11.08	11.75	12.50	14.50	15.13	15.75	16.75	17.38	18.25	19.50
1.80	1.64	2.92	4.00	4.80	5.50	6.38	7.25	8.81	9.38	10.00	10.63	11.50	12.50	13.25	14.25	15.25	16.50	17.50	18.75	19.75
1.90	1.58	2.81	3.88	4.80	5.50	6.33	7.50	8.50	9.25	9.75	10.42	11.50	12.50	13.63	14.58	15.50	16.38	17.50	18.50	20.00
2.00	1.61	2.79	3.94	5.00	5.63	6.42	7.50	8.50	9.00	10.00	11.08	12.25	13.25	14.25	15.50	16.50	17.63	19.00	20.00	20.75
2.10	1.60	2.78	3.95	5.00	5.70	6.50	7.50	8.19	9.25	10.42	11.63	12.75	14.00	15.25	16.50	17.63	18.63	19.75	20.75	21.50
2.20	1.56	2.82	3.95	5.00	5.88	6.75	7.50	8.63	9.67	10.88	12.25	13.50	14.75	15.75	17.00	18.00	18.88	19.75	20.50	21.25
2.30	1.53	2.72	3.94	5.00	6.00	6.69	7.63	8.83	10.00	11.50	12.75	13.75	14.75	15.88	16.88	18.00	18.75	19.63	20.42	21.50
2.40	1.50	2.80	3.88	4.92	5.92	6.75	7.88	9.00	10.38	11.38	12.25	13.00	14.50	15.63	16.75	18.00	19.00	20.00	21.00	22.00
2.50	1.48	2.82	3.92	5.00	5.88	7.00	8.00	9.25	10.25	11.00	11.75	13.75	14.63	15.75	17.25	18.00	18.88	20.50	21.50	22.50
2.60	1.46	2.84	4.00	4.92	5.88	7.25	8.25	9.08	10.00	11.00	12.75	13.88	15.25	16.50	17.25	18.25	19.25	20.75	22.25	23.75
2.70	1.46	2.84	3.88	4.88	5.92	6.75	8.00	8.92	9.75	11.25	13.00	14.50	15.50	16.25	17.13	18.50	19.75	21.00	22.25	23.25
2.80	1.42	2.85	3.83	4.83	6.00	6.75	7.75	8.75	10.13	11.50	12.75	13.75	14.75	15.88	17.00	18.00	19.25	20.33	21.25	22.50
2.90	1.40	2.83	3.75	4.75	5.63	6.67	8.00	9.25	10.38	11.50	12.50	13.63	14.63	15.67	16.63	17.67	18.75	19.67	20.75	22.00
3.00	1.42	2.83	3.67	4.67	5.63	6.67	8.25	9.38	10.75	11.63	12.50	13.50	14.63	15.50	16.50	17.50	18.42	19.42	21.75	24.00
3.10	1.50	2.88	3.71	4.67	5.63	7.00	8.25	9.50	11.00	11.88	12.83	13.75	14.63	15.50	16.75	17.50	19.25	22.00	23.50	24.00
3.20	1.50	2.81	3.71	4.63	5.75	7.13	8.50	9.75	11.00	11.88	12.75	14.00	14.67	15.75	16.75	18.75	21.50	22.50	23.50	24.00
3.30	1.46	2.83	3.81	4.63	5.75	7.25	8.42	9.75	10.88	11.75	13.00	13.75	14.88	15.75	18.00	20.25	21.25	22.50	23.50	24.75
3.40	1.46	2.83	3.65	4.63	6.00	7.25	8.38	9.75	10.63	11.50	12.75	14.25	15.00	16.75	19.00	20.00	21.00	22.08	23.25	24.50
3.50	1.46	2.82	3.75	4.71	6.13	7.25	8.38	9.58	10.25	11.25	13.50	14.50	15.50	17.75	18.75	19.75	20.88	21.88	23.00	24.00
3.60	1.43	2.83	3.75	4.88	6.38	7.25	8.50	9.38	10.25	11.50	14.00	14.75	16.58	17.63	18.50	19.63	20.63	21.63	22.63	24.00
3.70	1.46	2.85	3.75	5.08	6.50	7.38	8.38	9.33	10.50	12.00	14.25	15.50	16.63	17.67	18.63	19.75	20.50	21.50	22.63	24.00
3.80	1.39	2.75	3.88	5.25	6.50	7.50	8.50	9.25	11.00	13.00	14.38	15.42	16.50	17.50	18.50	19.75	20.75	21.75	22.67	24.08
3.90	1.43	2.69	3.75	5.50	6.50	7.58	8.38	10.00	11.50	13.50	14.50	15.83	16.63	17.50	18.38	19.63	20.88	22.00	23.50	24.25
4.00	1.44	2.75	3.88	5.75	6.63	7.67	8.50	10.50	11.75	13.50	14.75	15.88	17.00	17.75	18.75	20.00	21.25	22.75	24.25	25.00

Table 4-6 Average reduction factors (r) for systems with α=0.2 and ξ=25%

Fundamental	μ_T																			
Period (T_{el})	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	0.95	1.48	1.89	2.06	2.33	2.50	2.63	2.78	2.86	3.00	3.04	3.17	3.25	3.25	3.33	3.45	3.50	3.56	3.67	3.75
0.20	1.84	2.72	3.31	3.85	4.17	4.50	4.94	5.25	5.55	5.75	5.92	6.00	6.25	6.42	6.63	6.75	6.90	7.13	7.15	7.44
0.30	1.96	3.00	4.04	4.50	4.92	5.38	5.75	6.08	6.44	6.88	7.13	7.50	7.75	8.00	8.42	8.88	9.25	10.00	10.25	10.50
0.40	2.25	3.30	4.15	4.75	5.38	6.00	6.50	7.25	7.50	8.25	8.75	9.25	9.88	10.33	10.88	11.25	11.75	12.15	12.63	12.94
0.50	2.38	4.06	4.83	5.50	6.25	6.75	7.50	8.13	9.25	10.00	10.75	11.31	11.75	12.33	13.00	13.63	14.13	14.75	15.13	15.75
0.60	2.21	3.63	4.58	5.15	5.71	6.75	7.88	8.50	9.08	9.58	10.25	10.92	11.58	12.13	12.75	13.50	14.25	14.83	15.38	16.00
0.70	2.21	3.50	4.50	5.44	6.38	7.08	7.75	8.63	9.75	10.75	11.25	12.00	12.50	13.25	13.75	14.38	14.95	15.75	16.25	16.88
0.80	1.97	3.25	4.25	5.13	6.00	7.00	8.00	8.75	9.42	10.00	10.80	11.44	12.13	13.00	13.50	14.13	14.50	15.00	15.50	16.08
0.90	1.93	3.33	4.33	5.25	6.06	7.25	8.36	9.00	9.50	10.75	11.25	11.75	12.33	12.92	13.50	13.94	14.75	15.42	16.25	16.75
1.00	1.93	3.13	4.30	5.13	5.75	7.00	8.00	8.75	9.42	10.08	10.69	11.38	12.25	12.92	13.83	14.50	15.25	16.25	17.25	17.75
1.10	1.89	3.29	4.38	5.13	6.25	7.25	7.92	8.75	9.44	10.15	11.06	11.75	12.50	13.25	14.00	14.50	15.25	15.75	16.50	17.25
1.20	2.00	3.38	4.42	5.50	6.50	7.40	8.25	9.25	10.00	10.75	11.50	12.17	12.92	13.75	14.63	15.75	16.50	17.50	18.00	18.69
1.30	2.00	3.42	4.58	5.50	6.42	7.50	8.50	9.25	10.17	10.75	11.63	12.75	13.75	14.75	15.38	15.88	16.88	17.50	18.75	19.75
1.40	2.03	3.40	4.45	5.60	6.75	7.75	8.58	9.45	10.25	11.75	12.50	13.38	14.13	14.83	15.50	16.58	17.38	18.33	19.75	20.75
1.50	2.00	3.56	4.50	5.60	6.55	7.44	8.63	10.00	11.00	11.50	12.50	13.17	14.08	14.67	15.50	16.50	17.75	19.13	20.13	21.50
1.60	1.91	3.34	4.33	5.30	6.15	7.25	8.50	9.25	10.75	11.50	12.00	12.75	13.50	14.25	15.38	16.63	18.00	18.88	20.00	21.00
1.70	1.79	3.25	4.17	5.25	6.08	7.00	8.13	9.50	10.25	10.75	11.50	12.25	13.00	14.50	15.50	16.63	17.50	18.25	18.75	19.75
1.80	1.72	3.00	4.13	5.08	5.88	6.69	7.75	9.25	9.83	10.38	10.92	11.75	13.00	14.00	14.88	15.75	16.50	17.75	19.00	20.00
1.90	1.70	3.00	4.08	4.90	5.71	6.50	7.50	9.00	9.50	10.00	10.69	11.75	12.75	14.00	14.83	16.00	17.00	18.00	19.00	20.13
2.00	1.71	3.00	4.15	5.06	5.88	6.75	8.00	9.00	9.50	10.25	11.25	12.63	13.63	14.58	15.50	16.50	17.50	19.25	20.50	21.25
2.10	1.71	3.00	4.19	5.17	5.92	6.67	8.00	9.00	9.44	10.50	12.06	13.10	14.25	15.25	16.50	17.50	19.25	20.00	20.88	22.00
2.20	1.66	2.91	4.13	5.00	6.00	6.92	7.88	8.67	9.88	11.13	12.33	13.50	15.00	16.00	17.50	18.50	19.50	20.50	21.25	22.00
2.30	1.64	2.88	4.00	5.17	6.17	7.25	7.92	9.00	10.17	11.50	12.75	14.00	15.50	16.63	17.75	18.63	19.38	20.25	21.13	22.13
2.40	1.64	2.82	4.13	5.17	6.19	7.13	8.00	9.08	10.58	11.75	13.00	14.25	15.50	16.50	17.75	18.50	19.38	20.25	21.50	22.50
2.50	1.61	2.85	4.25	5.25	6.25	7.00	8.25	9.50	10.81	11.58	12.50	14.25	15.25	16.25	17.50	18.50	20.00	21.00	22.00	23.00
2.60	1.63	3.04	4.17	5.25	6.25	7.25	8.17	9.58	10.50	11.38	13.25	14.25	15.38	16.75	17.75	18.50	19.75	21.25	22.38	23.63
2.70	1.59	3.00	4.06	5.14	6.25	7.00	8.31	9.25	10.13	11.75	13.25	14.38	15.75	16.50	17.42	18.75	20.00	21.38	22.75	23.88
2.80	1.54	3.04	3.93	4.96	6.08	6.92	8.00	8.88	9.88	12.00	13.25	14.25	15.25	16.25	17.50	18.50	19.75	21.00	22.00	23.25
2.90	1.50	2.92	3.92	5.00	6.00	6.88	7.88	9.00	10.75	11.88	12.88	13.88	14.92	16.00	17.00	18.25	19.33	20.25	21.50	22.00
3.00	1.53	2.88	3.92	4.88	6.00	6.88	8.00	9.50	10.75	11.75	12.75	13.75	14.92	15.88	16.88	18.08	19.00	20.00	21.25	22.25
3.10	1.50	2.88	3.92	4.75	5.75	6.75	8.50	9.88	11.00	12.00	13.00	14.00	15.00	16.00	17.13	18.00	19.13	20.00	22.00	24.00
3.20	1.54	2.92	3.83	4.75	5.92	7.50	8.75	10.13	11.25	12.13	13.08	14.00	14.92	16.25	17.00	18.00	19.17	21.75	24.00	24.00
3.30	1.56	2.88	3.90	4.75	6.00	7.50	8.75	10.00	11.00	12.08	13.00	14.00	15.00	16.00	17.00	18.50	20.88	23.00	24.00	24.00
3.40	1.58	2.83	3.95	4.75	6.25	7.25	8.67	10.00	10.83	11.75	13.00	14.25	15.00	15.88	17.25	19.75	21.50	22.75	23.75	24.75
3.50	1.56	2.88	3.90	4.83	6.33	7.00	8.50	9.75	10.58	11.50	13.50	14.25	15.00	16.00	18.63	20.25	21.25	22.25	23.38	24.50
3.60	1.56	2.88	3.90	5.08	6.38	7.25	8.50	9.58	10.50	11.63	13.75	14.50	15.50	17.63	19.00	20.00	21.00	22.13	23.25	24.25
3.70	1.57	2.88	4.00	5.25	6.38	7.25	8.50	9.38	10.58	12.50	14.00	15.00	16.50	17.88	18.88	20.00	21.00	22.13	23.25	24.50
3.80	1.55	2.92	4.08	5.33	6.63	7.38	8.50	9.44	10.50	13.25	14.50	15.25	16.75	17.75	18.88	20.00	21.00	22.13	23.25	24.50
3.90	1.50	2.88	4.00	5.58	6.63	7.58	8.58	9.75	11.25	13.00	14.38	15.88	16.88	18.00	19.00	20.25	21.25	22.17	23.50	24.50
4.00	1.58	2.88	4.00	5.75	6.67	7.83	8.67	10.00	11.75	13.50	15.00	16.13	17.13	18.00	19.13	20.38	21.50	22.50	23.83	25.00

Table 4-7 Average reduction factors (*r*) for systems with α=0.2 and ξ=30%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	0.97	1.61	1.93	2.25	2.46	2.75	2.83	3.00	3.07	3.15	3.25	3.30	3.38	3.45	3.50	3.60	3.65	3.70	3.81	3.81
0.20	2.13	2.93	3.46	4.06	4.45	4.90	5.18	5.50	5.70	5.92	6.17	6.42	6.50	6.75	6.88	7.06	7.20	7.40	7.63	7.75
0.30	2.17	3.25	4.29	4.81	5.17	5.63	6.08	6.38	6.75	7.17	7.44	7.75	8.17	8.50	9.00	9.25	9.75	10.25	10.75	11.13
0.40	2.42	3.58	4.40	5.15	5.71	6.38	7.00	7.50	8.25	8.50	9.25	9.75	10.25	10.92	11.38	11.81	12.25	12.75	13.13	13.63
0.50	2.65	4.25	5.25	6.00	6.50	7.00	8.00	8.63	9.63	10.63	11.17	11.83	12.38	12.88	13.50	14.25	14.75	15.25	15.75	16.25
0.60	2.44	3.92	4.88	5.56	6.08	7.00	8.25	8.88	9.44	10.08	10.58	11.42	12.00	12.75	13.20	13.92	14.58	15.50	16.00	16.44
0.70	2.39	3.75	4.88	5.81	6.67	7.56	8.19	9.06	10.00	11.25	11.75	12.50	13.00	13.75	14.38	15.06	15.58	16.15	16.75	17.50
0.80	2.18	3.50	4.57	5.50	6.33	7.17	8.50	9.13	9.67	10.75	11.25	12.00	12.63	13.38	13.92	14.50	15.13	15.75	16.31	16.81
0.90	2.11	3.40	4.58	5.55	6.50	7.50	8.75	9.44	10.00	10.63	12.00	12.25	12.83	13.38	14.00	14.58	15.08	15.75	16.38	17.25
1.00	1.98	3.33	4.42	5.50	6.13	7.50	8.17	9.19	10.00	10.50	11.17	11.83	12.42	13.25	13.92	14.88	15.50	16.13	16.75	17.25
1.10	2.00	3.44	4.50	5.50	6.75	7.50	8.38	9.17	9.81	10.44	11.38	12.17	12.75	13.50	14.50	15.25	16.00	16.75	17.50	18.50
1.20	2.10	3.54	4.67	5.63	7.00	7.80	8.69	9.40	10.25	11.00	11.70	12.50	13.38	14.00	15.00	16.00	17.00	17.88	18.75	19.25
1.30	2.04	3.60	4.75	5.67	6.81	7.75	8.75	9.50	10.38	11.13	11.92	12.75	13.75	15.00	15.63	16.63	17.25	18.00	18.75	20.00
1.40	2.11	3.69	4.75	5.88	6.75	7.88	8.90	9.75	10.75	11.75	12.63	13.67	14.58	15.25	16.25	16.88	18.00	18.75	19.75	21.00
1.50	2.13	3.69	4.71	5.83	6.81	8.00	9.00	9.75	11.50	12.25	13.00	13.75	14.42	15.13	16.25	17.25	18.13	19.25	20.50	21.50
1.60	2.08	3.53	4.50	5.54	6.50	7.63	8.42	10.00	11.00	11.75	12.25	13.17	14.00	14.75	16.00	17.00	18.13	19.25	20.50	21.75
1.70	2.00	3.35	4.38	5.40	6.31	7.33	8.56	9.75	10.50	11.00	11.67	12.75	13.75	14.75	15.75	17.00	17.92	19.00	19.63	20.00
1.80	1.83	3.21	4.30	5.30	6.15	7.13	8.25	9.50	10.13	10.75	11.25	12.13	13.50	14.42	15.63	16.38	16.92	18.00	19.00	20.00
1.90	1.85	3.20	4.38	5.19	5.93	6.94	7.75	9.38	9.88	10.50	11.00	12.13	13.25	14.25	15.25	16.00	17.25	18.50	19.50	20.00
2.00	1.83	3.20	4.50	5.25	6.08	7.25	8.50	9.38	9.88	10.50	11.50	13.00	14.00	14.83	16.00	17.00	17.75	18.92	20.75	22.00
2.10	1.82	3.15	4.42	5.40	6.15	7.00	8.50	9.25	10.00	10.69	12.25	13.50	14.42	15.50	16.50	17.63	19.00	20.50	21.25	22.13
2.20	1.82	3.14	4.33	5.38	6.25	7.25	8.38	9.25	9.94	11.33	12.75	13.63	15.00	16.25	17.25	19.25	19.75	20.75	21.75	23.00
2.30	1.79	3.09	4.25	5.38	6.38	7.25	8.25	9.17	10.50	11.63	12.75	14.00	15.50	16.75	18.25	19.13	20.00	21.00	22.00	22.75
2.40	1.79	3.04	4.33	5.44	6.50	7.38	8.25	9.38	10.50	11.88	13.00	14.75	16.25	17.25	18.25	19.13	19.92	20.88	21.88	22.88
2.50	1.73	3.04	4.33	5.42	6.50	7.38	8.25	9.50	11.00	12.25	13.25	15.00	16.00	16.88	18.00	19.00	20.00	21.25	22.50	23.25
2.60	1.75	3.05	4.25	5.40	6.50	7.50	8.38	9.88	11.00	12.00	13.75	15.00	16.00	17.13	18.00	19.00	20.75	21.75	22.75	24.00
2.70	1.66	3.06	4.17	5.35	6.50	7.25	8.25	9.63	10.63	11.42	13.75	14.75	16.00	16.88	17.92	19.00	20.50	21.75	22.88	24.13
2.80	1.61	3.00	4.10	5.30	6.13	6.88	8.25	9.25	10.13	12.50	13.50	14.63	15.50	16.63	18.00	19.00	20.25	21.56	22.50	23.75
2.90	1.59	3.04	4.00	5.19	6.00	7.00	8.00	9.00	11.00	12.25	13.25	14.33	15.33	16.50	17.50	18.75	19.75	21.00	22.00	23.25
3.00	1.63	3.04	4.08	5.13	6.25	7.00	8.13	9.50	11.25	12.25	13.25	14.13	15.38	16.38	17.38	18.50	19.50	20.75	21.75	23.00
3.10	1.63	3.00	4.06	5.00	6.00	7.00	8.25	10.00	11.13	12.25	13.25	14.17	15.25	16.50	17.63	18.50	19.50	20.75	22.00	23.00
3.20	1.63	3.00	4.06	5.00	6.00	7.00	8.75	10.13	11.25	12.25	13.25	14.25	15.25	16.50	17.63	18.63	19.63	20.75	22.75	23.88
3.30	1.61	3.00	4.08	5.00	6.00	7.50	8.88	10.25	11.25	12.25	13.25	14.20	15.25	16.50	17.38	18.38	19.75	22.00	23.50	24.00
3.40	1.64	2.92	3.95	5.00	6.25	7.50	8.88	10.25	11.00	12.08	13.00	14.25	15.00	16.00	17.13	18.50	20.75	23.25	24.00	24.00
3.50	1.64	2.85	3.95	5.00	6.50	7.50	8.75	10.00	10.75	11.75	13.50	14.25	15.13	16.00	17.25	19.50	21.50	22.75	23.75	24.75
3.60	1.65	2.92	3.89	4.94	6.38	7.50	9.00	9.75	10.63	11.75	13.75	14.50	15.25	16.13	18.75	20.25	21.50	22.50	23.63	25.00
3.70	1.65	2.92	4.00	5.25	6.38	7.50	8.75	9.75	10.67	12.25	14.00	14.75	15.75	17.75	19.25	20.25	21.25	22.63	23.75	25.00
3.80	1.67	2.95	4.00	5.42	6.50	7.50	8.75	9.50	10.63	12.75	14.25	15.25	16.50	18.13	19.13	20.25	21.38	22.63	24.00	25.00
3.90	1.66	3.00	4.25	5.56	6.69	7.75	8.75	9.67	10.88	13.50	14.75	15.50	17.13	18.25	19.38	20.50	21.50	22.75	24.25	25.25
4.00	1.71	3.00	4.25	5.67	6.88	8.00	8.88	10.00	11.25	13.50	14.50	16.00	17.25	18.58	19.58	20.75	21.75	23.00	24.25	25.25

Table 4-8 Average reduction factors (r) for systems with α=0.2 and ξ=35%

Fundamental Period (T_{el})	μ_T																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.00	1.73	2.10	2.50	2.75	2.83	3.00	3.08	3.15	3.30	3.38	3.46	3.50	3.63	3.67	3.75	3.81	3.88	3.94	4.06
0.20	2.21	3.13	3.69	4.30	4.80	5.15	5.42	5.75	5.96	6.18	6.43	6.67	6.92	7.08	7.25	7.40	7.63	7.83	8.00	8.15
0.30	2.38	3.56	4.54	5.00	5.50	5.92	6.50	6.75	7.13	7.46	7.88	8.20	8.50	9.00	9.50	9.75	10.25	10.75	11.00	11.50
0.40	2.63	3.83	4.75	5.46	6.00	6.75	7.25	8.00	8.50	9.00	9.58	10.00	10.75	11.25	11.88	12.38	12.75	13.19	13.75	14.13
0.50	2.90	4.50	5.50	6.25	7.00	7.50	8.25	9.13	10.00	11.25	11.75	12.38	13.00	13.56	14.17	14.69	15.25	15.75	16.38	16.83
0.60	2.58	4.17	5.17	5.83	6.50	7.17	8.58	9.33	10.00	10.63	11.13	11.75	12.50	13.25	14.00	14.50	15.13	15.75	16.50	17.00
0.70	2.63	4.00	5.08	5.92	7.08	8.00	8.75	9.38	10.25	11.50	12.25	12.75	13.50	14.00	15.08	15.56	16.10	16.63	17.25	17.88
0.80	2.39	3.75	4.67	5.85	6.69	7.50	8.75	9.50	10.17	11.25	11.75	12.50	13.00	13.75	14.50	15.06	15.75	16.33	16.88	17.38
0.90	2.21	3.55	4.69	5.83	6.63	7.75	8.75	9.69	10.38	11.13	12.00	12.75	13.25	13.88	14.50	15.25	15.58	16.25	17.00	17.58
1.00	2.16	3.44	4.58	5.75	6.50	7.88	8.63	9.50	10.50	11.00	11.58	12.25	12.83	13.38	14.25	15.00	15.75	16.25	16.75	17.63
1.10	2.18	3.67	4.56	5.75	7.00	8.00	8.83	9.50	10.13	10.85	11.50	12.42	13.38	14.13	14.75	15.75	16.50	17.25	17.75	18.00
1.20	2.25	3.67	4.75	5.92	7.13	8.17	9.00	9.75	10.50	11.75	12.00	12.75	13.58	14.50	15.25	16.25	17.00	18.25	18.75	19.25
1.30	2.21	3.80	4.75	5.90	7.00	8.00	9.00	9.75	10.50	11.44	12.38	13.00	13.88	15.50	16.50	16.75	17.44	18.25	19.08	20.00
1.40	2.25	3.80	5.00	6.13	7.19	8.00	9.17	10.00	11.00	12.00	12.75	14.00	15.00	15.92	16.63	17.17	17.92	19.25	20.25	21.25
1.50	2.25	3.84	5.06	6.10	6.97	8.25	9.00	10.25	11.13	12.50	13.50	14.25	14.88	15.75	16.50	17.50	18.38	19.50	20.75	22.00
1.60	2.14	3.69	4.71	5.75	6.75	7.75	8.88	10.25	11.25	12.00	12.75	13.42	14.25	15.25	16.00	17.00	18.25	19.63	20.50	21.50
1.70	2.08	3.50	4.58	5.57	6.50	7.56	9.06	10.00	10.75	11.50	11.92	12.88	14.00	15.25	16.25	17.25	18.38	19.25	20.50	21.25
1.80	2.00	3.42	4.46	5.55	6.50	7.50	8.50	9.75	10.25	11.25	12.00	12.50	13.75	14.75	16.00	17.08	17.67	18.17	19.00	20.00
1.90	1.95	3.31	4.50	5.40	6.25	7.38	8.50	9.63	10.25	11.00	11.50	12.38	13.75	14.75	15.75	16.25	17.25	18.50	19.50	20.00
2.00	2.00	3.25	4.75	5.42	6.33	7.38	8.50	9.75	10.25	10.88	11.63	13.00	14.25	15.25	16.00	17.00	18.25	19.50	20.50	22.00
2.10	1.96	3.30	4.63	5.60	6.42	7.50	9.00	9.63	10.25	10.94	12.25	13.75	14.75	15.63	16.75	17.75	18.88	20.75	22.00	22.75
2.20	1.96	3.33	4.63	5.63	6.50	7.50	8.75	9.50	10.13	11.25	13.00	14.00	15.00	16.25	17.25	18.63	20.50	21.25	22.00	23.00
2.30	1.91	3.29	4.50	5.63	6.50	7.75	8.75	9.50	10.40	12.00	13.13	14.25	15.50	16.75	18.50	19.50	20.38	21.50	22.75	23.75
2.40	1.86	3.18	4.50	5.63	6.75	7.50	8.50	9.42	10.88	12.13	13.17	14.50	16.00	17.75	18.63	19.63	20.50	21.50	22.50	23.38
2.50	1.86	3.17	4.56	5.65	6.75	7.75	8.58	9.58	11.00	12.17	13.50	15.00	16.63	17.75	18.63	19.50	20.63	21.50	22.50	23.75
2.60	1.84	3.19	4.42	5.63	6.75	7.50	8.44	9.88	11.25	12.50	13.50	15.50	16.50	17.50	18.67	20.00	21.00	22.25	23.25	24.25
2.70	1.79	3.17	4.25	5.58	6.56	7.25	8.50	10.00	11.00	12.00	14.00	15.25	16.25	17.38	18.25	19.50	21.00	22.08	23.25	24.50
2.80	1.75	3.00	4.20	5.44	6.25	7.00	8.50	9.50	10.50	12.75	14.00	15.00	16.00	16.88	18.13	19.50	20.63	21.75	22.88	23.88
2.90	1.69	3.10	4.10	5.33	6.17	7.13	8.25	9.25	11.25	12.75	13.75	14.63	15.83	16.88	18.00	19.25	20.25	21.50	22.50	23.75
3.00	1.67	3.06	4.13	5.25	6.25	7.13	8.25	9.25	11.50	12.50	13.50	14.38	15.63	16.63	17.75	18.92	20.00	21.25	22.25	23.50
3.10	1.69	3.06	4.13	5.25	6.25	7.00	8.25	10.25	11.50	12.50	13.50	14.50	15.63	16.83	17.92	19.00	20.00	21.25	22.50	23.50
3.20	1.72	3.06	4.25	5.25	6.17	7.25	9.00	10.25	11.50	12.50	13.50	14.63	15.75	16.88	18.00	19.00	20.25	21.50	22.50	23.88
3.30	1.72	3.05	4.19	5.25	6.25	7.17	9.00	10.25	11.25	12.25	13.38	14.58	15.75	16.75	17.88	18.88	20.00	21.25	23.25	24.00
3.40	1.69	3.05	4.08	5.08	6.25	7.75	9.08	10.25	11.25	12.33	13.35	14.50	15.50	16.63	17.63	18.63	20.13	22.00	23.00	24.00
3.50	1.67	3.00	4.00	5.08	6.50	7.63	9.00	10.25	11.00	12.08	13.25	14.25	15.25	16.38	17.38	18.75	20.75	22.00	24.00	24.00
3.60	1.69	3.00	4.00	5.08	6.75	7.75	9.00	10.00	11.00	12.00	13.75	14.50	15.25	16.25	17.50	20.00	21.25	22.75	24.25	25.25
3.70	1.69	2.95	4.00	5.25	6.75	7.75	9.00	10.00	10.92	12.00	14.00	14.75	15.50	16.50	19.00	20.50	21.75	23.00	24.00	25.50
3.80	1.70	2.92	4.00	5.38	6.63	8.00	9.00	9.75	11.00	12.25	14.25	15.00	16.00	18.00	19.38	20.50	21.75	23.08	24.50	25.50
3.90	1.70	3.00	4.00	5.58	6.67	8.00	9.00	9.88	11.00	13.00	14.75	15.50	16.63	18.25	19.50	20.50	22.13	23.00	24.50	25.50
4.00	1.75	3.00	4.33	5.75	6.88	8.25	9.00	10.00	11.13	13.75	15.00	16.00	17.25	18.75	19.83	21.00	22.13	23.50	24.75	25.75

Table 4-9 Average reduction factors (*r*) for systems with *α*=0.4 and *ξ*=5%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	0.90	1.33	1.48	1.58	1.69	1.75	1.75	1.92	1.96	1.98	2.17	2.31	2.38	2.44	2.58	2.64	2.68	2.81	2.88	2.92
0.20	0.95	1.75	2.35	2.69	3.06	3.31	3.57	3.75	4.08	4.19	4.44	4.63	4.88	5.00	5.25	5.42	5.67	5.88	6.33	6.75
0.30	1.00	2.11	2.79	3.25	3.56	4.08	4.75	5.13	5.38	5.63	5.75	6.25	6.50	6.83	7.17	7.50	7.92	8.42	8.67	9.00
0.40	1.15	2.14	2.85	3.56	4.08	4.58	5.00	5.50	6.00	6.50	7.13	7.75	8.00	8.50	8.88	9.25	9.63	10.08	10.56	11.06
0.50	1.23	2.43	3.38	4.06	4.58	5.13	6.25	7.00	7.50	8.00	8.63	9.17	9.88	10.25	10.75	11.56	12.00	12.50	13.13	13.50
0.60	1.21	2.33	3.10	3.88	4.83	5.50	6.33	6.75	7.50	8.00	8.58	8.94	9.50	10.25	10.83	11.50	12.00	12.50	13.38	13.88
0.70	1.25	2.29	3.25	4.19	4.96	5.75	6.50	7.13	8.00	9.00	9.63	10.00	10.75	11.25	12.00	12.50	12.88	13.33	13.75	15.00
0.80	1.21	2.32	3.07	4.00	4.83	5.88	6.63	7.25	8.00	8.50	9.25	10.25	10.67	11.20	11.75	12.50	13.00	13.50	14.00	14.25
0.90	1.22	2.21	3.25	4.15	5.00	5.88	7.00	7.75	8.38	9.00	9.75	10.25	10.88	11.42	12.75	13.25	13.75	14.38	14.88	15.42
1.00	1.16	2.25	3.38	4.05	4.88	5.88	6.68	7.42	8.00	8.83	9.67	10.25	10.88	11.42	11.88	12.50	13.50	14.13	14.75	15.00
1.10	1.11	2.30	3.33	4.00	5.06	5.92	6.75	7.63	8.38	9.25	10.00	10.63	11.25	12.13	12.88	13.67	14.38	15.00	15.67	16.25
1.20	1.16	2.43	3.50	4.21	5.25	6.00	7.00	8.00	8.75	9.75	10.50	11.25	12.00	12.67	13.63	14.25	15.00	15.75	16.75	17.75
1.30	1.13	2.46	3.41	4.25	5.25	6.25	7.13	7.92	8.75	9.75	10.19	11.50	12.33	13.00	13.63	14.75	15.50	16.13	17.13	18.25
1.40	1.14	2.56	3.46	4.38	5.33	6.20	7.38	8.50	9.25	10.13	11.13	11.92	12.58	13.50	14.50	15.25	16.50	17.25	17.75	19.50
1.50	1.11	2.50	3.45	4.50	5.50	6.50	7.50	8.75	9.81	10.56	11.08	11.63	12.50	13.75	15.25	15.75	16.75	17.13	18.25	19.33
1.60	1.14	2.50	3.50	4.58	5.30	6.13	7.25	8.50	9.33	10.06	10.67	11.17	12.25	13.50	14.00	15.00	15.50	16.75	18.00	18.00
1.70	1.18	2.42	3.54	4.25	5.00	5.69	6.88	8.00	8.75	9.25	9.92	10.50	11.63	12.75	13.50	14.17	16.00	16.75	17.50	18.00
1.80	1.12	2.25	3.31	4.13	4.71	5.75	6.63	7.13	8.00	8.75	9.75	10.38	11.75	12.58	13.38	14.38	15.13	15.75	16.50	18.00
1.90	1.05	2.25	3.25	4.13	4.88	5.75	6.25	7.13	8.08	9.00	9.75	10.50	11.63	12.50	13.13	14.00	14.50	17.00	18.00	18.00
2.00	1.06	2.31	3.50	4.50	5.13	5.69	6.50	7.38	8.50	9.50	10.25	10.92	12.25	12.88	13.50	15.25	16.00	17.50	18.00	18.00
2.10	1.00	2.35	3.50	4.50	5.13	5.88	7.00	8.00	8.50	9.50	10.33	11.25	12.25	13.00	14.25	15.50	16.25	17.25	18.13	19.50
2.20	1.00	2.36	3.38	4.25	5.17	6.25	7.13	7.75	8.38	9.50	10.13	11.13	12.13	13.25	14.25	15.19	16.50	18.08	19.25	20.00
2.30	0.97	2.38	3.31	4.25	5.25	6.25	7.00	7.63	8.75	9.50	10.42	11.38	12.38	13.50	15.00	16.50	17.50	18.50	19.75	21.00
2.40	0.96	2.32	3.25	4.30	5.25	6.00	6.75	7.81	8.75	9.75	10.63	11.75	13.25	15.00	16.25	17.25	18.00	19.00	20.25	21.50
2.50	0.94	2.25	3.25	4.25	5.33	6.00	7.00	8.13	9.00	9.88	10.88	12.50	15.00	15.75	16.75	17.50	18.50	19.25	20.25	21.50
2.60	0.92	2.33	3.35	4.40	5.25	6.19	7.33	8.25	9.00	10.75	12.00	13.50	14.50	15.25	16.50	17.75	18.75	19.50	21.50	22.50
2.70	0.94	2.38	3.31	4.38	5.30	6.00	7.00	8.50	9.75	11.33	12.17	12.94	14.75	15.50	16.50	17.50	19.00	20.75	21.50	23.00
2.80	1.05	2.25	3.25	4.20	5.06	6.00	7.25	8.75	10.25	11.50	12.25	13.25	14.13	15.25	16.50	18.00	19.25	21.00	22.00	23.25
2.90	1.03	2.50	3.25	4.20	5.00	6.00	7.75	9.00	10.25	11.25	12.50	13.38	14.33	15.17	17.00	18.50	19.50	20.50	21.50	22.00
3.00	1.05	2.41	3.25	4.15	5.13	6.38	7.88	9.25	10.38	11.25	12.38	13.38	14.42	16.00	17.38	18.38	19.25	20.13	21.25	22.38
3.10	1.06	2.43	3.25	4.25	5.38	6.63	8.25	9.50	10.50	11.38	12.63	13.50	14.83	16.75	17.50	18.50	19.50	20.38	21.25	22.50
3.20	1.04	2.46	3.42	4.25	5.75	6.75	8.25	9.38	10.50	11.50	12.58	13.75	16.00	16.75	17.75	18.88	19.63	20.63	21.50	22.58
3.30	1.06	2.41	3.35	4.50	5.58	7.06	8.13	9.25	10.38	11.25	12.75	15.00	15.75	17.00	17.88	18.92	19.83	20.63	21.50	22.50
3.40	1.00	2.30	3.38	4.38	5.75	7.00	7.92	9.13	10.00	11.25	14.00	15.00	16.00	17.00	17.75	18.50	19.67	20.63	21.50	22.38
3.50	1.03	2.30	3.38	4.42	5.75	7.00	7.63	8.92	10.00	11.75	14.00	15.08	16.08	16.83	17.75	18.50	19.42	20.75	22.25	22.75
3.60	1.00	2.35	3.42	4.25	5.50	6.75	7.75	9.06	10.25	12.63	14.13	15.13	16.00	17.00	17.75	18.75	20.00	21.50	22.75	23.50
3.70	1.00	2.42	3.50	4.50	5.75	6.58	7.75	9.25	11.50	12.75	14.13	15.25	16.25	17.00	18.00	19.00	20.00	21.50	22.50	23.50
3.80	1.04	2.25	3.50	4.50	5.75	6.83	8.25	9.25	11.58	12.75	14.13	15.50	16.25	17.25	18.75	19.50	20.50	21.50	22.50	23.38
3.90	1.04	2.38	3.63	4.63	5.85	7.00	8.25	10.00	11.75	13.00	14.25	15.50	16.50	17.75	18.75	19.50	20.50	21.75	22.75	24.00
4.00	1.03	2.21	3.50	4.63	5.92	7.33	8.25	10.50	11.63	13.00	14.50	15.75	16.75	17.75	18.75	19.75	20.75	22.50	23.50	25.13

Table 4-10 **Average reduction factors (*r*) for systems with α=0.4 and ξ=10%**

Fundamental Period (T_{el})	μ_T																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	0.85	1.34	1.50	1.60	1.72	1.88	2.00	2.06	2.13	2.25	2.45	2.57	2.67	2.79	2.84	2.88	3.00	3.04	3.08	3.13
0.20	1.23	2.13	2.65	3.13	3.42	3.81	3.95	4.25	4.56	4.83	5.00	5.25	5.25	5.50	6.00	6.13	6.33	6.50	6.81	7.00
0.30	1.34	2.40	3.17	3.63	4.00	4.75	5.00	5.42	5.63	5.88	6.17	6.42	7.25	7.50	7.75	8.00	8.42	8.92	9.25	9.58
0.40	1.39	2.45	3.33	3.94	4.50	4.94	5.58	6.25	6.75	7.13	7.67	8.25	8.75	9.25	9.50	9.92	10.39	10.81	11.25	11.81
0.50	1.58	2.83	3.70	4.50	5.00	5.75	6.50	7.25	7.83	8.75	9.25	9.88	10.50	11.13	11.75	12.25	12.75	13.25	14.25	14.50
0.60	1.47	2.63	3.39	4.25	5.00	6.25	6.75	7.50	8.00	8.42	9.17	9.75	10.50	11.06	11.58	12.25	13.00	13.50	14.00	14.75
0.70	1.47	2.63	3.50	4.67	5.42	6.25	7.13	7.63	8.38	9.50	10.25	10.67	11.50	12.13	12.63	13.25	13.75	14.25	14.67	15.17
0.80	1.43	2.59	3.38	4.21	5.25	6.33	7.06	7.70	8.55	9.13	10.13	10.75	11.25	11.75	12.38	12.75	13.50	14.13	14.63	15.00
0.90	1.44	2.50	3.50	4.50	5.33	6.33	7.25	8.00	8.75	9.50	10.17	11.00	11.50	12.00	12.63	13.75	14.25	15.25	16.00	16.50
1.00	1.40	2.38	3.63	4.50	5.38	6.50	7.17	7.90	8.50	9.25	10.25	10.83	11.75	12.50	13.25	13.75	14.00	14.63	15.50	16.00
1.10	1.45	2.60	3.63	4.42	5.50	6.38	7.13	8.06	8.94	9.75	10.25	10.88	11.75	12.50	13.50	14.13	14.67	15.50	16.25	17.00
1.20	1.50	2.71	3.75	4.58	5.75	6.63	7.63	8.50	9.50	10.25	10.75	11.63	12.75	13.75	14.25	15.00	15.88	16.63	17.50	18.38
1.30	1.50	2.69	3.60	4.58	5.67	6.75	7.50	8.50	9.25	10.38	11.25	12.25	13.06	13.81	14.33	15.50	16.25	17.25	18.00	18.88
1.40	1.44	2.82	3.75	4.70	5.88	6.75	7.75	9.00	9.88	10.75	11.81	12.50	13.13	13.75	15.00	16.00	17.00	18.00	19.00	20.00
1.50	1.39	2.81	3.75	4.75	5.88	6.75	7.92	8.88	10.00	11.00	11.75	12.38	12.88	14.25	15.75	16.75	17.25	18.50	19.58	20.00
1.60	1.41	2.75	3.70	4.70	5.75	6.63	7.50	9.00	9.75	10.50	11.25	11.88	12.50	14.00	15.13	16.00	16.75	17.75	18.75	19.75
1.70	1.38	2.67	3.75	4.55	5.38	6.20	7.13	8.75	9.38	10.00	10.58	11.17	12.25	13.50	14.33	15.25	16.50	17.50	18.75	19.75
1.80	1.30	2.46	3.50	4.42	5.14	6.25	7.13	8.25	8.75	9.25	10.08	10.88	12.00	13.25	14.25	15.25	16.25	17.00	18.00	18.75
1.90	1.25	2.44	3.50	4.38	5.50	6.08	6.75	7.75	8.25	9.13	10.25	11.25	12.42	13.25	14.25	15.25	16.25	17.00	18.25	19.00
2.00	1.18	2.44	3.46	4.50	5.50	6.00	6.75	7.67	8.75	9.83	10.75	12.00	13.00	14.00	14.75	15.50	17.50	18.25	19.25	20.00
2.10	1.20	2.57	3.63	5.00	5.50	6.13	6.94	8.25	9.13	10.50	11.50	12.38	13.25	14.00	15.75	16.75	17.75	18.50	19.50	20.50
2.20	1.18	2.61	3.46	4.75	5.50	6.25	7.50	8.50	9.50	10.50	11.50	12.58	13.50	14.75	15.50	16.63	17.75	18.75	19.63	20.50
2.30	1.20	2.60	3.67	4.60	5.75	6.44	7.58	8.25	9.25	10.75	11.75	12.50	13.58	14.67	15.88	17.25	18.33	19.50	20.75	21.75
2.40	1.17	2.46	3.65	4.58	5.75	6.63	7.42	8.38	9.50	10.25	11.25	12.63	14.00	15.50	16.75	17.75	18.75	19.63	21.50	22.00
2.50	1.16	2.54	3.55	4.60	5.75	6.50	7.25	8.38	9.20	10.38	11.75	13.50	15.00	16.25	17.25	18.00	19.00	19.75	21.50	22.00
2.60	1.15	2.55	3.60	4.63	5.75	6.42	7.44	8.63	9.75	10.83	12.50	14.25	15.25	16.25	17.25	18.25	19.25	20.50	22.25	23.25
2.70	1.16	2.58	3.55	4.55	5.50	6.38	7.75	8.88	9.88	11.19	12.50	13.75	14.75	16.00	17.25	18.00	18.94	20.75	21.75	23.00
2.80	1.16	2.63	3.50	4.56	5.38	6.50	7.83	8.67	10.25	11.38	12.17	13.38	14.75	16.00	16.88	17.75	19.25	20.75	21.50	23.50
2.90	1.18	2.63	3.55	4.40	5.33	6.50	7.88	9.17	10.75	11.75	12.50	13.63	14.75	15.63	16.50	18.00	19.50	21.00	22.75	24.00
3.00	1.20	2.63	3.50	4.40	5.50	6.50	8.08	9.50	10.75	11.75	12.63	13.75	14.63	15.63	17.25	18.75	20.00	21.25	22.25	23.75
3.10	1.25	2.63	3.50	4.50	5.50	6.75	8.25	9.58	10.75	11.88	12.88	13.88	14.75	16.50	17.75	19.00	19.88	21.00	22.13	24.00
3.20	1.22	2.58	3.50	4.58	5.75	7.00	8.50	9.75	10.75	12.00	12.75	13.88	15.50	17.25	18.25	19.00	20.00	21.75	22.75	24.00
3.30	1.18	2.57	3.63	4.50	5.88	7.25	8.50	9.75	10.75	11.75	12.83	14.25	16.25	17.25	18.25	19.75	20.75	21.75	22.75	23.75
3.40	1.19	2.65	3.60	4.81	6.00	7.25	8.25	9.50	10.50	11.58	13.00	15.13	16.25	17.00	18.50	19.50	20.25	21.25	22.25	23.50
3.50	1.17	2.58	3.50	4.67	6.00	7.25	8.38	9.33	10.25	11.38	13.75	15.13	16.33	17.25	18.25	19.25	20.13	21.00	22.13	23.25
3.60	1.18	2.63	3.63	4.44	6.00	7.13	8.33	9.17	10.25	11.17	14.33	15.38	16.25	17.17	18.08	19.25	20.50	21.38	22.25	23.25
3.70	1.19	2.50	3.50	4.50	6.08	7.17	8.33	9.13	10.13	12.50	14.50	15.50	16.38	17.33	18.17	19.15	20.50	22.00	22.75	23.50
3.80	1.18	2.50	3.63	4.75	6.25	7.13	8.38	9.38	11.00	13.33	14.50	15.50	16.50	17.25	18.50	19.50	21.00	22.25	23.25	24.00
3.90	1.18	2.50	3.75	4.75	6.25	7.13	8.40	9.38	12.00	13.38	14.50	15.75	16.50	17.63	19.00	20.00	21.25	22.25	23.25	24.50
4.00	1.18	2.46	4.00	4.92	6.25	7.38	8.42	9.38	12.00	13.42	14.75	16.00	16.75	18.25	19.25	20.25	21.50	22.50	23.75	25.25

Table 4-11 Average reduction factors (r) for systems with α=0.4 and ξ=15%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	0.81	1.41	1.59	1.78	1.91	2.06	2.10	2.31	2.44	2.46	2.63	2.75	2.85	2.96	3.09	3.13	3.19	3.25	3.44	3.44
0.20	1.40	2.30	2.80	3.35	3.75	4.13	4.50	4.80	5.00	5.25	5.50	5.50	5.75	6.00	6.31	6.56	6.75	6.88	7.13	7.38
0.30	1.54	2.63	3.50	4.00	4.25	4.75	5.38	5.75	6.00	6.25	6.50	6.92	7.25	8.00	8.50	8.75	9.00	9.38	9.75	10.25
0.40	1.65	2.88	3.63	4.21	4.88	5.38	6.00	6.50	7.30	7.75	8.25	8.88	9.50	10.00	10.31	10.69	11.17	11.63	12.00	12.55
0.50	1.95	3.25	4.15	5.00	5.50	6.13	6.88	7.75	8.50	9.25	10.00	10.50	11.50	12.06	12.58	13.25	13.75	14.25	14.69	15.63
0.60	1.68	2.94	3.81	4.56	5.42	6.58	7.38	7.92	8.42	9.00	9.88	10.50	11.25	11.83	12.58	13.13	13.81	14.33	14.92	15.50
0.70	1.71	2.92	3.88	5.00	5.75	6.63	7.60	8.25	9.00	10.00	10.88	11.50	12.25	12.88	13.38	14.25	14.83	15.38	15.75	16.13
0.80	1.63	2.80	3.68	4.58	5.69	6.63	7.50	8.17	9.00	9.75	10.50	11.50	11.88	12.50	13.00	13.38	14.08	14.75	15.38	16.00
0.90	1.65	2.80	3.85	4.75	5.65	6.75	7.58	8.25	9.25	10.08	10.63	11.20	12.00	12.67	13.25	14.25	14.75	15.25	15.92	16.50
1.00	1.50	2.64	3.85	4.75	5.67	6.75	7.69	8.35	8.94	9.75	10.58	11.50	12.00	12.75	13.75	14.25	14.75	15.25	15.75	16.38
1.10	1.58	2.83	4.00	4.75	5.75	6.75	7.58	8.56	9.44	9.94	10.67	11.25	11.94	12.67	13.25	14.50	15.13	15.75	16.50	17.25
1.20	1.67	2.96	4.13	4.83	6.05	7.00	8.13	8.90	9.67	10.58	11.33	12.00	13.25	14.38	14.92	15.75	16.75	17.50	18.17	19.00
1.30	1.75	2.96	3.95	5.00	6.00	7.17	7.92	9.00	9.75	10.75	11.50	12.75	13.75	14.38	15.00	16.00	16.92	17.75	18.75	19.50
1.40	1.70	3.11	4.06	5.00	6.38	7.17	8.33	9.25	10.50	11.25	12.25	13.13	13.83	14.50	15.50	16.50	17.63	18.50	19.63	20.50
1.50	1.61	3.04	4.11	5.13	6.17	7.17	8.25	9.38	10.25	11.50	12.35	13.13	13.67	14.63	15.75	17.00	18.25	19.00	20.50	22.00
1.60	1.53	3.04	3.96	5.00	6.00	7.08	8.00	9.25	10.31	11.00	11.75	12.50	13.00	14.25	15.50	16.83	18.25	18.75	19.75	20.25
1.70	1.47	2.85	3.88	4.88	5.81	6.63	7.75	8.88	9.75	10.42	11.13	12.00	13.50	14.50	15.25	16.00	16.75	17.75	19.00	20.00
1.80	1.47	2.78	3.75	4.65	5.42	6.56	7.50	8.83	9.50	10.00	10.75	11.42	12.63	14.00	14.63	15.75	17.00	18.25	19.25	20.00
1.90	1.40	2.60	3.72	4.65	5.50	6.33	7.75	8.38	8.88	9.50	10.50	11.67	13.00	13.88	15.00	15.75	16.75	17.75	18.75	20.00
2.00	1.39	2.63	3.75	4.67	5.50	6.38	7.25	8.13	8.92	10.00	11.50	12.42	13.50	14.50	15.50	16.50	17.63	19.25	20.00	20.75
2.10	1.34	2.71	3.71	5.00	5.69	6.50	7.25	8.25	9.58	10.63	11.75	13.25	14.50	15.50	16.50	17.50	18.58	19.50	20.50	21.75
2.20	1.34	2.69	4.00	5.13	5.88	6.63	7.42	8.75	9.75	11.25	12.38	13.75	14.75	15.75	17.00	18.08	19.00	19.75	20.75	21.75
2.30	1.34	2.69	4.00	5.00	5.88	6.63	7.88	9.00	10.25	11.56	12.50	13.75	14.75	15.88	17.25	18.25	19.13	20.13	21.25	22.00
2.40	1.33	2.61	4.00	4.88	6.00	6.75	8.25	9.00	10.25	11.25	12.25	13.50	14.75	16.00	17.50	18.50	19.75	21.00	22.00	22.00
2.50	1.28	2.69	3.88	4.88	5.92	7.00	7.88	9.00	9.92	11.13	12.50	14.00	15.50	16.75	17.75	18.50	19.75	21.25	22.50	23.75
2.60	1.32	2.80	3.80	5.00	6.08	6.88	8.00	8.88	10.00	11.50	13.25	14.75	16.00	17.00	17.75	18.75	19.63	21.50	22.63	23.75
2.70	1.33	2.81	3.80	4.83	5.88	6.67	7.70	8.88	10.25	12.00	13.25	14.25	15.50	16.50	17.63	18.75	19.75	21.50	22.50	23.50
2.80	1.33	2.83	3.79	4.83	5.75	6.50	7.75	8.88	10.63	11.88	12.88	14.00	15.13	16.25	17.17	18.25	19.50	21.00	21.75	22.75
2.90	1.34	2.79	3.63	4.63	5.58	6.42	7.75	9.50	10.67	11.75	12.75	14.00	15.13	16.00	17.00	18.25	19.25	20.25	21.75	23.75
3.00	1.35	2.80	3.63	4.58	5.58	6.58	8.25	9.75	11.00	11.88	13.00	14.13	15.00	16.00	17.13	18.13	19.75	21.00	23.50	24.00
3.10	1.36	2.83	3.65	4.50	5.75	6.75	8.50	9.75	11.00	12.25	13.13	14.25	15.00	16.25	17.13	19.25	21.00	22.25	23.50	24.00
3.20	1.38	2.82	3.67	4.63	5.88	7.25	8.50	9.88	11.25	12.25	13.13	14.25	15.25	16.38	18.50	19.75	20.75	22.00	23.25	24.75
3.30	1.39	2.75	3.64	4.81	6.13	7.50	8.75	9.75	11.13	12.08	13.25	14.38	15.25	17.25	18.50	19.50	20.63	22.25	24.00	25.00
3.40	1.35	2.75	3.67	4.75	6.00	7.50	8.50	9.75	10.83	11.75	13.25	14.75	15.88	17.50	18.25	19.50	21.50	22.50	23.50	24.00
3.50	1.34	2.75	3.69	5.00	6.17	7.50	8.58	9.63	10.67	11.88	13.25	14.75	16.25	17.25	18.75	20.00	21.00	22.00	23.00	24.00
3.60	1.33	2.70	3.75	5.00	6.25	7.38	8.50	9.60	10.58	12.00	13.75	15.50	16.50	17.75	18.75	19.75	20.75	22.00	23.00	24.00
3.70	1.38	2.67	3.75	5.00	6.55	7.50	8.50	9.42	10.75	12.25	14.50	15.58	16.75	17.75	18.75	19.88	20.88	22.00	23.25	24.25
3.80	1.38	2.70	3.75	5.13	6.58	7.50	8.63	9.50	10.75	13.25	14.67	15.81	16.75	17.75	18.75	20.25	21.25	22.25	23.25	24.50
3.90	1.35	2.58	3.67	5.50	6.58	7.50	8.63	9.75	11.13	13.63	15.00	16.00	16.88	17.88	18.92	20.25	21.75	23.00	23.75	25.00
4.00	1.29	2.63	3.75	5.56	6.58	7.75	8.67	10.00	11.75	13.75	15.00	16.13	17.00	18.00	19.50	21.00	22.25	23.25	24.38	25.50

Table 4-12 Average reduction factors (*r*) for systems with α=0.4 and ξ=20%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	0.95	1.44	1.75	1.88	2.00	2.20	2.31	2.46	2.65	2.71	2.94	3.04	3.13	3.21	3.25	3.40	3.44	3.56	3.67	3.75
0.20	1.50	2.54	3.13	3.63	4.00	4.58	4.83	5.10	5.38	5.50	5.75	6.08	6.31	6.50	6.58	6.83	7.08	7.25	7.38	7.63
0.30	1.72	3.00	3.81	4.25	4.75	5.25	5.67	6.00	6.25	6.75	7.13	7.38	7.75	8.17	9.00	9.25	9.75	10.00	10.38	11.00
0.40	1.88	3.13	3.81	4.50	5.13	5.75	6.50	7.00	7.80	8.31	9.00	9.60	10.13	10.63	11.08	11.56	12.00	12.45	12.81	13.25
0.50	2.15	3.75	4.58	5.25	6.00	6.75	7.38	8.42	9.50	10.13	10.75	11.17	12.00	12.69	13.33	13.92	14.44	15.08	15.58	16.25
0.60	1.96	3.20	4.25	4.88	5.69	7.00	7.75	8.50	9.06	9.75	10.56	11.13	11.92	12.63	13.25	14.00	14.58	15.33	15.88	16.50
0.70	1.92	3.25	4.25	5.25	6.25	7.08	8.05	8.75	9.75	10.50	11.38	12.13	13.00	13.60	14.13	14.75	15.50	16.25	16.75	17.25
0.80	1.80	3.11	4.08	5.00	5.94	7.25	8.00	8.67	9.44	10.17	11.00	11.75	12.50	13.25	13.75	14.33	14.83	15.25	15.88	16.50
0.90	1.81	3.08	4.19	5.10	6.08	7.13	8.25	8.94	9.75	10.79	11.38	12.00	12.50	13.08	14.00	14.75	15.25	15.75	16.50	17.13
1.00	1.69	3.05	4.17	4.94	6.10	7.00	8.00	8.81	9.38	10.25	11.00	12.00	13.00	13.50	14.13	14.75	15.38	16.25	17.50	18.00
1.10	1.66	3.07	4.25	5.25	6.25	7.19	8.00	8.88	9.81	10.58	11.13	11.75	13.00	13.75	14.50	15.00	15.50	16.75	17.50	18.00
1.20	1.86	3.21	4.38	5.25	6.50	7.42	8.50	9.50	10.13	10.88	11.75	12.50	13.50	14.63	15.58	16.58	17.25	18.25	19.13	19.63
1.30	1.82	3.21	4.42	5.36	6.25	7.63	8.50	9.33	10.25	11.17	11.88	13.50	14.38	14.88	15.50	17.00	17.50	18.42	19.50	20.38
1.40	1.86	3.30	4.33	5.44	6.75	7.63	8.63	9.67	11.00	11.75	12.88	13.81	14.50	15.25	16.25	17.13	18.00	19.00	20.00	21.00
1.50	1.79	3.35	4.42	5.45	6.56	7.55	8.63	10.00	10.88	12.13	12.88	13.67	14.50	15.38	16.50	17.50	18.75	19.75	20.50	22.00
1.60	1.72	3.29	4.18	5.25	6.21	7.38	8.50	9.63	10.81	11.50	12.38	13.00	13.75	15.25	16.25	17.38	18.50	20.25	21.00	21.50
1.70	1.61	3.11	4.10	5.13	6.13	7.00	8.00	9.38	10.25	10.83	11.67	12.75	14.00	15.08	16.13	17.25	18.00	18.50	19.25	20.00
1.80	1.58	2.94	4.08	4.93	5.81	6.67	7.88	9.25	9.88	10.38	11.00	12.25	13.50	14.50	15.25	16.13	17.25	18.25	19.50	20.00
1.90	1.56	2.79	4.13	4.92	5.67	6.50	7.50	9.00	9.50	10.25	10.94	12.00	13.50	14.50	15.25	16.50	17.75	18.75	19.75	20.75
2.00	1.56	2.86	4.00	5.00	5.75	6.67	8.25	8.75	9.50	10.33	11.67	12.88	14.00	15.00	16.00	16.88	17.92	19.50	20.75	21.63
2.10	1.50	2.82	4.06	5.25	5.92	6.75	7.88	8.63	9.75	11.00	12.33	13.42	14.50	15.50	16.75	18.25	19.50	20.25	21.00	21.92
2.20	1.46	2.84	3.96	5.13	6.13	7.08	7.75	9.00	10.13	11.42	12.83	14.00	15.25	16.25	18.00	18.75	19.75	20.63	21.50	22.75
2.30	1.46	2.84	4.00	5.25	6.17	6.94	8.00	9.25	10.50	11.83	13.00	14.50	15.50	17.00	18.00	19.00	20.00	20.88	22.00	22.88
2.40	1.44	2.86	4.13	5.13	6.25	7.00	8.38	9.50	10.88	12.00	13.00	14.25	15.75	17.00	18.13	19.25	20.25	21.50	22.50	23.50
2.50	1.44	2.88	4.13	5.25	6.13	7.25	8.50	9.75	10.88	11.67	13.00	14.63	15.75	17.25	18.25	19.25	20.75	22.00	23.00	24.00
2.60	1.43	2.96	4.10	5.25	6.25	7.25	8.50	9.75	10.75	12.00	13.75	15.00	16.25	17.25	18.25	19.17	21.25	22.25	23.25	24.25
2.70	1.43	2.95	4.00	5.10	6.25	7.00	8.38	9.38	10.50	12.50	13.75	15.00	16.00	17.00	18.00	19.00	20.75	22.00	23.13	24.13
2.80	1.43	2.89	4.00	5.08	6.00	6.92	8.13	9.38	11.25	12.38	13.38	14.50	15.63	16.63	17.75	18.88	20.25	21.50	22.50	23.50
2.90	1.39	2.88	3.88	5.00	5.88	7.00	8.13	10.00	11.25	12.25	13.25	14.25	15.38	16.50	17.50	18.75	20.25	21.00	21.88	22.92
3.00	1.46	2.94	3.85	4.88	5.88	6.92	8.25	10.00	11.00	12.25	13.25	14.33	15.50	16.50	17.63	18.75	19.63	20.63	21.88	23.75
3.10	1.50	2.90	3.83	4.71	6.00	7.25	8.75	10.13	11.33	12.25	13.25	14.50	15.50	16.75	17.75	18.75	19.75	21.63	23.50	24.00
3.20	1.55	2.88	3.83	4.75	6.00	7.75	8.92	10.25	11.50	12.58	13.50	14.50	15.63	16.75	17.75	19.00	21.00	23.00	24.00	24.00
3.30	1.50	2.92	3.75	4.75	6.00	7.75	8.88	10.13	11.38	12.50	13.50	14.58	15.50	16.50	18.25	20.25	21.50	22.50	23.75	25.25
3.40	1.47	2.88	3.81	4.88	6.13	7.63	8.83	10.13	11.13	12.25	13.38	14.75	15.50	17.00	18.88	19.75	21.00	22.13	23.75	25.50
3.50	1.47	2.80	3.83	4.95	6.13	7.50	8.83	9.88	11.00	11.88	14.00	14.75	15.75	17.50	18.63	19.63	21.00	23.00	24.00	25.00
3.60	1.46	2.83	3.88	5.00	6.50	7.50	8.75	9.88	10.83	12.00	14.00	15.00	16.50	17.50	18.63	20.25	21.75	22.75	24.00	25.00
3.70	1.46	2.83	4.00	5.33	6.75	7.58	8.75	9.75	11.00	12.50	14.00	15.50	16.75	17.75	19.50	20.50	21.75	22.75	23.75	25.00
3.80	1.43	2.75	4.00	5.38	6.75	7.63	8.75	9.83	11.25	12.88	14.50	16.00	16.88	18.25	19.50	20.50	21.50	22.75	23.75	25.00
3.90	1.50	2.71	3.92	5.58	6.83	7.75	9.00	10.25	11.50	13.50	15.00	16.17	17.25	18.25	19.50	20.50	21.75	23.00	24.25	25.50
4.00	1.54	2.88	3.88	5.85	6.88	7.88	9.00	10.50	11.75	14.00	15.31	16.50	17.50	18.58	19.75	21.00	22.33	23.50	24.75	26.00

Table 4-13 Average reduction factors (*r*) for systems with α=0.4 and ξ=25%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	0.98	1.55	1.78	1.97	2.25	2.36	2.50	2.79	2.88	3.04	3.10	3.25	3.31	3.44	3.44	3.58	3.69	3.75	3.85	4.00
0.20	1.71	2.75	3.25	3.85	4.25	4.88	5.15	5.44	5.69	6.00	6.19	6.40	6.65	6.83	7.06	7.19	7.44	7.67	7.92	8.00
0.30	1.94	3.25	4.06	4.50	5.13	5.63	5.95	6.44	6.70	7.13	7.42	7.92	8.25	9.00	9.25	9.75	10.25	10.75	11.00	11.38
0.40	2.17	3.38	4.17	4.90	5.58	6.25	7.00	7.50	8.13	8.83	9.42	10.08	10.83	11.25	11.75	12.21	12.75	13.20	13.63	14.06
0.50	2.31	4.00	5.06	5.50	6.25	7.25	8.08	9.08	10.13	10.75	11.38	12.00	12.67	13.50	14.00	14.81	15.31	15.88	16.38	16.88
0.60	2.18	3.63	4.67	5.25	6.08	7.25	8.38	9.13	9.75	10.33	11.08	11.75	12.63	13.25	14.00	14.50	15.50	16.25	16.83	17.38
0.70	2.15	3.63	4.50	5.67	6.75	7.56	8.42	9.38	10.25	11.00	11.88	12.69	13.63	14.33	14.75	15.42	16.25	17.00	17.63	18.33
0.80	1.93	3.31	4.42	5.38	6.35	7.58	8.50	9.13	10.00	10.81	11.50	12.42	13.25	13.83	14.50	15.10	15.63	16.13	16.69	17.25
0.90	1.93	3.33	4.44	5.44	6.55	7.63	8.75	9.44	10.25	11.13	11.85	12.58	13.25	13.75	14.42	15.38	16.00	16.63	17.00	17.63
1.00	1.89	3.30	4.38	5.38	6.42	7.33	8.56	9.25	9.94	11.00	11.50	12.50	13.33	14.13	14.75	15.38	15.88	16.42	17.50	18.00
1.10	1.89	3.32	4.50	5.63	6.63	7.50	8.44	9.33	10.08	11.08	11.83	12.25	13.00	13.75	15.00	15.75	17.25	18.00	18.88	19.58
1.20	1.96	3.50	4.75	5.63	6.83	7.75	8.88	9.88	10.63	11.31	12.08	12.92	13.88	14.75	15.75	16.75	18.00	19.00	19.58	20.13
1.30	1.94	3.54	4.63	5.69	6.75	8.00	8.88	9.67	10.56	11.42	12.42	13.75	14.88	16.00	17.00	17.25	18.13	19.08	20.00	21.13
1.40	1.97	3.46	4.67	5.70	7.00	8.08	9.08	9.94	11.25	12.38	13.50	14.38	15.00	16.25	16.75	17.83	18.50	19.50	20.75	21.75
1.50	1.96	3.57	4.67	5.85	6.92	7.88	9.13	10.50	11.75	12.63	13.50	14.25	15.00	16.00	16.88	17.75	19.00	20.50	21.25	22.13
1.60	1.91	3.50	4.50	5.46	6.50	7.63	8.88	10.08	11.25	12.00	12.75	13.63	14.38	15.50	16.50	17.63	19.00	20.00	22.00	22.00
1.70	1.80	3.25	4.32	5.38	6.42	7.75	8.67	10.00	10.63	11.50	12.25	12.92	14.25	15.25	16.38	17.63	19.00	19.75	20.38	20.92
1.80	1.67	3.08	4.25	5.25	6.25	7.08	8.25	9.50	10.33	10.88	11.63	12.63	14.75	15.50	16.25	16.88	17.50	18.50	19.75	20.00
1.90	1.67	3.05	4.38	5.17	5.96	6.88	8.50	9.38	10.00	10.63	11.38	12.75	14.00	15.25	15.75	16.75	18.00	19.25	20.50	21.75
2.00	1.66	3.07	4.38	5.25	6.08	7.25	8.75	9.25	10.00	10.70	11.88	13.25	14.50	15.38	16.50	17.75	19.00	20.00	21.13	22.50
2.10	1.64	3.03	4.38	5.25	6.25	7.00	8.75	9.25	10.00	11.17	12.50	14.00	14.92	16.00	17.00	18.13	20.25	21.38	22.25	23.00
2.20	1.64	2.94	4.17	5.25	6.25	7.50	8.50	9.17	10.33	11.75	13.25	14.33	15.25	16.50	18.00	19.63	20.63	21.50	22.25	23.25
2.30	1.63	2.96	4.18	5.50	6.63	7.38	8.38	9.44	10.69	12.17	13.33	14.50	15.75	18.00	18.75	19.75	20.75	21.75	22.75	23.75
2.40	1.58	3.05	4.25	5.50	6.50	7.25	8.58	9.83	11.13	12.38	13.50	14.75	16.75	17.75	18.83	19.75	20.75	21.88	22.88	24.00
2.50	1.58	3.10	4.25	5.56	6.42	7.25	8.63	9.92	11.38	12.50	13.63	15.00	16.50	17.63	18.75	19.88	21.00	22.25	23.50	24.00
2.60	1.61	3.13	4.25	5.56	6.38	7.50	8.75	10.30	11.25	12.25	14.00	15.50	16.63	17.83	18.92	20.25	21.50	22.75	23.75	24.75
2.70	1.57	3.13	4.17	5.44	6.38	7.25	8.83	9.88	11.00	12.75	14.13	15.38	16.50	17.63	18.50	20.00	21.50	22.50	23.63	25.00
2.80	1.56	3.07	4.11	5.25	6.25	7.17	8.50	9.63	11.00	12.63	14.00	15.00	16.08	17.25	18.33	19.50	20.88	21.88	23.13	24.13
2.90	1.53	3.04	4.06	5.19	6.25	7.31	8.25	9.50	11.25	12.63	13.75	14.75	15.92	17.00	18.13	19.17	20.75	21.50	22.50	23.63
3.00	1.56	3.00	4.05	5.13	6.25	7.38	8.42	10.25	11.50	12.50	13.75	14.75	15.75	17.00	18.00	19.50	20.50	21.38	22.42	23.50
3.10	1.59	2.95	4.00	5.00	6.25	7.50	9.00	10.38	11.50	12.75	13.50	15.00	16.00	17.25	18.50	19.38	20.38	21.25	22.75	23.75
3.20	1.64	2.96	3.93	5.00	6.25	7.75	9.25	10.58	11.63	12.75	13.63	15.00	16.25	17.38	18.25	19.25	20.50	22.00	23.50	24.00
3.30	1.63	2.93	3.96	5.17	6.25	8.00	9.25	10.50	11.63	12.75	14.00	14.88	16.00	17.25	18.25	19.50	21.00	22.75	24.00	24.00
3.40	1.63	2.90	3.92	5.10	6.38	7.75	9.13	10.38	11.50	12.50	13.75	14.75	15.75	16.75	18.50	20.00	22.00	23.00	24.25	25.75
3.50	1.55	2.83	3.95	5.13	6.75	7.75	9.00	10.25	11.25	12.25	14.00	14.75	15.63	17.00	18.75	20.25	21.50	22.50	24.00	25.38
3.60	1.58	2.85	3.95	5.33	6.75	7.75	9.00	10.13	11.25	12.38	14.25	15.00	16.00	17.75	19.25	20.25	21.25	22.75	24.75	25.75
3.70	1.61	2.89	4.05	5.38	6.75	7.75	9.00	10.13	11.17	12.75	14.50	15.25	16.75	18.00	19.00	20.13	22.00	23.50	24.75	25.75
3.80	1.58	2.89	4.13	5.58	6.75	7.88	9.00	9.94	11.13	13.25	14.50	15.83	17.00	18.00	19.25	21.25	22.25	23.50	24.50	25.75
3.90	1.61	2.88	4.13	5.75	6.75	7.92	8.92	10.13	11.50	13.25	14.63	16.00	17.25	18.50	20.25	21.25	22.50	23.50	24.50	25.75
4.00	1.63	2.90	4.17	6.00	6.92	8.08	9.25	10.50	12.00	13.75	15.50	16.63	17.75	19.00	20.25	21.50	22.50	23.75	24.88	26.50

Table 4-14 Average reduction factors (r) for systems with α=0.4 and ξ=30%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.00	1.61	1.91	2.14	2.46	2.68	2.81	3.00	3.08	3.25	3.25	3.50	3.50	3.58	3.67	3.80	3.88	4.05	4.06	4.29
0.20	1.89	2.95	3.46	4.13	4.67	5.13	5.44	5.81	6.13	6.25	6.50	6.69	6.95	7.20	7.44	7.69	7.92	8.13	8.38	8.63
0.30	2.13	3.38	4.31	4.88	5.42	5.92	6.25	6.70	7.19	7.46	7.95	8.50	9.00	9.50	9.75	10.25	10.88	11.25	11.58	12.08
0.40	2.31	3.71	4.44	5.31	6.00	6.75	7.50	8.00	8.63	9.38	10.00	10.67	11.33	11.94	12.54	13.00	13.40	13.94	14.33	14.81
0.50	2.56	4.31	5.35	6.00	6.75	7.75	8.63	9.63	10.63	11.50	12.13	12.75	13.50	14.08	14.75	15.25	16.00	16.75	17.38	17.88
0.60	2.35	3.95	5.00	5.63	6.33	7.75	8.75	9.55	10.33	11.06	11.69	12.50	13.25	14.00	14.50	15.25	16.25	17.00	17.63	18.25
0.70	2.33	4.00	5.00	5.94	7.17	8.08	9.06	10.00	10.70	11.63	12.50	13.25	14.25	15.00	15.50	16.00	16.75	18.00	18.75	19.25
0.80	2.13	3.58	4.75	5.92	6.83	8.00	8.88	9.63	10.50	11.35	12.17	12.94	13.75	14.50	15.00	15.67	16.38	17.06	17.63	18.25
0.90	2.05	3.50	4.67	5.82	6.88	8.00	9.08	9.88	10.92	11.75	12.50	13.08	13.83	14.50	15.00	15.88	16.75	17.50	17.75	18.42
1.00	2.00	3.50	4.58	5.63	6.75	7.75	8.88	9.82	10.65	11.50	12.13	12.75	13.75	14.56	15.00	15.75	16.25	17.00	17.50	18.25
1.10	2.00	3.55	4.75	5.88	6.88	8.00	8.92	9.75	10.50	11.50	12.50	13.17	14.00	14.75	15.50	16.25	17.25	18.00	18.50	20.00
1.20	2.04	3.58	4.88	6.06	7.25	8.25	9.19	10.25	11.25	12.00	12.58	13.38	14.50	15.75	16.38	17.25	18.75	19.17	20.13	21.13
1.30	2.08	3.75	4.83	5.94	7.15	8.25	9.38	10.25	10.92	11.75	12.75	14.00	15.00	16.25	17.25	18.00	18.75	19.75	20.50	21.50
1.40	2.09	3.66	5.00	6.15	7.25	8.42	9.38	10.17	11.25	12.50	14.00	15.00	15.75	16.50	17.25	18.25	19.50	20.38	21.38	22.50
1.50	2.13	3.75	4.94	6.19	7.25	8.25	9.19	10.75	12.00	13.30	14.00	15.00	16.00	16.50	17.50	18.25	19.38	20.75	22.00	23.00
1.60	2.00	3.60	4.63	5.75	6.81	8.00	9.25	10.75	11.63	12.50	13.50	14.25	15.08	15.88	16.75	18.00	19.25	20.50	21.75	23.25
1.70	1.94	3.44	4.57	5.65	6.67	7.69	9.13	10.25	11.13	12.00	12.75	13.50	14.50	15.50	16.63	18.00	19.25	20.75	22.00	22.00
1.80	1.85	3.25	4.55	5.50	6.50	7.75	8.50	10.00	10.63	11.25	12.25	13.00	14.75	16.00	17.00	18.00	18.75	19.25	20.13	21.25
1.90	1.80	3.18	4.67	5.44	6.38	7.38	8.75	9.75	10.50	11.00	11.75	14.00	14.50	15.75	16.50	17.13	18.25	19.25	20.50	21.75
2.00	1.81	3.21	4.75	5.56	6.42	7.38	9.00	9.88	10.38	11.13	12.50	13.75	15.25	16.00	16.88	18.00	19.25	20.75	22.00	23.25
2.10	1.79	3.14	4.63	5.60	6.44	7.75	9.00	9.75	10.38	11.58	13.00	14.50	15.25	16.25	17.75	19.00	20.00	21.25	22.88	24.00
2.20	1.73	3.18	4.50	5.67	6.63	7.75	9.00	9.75	10.50	12.00	13.50	14.63	15.75	17.00	18.13	20.00	21.50	22.50	23.50	24.00
2.30	1.73	3.19	4.42	5.75	7.00	7.75	9.00	9.65	10.92	12.50	13.75	14.88	16.17	17.75	19.38	20.50	21.75	22.75	23.75	24.00
2.40	1.71	3.15	4.42	5.80	6.88	7.75	8.88	9.92	11.42	12.75	14.00	15.17	16.50	18.50	19.50	20.75	21.75	22.75	23.75	25.00
2.50	1.71	3.19	4.50	5.83	6.75	7.67	9.00	10.19	11.67	12.92	14.13	15.38	17.25	18.50	19.50	20.50	21.75	22.75	23.75	24.88
2.60	1.68	3.20	4.50	5.75	6.69	7.63	9.00	10.50	11.83	13.00	14.17	15.75	17.50	18.38	19.50	20.50	21.75	22.88	24.00	25.25
2.70	1.65	3.18	4.41	5.69	6.58	7.50	9.00	10.50	11.42	12.75	14.38	16.00	17.00	18.13	19.33	20.63	21.75	23.00	24.13	25.25
2.80	1.64	3.17	4.30	5.44	6.42	7.38	8.75	10.00	11.25	12.88	14.25	15.50	16.58	17.75	19.00	20.25	21.38	22.50	23.75	25.00
2.90	1.61	3.14	4.21	5.38	6.25	7.42	8.75	9.88	11.50	12.88	14.13	15.17	16.38	17.50	18.63	20.00	21.13	22.13	23.25	24.25
3.00	1.63	3.10	4.20	5.50	6.50	7.56	8.75	10.50	11.75	13.00	14.00	15.25	16.50	17.50	18.63	20.00	21.00	22.00	23.08	24.00
3.10	1.65	3.13	4.21	5.50	6.38	7.63	8.88	10.75	12.00	13.00	14.00	15.25	16.50	17.63	19.00	20.00	21.13	22.00	23.08	24.25
3.20	1.65	3.05	4.21	5.50	6.50	7.75	9.25	10.75	12.00	13.00	14.00	15.50	16.63	17.88	19.00	20.00	21.00	22.13	23.38	24.50
3.30	1.67	3.05	4.19	5.33	6.50	8.13	9.42	10.83	11.88	12.75	14.25	15.50	16.63	17.75	18.75	19.75	20.92	22.25	23.25	25.25
3.40	1.71	2.93	4.13	5.38	6.63	8.25	9.50	10.75	11.75	12.75	14.08	15.25	16.25	17.50	18.50	19.75	21.25	22.38	24.50	26.00
3.50	1.71	2.92	4.00	5.25	7.00	8.00	9.38	10.50	11.50	12.75	13.88	15.00	16.00	17.00	18.50	20.00	21.25	23.50	24.75	26.00
3.60	1.69	2.92	4.00	5.50	7.00	7.92	9.25	10.38	11.50	12.67	14.25	15.00	15.88	17.50	19.00	20.25	22.25	23.25	24.50	26.00
3.70	1.67	2.94	4.06	5.63	7.00	8.00	9.25	10.38	11.50	13.00	14.50	15.25	16.25	18.00	19.38	21.00	22.00	23.00	24.75	26.50
3.80	1.67	3.00	4.15	5.81	7.00	8.00	9.25	10.25	11.38	13.50	14.75	15.50	17.25	18.25	19.75	20.75	22.13	24.00	25.50	26.50
3.90	1.64	2.96	4.35	6.00	7.00	8.13	9.25	10.25	11.44	13.75	15.25	16.13	17.50	18.75	19.75	21.25	23.25	24.25	25.50	26.75
4.00	1.68	3.00	4.38	6.00	7.00	8.25	9.25	10.42	11.88	14.00	15.25	16.50	17.75	18.92	20.50	22.00	23.25	24.50	25.50	26.88

Table 4-15 Average reduction factors (r) for systems with α=0.4 and ξ=35%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.12	1.69	2.05	2.36	2.67	2.88	3.00	3.17	3.25	3.50	3.50	3.58	3.75	3.81	4.00	4.05	4.08	4.31	4.31	4.54
0.20	2.11	3.18	3.75	4.50	4.92	5.33	5.83	6.17	6.50	6.63	6.88	7.08	7.33	7.50	7.75	8.00	8.25	8.63	8.88	9.00
0.30	2.33	3.50	4.58	5.25	5.75	6.25	6.65	7.08	7.67	8.00	8.75	9.25	9.75	10.00	10.50	11.00	11.50	12.00	12.33	12.63
0.40	2.56	4.00	4.88	5.63	6.25	7.00	8.00	8.50	9.25	10.00	10.63	11.25	11.92	12.58	13.15	13.63	14.06	14.58	15.08	15.58
0.50	2.75	4.50	5.80	6.58	7.25	8.00	9.17	10.25	11.25	12.08	12.75	13.35	14.08	14.75	15.38	16.00	16.50	17.25	18.00	18.50
0.60	2.55	4.19	5.42	6.00	6.75	8.13	9.25	10.00	10.75	11.63	12.44	13.10	13.83	14.50	15.25	16.00	16.75	17.50	18.25	19.00
0.70	2.50	4.19	5.25	6.38	7.56	8.60	9.58	10.50	11.33	12.17	13.13	14.13	15.00	15.63	16.50	17.00	17.50	18.50	19.25	19.75
0.80	2.23	3.83	5.08	6.30	7.25	8.25	9.25	10.08	11.00	11.88	12.83	13.58	14.25	15.13	15.75	16.50	17.00	17.75	18.38	19.00
0.90	2.21	3.63	5.00	6.17	7.18	8.19	9.33	10.25	11.33	12.25	13.00	13.75	14.38	15.13	15.75	16.50	17.13	18.00	19.00	19.50
1.00	2.17	3.67	4.75	6.05	7.17	8.31	9.38	10.30	11.08	11.88	12.50	13.17	14.13	15.06	15.75	16.33	16.75	17.50	18.00	18.63
1.10	2.13	3.67	4.88	6.13	7.13	8.25	9.40	10.33	11.13	12.00	12.92	13.75	14.50	15.25	16.00	16.50	17.13	17.75	18.50	19.38
1.20	2.25	3.81	5.08	6.38	7.50	8.67	9.75	10.50	11.75	12.75	13.38	14.00	15.00	16.25	17.25	17.75	18.63	20.00	21.25	22.00
1.30	2.19	3.85	5.00	6.19	7.50	8.69	9.83	10.75	11.50	12.13	13.10	14.25	15.25	16.25	17.25	18.25	19.25	20.50	21.50	22.13
1.40	2.25	3.90	5.25	6.50	7.67	8.69	9.69	10.63	11.42	12.88	14.00	15.00	16.50	17.50	18.25	18.75	19.69	20.88	22.08	23.25
1.50	2.32	3.95	5.25	6.50	7.50	8.58	9.63	10.63	12.25	13.50	14.75	15.58	16.50	17.25	18.00	19.25	20.00	21.13	22.50	23.75
1.60	2.19	3.75	4.89	6.13	7.19	8.13	9.42	10.92	12.17	13.00	14.00	15.00	15.50	16.50	17.25	18.25	19.50	21.00	22.25	23.50
1.70	2.06	3.56	4.83	5.85	6.90	8.08	9.50	10.75	11.50	12.50	13.25	14.00	14.83	15.75	16.88	18.25	19.75	21.00	22.00	22.00
1.80	1.97	3.43	4.75	5.70	6.75	8.25	9.13	10.25	11.13	11.88	12.63	13.50	14.75	16.00	17.25	18.50	19.75	20.75	21.75	22.00
1.90	1.91	3.39	4.79	5.65	6.75	8.00	9.00	10.00	11.00	11.75	12.50	14.50	15.75	16.50	17.38	18.00	18.75	19.50	20.75	22.00
2.00	1.94	3.40	5.00	5.75	6.75	8.00	9.25	10.25	11.00	11.75	13.00	14.50	15.50	16.75	17.25	18.25	19.50	20.75	22.00	22.00
2.10	1.91	3.39	5.00	5.75	6.88	8.13	9.38	10.13	10.88	11.75	13.25	14.50	16.00	16.75	17.75	19.25	20.50	22.00	23.25	24.25
2.20	1.88	3.39	4.75	6.00	7.00	8.25	9.38	10.13	10.94	12.25	14.00	15.25	16.00	17.25	18.50	20.00	21.38	22.75	24.25	25.50
2.30	1.89	3.38	4.75	6.05	7.25	8.25	9.25	10.00	11.17	12.75	14.25	15.25	16.50	17.88	19.25	20.75	22.25	23.50	24.50	25.50
2.40	1.88	3.25	4.67	6.00	7.00	8.00	9.00	10.08	11.50	13.25	14.38	15.63	17.08	18.50	20.08	21.25	22.50	23.75	24.75	25.75
2.50	1.80	3.22	4.65	6.06	7.13	8.06	9.25	10.50	12.00	13.25	14.50	16.00	17.50	19.08	20.25	21.50	22.50	23.75	24.75	25.75
2.60	1.79	3.31	4.69	6.13	7.00	7.90	9.25	10.83	12.25	13.38	14.75	16.50	18.00	19.25	20.50	21.50	22.50	23.75	24.75	25.75
2.70	1.75	3.35	4.60	5.88	6.83	7.75	9.15	10.75	11.92	13.33	14.69	16.25	17.50	18.75	20.00	21.25	22.25	23.38	24.50	25.67
2.80	1.69	3.21	4.42	5.69	6.63	7.50	9.08	10.50	11.63	13.13	14.50	16.00	16.94	18.17	19.50	20.63	21.88	23.00	24.25	25.50
2.90	1.69	3.20	4.44	5.63	6.50	7.63	8.75	10.25	11.50	13.13	14.50	15.63	16.75	17.92	19.25	20.42	21.50	22.75	23.75	25.00
3.00	1.69	3.19	4.40	5.50	6.50	7.67	9.00	10.13	11.75	13.25	14.25	15.50	16.75	18.00	19.38	20.38	21.42	22.63	23.63	24.83
3.10	1.72	3.19	4.40	5.75	6.50	7.83	9.00	10.75	12.13	13.25	14.38	15.75	17.00	18.25	19.50	20.63	21.63	22.63	23.75	24.75
3.20	1.75	3.19	4.38	5.75	6.63	8.00	9.50	11.00	12.25	13.25	14.42	16.00	17.13	18.50	19.63	20.63	21.63	22.75	23.75	25.25
3.30	1.81	3.13	4.35	5.50	6.75	8.13	9.75	11.00	12.25	13.25	14.50	15.88	17.13	18.25	19.25	20.50	21.50	22.63	23.75	25.00
3.40	1.78	3.10	4.25	5.42	6.63	8.38	9.63	11.00	12.00	13.00	14.58	15.63	16.75	18.00	19.00	20.00	21.42	22.63	23.63	24.88
3.50	1.75	2.96	4.19	5.42	7.00	8.25	9.67	10.88	11.88	13.00	14.38	15.50	16.50	17.50	18.75	20.00	21.25	22.25	24.00	26.00
3.60	1.72	2.94	4.14	5.75	7.25	8.25	9.50	10.63	11.75	13.17	14.17	15.25	16.50	17.50	19.00	20.25	21.25	23.00	25.25	26.00
3.70	1.72	2.97	4.10	5.75	7.25	8.25	9.50	10.63	11.75	13.00	14.50	15.38	16.25	18.00	19.25	20.50	22.25	24.00	25.25	26.00
3.80	1.72	3.00	4.13	5.88	7.00	8.38	9.50	10.50	11.75	13.25	14.75	15.50	16.75	18.50	19.50	21.25	22.75	23.75	25.25	26.75
3.90	1.70	3.08	4.38	6.00	7.13	8.50	9.50	10.63	11.69	13.75	15.00	16.00	17.50	18.75	20.25	21.75	22.75	24.13	26.00	27.50
4.00	1.75	3.13	4.50	6.25	7.25	8.50	9.50	10.75	12.00	14.25	15.50	16.75	18.00	19.25	20.75	21.75	23.50	25.25	26.50	27.75

Table 4-16 Average reduction factors (r) for systems with α=0.6 and ξ=5%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.00	1.38	1.54	1.67	1.75	1.75	1.88	2.00	2.00	2.08	2.21	2.31	2.33	2.50	2.58	2.68	2.68	2.84	2.88	2.92
0.20	0.98	1.75	2.33	2.69	3.06	3.33	3.58	3.75	4.13	4.25	4.50	4.67	4.92	5.25	5.25	5.46	5.75	6.00	6.38	6.75
0.30	0.99	2.09	2.69	3.17	3.58	4.25	4.83	5.13	5.42	5.63	6.00	6.38	6.75	7.08	7.33	7.81	8.30	8.63	9.00	9.25
0.40	1.13	2.13	2.95	3.63	4.19	4.63	5.25	5.75	6.25	6.69	7.50	7.75	8.25	8.75	9.13	9.56	10.00	10.50	11.06	11.55
0.50	1.20	2.46	3.38	4.06	4.58	5.35	6.25	7.00	7.75	8.25	8.88	9.50	10.08	10.50	11.25	11.83	12.25	13.13	13.50	14.00
0.60	1.19	2.33	3.19	3.93	4.88	5.75	6.38	6.75	7.50	8.25	8.83	9.25	9.92	10.58	11.00	11.67	12.50	13.13	13.75	14.17
0.70	1.25	2.30	3.25	4.35	5.13	6.00	6.75	7.50	8.25	9.00	9.67	10.25	11.00	11.50	12.17	12.63	13.13	13.67	14.25	15.33
0.80	1.17	2.32	3.11	4.00	5.00	6.05	6.75	7.42	8.13	8.75	9.25	10.33	10.92	11.50	12.25	12.75	13.25	13.75	14.25	14.75
0.90	1.17	2.25	3.31	4.15	5.08	6.00	7.25	7.83	8.58	9.25	9.75	10.58	11.13	11.75	12.42	13.50	14.13	14.63	15.17	15.75
1.00	1.13	2.21	3.50	4.05	5.00	6.00	6.80	7.50	8.25	9.25	9.88	10.50	11.08	11.69	12.25	13.00	13.88	14.50	15.25	15.63
1.10	1.11	2.29	3.38	4.13	5.10	6.06	6.75	7.75	8.75	9.25	10.25	10.75	11.50	12.42	13.25	14.00	14.75	15.38	16.08	16.75
1.20	1.13	2.45	3.50	4.20	5.38	6.25	7.13	8.25	9.00	10.00	10.75	11.50	12.38	13.25	14.00	14.63	15.75	16.25	17.25	18.00
1.30	1.13	2.47	3.43	4.33	5.38	6.33	7.25	8.13	9.00	9.75	10.50	11.83	12.50	13.50	14.00	15.00	15.88	16.67	17.63	18.50
1.40	1.11	2.50	3.55	4.54	5.50	6.42	7.58	8.58	9.38	10.38	11.33	12.00	12.75	13.50	14.75	15.50	16.75	17.50	18.25	19.75
1.50	1.10	2.50	3.44	4.67	5.65	6.58	7.88	8.75	10.00	10.63	11.25	11.88	13.00	14.00	15.50	16.00	17.25	17.75	19.00	19.88
1.60	1.14	2.56	3.55	4.63	5.33	6.25	7.75	8.75	9.38	10.25	10.88	11.50	13.00	13.75	14.38	15.25	16.00	17.75	18.00	18.00
1.70	1.13	2.38	3.58	4.25	5.00	5.75	7.13	8.25	8.92	9.38	10.25	10.88	11.88	13.25	13.75	15.50	16.50	17.25	18.25	18.75
1.80	1.13	2.38	3.35	4.25	4.92	6.00	6.75	7.75	8.00	9.00	10.00	10.50	12.00	13.00	13.88	14.83	15.58	16.50	17.25	18.50
1.90	1.09	2.29	3.25	4.17	4.92	5.83	6.42	7.25	8.25	9.25	9.88	11.00	12.00	12.75	13.38	14.25	15.13	17.25	18.00	18.00
2.00	1.08	2.35	3.50	4.50	5.17	5.75	6.56	7.63	8.63	9.75	10.50	11.38	12.50	13.25	13.75	15.75	16.75	18.00	18.75	19.75
2.10	1.04	2.39	3.42	4.50	5.15	5.94	7.25	8.13	8.75	9.83	10.63	11.63	12.63	13.38	14.50	15.50	16.75	17.75	18.75	20.00
2.20	1.04	2.41	3.38	4.33	5.17	6.25	7.38	7.92	8.67	9.75	10.63	11.63	12.63	13.75	14.75	15.75	17.25	18.50	19.50	20.63
2.30	1.00	2.36	3.35	4.25	5.33	6.42	7.13	7.75	9.00	9.75	10.63	11.75	12.63	14.00	15.50	16.75	17.75	18.88	20.00	21.75
2.40	0.97	2.28	3.21	4.38	5.25	6.25	7.00	7.88	9.00	10.00	11.00	12.00	13.75	15.50	16.50	17.25	18.25	19.25	20.50	22.13
2.50	0.98	2.29	3.25	4.42	5.50	6.25	7.08	8.25	9.13	10.13	11.63	13.50	15.25	16.00	17.00	17.75	18.75	19.50	20.88	22.25
2.60	0.98	2.33	3.35	4.42	5.50	6.20	7.38	8.50	9.50	10.88	12.50	13.75	14.75	15.75	16.88	18.25	19.00	20.00	21.88	23.00
2.70	1.06	2.38	3.31	4.40	5.30	6.00	7.75	8.50	9.88	11.42	12.38	13.25	15.00	16.00	17.00	18.00	19.13	20.75	21.88	23.50
2.80	1.08	2.40	3.31	4.25	5.13	6.00	7.75	8.88	10.50	11.50	12.50	13.50	14.50	15.75	16.75	18.13	19.50	21.50	22.50	23.75
2.90	1.08	2.45	3.25	4.25	5.00	6.50	7.92	9.25	10.75	11.50	12.75	13.50	14.63	15.58	17.25	18.75	19.75	20.75	21.75	22.00
3.00	1.06	2.45	3.31	4.31	5.13	6.75	8.25	9.42	10.63	11.50	12.63	13.63	14.50	16.00	17.63	18.50	19.50	20.50	22.00	23.25
3.10	1.07	2.50	3.38	4.25	5.50	6.75	8.50	9.75	10.75	11.75	12.83	13.75	15.00	16.75	17.50	18.50	20.00	21.00	22.00	23.00
3.20	1.08	2.53	3.38	4.50	5.83	7.00	8.50	9.75	10.83	11.75	12.88	14.25	16.00	16.75	18.25	19.25	20.00	21.00	22.00	23.25
3.30	1.08	2.50	3.40	4.75	5.75	7.25	8.25	9.75	10.75	11.75	13.25	15.13	16.13	17.50	18.25	19.25	20.00	21.00	22.13	23.00
3.40	1.11	2.39	3.38	4.56	5.75	7.25	8.13	9.50	10.25	11.38	13.88	15.25	16.17	17.13	17.88	19.00	20.08	21.08	22.00	23.00
3.50	1.07	2.39	3.38	4.44	5.75	7.25	7.92	9.25	10.00	12.25	14.25	15.25	16.13	16.88	17.88	18.75	19.92	21.25	22.00	22.88
3.60	1.04	2.42	3.38	4.25	5.75	6.75	7.88	9.25	10.75	12.75	14.25	15.33	16.13	17.13	17.92	18.88	20.50	22.00	22.75	23.50
3.70	1.07	2.42	3.50	4.50	5.88	6.67	8.25	9.38	11.25	13.00	14.33	15.25	16.25	17.17	18.50	19.25	21.00	22.00	23.00	24.00
3.80	1.10	2.35	3.50	4.58	5.83	6.88	8.00	9.75	11.75	13.13	14.25	15.50	16.25	17.50	19.00	19.75	21.00	22.00	22.75	24.00
3.90	1.13	2.42	3.75	4.75	5.88	7.13	8.25	10.50	12.00	13.25	14.50	15.50	16.63	18.00	19.00	20.00	21.25	22.25	23.25	24.75
4.00	1.10	2.33	3.83	4.83	6.10	7.50	8.50	10.25	12.00	13.25	14.75	15.75	17.00	18.00	19.00	20.00	22.00	22.75	24.25	25.63

Table 4-17 Average reduction factors (r) for systems with α=0.6 and ξ=10%

Fundamental	μ_T																			
Period (T_{el})	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.00	1.43	1.55	1.67	1.75	1.89	2.04	2.15	2.17	2.30	2.45	2.55	2.71	2.81	2.84	2.93	3.04	3.04	3.17	3.25
0.20	1.16	2.13	2.64	3.17	3.50	3.80	4.06	4.31	4.63	4.88	5.00	5.25	5.50	5.88	6.13	6.38	6.63	6.88	7.25	7.50
0.30	1.29	2.38	3.13	3.63	4.00	4.75	5.17	5.50	5.75	6.00	6.42	7.00	7.25	7.75	8.00	8.50	8.94	9.50	9.83	10.25
0.40	1.38	2.46	3.35	4.05	4.60	5.00	5.75	6.25	6.92	7.50	8.25	8.75	9.25	9.63	10.10	10.63	11.00	11.44	12.00	12.50
0.50	1.58	2.88	3.70	4.38	5.13	5.92	6.83	7.58	8.25	9.00	9.75	10.50	11.13	11.75	12.38	12.88	13.42	14.00	14.75	15.38
0.60	1.44	2.58	3.39	4.33	5.42	6.25	7.00	7.63	8.25	8.88	9.75	10.25	11.08	11.63	12.25	13.00	13.56	14.25	14.75	15.42
0.70	1.43	2.56	3.75	4.67	5.63	6.56	7.42	8.13	9.25	10.00	10.63	11.33	12.00	12.63	13.25	13.75	14.25	14.75	15.50	16.25
0.80	1.45	2.63	3.42	4.38	5.50	6.58	7.33	8.08	8.88	9.38	10.50	11.13	11.75	12.17	12.90	13.58	14.25	14.75	15.13	16.00
0.90	1.44	2.47	3.67	4.63	5.50	6.63	7.50	8.42	9.13	9.88	10.75	11.38	11.92	12.63	13.63	14.25	15.00	15.50	16.25	17.00
1.00	1.38	2.39	3.75	4.58	5.75	6.50	7.38	8.17	8.92	9.88	10.50	11.25	11.88	13.00	13.50	14.00	14.50	15.25	17.00	17.50
1.10	1.38	2.57	3.67	4.42	5.75	6.58	7.42	8.42	9.19	10.00	10.63	11.25	12.00	12.75	13.75	14.50	15.25	15.88	16.75	17.63
1.20	1.43	2.75	3.75	4.63	5.75	6.75	7.75	8.67	9.75	10.50	11.33	12.75	13.50	14.13	15.00	15.88	16.75	17.63	18.50	19.38
1.30	1.39	2.75	3.69	4.75	5.85	7.00	7.75	8.75	9.63	10.75	11.75	12.75	13.56	14.25	15.00	16.50	17.13	17.88	18.63	19.50
1.40	1.42	2.82	3.81	4.85	6.25	7.00	8.13	9.25	10.13	11.50	12.13	12.83	13.63	14.63	15.75	16.75	17.75	18.75	19.75	20.50
1.50	1.39	2.88	3.85	5.00	6.00	6.96	8.25	9.17	10.75	11.33	12.00	12.75	13.63	15.00	16.25	17.25	18.13	19.00	20.75	21.25
1.60	1.39	2.81	3.80	4.90	6.00	7.00	8.25	9.25	10.00	10.67	11.75	12.25	13.19	14.50	15.63	17.00	17.63	18.50	19.38	20.00
1.70	1.40	2.75	3.75	4.60	5.50	6.63	7.50	8.88	9.75	10.25	11.00	11.75	13.00	14.25	15.00	16.00	17.25	18.25	19.50	20.00
1.80	1.36	2.54	3.50	4.50	5.20	6.25	7.50	8.38	9.00	9.63	10.50	11.50	12.75	13.88	15.00	16.25	17.25	18.25	19.00	20.00
1.90	1.21	2.42	3.50	4.44	5.50	6.13	7.00	8.00	8.63	9.50	10.75	12.00	13.00	14.00	14.88	15.75	16.75	17.75	18.75	20.00
2.00	1.19	2.47	3.63	4.50	5.58	6.25	6.95	8.00	9.13	10.25	11.50	12.75	13.50	14.75	15.50	16.50	17.75	18.75	19.88	21.00
2.10	1.18	2.60	4.00	5.00	5.67	6.38	7.25	8.50	9.63	11.00	12.00	13.08	14.00	15.00	16.50	17.50	18.50	19.56	20.50	21.50
2.20	1.17	2.64	4.00	4.75	5.75	6.44	7.63	8.75	10.00	11.00	12.25	13.25	14.38	15.50	16.50	17.75	18.50	19.63	20.50	21.50
2.30	1.21	2.63	3.75	4.69	5.75	6.75	7.88	8.63	9.75	11.25	12.25	13.25	14.58	15.88	17.25	18.25	19.25	20.50	21.50	22.00
2.40	1.14	2.57	3.69	4.88	6.00	7.00	7.67	8.67	9.88	10.88	12.25	13.38	15.25	16.50	17.50	18.50	19.75	21.25	22.50	23.50
2.50	1.14	2.57	3.63	4.85	5.83	6.67	7.63	8.75	9.75	11.25	12.25	14.50	15.75	16.75	17.75	18.75	19.63	21.50	22.50	23.50
2.60	1.14	2.63	3.67	4.75	5.88	6.60	7.67	8.92	10.25	11.25	13.50	14.50	15.75	16.75	18.00	19.00	20.00	21.38	22.50	23.50
2.70	1.15	2.78	3.64	4.83	5.75	6.50	7.88	9.17	10.42	12.25	13.00	14.00	15.38	16.75	17.75	18.75	19.88	21.25	22.25	24.00
2.80	1.16	2.65	3.60	4.63	5.50	6.75	8.08	9.25	10.75	11.50	12.63	14.00	15.75	16.50	17.50	18.50	20.00	21.00	22.50	24.00
2.90	1.16	2.63	3.55	4.50	5.50	7.00	8.25	9.50	10.75	11.75	12.88	14.50	15.38	16.38	17.25	18.75	20.00	22.00	23.50	24.00
3.00	1.22	2.65	3.50	4.50	5.63	6.75	8.38	10.00	11.25	12.00	13.13	14.38	15.38	16.25	17.42	19.50	20.50	21.75	23.25	24.00
3.10	1.28	2.65	3.58	4.75	5.67	7.00	8.75	10.00	11.25	12.50	13.42	14.33	15.38	16.75	18.13	19.25	20.25	21.50	23.50	25.00
3.20	1.25	2.67	3.67	4.83	5.92	7.25	8.75	10.13	11.38	12.38	13.42	14.50	15.75	17.25	18.25	19.25	21.00	23.00	24.00	25.00
3.30	1.25	2.64	3.75	4.88	6.13	7.50	8.75	10.00	11.00	12.13	13.38	14.75	16.25	17.25	18.38	20.50	21.75	22.75	23.75	24.75
3.40	1.20	2.68	3.65	4.92	6.25	7.58	8.63	9.75	10.88	12.00	13.75	15.33	16.25	17.75	19.25	20.25	21.50	22.50	23.50	24.75
3.50	1.22	2.69	3.67	4.75	6.13	7.42	8.63	9.75	10.75	11.63	14.00	15.38	17.00	18.00	19.00	20.00	21.00	22.25	23.38	24.38
3.60	1.25	2.63	3.75	4.63	6.50	7.38	8.63	9.58	10.63	12.00	14.50	15.63	16.75	17.75	18.75	20.00	21.13	22.25	23.38	24.38
3.70	1.25	2.63	3.75	4.75	6.25	7.25	8.50	9.50	10.75	13.25	14.75	16.00	16.75	17.75	19.00	20.25	21.25	22.25	23.38	24.38
3.80	1.25	2.58	3.75	4.75	6.25	7.42	8.50	9.44	12.00	13.50	15.00	15.75	16.75	17.88	19.25	20.75	21.75	22.75	23.63	24.88
3.90	1.25	2.50	4.00	5.00	6.50	7.56	8.69	9.63	12.50	13.75	15.00	15.88	16.92	18.50	19.50	21.00	22.00	23.00	24.17	25.50
4.00	1.18	2.47	4.25	5.13	6.50	7.63	8.75	10.25	12.75	13.88	15.13	16.13	17.50	19.00	20.00	21.13	22.13	23.38	24.88	26.38

Table 4-18 Average reduction factors (r) for systems with α=0.6 and ξ=15%

Fundamental	μ_T																			
Period (T_{el})	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.06	1.45	1.63	1.75	1.91	2.06	2.13	2.30	2.40	2.56	2.69	2.82	2.91	3.00	3.11	3.19	3.25	3.30	3.38	3.69
0.20	1.38	2.31	2.94	3.38	3.88	4.19	4.50	4.83	5.25	5.50	5.63	5.83	6.08	6.50	6.75	6.88	7.00	7.25	7.63	8.00
0.30	1.48	2.63	3.50	4.00	4.50	5.00	5.50	5.75	6.25	6.50	6.92	7.50	8.00	8.50	8.75	9.25	9.75	10.13	10.58	11.00
0.40	1.65	2.89	3.65	4.42	4.94	5.50	6.33	7.00	7.63	8.17	9.06	9.75	10.13	10.50	11.13	11.50	12.00	12.50	12.92	13.50
0.50	1.84	3.25	4.17	5.00	5.50	6.44	7.25	8.38	9.13	9.75	10.50	11.25	12.19	12.85	13.50	14.08	14.63	15.30	16.00	16.50
0.60	1.64	2.88	3.88	4.75	5.88	6.92	7.63	8.25	9.00	9.75	10.44	11.38	12.00	12.75	13.50	14.25	14.88	15.38	15.95	16.58
0.70	1.67	2.93	4.17	5.19	6.08	7.13	7.95	8.88	9.88	10.58	11.50	12.25	12.92	13.44	14.17	14.88	15.67	16.50	17.17	17.83
0.80	1.59	2.85	3.83	5.00	6.00	7.06	7.90	8.75	9.42	10.38	11.13	11.83	12.50	13.25	13.75	14.33	15.00	15.63	16.38	17.13
0.90	1.64	2.81	4.00	5.00	5.92	7.25	8.13	8.94	9.81	10.63	11.25	12.06	12.75	13.58	14.25	14.75	15.42	16.17	16.75	17.38
1.00	1.50	2.68	4.00	4.92	6.00	7.00	8.00	8.75	9.50	10.56	11.25	12.00	12.50	13.25	13.75	14.75	15.38	15.94	16.75	18.00
1.10	1.57	2.80	3.92	4.88	6.30	7.06	7.88	9.25	9.92	10.44	11.13	12.00	13.00	14.00	14.50	15.75	16.50	17.63	18.33	19.25
1.20	1.63	3.06	4.10	5.08	6.25	7.50	8.50	9.25	10.25	11.13	12.13	13.13	14.25	15.25	16.00	17.00	17.75	18.50	19.58	20.38
1.30	1.66	3.04	4.06	5.13	6.25	7.50	8.33	9.38	10.38	11.33	12.88	13.58	14.25	14.92	16.00	17.19	18.25	19.25	20.50	21.25
1.40	1.66	3.13	4.13	5.17	6.63	7.38	8.63	9.69	10.88	12.00	13.00	13.75	14.58	15.50	16.50	17.50	18.63	19.50	20.67	21.67
1.50	1.61	3.11	4.21	5.33	6.38	7.58	8.75	10.00	11.25	12.00	12.88	13.75	14.75	15.83	17.00	18.25	19.25	19.88	20.75	22.00
1.60	1.50	3.05	4.06	5.20	6.30	7.50	8.75	10.00	10.63	11.42	12.25	13.25	14.50	15.50	16.75	17.63	18.75	20.50	21.00	21.50
1.70	1.50	2.95	4.04	4.94	6.06	7.25	8.50	9.50	10.13	10.92	11.75	12.75	14.25	15.17	16.25	17.13	17.88	18.75	20.00	20.00
1.80	1.43	2.81	3.81	4.75	5.75	6.75	8.00	9.25	9.75	10.25	11.25	12.50	13.75	14.75	15.63	16.75	17.75	19.00	20.00	20.00
1.90	1.41	2.64	3.75	4.69	5.75	6.63	8.13	8.75	9.38	10.17	11.25	12.38	13.75	14.75	16.00	17.25	18.50	19.50	20.25	21.25
2.00	1.39	2.67	4.00	4.88	5.83	6.50	7.75	8.50	9.50	10.63	11.88	13.25	14.50	15.25	16.25	17.50	18.63	19.75	21.50	22.25
2.10	1.36	2.79	3.92	4.94	6.00	6.63	7.75	8.88	10.00	11.38	12.58	13.83	14.75	15.75	17.00	18.75	19.63	20.63	21.63	22.63
2.20	1.33	2.81	4.00	5.13	6.00	6.88	7.92	9.25	10.58	11.75	13.08	14.25	15.38	16.75	18.13	19.13	20.13	21.00	22.00	23.00
2.30	1.28	2.75	4.00	5.25	6.00	7.00	8.33	9.50	11.00	12.13	13.25	14.63	16.00	17.25	18.25	19.25	20.25	21.25	22.38	23.50
2.40	1.28	2.83	4.00	5.17	6.13	7.25	8.50	9.50	10.75	12.00	13.25	14.81	16.00	17.13	18.50	19.50	20.50	21.50	22.75	24.00
2.50	1.33	2.75	3.94	5.25	6.17	7.25	8.38	9.75	10.75	12.00	13.50	15.00	16.00	17.25	18.50	19.75	20.75	22.00	23.00	24.00
2.60	1.35	2.85	3.96	5.30	6.25	7.19	8.42	9.50	11.00	12.50	14.00	15.25	16.50	17.50	18.75	20.00	21.25	22.25	23.38	24.63
2.70	1.36	2.81	3.95	5.08	6.00	7.08	8.25	9.50	10.75	12.75	13.75	15.00	16.00	17.25	18.50	19.75	21.00	22.00	23.17	25.00
2.80	1.35	2.86	3.94	4.92	5.92	6.92	8.33	9.50	11.25	12.38	13.38	14.63	16.00	17.13	18.25	19.63	20.63	21.75	23.75	24.75
2.90	1.36	2.86	3.90	4.83	5.83	6.88	8.25	10.06	11.13	12.25	13.38	14.88	16.00	17.00	18.25	19.50	20.50	22.00	22.75	23.75
3.00	1.38	2.85	3.85	4.70	5.88	7.08	8.75	10.25	11.25	12.25	13.63	14.88	16.00	17.25	18.38	19.38	20.50	21.50	23.00	24.00
3.10	1.43	2.88	3.85	4.75	6.00	7.38	9.00	10.33	11.50	12.63	13.83	15.00	16.25	17.50	18.50	19.63	20.75	22.50	24.25	26.00
3.20	1.43	2.88	3.83	5.00	6.13	7.75	9.00	10.38	11.63	12.88	14.00	15.25	16.33	17.50	18.63	20.25	21.50	23.00	24.25	25.88
3.30	1.39	2.83	3.83	5.06	6.50	7.75	9.08	10.25	11.63	12.83	13.88	15.08	16.50	17.63	18.75	20.00	21.38	23.25	25.00	26.00
3.40	1.38	2.86	3.92	5.00	6.50	7.75	9.00	10.13	11.38	12.56	14.00	15.25	16.25	17.50	18.67	20.50	22.25	24.00	25.00	26.00
3.50	1.36	2.81	3.88	5.13	6.38	7.50	8.88	10.00	11.17	12.42	14.25	15.25	16.25	17.50	19.50	21.25	22.50	23.75	24.75	25.75
3.60	1.36	2.88	3.94	5.13	6.63	7.63	8.88	9.92	11.13	12.38	14.50	15.58	17.00	18.50	20.00	21.25	22.25	23.50	24.75	26.00
3.70	1.36	2.75	3.94	5.17	6.80	7.70	8.83	10.00	11.00	12.75	14.75	16.13	17.75	18.75	20.00	21.25	22.25	23.63	24.88	25.88
3.80	1.36	2.71	3.88	5.33	6.83	7.70	8.75	9.88	11.00	13.50	15.00	16.50	17.75	18.75	20.00	21.13	22.50	23.75	24.75	26.00
3.90	1.36	2.71	3.88	5.38	6.83	7.88	8.88	9.88	12.00	14.08	15.38	16.75	17.75	18.75	20.13	21.75	22.75	24.00	25.00	26.50
4.00	1.34	2.71	4.00	5.58	6.88	7.94	9.13	10.13	12.75	14.38	15.75	16.75	17.88	19.25	20.50	21.75	23.00	24.25	25.50	27.25

Table 4-19 Average reduction factors (r) for systems with α=0.6 and ξ=20%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.09	1.50	1.69	1.91	2.00	2.17	2.38	2.45	2.65	2.75	2.90	3.04	3.10	3.28	3.38	3.44	3.67	3.69	3.75	3.94
0.20	1.50	2.46	3.11	3.69	4.10	4.58	5.00	5.30	5.50	5.75	6.06	6.38	6.63	7.00	7.25	7.50	7.75	8.00	8.33	8.58
0.30	1.67	2.92	3.75	4.38	4.88	5.44	5.94	6.25	6.67	7.00	7.42	7.92	8.75	9.25	9.50	10.13	10.50	11.00	11.31	11.69
0.40	1.85	3.17	4.00	4.69	5.38	6.13	6.88	7.75	8.31	8.94	9.67	10.38	11.00	11.50	12.00	12.50	13.05	13.55	14.04	14.55
0.50	2.10	3.75	4.63	5.50	6.00	7.08	8.13	9.25	10.05	10.65	11.33	12.15	12.88	13.67	14.42	15.08	15.69	16.38	17.13	18.00
0.60	1.89	3.20	4.33	5.13	6.19	7.50	8.25	9.00	9.80	10.50	11.50	12.00	13.00	14.00	14.75	15.50	16.25	16.63	17.13	17.69
0.70	1.90	3.25	4.50	5.67	6.75	7.65	8.75	9.58	10.50	11.38	12.33	13.25	14.00	14.50	15.25	16.25	17.00	17.50	18.25	18.92
0.80	1.75	3.13	4.33	5.25	6.45	7.58	8.45	9.38	10.17	11.25	12.00	12.75	13.25	13.92	14.60	15.19	16.08	16.88	17.50	18.25
0.90	1.79	3.14	4.33	5.35	6.44	7.75	8.70	9.75	10.63	11.25	12.00	12.63	13.50	14.38	15.13	15.75	16.25	16.88	17.50	18.25
1.00	1.68	3.10	4.30	5.25	6.42	7.44	8.57	9.31	10.13	11.17	12.17	12.88	13.63	14.25	14.88	15.63	16.25	16.75	17.69	19.00
1.10	1.64	3.14	4.25	5.38	6.63	7.58	8.50	9.58	10.50	11.25	11.88	12.50	13.25	14.25	15.00	16.00	18.00	18.88	20.00	20.63
1.20	1.82	3.32	4.50	5.40	6.81	7.88	9.00	10.00	10.75	11.50	12.58	13.50	14.75	15.25	17.25	18.00	18.88	19.75	20.42	21.50
1.30	1.75	3.32	4.50	5.56	6.75	8.00	9.00	9.75	10.88	11.92	13.50	14.50	15.25	16.00	17.13	18.08	19.00	20.33	21.50	22.00
1.40	1.88	3.38	4.55	5.50	7.00	8.08	9.13	10.25	11.50	12.75	13.63	14.50	15.50	16.50	17.38	18.63	19.75	21.25	22.50	23.50
1.50	1.75	3.46	4.50	5.69	7.00	7.83	9.25	10.58	11.75	12.75	13.50	14.38	15.75	16.75	17.67	18.75	20.00	21.13	22.13	23.25
1.60	1.71	3.33	4.25	5.42	6.50	7.75	9.13	10.50	11.25	12.00	12.88	14.00	15.00	16.13	17.25	18.75	19.50	20.50	22.50	23.75
1.70	1.66	3.17	4.21	5.31	6.42	7.63	8.75	10.00	10.75	11.50	12.25	13.50	14.75	15.88	17.00	18.25	20.00	20.50	21.00	21.88
1.80	1.63	3.00	4.29	5.08	6.17	7.25	8.50	9.50	10.38	10.94	12.00	13.75	14.75	15.75	16.67	17.42	18.25	19.50	20.75	22.00
1.90	1.50	2.84	4.25	5.00	5.92	7.00	8.50	9.38	10.00	10.69	12.00	13.00	14.50	15.50	16.38	17.75	19.00	20.25	21.50	22.00
2.00	1.46	2.88	4.25	5.25	6.00	7.25	8.75	9.50	10.00	11.00	12.38	13.75	15.00	16.00	17.50	18.75	20.00	21.25	22.25	23.25
2.10	1.48	3.00	4.25	5.25	6.25	7.25	8.38	9.19	10.42	11.50	13.13	14.50	15.75	16.75	17.75	19.25	20.75	22.25	23.75	24.00
2.20	1.45	2.88	4.13	5.38	6.42	7.25	8.38	9.63	10.88	12.25	13.67	14.88	16.00	17.75	19.25	20.75	22.00	23.00	24.00	24.00
2.30	1.42	2.91	4.17	5.50	6.50	7.38	8.75	9.92	11.38	12.67	14.00	15.50	17.00	18.58	19.50	20.75	21.75	22.75	24.00	25.25
2.40	1.42	3.00	4.25	5.50	6.42	7.63	8.92	10.33	11.50	12.92	14.25	15.75	17.50	18.50	19.50	20.50	21.50	22.75	24.00	25.00
2.50	1.43	2.95	4.31	5.50	6.38	7.63	9.00	10.50	11.75	13.00	14.75	15.75	17.00	18.50	19.50	20.63	21.63	22.75	24.00	25.00
2.60	1.44	3.00	4.25	5.50	6.50	7.75	9.13	10.42	11.75	13.25	14.75	15.75	17.00	18.38	19.63	20.67	21.75	23.00	24.25	25.50
2.70	1.44	3.05	4.25	5.38	6.42	7.75	9.06	10.13	11.75	13.25	14.50	15.75	16.83	18.13	19.38	20.50	21.75	23.08	24.25	25.50
2.80	1.44	3.04	4.19	5.30	6.25	7.56	8.67	10.25	12.00	13.00	14.13	15.38	16.63	17.81	19.08	20.25	21.50	22.50	23.88	25.25
2.90	1.44	3.04	4.13	5.15	6.08	7.44	8.75	10.58	11.63	12.75	14.13	15.25	16.38	17.50	19.00	20.00	21.25	22.63	24.25	25.25
3.00	1.50	3.00	4.11	4.96	6.08	7.50	9.25	10.50	11.50	12.75	14.13	15.38	16.38	17.63	19.00	20.25	21.50	22.75	23.75	24.75
3.10	1.53	3.00	4.13	5.00	6.20	7.83	9.38	10.58	11.75	13.00	14.33	15.50	16.75	18.00	19.25	20.50	21.63	22.50	23.75	24.88
3.20	1.53	3.05	4.08	5.13	6.25	8.25	9.50	10.75	12.06	13.25	14.50	15.75	17.08	18.13	19.38	20.42	21.50	22.63	24.25	25.75
3.30	1.54	2.95	4.17	5.19	6.42	8.25	9.50	10.75	12.06	13.25	14.50	15.81	16.92	18.13	19.13	20.25	21.75	23.50	25.00	26.00
3.40	1.47	3.00	4.08	5.30	6.42	8.00	9.38	10.50	11.83	13.00	14.42	15.50	16.75	17.75	19.00	20.50	22.00	23.50	25.00	26.38
3.50	1.47	2.92	4.00	5.25	6.58	7.88	9.13	10.38	11.63	13.00	14.50	15.75	16.75	17.88	19.25	20.50	22.00	23.75	25.25	26.00
3.60	1.46	2.92	4.00	5.38	6.75	7.88	9.13	10.38	11.58	13.00	14.75	15.83	16.75	18.00	19.50	21.00	22.75	24.50	26.00	27.00
3.70	1.45	2.88	4.10	5.50	6.88	7.92	9.00	10.25	11.58	13.25	14.75	16.00	17.17	18.50	20.25	22.00	23.75	24.75	26.00	27.25
3.80	1.44	2.90	4.06	5.50	6.92	8.06	9.13	10.42	11.63	13.75	15.00	16.17	17.50	19.25	21.00	22.25	23.50	24.75	26.00	27.30
3.90	1.50	2.92	4.00	5.75	7.06	8.13	9.25	10.50	11.67	14.25	15.25	16.83	18.25	20.00	21.25	22.25	23.50	25.00	26.25	27.50
4.00	1.50	2.88	4.13	6.00	7.13	8.25	9.25	10.50	12.38	14.50	15.75	17.50	18.75	20.00	21.25	22.63	24.25	25.25	26.50	28.00

Table 4-20 Average reduction factors (*r*) for systems with α=0.6 and ξ=25%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.13	1.58	1.78	1.96	2.17	2.33	2.46	2.70	2.81	3.00	3.10	3.30	3.42	3.58	3.67	3.75	3.92	3.92	4.06	4.17
0.20	1.75	2.68	3.38	4.00	4.38	4.75	5.31	5.63	5.94	6.19	6.50	6.83	7.25	7.50	7.63	7.88	8.25	8.58	8.88	9.17
0.30	1.88	3.30	4.13	4.75	5.33	5.75	6.21	6.75	7.17	7.63	8.00	8.75	9.25	10.00	10.25	10.75	11.25	11.75	12.10	12.42
0.40	2.05	3.41	4.25	5.13	5.75	6.75	7.38	8.25	9.08	9.67	10.42	11.13	11.75	12.33	12.83	13.50	14.00	14.50	15.00	15.25
0.50	2.30	4.00	5.13	5.75	6.67	7.63	9.13	10.00	10.92	11.46	12.25	12.94	13.75	14.50	15.25	16.00	16.63	17.25	17.88	18.50
0.60	2.13	3.63	4.75	5.50	6.75	8.00	8.88	9.63	10.38	11.56	12.25	12.92	13.75	15.00	15.75	16.81	17.38	17.88	18.38	19.00
0.70	2.00	3.50	4.83	6.00	7.25	8.25	9.33	10.38	11.31	12.42	13.50	14.25	15.00	15.50	16.13	17.25	18.00	18.75	19.25	20.13
0.80	1.93	3.40	4.83	5.92	7.00	8.13	9.13	10.00	11.00	12.00	12.63	13.50	14.00	14.69	15.44	16.25	17.13	17.88	18.50	19.38
0.90	1.89	3.42	4.70	5.82	7.05	8.25	9.38	10.25	11.25	11.88	12.63	13.38	14.25	15.00	15.75	16.50	17.25	18.00	18.63	19.25
1.00	1.88	3.45	4.60	5.75	6.88	8.00	9.08	9.92	10.83	11.83	12.67	13.50	14.31	15.00	15.75	16.25	17.00	17.75	18.58	19.38
1.10	1.83	3.42	4.75	5.83	7.06	8.00	9.00	10.13	11.25	12.00	13.00	14.00	14.50	15.25	15.75	16.67	18.00	19.75	20.75	21.50
1.20	1.92	3.55	4.88	5.88	7.25	8.50	9.63	10.50	11.38	12.17	13.13	14.00	15.38	16.63	17.88	19.13	20.13	21.25	22.00	22.75
1.30	1.96	3.55	4.67	6.05	7.25	8.44	9.50	10.38	11.20	12.25	13.50	14.63	15.75	17.25	18.08	18.88	19.92	21.00	22.13	23.13
1.40	1.94	3.63	5.00	6.00	7.42	8.58	9.58	10.58	12.08	13.50	14.50	15.25	16.25	17.25	18.38	19.75	20.63	22.00	23.25	24.00
1.50	1.96	3.75	4.85	6.19	7.33	8.25	9.50	11.25	12.58	13.42	14.25	15.25	16.50	17.50	18.58	19.88	21.25	22.50	23.75	24.00
1.60	1.79	3.58	4.60	5.85	6.88	8.17	9.67	11.00	12.00	13.00	13.75	14.75	15.75	16.67	18.00	19.38	20.75	21.75	22.75	24.25
1.70	1.75	3.31	4.50	5.69	6.75	8.25	9.25	10.58	11.50	12.25	13.17	14.13	15.25	16.38	17.75	19.25	20.00	22.00	23.25	24.50
1.80	1.70	3.20	4.54	5.45	6.50	7.67	9.00	10.08	10.83	11.63	12.67	13.88	15.17	16.25	17.83	19.25	20.00	20.75	21.50	22.75
1.90	1.64	3.05	4.56	5.40	6.38	7.50	9.00	9.88	10.63	11.38	13.50	14.50	15.38	16.50	17.25	18.13	19.25	20.50	21.75	23.25
2.00	1.63	3.11	4.58	5.50	6.45	7.75	9.25	9.88	10.63	12.00	13.00	14.50	16.00	16.75	17.75	19.25	20.50	21.75	23.25	24.00
2.10	1.64	3.13	4.50	5.67	6.63	8.00	9.25	9.88	10.92	12.13	13.50	15.25	16.13	17.25	18.75	20.25	21.50	22.75	24.08	25.25
2.20	1.61	3.08	4.44	5.63	7.00	7.88	9.00	9.92	11.17	12.63	14.25	15.38	16.75	18.00	19.38	21.00	22.25	23.75	25.00	26.00
2.30	1.56	3.11	4.42	5.75	7.00	7.92	9.00	10.25	11.63	13.13	14.63	15.88	17.25	18.75	20.50	22.00	23.25	24.25	25.50	26.00
2.40	1.57	3.13	4.46	5.92	6.88	8.00	9.50	10.75	12.00	13.58	15.00	16.50	18.00	19.50	20.75	22.00	23.25	24.25	25.50	26.00
2.50	1.54	3.14	4.46	5.88	6.70	7.92	9.50	10.92	12.38	13.75	15.50	17.00	18.33	19.38	20.50	21.75	23.00	24.25	25.50	26.00
2.60	1.57	3.21	4.46	5.75	6.75	8.00	9.63	11.13	12.44	13.67	15.75	17.00	18.25	19.38	20.50	21.75	23.25	24.25	25.50	26.75
2.70	1.57	3.20	4.46	5.67	6.69	8.13	9.63	11.00	12.25	14.00	15.50	16.50	17.75	19.00	20.17	21.50	22.75	24.00	25.25	26.50
2.80	1.56	3.15	4.42	5.58	6.45	8.00	9.50	10.75	12.25	13.75	14.75	16.00	17.33	18.50	19.75	21.00	22.25	23.50	24.75	26.00
2.90	1.56	3.08	4.35	5.50	6.50	7.88	9.25	10.75	12.25	13.38	14.50	16.00	17.00	18.50	19.75	21.00	22.25	23.25	24.50	25.75
3.00	1.56	3.10	4.30	5.50	6.50	7.88	9.50	11.00	12.13	13.17	14.75	15.75	16.92	18.50	19.75	21.00	22.25	23.50	24.75	26.00
3.10	1.61	3.08	4.38	5.38	6.58	8.13	9.83	11.00	12.00	13.38	14.75	15.88	17.25	18.75	20.00	21.25	22.50	23.75	24.88	26.00
3.20	1.61	3.13	4.38	5.56	6.63	8.42	9.88	11.00	12.25	13.50	14.88	16.13	17.50	18.75	20.00	21.50	22.63	23.75	24.75	26.00
3.30	1.64	3.13	4.33	5.56	7.00	8.56	9.88	11.25	12.38	13.50	14.88	16.25	17.50	18.88	20.13	21.25	22.25	23.50	24.50	25.88
3.40	1.65	3.06	4.25	5.50	7.00	8.42	9.75	11.08	12.33	13.50	14.88	16.13	17.38	18.50	19.75	20.75	22.00	23.13	24.75	26.00
3.50	1.56	3.00	4.20	5.44	7.00	8.33	9.63	10.88	12.13	13.50	14.75	16.00	17.13	18.13	19.38	20.50	22.00	23.75	25.50	27.25
3.60	1.58	3.04	4.20	5.63	7.00	8.25	9.50	10.83	12.00	13.58	14.75	16.00	17.00	18.25	19.38	21.00	22.75	24.00	25.50	27.13
3.70	1.61	3.04	4.30	5.67	7.00	8.25	9.50	10.75	12.25	13.75	15.13	16.25	17.25	18.44	20.00	21.38	23.00	24.50	26.25	27.75
3.80	1.61	3.05	4.31	5.83	7.00	8.25	9.50	10.69	12.13	13.88	15.38	16.50	17.50	18.88	20.25	22.00	23.75	25.50	26.75	28.00
3.90	1.50	3.00	4.31	5.92	7.17	8.38	9.50	10.88	12.25	14.25	15.50	16.63	17.88	19.50	21.25	22.75	24.50	25.75	27.00	28.38
4.00	1.50	3.10	4.33	6.08	7.38	8.50	9.75	11.13	12.63	14.50	15.75	17.00	18.50	20.25	22.00	23.50	24.75	26.00	27.38	28.92

Table 4-21 Average reduction factors (*r*) for systems with α=0.6 and ξ=30%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.13	1.63	1.89	2.13	2.35	2.54	2.75	2.96	3.06	3.30	3.42	3.58	3.67	3.81	3.92	3.92	4.08	4.17	4.33	4.50
0.20	1.83	2.88	3.75	4.21	4.75	5.25	5.67	5.92	6.33	6.63	6.88	7.25	7.63	7.92	8.25	8.50	8.75	9.13	9.50	9.88
0.30	2.00	3.55	4.38	4.92	5.67	6.15	6.69	7.17	7.69	8.25	9.25	9.75	10.00	10.63	11.13	11.50	12.08	12.56	12.88	13.25
0.40	2.30	3.75	4.69	5.50	6.50	7.25	8.13	9.06	9.75	10.50	11.25	11.88	12.58	13.00	13.75	14.25	14.75	15.33	15.75	16.33
0.50	2.50	4.25	5.50	6.25	7.25	8.38	9.50	10.63	11.63	12.42	13.13	13.75	14.75	15.50	16.00	16.75	17.44	18.19	18.88	19.63
0.60	2.35	4.06	5.25	5.92	7.38	8.38	9.25	10.25	11.31	12.17	13.17	13.88	14.63	15.67	17.00	17.75	18.25	18.88	19.50	20.25
0.70	2.21	4.06	5.13	6.39	7.63	8.88	10.00	11.25	12.25	13.25	14.25	15.25	15.75	16.50	17.25	18.00	19.00	19.88	20.75	21.75
0.80	1.98	3.58	5.13	6.42	7.40	8.75	9.63	10.63	11.75	12.50	13.38	14.17	14.88	15.63	16.50	17.25	18.25	19.13	20.00	20.75
0.90	2.07	3.65	4.90	6.17	7.44	8.67	9.75	11.00	12.00	12.50	13.25	14.25	15.06	15.88	16.58	17.50	18.25	19.00	19.75	20.50
1.00	2.00	3.63	4.75	6.00	7.38	8.50	9.63	10.42	11.25	12.31	13.19	13.92	14.83	15.63	16.25	16.88	17.67	18.38	19.25	20.25
1.10	2.00	3.58	4.92	6.13	7.33	8.50	9.63	10.58	11.63	12.88	13.75	14.50	15.25	15.75	16.63	17.38	18.25	19.13	20.50	21.88
1.20	2.11	3.75	5.08	6.25	7.75	8.83	10.00	11.13	12.00	13.13	14.25	15.25	16.00	17.00	18.25	19.50	21.00	22.25	23.25	24.00
1.30	2.13	3.83	5.00	6.38	7.67	9.08	10.25	11.00	11.83	12.75	14.00	15.25	16.75	17.75	19.00	20.00	21.00	22.17	23.25	24.38
1.40	2.14	3.75	5.29	6.50	7.83	9.00	10.08	11.08	12.17	13.58	14.75	16.25	17.00	18.25	19.50	20.75	21.63	22.63	24.00	25.50
1.50	2.13	3.83	5.14	6.60	7.69	8.67	9.88	11.17	13.06	14.13	15.25	16.33	17.31	18.33	19.56	20.63	21.75	23.25	24.75	26.00
1.60	2.00	3.68	4.90	6.19	7.33	8.50	10.00	11.58	12.75	13.75	15.00	15.63	16.75	17.50	19.00	20.25	21.50	23.00	24.00	25.00
1.70	1.93	3.50	4.79	5.90	7.06	8.38	9.88	11.13	12.25	13.00	14.00	15.00	15.88	17.00	18.50	20.25	21.25	22.50	23.75	25.50
1.80	1.85	3.40	4.80	5.81	6.92	8.25	9.38	10.67	11.58	12.38	13.25	14.25	15.50	17.00	18.25	19.63	21.25	22.50	24.00	25.13
1.90	1.73	3.25	4.83	5.75	7.00	7.92	9.50	10.25	11.25	12.13	13.50	14.75	16.00	17.38	18.75	19.75	20.75	21.50	22.75	24.00
2.00	1.79	3.30	5.00	5.88	7.00	8.25	9.58	10.50	11.25	13.25	14.50	15.25	16.50	17.75	18.75	19.63	21.00	22.25	23.50	25.00
2.10	1.73	3.38	5.00	5.94	7.00	8.50	9.63	10.38	11.75	12.75	14.25	15.75	17.00	17.88	19.00	20.50	22.00	23.25	24.75	26.00
2.20	1.73	3.25	4.75	6.00	7.25	8.58	9.50	10.42	11.69	13.00	14.75	16.00	17.13	18.50	19.75	21.25	22.75	24.33	25.75	27.00
2.30	1.73	3.33	4.67	6.00	7.25	8.50	9.44	10.63	12.00	13.50	15.25	16.17	17.63	19.25	20.75	22.25	23.75	25.00	26.00	27.50
2.40	1.68	3.33	4.69	6.13	7.17	8.35	9.75	11.25	12.50	14.00	15.25	17.00	18.25	19.75	21.38	22.75	24.00	25.25	26.50	27.75
2.50	1.68	3.35	4.75	6.25	7.25	8.25	10.25	11.50	13.00	14.50	16.00	17.25	19.00	20.25	21.38	22.50	23.75	25.25	26.50	28.00
2.60	1.69	3.35	4.70	6.13	7.08	8.50	10.25	11.50	13.00	14.75	16.13	17.88	19.00	20.13	21.25	22.50	24.00	25.25	26.75	28.00
2.70	1.68	3.38	4.67	5.94	6.94	8.50	10.00	11.55	13.00	14.50	16.25	17.50	18.50	19.75	20.92	22.25	23.50	25.00	26.25	27.50
2.80	1.67	3.25	4.58	5.75	6.75	8.31	9.88	11.17	12.75	14.50	15.63	17.00	18.13	19.25	20.50	21.75	23.13	24.38	25.63	27.00
2.90	1.67	3.20	4.57	5.63	6.69	8.25	9.81	11.00	12.75	14.13	15.25	16.58	17.75	19.25	20.50	21.75	23.00	24.25	25.50	26.75
3.00	1.64	3.20	4.56	5.75	6.75	8.31	9.75	11.50	12.50	14.00	15.25	16.50	18.00	19.25	20.50	21.75	23.00	24.25	25.50	26.75
3.10	1.65	3.25	4.56	5.75	6.88	8.38	10.00	11.50	12.63	14.00	15.25	16.50	18.25	19.50	20.75	22.00	23.25	24.50	25.75	27.00
3.20	1.71	3.25	4.63	5.75	7.00	8.58	10.25	11.50	12.63	14.00	15.25	16.75	18.25	19.50	20.75	22.00	23.25	24.75	26.00	27.00
3.30	1.75	3.19	4.50	5.75	6.94	8.88	10.33	11.50	12.75	14.00	15.50	17.00	18.25	19.50	20.75	22.08	23.25	24.50	25.75	27.00
3.40	1.72	3.20	4.42	5.58	7.50	8.81	10.13	11.55	12.58	14.00	15.50	16.75	18.00	19.25	20.63	21.75	23.00	24.25	25.25	26.63
3.50	1.69	3.15	4.25	5.63	7.50	8.67	10.08	11.31	12.50	13.92	15.13	16.38	17.67	18.88	20.00	21.25	22.50	23.75	25.00	26.75
3.60	1.68	3.13	4.25	5.75	7.25	8.67	10.00	11.25	12.50	14.00	15.17	16.38	17.63	18.75	20.00	21.25	22.38	23.75	25.50	27.13
3.70	1.64	3.13	4.40	6.00	7.25	8.75	9.88	11.13	12.75	14.13	15.25	16.50	17.63	18.88	19.92	21.19	22.75	24.50	26.25	28.00
3.80	1.64	3.11	4.42	6.25	7.42	8.75	9.75	11.17	12.75	14.25	15.58	16.63	17.88	19.00	20.17	21.75	23.50	25.25	26.75	28.25
3.90	1.69	3.20	4.56	6.25	7.42	8.75	9.75	11.50	12.75	14.50	16.00	17.00	18.13	19.25	21.00	22.38	24.00	25.75	27.25	29.00
4.00	1.68	3.20	4.67	6.25	7.58	8.75	10.00	11.50	12.88	14.83	16.13	17.50	18.50	19.92	21.50	23.25	25.00	26.75	28.00	29.50

Table 4-22 Average reduction factors (r) for systems with α=0.6 and ξ=35%

Fundamental	μ_T																			
Period (T_{el})	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.15	1.67	1.97	2.25	2.53	2.68	2.93	3.13	3.38	3.50	3.63	3.75	3.88	3.92	4.13	4.30	4.38	4.54	4.67	4.79
0.20	2.00	3.08	3.88	4.50	5.00	5.58	6.00	6.42	6.67	7.00	7.38	7.88	8.00	8.38	8.75	9.13	9.50	9.88	10.17	10.50
0.30	2.21	3.80	4.63	5.50	6.08	6.58	7.17	7.75	8.50	9.25	9.75	10.25	10.75	11.38	12.00	12.38	12.88	13.33	13.67	14.00
0.40	2.42	4.00	5.00	6.00	6.75	7.83	8.75	9.63	10.38	11.17	12.00	12.58	13.25	14.00	14.50	15.00	15.50	16.00	16.50	17.13
0.50	2.71	4.58	5.92	6.75	7.81	9.00	10.25	11.13	12.20	13.17	14.00	14.75	15.63	16.50	17.13	17.88	18.58	19.38	20.13	20.75
0.60	2.46	4.30	5.50	6.38	7.63	8.88	10.00	10.88	12.00	13.00	13.75	15.00	16.25	17.00	17.75	18.75	19.50	20.25	21.00	21.50
0.70	2.42	4.25	5.50	6.81	8.25	9.50	10.63	11.88	13.00	14.00	15.13	16.00	16.75	17.50	18.38	19.25	20.33	21.25	22.25	23.00
0.80	2.20	3.92	5.44	6.88	7.83	9.13	10.19	11.33	12.50	13.25	14.08	15.00	15.75	16.83	17.50	18.50	19.25	20.25	21.50	22.00
0.90	2.19	3.89	5.33	6.50	7.75	9.13	10.33	11.50	12.50	13.25	14.00	15.00	15.88	16.75	17.50	18.38	19.25	20.08	20.88	21.75
1.00	2.16	3.86	4.96	6.42	7.75	9.00	10.25	11.13	11.92	12.75	13.75	14.50	15.50	16.38	17.25	18.00	18.50	19.17	20.13	21.00
1.10	2.15	3.75	5.13	6.42	7.75	9.06	10.17	11.13	12.25	13.19	14.13	15.05	15.83	16.50	17.17	17.95	19.00	19.88	21.00	22.13
1.20	2.25	3.92	5.33	6.75	8.10	9.38	10.44	12.00	13.25	14.00	15.00	16.00	16.50	17.50	18.17	19.19	21.00	22.75	24.08	25.25
1.30	2.29	4.00	5.25	6.75	8.13	9.63	10.63	11.42	12.50	13.50	14.88	16.25	17.00	18.00	20.00	21.25	22.75	24.00	25.00	26.00
1.40	2.29	4.00	5.50	6.92	8.33	9.42	10.50	11.67	12.75	13.75	15.13	16.25	17.50	19.00	20.38	21.50	22.50	23.63	24.88	26.00
1.50	2.29	4.08	5.50	6.96	8.13	9.17	10.50	11.42	13.00	14.50	16.00	16.92	18.08	19.25	20.58	21.63	22.67	24.00	25.25	26.00
1.60	2.16	3.85	5.21	6.60	7.75	9.00	10.00	11.75	13.31	14.50	15.50	16.50	17.75	18.50	19.75	21.00	22.13	23.50	25.00	26.00
1.70	2.00	3.63	5.13	6.25	7.50	8.81	10.25	11.50	13.00	13.88	14.75	15.75	16.75	17.75	19.50	20.75	22.00	23.25	24.50	25.75
1.80	1.93	3.55	5.08	6.06	7.17	8.75	9.92	11.25	12.33	13.25	14.13	15.00	16.13	17.75	19.50	20.50	21.75	23.25	24.75	26.00
1.90	1.91	3.46	5.00	6.00	7.25	8.63	9.75	10.63	11.50	12.75	13.69	14.88	16.25	17.75	19.25	20.75	22.00	23.50	25.00	26.00
2.00	1.90	3.45	5.13	6.00	7.50	8.63	9.75	11.00	12.00	13.50	14.75	16.00	17.38	18.88	20.25	21.25	22.25	23.25	24.75	26.00
2.10	1.88	3.56	5.13	6.13	7.58	8.75	10.00	11.00	12.25	14.25	15.25	16.38	18.00	19.00	19.75	20.88	22.25	23.75	25.00	26.63
2.20	1.86	3.43	5.00	6.38	7.63	8.88	10.00	10.92	12.75	13.63	15.25	17.00	18.00	18.88	20.25	21.75	23.00	24.50	26.25	27.50
2.30	1.75	3.44	5.00	6.33	7.75	8.88	9.88	11.17	12.50	14.00	15.75	17.00	17.92	19.38	20.88	22.50	24.25	25.56	26.88	28.25
2.40	1.75	3.45	5.00	6.38	7.63	8.75	10.00	11.50	13.00	14.50	15.88	17.25	18.75	20.50	22.00	23.25	24.50	25.75	27.00	28.00
2.50	1.75	3.46	5.00	6.42	7.63	8.83	10.25	12.00	13.50	14.88	16.50	18.00	19.50	20.75	22.17	23.50	24.75	26.00	27.50	28.75
2.60	1.80	3.56	4.92	6.42	7.50	9.00	10.75	12.25	13.63	15.25	17.00	18.25	19.75	20.92	22.17	23.38	24.75	26.25	27.50	29.00
2.70	1.73	3.50	4.88	6.19	7.25	8.75	10.50	12.00	13.75	15.25	16.88	18.17	19.38	20.63	21.75	23.13	24.38	25.75	27.00	28.50
2.80	1.70	3.38	4.71	6.00	7.00	8.63	10.17	11.75	13.25	15.00	16.42	17.63	19.00	20.25	21.50	22.75	23.88	25.13	26.38	27.88
2.90	1.69	3.38	4.65	5.90	7.00	8.50	10.17	11.63	13.25	14.63	16.08	17.50	18.75	20.00	21.25	22.50	23.75	25.00	26.25	27.50
3.00	1.69	3.35	4.70	5.83	7.00	8.56	10.13	11.42	13.25	14.75	16.00	17.50	18.75	20.00	21.25	22.50	23.75	25.00	26.25	27.50
3.10	1.75	3.35	4.69	5.88	7.17	8.75	10.25	11.75	13.13	14.75	16.00	17.50	19.00	20.25	21.50	23.00	24.25	25.50	26.50	27.50
3.20	1.81	3.35	4.75	5.88	7.25	8.83	10.50	12.00	13.17	14.63	16.00	17.75	19.00	20.50	21.75	23.00	24.25	25.50	26.50	27.75
3.30	1.82	3.35	4.70	5.83	7.33	8.88	10.50	11.88	13.13	14.63	16.25	17.75	19.00	20.25	21.50	22.75	24.00	25.25	26.75	28.00
3.40	1.83	3.29	4.63	5.75	7.50	9.08	10.42	11.75	13.08	14.75	16.00	17.25	18.75	20.00	21.25	22.50	23.75	25.00	26.25	27.63
3.50	1.75	3.29	4.46	5.83	7.50	9.05	10.50	11.58	12.94	14.50	15.75	17.00	18.25	19.50	20.83	22.25	23.50	24.75	26.13	27.50
3.60	1.75	3.25	4.42	5.88	7.75	9.08	10.50	11.75	13.08	14.38	15.50	16.75	18.25	19.50	20.75	22.00	23.25	24.63	26.25	27.75
3.70	1.73	3.20	4.50	6.00	7.75	9.00	10.38	11.63	13.00	14.50	15.67	16.83	18.13	19.25	20.75	22.00	23.38	25.00	26.50	28.00
3.80	1.75	3.21	4.63	6.25	7.75	9.00	10.25	11.63	13.00	14.63	15.75	16.92	18.17	19.42	20.75	22.25	23.75	25.25	27.00	28.75
3.90	1.75	3.29	4.67	6.50	7.75	9.00	10.25	12.00	13.25	14.50	16.00	17.25	18.50	19.75	21.00	22.50	24.25	25.75	27.50	29.25
4.00	1.75	3.25	4.83	6.50	7.92	9.00	10.38	12.25	13.38	15.00	16.50	17.75	19.00	20.25	21.50	23.13	24.88	26.58	28.33	30.00

Table 4-23 Average reduction factors (r) for systems with α=0.8 and ξ=5%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.04	1.42	1.55	1.68	1.75	1.88	2.00	2.00	2.06	2.17	2.25	2.25	2.40	2.50	2.61	2.71	2.79	2.83	2.91	3.00
0.20	1.05	1.85	2.38	2.83	3.13	3.54	3.67	3.88	4.13	4.33	4.50	4.69	5.00	5.25	5.50	5.75	6.00	6.38	6.63	7.00
0.30	1.11	2.09	2.75	3.25	3.63	4.33	4.81	5.25	5.50	5.88	6.17	6.50	6.88	7.20	7.58	8.00	8.50	9.00	9.38	9.75
0.40	1.11	2.13	2.94	3.63	4.25	4.65	5.31	5.67	6.25	7.00	7.63	8.13	8.63	9.08	9.44	9.92	10.55	11.05	11.56	12.00
0.50	1.20	2.46	3.33	4.08	4.63	5.50	6.38	7.13	8.00	8.50	9.50	9.75	10.44	11.25	11.63	12.19	13.06	13.58	14.00	14.50
0.60	1.17	2.31	3.25	4.00	4.88	5.75	6.50	6.92	7.83	8.38	9.00	9.75	10.38	10.75	11.25	12.00	13.00	13.75	14.13	14.88
0.70	1.25	2.29	3.33	4.38	5.19	6.15	7.00	7.75	8.63	9.13	10.00	10.75	11.25	11.94	12.50	13.00	13.75	14.25	15.25	16.00
0.80	1.15	2.29	3.13	4.00	5.10	6.06	6.88	7.75	8.25	9.00	10.06	10.58	11.13	12.00	12.55	13.00	13.50	14.00	14.63	15.75
0.90	1.15	2.25	3.35	4.35	5.13	6.00	7.25	8.00	8.75	9.50	10.17	10.83	11.50	12.25	13.00	13.75	14.33	14.88	15.50	16.25
1.00	1.13	2.21	3.44	4.08	5.17	6.17	7.00	7.75	8.63	9.38	10.00	10.75	11.50	12.50	13.00	13.50	14.50	15.13	15.83	16.58
1.10	1.10	2.33	3.38	4.25	5.17	6.19	7.00	8.00	9.00	9.50	10.25	11.13	12.00	12.88	13.75	14.50	15.00	15.88	16.63	17.42
1.20	1.15	2.42	3.45	4.21	5.42	6.50	7.33	8.33	9.25	10.13	11.17	11.88	12.88	13.58	14.50	15.50	16.50	17.38	18.00	18.75
1.30	1.15	2.50	3.42	4.42	5.58	6.56	7.42	8.42	9.19	10.13	11.25	12.13	12.88	13.63	14.75	16.00	16.75	17.25	18.00	19.00
1.40	1.16	2.50	3.58	4.58	5.75	6.75	7.88	8.81	9.67	10.75	11.63	12.25	13.00	14.25	15.25	16.00	17.25	18.13	19.25	20.00
1.50	1.16	2.47	3.46	4.80	5.83	6.70	7.88	9.00	10.13	10.88	11.38	12.25	13.50	14.75	15.75	16.50	17.50	18.75	19.75	20.63
1.60	1.17	2.55	3.55	4.75	5.50	6.50	8.00	8.75	9.50	10.42	11.25	11.75	13.25	14.30	14.92	16.00	17.00	18.25	19.50	20.00
1.70	1.19	2.42	3.63	4.25	5.08	5.75	7.50	8.50	9.25	9.75	10.42	11.25	12.50	13.50	14.75	16.00	17.25	18.25	19.00	19.75
1.80	1.18	2.39	3.42	4.35	5.00	6.00	6.75	7.75	8.25	9.25	10.25	11.25	12.42	13.50	14.50	15.50	16.25	17.00	18.00	19.25
1.90	1.15	2.31	3.31	4.25	5.00	5.88	6.50	7.42	8.38	9.50	10.50	11.75	12.58	13.25	14.00	14.88	15.75	17.50	18.50	19.50
2.00	1.13	2.30	3.50	4.75	5.31	5.88	6.75	7.92	9.00	10.25	11.00	11.88	12.75	13.63	14.50	16.50	17.75	18.75	19.50	20.50
2.10	1.11	2.42	3.50	4.50	5.19	6.13	7.25	8.25	9.00	10.13	11.17	12.00	13.00	13.88	14.88	16.00	17.25	18.50	19.75	20.75
2.20	1.05	2.41	3.45	4.38	5.50	6.38	7.58	8.13	9.13	10.25	11.13	12.25	13.25	14.25	15.25	16.58	17.75	19.25	20.13	21.50
2.30	1.04	2.39	3.38	4.33	5.50	6.50	7.17	8.17	9.25	10.17	11.25	12.13	13.50	14.50	16.25	17.50	18.25	20.00	21.25	22.50
2.40	1.05	2.36	3.21	4.44	5.42	6.33	7.13	8.17	9.25	10.38	11.38	12.75	13.75	16.00	17.00	17.75	18.75	19.88	22.00	23.00
2.50	1.07	2.25	3.25	4.42	5.50	6.25	7.13	8.38	9.42	10.75	11.88	12.88	15.50	16.50	17.25	18.25	19.25	20.25	21.75	23.08
2.60	1.07	2.38	3.39	4.50	5.50	6.25	7.50	8.75	9.88	11.50	12.25	14.25	15.25	16.25	17.38	18.63	19.63	21.25	23.00	24.63
2.70	1.13	2.40	3.40	4.44	5.38	6.00	8.00	9.00	10.50	11.50	12.75	13.75	15.50	16.75	17.50	18.50	19.75	22.25	24.00	24.00
2.80	1.13	2.42	3.30	4.21	5.25	6.00	7.75	9.13	10.50	11.75	12.63	14.25	15.25	16.25	17.25	18.92	21.00	22.00	23.25	24.00
2.90	1.13	2.46	3.30	4.25	5.08	6.25	8.00	9.50	11.00	12.00	13.00	14.13	15.13	16.13	17.63	19.25	20.25	21.50	22.50	24.00
3.00	1.18	2.50	3.25	4.33	5.25	6.83	8.17	9.67	11.00	12.00	13.25	14.08	15.00	16.50	18.00	19.00	20.00	22.00	23.25	24.00
3.10	1.21	2.50	3.50	4.55	5.75	7.00	8.58	9.83	11.13	12.25	13.25	14.25	15.75	16.75	17.75	19.25	21.00	22.00	23.25	24.13
3.20	1.21	2.57	3.44	4.63	5.75	7.25	8.75	10.00	11.30	12.25	13.50	14.75	16.00	17.00	18.75	20.00	21.00	22.00	23.25	24.00
3.30	1.25	2.54	3.44	4.75	6.00	7.38	8.50	10.00	11.00	12.00	13.25	15.38	16.75	18.00	18.75	19.75	21.00	22.08	23.13	24.00
3.40	1.14	2.46	3.50	4.50	5.83	7.25	8.50	9.81	10.67	11.75	14.25	15.50	16.75	17.75	18.75	19.50	20.88	21.88	22.88	24.00
3.50	1.14	2.46	3.44	4.40	5.88	7.25	8.38	9.50	10.25	12.50	14.50	15.50	16.38	17.38	18.50	19.63	20.63	21.63	22.50	23.88
3.60	1.15	2.50	3.50	4.25	5.88	7.00	8.33	9.33	10.75	13.00	14.50	15.50	16.50	17.50	19.00	20.25	21.25	22.25	23.00	24.00
3.70	1.15	2.44	3.58	4.50	5.92	7.00	8.50	9.25	11.50	13.55	14.58	15.50	16.50	18.00	19.50	20.50	21.50	22.50	23.63	24.75
3.80	1.17	2.40	3.50	4.75	5.88	7.13	8.38	9.88	12.13	13.58	14.58	15.63	16.75	18.50	19.25	20.50	21.50	22.63	24.00	25.25
3.90	1.25	2.38	3.75	4.83	6.06	7.13	8.50	10.75	12.25	13.63	14.75	15.75	17.50	18.50	19.50	20.75	21.75	23.25	24.25	25.50
4.00	1.18	2.42	4.00	5.00	6.19	7.58	8.50	11.00	12.38	13.63	15.00	16.50	17.50	18.50	19.50	21.25	22.50	23.50	24.88	26.00

Table 4-24 Average reduction factors (*r*) for systems with α=0.8 and ξ=10%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.09	1.47	1.63	1.75	1.86	2.00	2.00	2.18	2.29	2.36	2.50	2.63	2.71	2.82	2.92	3.00	3.04	3.07	3.25	3.25
0.20	1.18	2.08	2.69	3.13	3.58	3.75	4.06	4.33	4.63	4.88	5.13	5.50	5.88	6.00	6.25	6.63	7.00	7.25	7.75	8.00
0.30	1.21	2.31	3.13	3.58	4.08	4.81	5.25	5.75	6.00	6.25	7.00	7.25	7.75	8.08	8.50	9.17	9.75	10.25	10.50	10.75
0.40	1.38	2.46	3.35	4.13	4.67	5.25	5.83	6.50	7.25	8.00	8.88	9.25	9.69	10.13	10.88	11.33	11.83	12.35	12.92	13.50
0.50	1.53	2.85	3.75	4.42	5.42	6.17	7.25	7.88	8.75	9.38	10.33	11.13	11.88	12.56	13.13	13.83	14.44	15.08	15.88	16.63
0.60	1.41	2.58	3.63	4.63	5.75	6.50	7.33	8.00	8.63	9.42	10.25	11.00	11.75	12.38	13.00	13.88	14.50	15.08	15.69	16.50
0.70	1.43	2.60	3.88	4.88	5.88	6.90	7.75	8.75	9.63	10.42	11.19	11.92	12.63	13.31	13.94	14.75	15.75	16.31	17.00	17.63
0.80	1.42	2.59	3.60	4.63	5.75	6.81	7.67	8.50	9.38	10.13	10.92	11.63	12.25	13.00	13.63	14.25	15.00	15.75	16.50	17.25
0.90	1.38	2.50	3.71	4.83	5.75	7.05	8.06	8.69	9.75	10.50	11.15	11.75	12.63	13.50	14.13	14.75	15.50	16.25	17.00	17.83
1.00	1.32	2.41	3.80	4.80	5.75	6.88	7.58	8.50	9.25	10.25	11.00	11.67	12.38	12.94	14.00	14.63	16.25	17.00	17.75	18.50
1.10	1.36	2.60	3.69	4.50	5.86	6.75	7.67	9.00	9.50	10.25	11.25	12.00	12.75	13.88	15.00	15.63	16.50	17.50	18.38	19.25
1.20	1.38	2.82	3.83	4.75	6.00	7.00	8.00	9.00	10.25	11.25	12.25	13.33	14.13	15.00	15.75	16.88	18.00	19.00	20.00	20.75
1.30	1.36	2.82	3.69	5.00	6.08	7.25	8.08	9.13	10.13	11.42	12.42	13.25	14.06	15.25	16.75	17.75	18.75	19.50	20.25	21.00
1.40	1.39	2.83	3.83	5.06	6.25	7.25	8.38	9.58	10.50	11.75	12.50	13.38	14.75	16.25	17.00	17.63	18.63	20.00	20.92	22.00
1.50	1.34	2.85	3.93	5.17	6.13	7.38	8.50	9.63	11.00	11.75	12.38	13.38	15.06	15.92	17.00	18.25	18.88	20.00	21.25	23.25
1.60	1.36	2.83	3.91	5.08	6.25	7.25	8.75	9.50	10.25	11.25	12.25	13.38	14.50	15.75	16.50	17.50	19.00	19.50	20.50	21.75
1.70	1.39	2.79	3.85	4.75	5.69	7.00	8.25	9.13	10.00	10.75	11.75	13.00	14.25	15.17	16.00	17.00	18.25	19.50	20.75	22.00
1.80	1.35	2.55	3.50	4.63	5.44	6.58	8.08	9.00	9.50	10.25	11.25	12.38	13.63	14.75	16.00	17.25	18.50	19.75	21.00	22.00
1.90	1.31	2.45	3.75	4.63	5.75	6.38	7.50	8.25	9.00	10.25	11.50	12.75	13.88	15.25	16.25	17.25	18.25	19.50	20.50	22.00
2.00	1.23	2.57	3.75	4.63	5.63	6.42	7.50	8.42	9.58	11.00	12.13	13.25	14.25	15.25	16.38	17.50	19.25	20.50	21.50	22.50
2.10	1.22	2.63	4.00	5.00	5.75	6.75	7.67	9.13	10.13	11.50	12.75	14.00	15.00	16.13	17.38	18.50	19.75	20.88	21.88	23.00
2.20	1.25	2.64	4.00	5.00	5.88	6.75	8.13	9.25	10.50	11.67	13.00	14.13	15.38	16.63	17.81	18.88	19.92	20.92	22.13	23.25
2.30	1.25	2.64	3.75	5.00	6.00	7.00	8.25	9.38	10.50	12.00	13.13	14.50	15.50	17.00	18.00	19.13	20.25	21.38	22.63	23.75
2.40	1.15	2.59	3.75	5.00	6.13	7.17	8.25	9.42	10.63	11.88	13.00	14.13	15.42	16.75	18.25	19.63	20.67	21.88	23.00	24.25
2.50	1.16	2.56	3.75	4.92	6.08	7.25	8.25	9.50	10.63	11.75	12.92	14.17	16.00	17.50	18.75	19.88	21.13	22.25	23.50	24.75
2.60	1.15	2.67	3.75	4.92	6.00	7.00	8.38	9.63	10.75	12.00	13.25	15.25	16.50	17.75	18.75	20.13	21.25	22.44	23.67	25.00
2.70	1.15	2.75	3.80	4.88	5.83	7.06	8.38	9.58	10.75	12.00	13.50	14.88	16.13	17.50	18.75	20.13	22.00	23.25	24.50	25.75
2.80	1.17	2.75	3.81	4.75	5.71	7.00	8.42	9.88	11.00	12.25	13.63	15.00	16.25	17.50	18.67	20.75	22.50	23.50	24.50	25.50
2.90	1.16	2.63	3.65	4.69	5.75	7.00	8.60	9.88	11.25	12.25	13.75	15.00	16.08	17.38	19.25	20.50	21.50	22.50	24.00	25.50
3.00	1.25	2.67	3.67	4.65	6.00	7.50	8.75	10.25	11.50	12.50	13.88	15.08	16.25	18.00	19.00	20.00	21.50	23.25	24.50	26.00
3.10	1.25	2.79	3.67	4.83	6.00	7.75	8.83	10.38	11.75	12.88	14.08	15.25	16.50	17.63	19.00	20.25	21.50	23.50	25.75	26.00
3.20	1.29	2.75	3.80	5.00	6.50	7.75	8.92	10.50	11.88	13.00	14.17	15.42	16.58	17.67	18.88	21.00	23.25	24.75	25.50	26.50
3.30	1.29	2.75	3.88	5.08	6.50	8.00	9.00	10.50	11.75	12.88	14.50	15.38	16.50	18.25	20.25	22.25	23.50	24.25	25.38	26.75
3.40	1.30	2.71	3.85	5.06	6.50	7.88	9.00	10.25	11.58	12.67	14.50	15.67	17.25	19.50	20.75	22.00	23.00	24.00	25.50	26.63
3.50	1.25	2.67	3.70	4.88	6.50	7.75	8.92	10.13	11.25	12.50	14.63	16.25	18.00	19.25	20.50	21.75	22.83	24.00	25.00	26.50
3.60	1.28	2.67	3.75	4.88	6.75	7.81	9.00	10.13	11.25	13.25	15.25	16.75	18.00	19.00	20.25	21.75	22.83	24.00	25.25	26.38
3.70	1.31	2.69	3.75	4.88	6.50	7.75	8.75	9.95	12.25	13.50	15.38	16.63	17.75	19.13	20.50	21.75	22.88	24.08	25.33	26.42
3.80	1.31	2.67	3.75	5.25	6.63	7.83	8.75	10.50	12.50	13.75	15.50	16.75	17.88	19.13	20.50	21.75	22.92	24.13	25.33	26.50
3.90	1.30	2.69	4.00	5.25	6.63	8.00	9.08	11.25	12.75	14.25	15.75	16.75	18.25	19.25	20.63	21.88	23.13	24.25	25.50	26.75
4.00	1.31	2.56	4.25	5.50	6.75	8.00	9.17	11.75	13.00	14.63	15.75	17.25	18.50	19.75	21.08	22.25	23.58	24.88	26.25	27.25

Table 4-25 Average reduction factors (r) for systems with α=0.8 and ξ=15%

Fundamental	μ_T																			
Period (T_{el})	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.13	1.50	1.70	1.84	2.00	2.11	2.21	2.30	2.42	2.61	2.67	2.88	3.00	3.00	3.17	3.25	3.31	3.44	3.50	3.65
0.20	1.34	2.25	2.94	3.40	3.70	4.13	4.58	5.00	5.38	5.58	5.83	6.13	6.25	6.63	7.25	7.58	8.00	8.25	8.50	8.75
0.30	1.41	2.63	3.38	4.00	4.58	5.25	5.75	6.25	6.50	7.25	7.75	8.00	8.50	9.00	9.75	10.00	10.50	10.92	11.33	11.58
0.40	1.61	2.88	3.75	4.50	5.19	5.83	6.67	7.42	8.25	9.08	9.75	10.31	10.75	11.25	12.00	12.50	13.00	13.58	14.17	14.81
0.50	1.79	3.25	4.20	5.00	5.90	7.00	8.00	8.88	9.58	10.50	11.31	12.25	13.00	13.69	14.42	15.17	16.00	16.75	17.67	18.38
0.60	1.61	2.88	4.00	5.00	6.33	7.25	7.92	8.69	9.63	10.50	11.38	12.50	13.50	14.25	15.25	15.50	16.17	17.00	17.75	18.83
0.70	1.66	2.95	4.30	5.50	6.42	7.63	8.58	9.75	10.75	11.75	12.63	13.25	14.00	14.75	15.50	16.50	17.13	17.88	19.00	19.88
0.80	1.58	2.88	4.00	5.17	6.44	7.50	8.50	9.38	10.50	11.13	11.88	12.60	13.20	13.92	15.00	15.92	16.67	17.25	17.75	18.38
0.90	1.61	2.92	4.17	5.25	6.50	7.67	8.63	9.50	10.38	11.25	11.92	13.00	13.75	14.50	15.17	15.88	16.58	17.50	18.25	19.13
1.00	1.48	3.00	4.13	5.19	6.25	7.56	8.38	9.19	10.25	11.08	11.83	12.50	13.38	14.25	15.00	15.75	17.00	18.25	19.00	20.13
1.10	1.56	2.95	4.13	5.00	6.40	7.42	8.50	9.63	10.38	11.00	11.69	12.67	13.75	15.00	16.25	17.38	18.50	19.25	20.19	21.13
1.20	1.61	3.13	4.13	5.38	6.75	7.88	9.06	10.00	10.92	11.92	13.25	14.00	15.25	16.25	17.13	18.19	19.42	20.58	21.75	23.00
1.30	1.60	3.11	4.10	5.50	6.88	7.83	8.81	9.88	11.30	12.25	13.50	14.33	15.38	16.50	18.00	19.38	20.38	21.13	21.88	22.75
1.40	1.58	3.14	4.33	5.63	6.81	8.00	9.06	10.38	11.88	12.75	13.58	14.50	16.00	17.25	18.75	20.25	21.00	22.00	23.00	24.00
1.50	1.48	3.14	4.31	5.40	6.69	7.92	9.25	10.75	11.75	12.63	13.50	14.75	16.00	17.25	19.00	20.00	20.75	21.75	23.00	24.00
1.60	1.50	3.13	4.18	5.39	6.75	8.00	9.25	10.25	11.08	12.00	13.25	14.50	15.50	17.00	18.25	19.50	21.00	22.25	23.50	25.25
1.70	1.45	3.00	4.19	5.13	6.50	7.75	9.00	9.83	10.63	11.50	12.75	14.00	15.25	16.75	17.25	19.00	20.00	21.25	23.00	24.00
1.80	1.42	2.84	4.00	4.95	6.00	7.38	8.63	9.50	10.17	11.25	13.00	14.00	15.08	16.00	16.88	18.25	19.50	20.75	22.00	23.50
1.90	1.39	2.71	4.00	4.92	5.75	7.00	8.50	9.25	10.00	11.13	12.38	13.63	15.00	16.00	17.25	18.75	20.00	21.25	22.75	24.00
2.00	1.41	2.70	4.08	5.25	6.08	7.00	8.38	9.19	10.25	11.50	13.00	14.38	15.75	17.00	18.50	19.75	20.75	22.00	23.13	24.75
2.10	1.36	2.81	4.00	5.25	6.13	7.00	8.25	9.63	10.75	12.25	13.75	15.00	16.00	17.25	18.75	20.50	22.00	23.00	24.50	25.75
2.20	1.33	2.86	4.25	5.50	6.38	7.25	8.75	9.88	11.25	12.67	14.13	15.38	17.00	18.50	20.00	21.00	22.25	23.50	24.50	25.75
2.30	1.29	2.89	4.13	5.50	6.25	7.50	8.88	10.38	11.75	13.25	14.75	16.25	17.75	18.75	19.88	21.00	22.13	23.25	24.50	25.75
2.40	1.28	2.88	4.17	5.50	6.42	7.69	9.13	10.25	11.88	13.25	14.38	16.25	17.75	19.00	20.00	21.00	22.25	23.38	24.50	26.00
2.50	1.31	2.85	4.19	5.50	6.50	7.88	9.13	10.58	12.00	13.00	14.50	15.83	17.25	18.75	20.00	21.25	22.38	23.63	24.75	26.00
2.60	1.33	2.85	4.15	5.38	6.56	8.00	9.17	10.75	11.75	13.50	14.75	16.08	17.38	18.75	20.13	21.50	22.63	24.00	25.25	26.50
2.70	1.35	2.89	4.15	5.38	6.25	7.75	9.13	10.25	12.00	13.25	14.67	15.92	17.17	18.63	20.00	21.25	22.50	24.00	25.25	26.75
2.80	1.36	2.86	4.08	5.17	6.19	7.63	8.88	10.75	12.00	13.17	14.50	15.75	17.00	18.58	19.75	21.00	22.25	23.88	25.50	26.75
2.90	1.38	2.86	4.06	5.00	6.25	7.50	9.00	10.50	11.75	13.13	14.50	15.75	17.13	18.50	19.75	21.13	22.75	24.25	25.75	26.63
3.00	1.38	2.90	4.00	4.94	6.25	7.75	9.25	10.58	11.75	13.13	14.50	15.75	17.25	18.50	20.00	21.75	23.25	24.25	25.25	26.75
3.10	1.42	2.88	4.00	5.00	6.50	8.00	9.42	10.88	12.13	13.38	14.75	16.00	17.50	19.00	20.50	21.75	22.75	24.00	25.75	27.50
3.20	1.44	2.81	4.00	5.25	6.63	8.00	9.50	11.00	12.33	13.56	14.88	16.13	17.63	18.88	20.25	21.50	22.88	24.75	26.50	28.13
3.30	1.42	2.85	4.13	5.50	6.75	8.13	9.75	11.00	12.25	13.56	14.88	16.17	17.50	18.75	20.13	21.67	23.38	25.00	26.63	28.00
3.40	1.44	2.85	4.17	5.25	6.75	8.13	9.50	10.75	12.00	13.44	15.00	16.00	17.25	18.75	20.38	22.00	23.75	25.33	27.00	28.50
3.50	1.43	2.88	4.00	5.38	6.75	8.08	9.25	10.50	11.92	13.50	14.75	16.00	17.42	19.00	20.75	22.50	23.88	25.38	27.08	28.38
3.60	1.39	2.90	4.00	5.38	6.88	8.08	9.25	10.58	12.25	13.50	15.00	16.50	18.00	19.75	21.25	22.75	24.25	25.88	27.00	28.25
3.70	1.38	2.86	4.00	5.50	7.00	8.17	9.25	10.63	12.13	13.88	15.75	16.88	18.75	20.25	21.50	23.25	24.50	26.00	27.25	28.75
3.80	1.38	2.88	3.94	5.50	7.08	8.17	9.38	10.75	12.75	14.13	16.25	17.50	19.00	20.50	22.00	23.25	24.75	26.25	27.75	28.00
3.90	1.38	2.81	4.00	5.75	7.17	8.33	9.50	11.00	13.00	14.50	16.50	17.88	19.25	20.75	22.00	23.50	25.00	26.50	28.00	28.00
4.00	1.38	2.75	4.25	5.75	7.25	8.50	9.50	11.75	13.25	15.00	16.63	18.25	19.63	21.00	22.33	23.75	25.25	26.75	28.25	29.75

Table 4-26 Average reduction factors (*r*) for systems with α=0.8 and ξ=20%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.15	1.63	1.75	1.96	2.07	2.21	2.35	2.50	2.65	2.75	3.00	3.00	3.25	3.31	3.42	3.54	3.67	3.85	4.04	4.08
0.20	1.45	2.46	3.15	3.75	4.13	4.67	5.08	5.45	5.75	6.00	6.38	6.75	7.00	7.50	8.00	8.25	8.63	8.88	9.25	9.50
0.30	1.59	3.00	3.81	4.50	5.00	5.75	6.25	6.75	7.25	7.75	8.50	8.83	9.42	10.08	10.58	10.94	11.42	11.83	12.20	12.63
0.40	1.81	3.17	4.13	4.90	5.67	6.50	7.50	8.31	9.08	9.92	10.63	11.25	12.00	12.50	13.00	13.50	14.17	14.75	15.58	16.25
0.50	2.04	3.63	4.75	5.50	6.58	7.88	8.75	9.75	10.70	11.50	12.38	13.38	14.13	15.00	15.75	16.50	17.25	18.13	18.88	19.75
0.60	1.83	3.31	4.50	5.58	6.94	8.00	8.58	9.58	11.00	11.75	12.50	13.50	14.75	15.92	16.50	17.50	18.50	19.00	19.67	20.88
0.70	1.81	3.25	4.63	6.00	7.08	8.20	9.63	10.75	11.88	13.00	13.75	14.50	15.25	16.25	17.13	18.00	19.38	20.58	21.38	22.13
0.80	1.75	3.21	4.58	5.75	7.13	8.13	9.25	10.25	11.25	12.13	12.83	13.58	14.50	15.25	16.13	17.13	18.31	19.08	19.69	20.50
0.90	1.72	3.25	4.50	5.75	7.17	8.38	9.50	10.25	11.13	12.13	13.25	13.88	14.88	15.67	16.50	17.13	17.88	18.63	19.50	20.50
1.00	1.64	3.35	4.50	5.60	6.83	8.06	9.08	10.00	11.00	12.13	12.92	13.67	14.50	15.25	16.13	17.00	17.94	19.25	20.38	21.50
1.10	1.68	3.25	4.50	5.75	6.94	8.00	9.25	10.33	11.25	12.00	12.67	13.75	14.75	16.00	17.75	18.63	19.75	21.00	21.75	22.88
1.20	1.79	3.38	4.44	5.92	7.25	8.50	10.00	10.83	11.63	12.75	14.25	15.00	16.50	18.25	19.25	20.08	21.08	22.08	23.13	24.00
1.30	1.75	3.38	4.50	5.95	7.50	8.42	9.50	10.58	11.75	12.92	14.25	15.50	16.63	18.00	19.00	20.50	21.50	22.75	24.00	25.00
1.40	1.75	3.43	4.65	6.31	7.42	8.63	9.63	11.13	12.67	13.67	14.75	15.88	17.13	18.50	20.25	21.50	22.25	23.25	24.25	25.25
1.50	1.75	3.46	4.71	6.00	7.25	8.38	10.06	11.63	12.63	13.50	14.58	16.25	17.25	18.88	20.25	22.00	23.00	24.00	25.25	26.00
1.60	1.69	3.33	4.47	5.71	7.00	8.50	10.00	11.13	12.00	12.88	14.63	15.75	16.75	18.25	19.75	20.75	21.75	22.88	24.00	25.38
1.70	1.64	3.21	4.42	5.80	7.00	8.25	9.50	10.63	11.25	12.50	13.75	14.92	16.38	18.25	19.50	20.75	22.00	23.25	24.75	26.00
1.80	1.63	2.96	4.38	5.25	6.63	7.88	9.25	10.13	10.92	12.00	13.38	14.75	16.25	17.50	18.67	20.38	22.25	23.75	25.25	26.00
1.90	1.50	2.97	4.25	5.25	6.21	7.75	9.08	9.88	11.25	12.75	13.75	15.00	16.25	17.25	18.38	19.63	21.50	23.50	25.00	26.00
2.00	1.46	3.03	4.50	5.42	6.38	8.08	9.25	10.00	11.25	12.38	14.00	15.50	16.75	17.75	19.25	20.75	22.00	23.75	25.50	26.75
2.10	1.46	3.05	4.38	5.42	6.75	8.08	9.00	10.17	11.42	12.88	14.50	16.00	17.25	18.75	20.25	21.75	23.25	24.50	26.00	27.50
2.20	1.44	2.97	4.38	5.75	6.75	7.88	9.25	10.50	11.92	13.50	15.13	16.50	18.00	19.25	21.00	22.25	23.75	25.00	26.63	28.50
2.30	1.40	3.13	4.50	5.88	6.88	8.00	9.50	10.92	12.42	14.00	15.58	17.00	18.75	20.25	21.75	23.00	24.50	25.75	27.00	28.63
2.40	1.40	3.10	4.56	5.75	6.75	8.17	10.00	11.25	13.00	14.38	16.00	17.75	19.13	20.33	21.58	22.88	24.00	25.50	26.88	28.50
2.50	1.43	3.06	4.50	5.75	6.88	8.25	10.00	11.50	13.19	14.75	16.50	17.75	19.00	20.25	21.50	22.88	24.13	25.25	27.00	28.50
2.60	1.44	3.06	4.50	5.83	7.00	8.50	10.25	11.75	13.08	14.63	16.25	17.63	19.00	20.25	21.75	22.88	24.13	25.42	27.25	28.75
2.70	1.40	3.14	4.58	5.67	7.00	8.50	10.00	11.25	13.00	14.50	15.88	17.50	18.75	20.25	21.25	22.63	23.88	25.38	27.00	28.50
2.80	1.40	3.07	4.40	5.55	6.75	8.25	9.50	10.75	12.75	14.00	15.50	17.08	18.50	19.63	20.88	22.25	23.50	25.25	26.50	28.00
2.90	1.44	3.05	4.35	5.50	6.67	7.94	9.25	11.25	12.56	14.25	15.50	17.00	18.25	19.50	20.75	22.25	23.50	25.13	26.50	28.00
3.00	1.50	3.06	4.33	5.38	6.75	8.00	9.75	11.13	12.50	14.00	15.75	17.00	18.50	19.75	21.00	22.50	23.83	25.38	26.75	27.92
3.10	1.50	3.05	4.38	5.50	6.75	8.58	10.00	11.25	12.63	14.25	15.50	17.25	18.50	20.00	21.25	22.63	24.25	25.75	27.00	28.50
3.20	1.53	3.00	4.33	5.50	7.00	8.67	10.25	11.50	12.75	14.25	15.75	17.00	18.50	20.00	21.50	23.25	24.75	26.25	27.50	28.75
3.30	1.55	2.96	4.25	5.60	7.25	8.67	10.25	11.50	12.88	14.33	15.63	17.00	18.50	20.13	21.63	23.25	25.00	26.25	27.63	29.00
3.40	1.50	2.96	4.25	5.75	7.13	8.50	10.00	11.50	12.83	14.19	15.50	17.00	18.42	19.75	21.25	23.00	24.75	26.13	28.00	29.50
3.50	1.47	2.96	4.25	5.75	7.17	8.50	9.75	11.25	12.63	14.00	15.50	16.75	18.25	19.75	21.38	23.00	24.58	26.00	27.63	29.13
3.60	1.50	3.00	4.31	6.00	7.25	8.38	9.75	11.00	12.50	14.13	15.75	16.88	18.58	20.13	21.88	23.38	24.75	26.50	28.00	29.50
3.70	1.54	3.04	4.25	5.75	7.33	8.42	9.63	11.13	12.63	14.25	15.75	17.25	19.08	20.75	22.25	23.75	25.50	27.00	28.25	29.63
3.80	1.50	3.00	4.13	5.88	7.33	8.50	9.75	11.25	13.00	14.50	16.25	17.75	19.50	21.00	22.75	24.50	25.75	27.00	28.50	30.00
3.90	1.50	3.04	4.19	6.00	7.40	8.63	10.00	11.50	13.25	15.00	16.75	18.25	19.88	21.50	23.25	24.50	26.00	27.75	29.50	30.00
4.00	1.50	3.00	4.33	6.30	7.50	8.75	10.25	11.75	13.50	15.25	17.25	18.63	20.25	22.00	23.50	25.00	26.75	28.75	30.00	30.00

Table 4-27 Average reduction factors (*r*) for systems with α=0.8 and ξ=25%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.18	1.63	1.84	2.07	2.18	2.38	2.55	2.69	2.88	3.00	3.20	3.31	3.42	3.55	3.83	3.94	4.13	4.19	4.38	4.42
0.20	1.56	2.68	3.42	4.00	4.50	5.06	5.42	5.83	6.17	6.50	7.25	7.50	7.88	8.25	8.75	9.00	9.25	9.50	9.75	10.00
0.30	1.75	3.25	4.13	4.75	5.45	6.17	6.75	7.38	7.88	8.75	9.33	9.75	10.38	10.88	11.33	11.88	12.38	12.75	13.19	13.63
0.40	2.00	3.43	4.44	5.33	6.38	7.19	8.17	9.06	10.00	10.67	11.50	12.00	12.75	13.33	13.88	14.44	15.13	15.75	16.63	17.25
0.50	2.21	4.00	5.17	6.00	7.19	8.63	9.75	10.83	11.75	12.50	13.38	14.25	15.50	16.25	17.00	17.88	18.75	19.75	20.50	21.63
0.60	2.04	3.75	5.05	6.13	7.42	8.50	9.25	10.63	11.75	13.00	14.00	15.25	16.25	17.08	17.75	18.63	19.58	20.50	21.50	22.75
0.70	1.96	3.80	4.95	6.38	7.67	9.13	10.75	11.75	12.75	14.00	14.88	15.83	16.67	17.63	19.00	20.50	21.38	22.25	23.00	24.00
0.80	1.93	3.54	5.08	6.25	7.75	8.75	10.00	11.25	12.25	13.00	13.88	14.63	15.75	17.00	18.00	18.75	19.67	20.75	21.63	22.42
0.90	1.89	3.61	4.86	6.33	7.65	9.08	10.17	11.00	12.00	13.25	14.00	14.67	15.75	17.00	18.00	18.75	19.50	20.25	21.25	22.25
1.00	1.85	3.50	4.92	6.15	7.42	8.75	9.58	10.75	12.00	12.75	13.83	14.75	15.50	16.38	17.33	18.31	19.17	20.50	21.88	23.00
1.10	1.86	3.44	4.88	6.25	7.50	8.75	9.85	11.00	12.13	13.08	14.00	14.75	15.67	16.75	18.75	19.63	21.00	22.00	23.13	24.25
1.20	1.93	3.65	5.00	6.50	7.75	9.25	10.50	11.75	12.50	13.25	15.13	16.25	17.75	18.88	20.00	21.13	22.38	23.75	25.25	26.50
1.30	2.00	3.70	4.88	6.44	7.88	9.25	10.50	11.50	12.42	14.25	15.00	16.63	17.88	19.38	20.75	21.83	22.92	24.25	25.50	26.75
1.40	1.96	3.75	5.15	6.79	8.00	9.25	10.38	11.50	12.75	14.42	15.75	17.00	18.21	19.50	21.00	22.50	23.63	25.00	26.25	27.25
1.50	1.96	3.75	5.04	6.50	7.83	8.90	10.25	12.25	13.63	14.63	16.00	17.25	18.50	20.25	21.63	23.00	24.25	25.25	26.25	27.63
1.60	1.82	3.50	4.90	6.13	7.42	8.83	10.56	11.75	13.00	14.25	15.50	16.50	18.00	19.50	21.00	22.75	24.00	25.00	26.25	27.50
1.70	1.73	3.38	4.75	6.05	7.20	8.88	10.25	11.25	12.38	14.00	15.00	16.17	17.75	19.13	20.75	21.75	23.00	24.00	25.25	26.75
1.80	1.68	3.21	4.80	5.81	7.13	8.50	9.92	11.00	12.00	13.25	14.38	16.00	18.00	19.00	20.25	21.50	22.75	24.25	26.00	27.75
1.90	1.64	3.20	4.63	5.60	7.00	8.25	9.50	10.50	11.63	12.88	14.25	15.88	17.63	19.00	20.50	22.00	23.50	25.00	26.50	28.00
2.00	1.63	3.25	4.75	5.75	7.00	8.50	9.75	10.75	12.75	13.75	15.13	16.38	17.75	19.25	21.25	22.75	24.25	25.75	27.25	28.00
2.10	1.64	3.25	4.75	6.00	7.25	8.75	9.75	11.25	12.38	13.75	15.50	17.25	18.25	19.50	21.25	23.25	24.50	26.25	27.50	29.00
2.20	1.59	3.21	4.70	6.00	7.50	8.75	9.83	11.13	12.63	14.25	16.25	17.25	18.50	20.25	21.75	23.38	25.13	26.58	28.13	29.63
2.30	1.60	3.20	4.75	6.17	7.38	8.67	10.13	11.50	13.00	14.75	16.25	17.63	19.38	21.00	22.63	24.00	25.75	27.25	28.75	30.33
2.40	1.58	3.20	4.75	6.25	7.33	8.75	10.50	12.00	13.63	15.25	17.00	18.50	19.88	21.63	23.13	24.50	26.08	27.50	29.00	30.50
2.50	1.53	3.20	4.88	6.13	7.25	9.00	10.75	12.50	13.88	15.75	17.00	18.75	20.25	21.75	23.00	24.25	26.00	27.75	29.25	30.00
2.60	1.56	3.33	4.75	6.13	7.25	9.25	11.00	12.50	14.25	15.75	17.42	19.00	20.50	21.75	23.00	24.25	26.25	28.00	29.50	30.00
2.70	1.59	3.33	4.81	6.00	7.25	9.13	10.75	12.42	14.13	15.83	17.50	19.00	20.25	21.50	22.75	23.92	26.00	27.50	29.00	30.00
2.80	1.59	3.21	4.75	5.75	7.25	8.81	10.55	12.00	13.83	15.50	17.00	18.25	19.63	20.88	22.13	23.63	25.50	27.00	28.25	29.75
2.90	1.58	3.17	4.58	5.80	7.13	8.75	10.38	11.92	13.50	15.25	16.75	18.13	19.25	20.50	22.00	23.50	25.25	26.50	28.00	29.00
3.00	1.59	3.25	4.56	5.75	7.14	8.67	10.50	11.88	13.25	15.00	16.63	18.00	19.25	20.75	22.00	23.75	25.25	26.50	27.75	28.92
3.10	1.60	3.25	4.63	5.81	7.33	8.92	10.58	11.88	13.25	15.00	16.75	18.25	19.75	21.00	22.50	24.00	25.50	26.75	27.92	29.42
3.20	1.64	3.25	4.65	5.85	7.50	9.31	10.63	12.00	13.50	15.13	16.75	18.25	19.75	21.25	22.83	24.25	25.63	26.94	28.42	29.88
3.30	1.60	3.25	4.50	5.85	7.75	9.25	10.75	12.00	13.63	15.13	16.50	18.25	19.63	21.25	23.00	24.25	25.50	27.13	28.50	30.00
3.40	1.60	3.21	4.46	5.88	7.75	9.13	10.50	12.00	13.56	14.92	16.25	17.83	19.50	21.00	22.75	24.00	25.63	27.00	28.50	30.00
3.50	1.55	3.13	4.42	5.88	7.58	9.00	10.25	11.75	13.38	14.75	16.25	17.63	19.17	20.88	22.38	23.88	25.75	27.00	28.50	30.00
3.60	1.57	3.16	4.55	5.88	7.58	8.88	10.25	11.88	13.33	14.69	16.13	17.67	19.50	20.75	22.50	24.00	25.75	27.50	28.75	30.00
3.70	1.59	3.15	4.56	6.13	7.63	8.88	10.38	11.75	13.25	14.67	16.25	18.00	19.50	21.25	22.75	24.17	25.75	27.50	29.25	30.50
3.80	1.60	3.15	4.50	6.25	7.63	8.92	10.50	11.75	13.50	14.88	16.50	18.25	20.00	21.50	22.88	24.50	26.00	27.63	29.25	30.83
3.90	1.63	3.19	4.58	6.33	7.65	8.92	10.50	11.75	13.50	15.17	16.75	18.75	20.25	21.75	23.25	25.00	26.50	28.25	29.75	31.00
4.00	1.56	3.25	4.50	6.50	7.81	9.13	10.50	12.00	13.88	15.50	17.25	19.13	20.75	22.25	24.00	25.75	27.25	28.50	30.00	31.75

Table 4-28 Average reduction factors (r) for systems with α=0.8 and ξ=30%

Fundamental	μ_T																			
Period (T_{el})	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.22	1.67	1.93	2.14	2.35	2.50	2.75	3.00	3.00	3.29	3.42	3.54	3.80	3.92	4.10	4.25	4.42	4.50	4.75	4.75
0.20	1.69	2.81	3.65	4.25	4.88	5.38	5.83	6.08	6.58	7.13	7.75	8.25	8.63	9.00	9.25	9.63	10.13	10.50	11.00	11.25
0.30	1.96	3.55	4.50	5.25	5.94	6.75	7.25	8.00	8.75	9.50	10.00	10.67	11.25	11.75	12.19	12.67	13.33	14.00	14.50	15.00
0.40	2.17	3.75	4.75	5.83	7.00	8.08	8.88	9.88	10.75	11.38	12.50	13.00	13.50	14.13	14.88	15.50	16.33	17.13	18.00	19.00
0.50	2.38	4.33	5.58	6.63	8.00	9.50	10.50	11.75	12.75	13.50	14.50	15.38	16.38	17.75	19.00	19.75	20.50	21.63	22.50	23.50
0.60	2.25	4.17	5.50	6.50	7.88	9.00	10.42	11.56	12.50	14.00	15.50	16.50	17.25	18.13	19.25	20.00	20.75	21.69	22.88	24.13
0.70	2.17	4.00	5.50	6.94	8.50	10.06	11.25	12.75	13.67	15.00	16.13	17.00	18.50	19.50	20.63	21.88	23.00	24.00	24.75	25.75
0.80	1.98	3.80	5.50	6.83	8.31	9.63	11.00	12.00	13.06	14.13	15.00	16.00	17.13	18.75	19.50	20.50	21.50	22.50	23.50	24.38
0.90	2.11	3.86	5.25	6.75	8.19	9.75	10.83	11.88	13.00	14.25	15.25	16.25	17.25	18.00	19.54	20.38	21.25	22.25	23.25	24.13
1.00	2.00	3.75	5.13	6.63	8.00	9.31	10.20	11.75	12.75	13.75	14.75	15.83	16.75	17.63	18.42	19.38	20.50	21.50	23.00	24.25
1.10	2.00	3.65	5.17	6.63	8.08	9.20	10.58	11.67	12.88	13.88	14.83	15.69	16.63	17.88	19.33	20.75	22.00	23.50	24.88	26.00
1.20	2.13	3.85	5.25	6.83	8.25	9.80	11.08	12.50	13.75	14.75	15.88	17.25	18.50	20.25	21.50	22.50	23.75	25.00	26.38	28.00
1.30	2.05	3.92	5.25	6.94	8.50	10.00	11.00	12.13	13.50	15.25	16.75	17.50	19.00	20.25	21.75	23.13	24.63	26.13	27.25	28.50
1.40	2.16	4.00	5.58	7.20	8.42	9.75	11.13	12.38	13.75	15.17	16.75	18.00	19.50	21.08	22.50	24.00	25.50	27.00	28.50	30.00
1.50	2.13	4.00	5.53	7.00	8.25	9.50	11.00	12.50	13.92	15.63	16.94	18.25	19.63	21.00	22.75	24.25	25.50	26.75	28.00	29.38
1.60	2.00	3.75	5.22	6.58	8.00	9.50	11.25	12.42	14.00	15.25	16.50	18.00	19.25	20.75	22.25	23.75	25.00	26.00	27.25	28.63
1.70	1.92	3.60	5.00	6.38	7.63	9.25	10.81	12.25	13.75	15.00	16.25	17.50	18.67	20.25	21.75	23.50	24.75	26.00	27.25	28.00
1.80	1.84	3.46	5.00	6.25	7.50	9.13	10.58	12.00	13.13	14.25	15.50	17.25	18.50	20.00	21.50	23.00	24.25	25.50	26.63	28.00
1.90	1.75	3.35	5.00	6.13	7.75	8.88	10.13	11.25	12.50	13.88	15.50	17.25	18.75	20.13	21.50	22.63	24.00	25.75	27.50	28.00
2.00	1.71	3.45	5.08	6.13	7.63	9.25	10.17	11.50	12.75	14.25	16.25	18.00	19.50	21.00	22.25	23.75	25.50	27.00	28.75	30.00
2.10	1.69	3.50	5.13	6.38	7.75	9.13	10.38	12.25	13.75	15.00	16.50	18.33	19.75	21.50	23.13	24.63	26.00	27.75	29.25	30.00
2.20	1.69	3.45	5.00	6.42	8.00	9.25	10.75	12.00	13.50	15.25	17.00	18.50	20.00	22.25	23.50	25.00	26.50	28.00	29.50	31.00
2.30	1.68	3.39	5.08	6.50	7.83	9.25	10.75	12.25	13.75	15.75	17.25	18.38	20.00	22.00	23.75	25.25	26.75	28.25	30.25	32.00
2.40	1.65	3.39	5.00	6.44	7.83	9.50	11.00	12.75	14.38	16.00	17.50	19.00	20.75	22.50	24.25	25.75	27.17	28.88	30.38	31.88
2.50	1.67	3.40	5.13	6.50	7.75	9.75	11.50	13.25	14.75	16.50	18.25	20.00	21.50	23.00	24.42	25.92	27.25	29.00	30.50	32.00
2.60	1.68	3.50	5.00	6.45	7.69	9.75	12.00	13.50	15.00	17.00	18.50	20.13	21.50	23.08	24.58	26.00	27.50	28.88	30.38	31.88
2.70	1.68	3.43	5.00	6.33	7.63	9.75	11.50	13.00	15.25	16.75	18.50	19.75	21.33	22.75	24.25	25.75	27.00	28.50	30.00	31.38
2.80	1.65	3.39	5.00	6.15	7.63	9.25	11.13	13.00	15.00	16.50	18.00	19.33	20.83	22.13	23.75	25.00	26.50	27.75	29.25	30.58
2.90	1.65	3.42	4.88	6.08	7.58	9.25	11.25	13.00	14.50	16.25	17.63	19.08	20.50	21.75	23.50	24.75	26.00	27.50	28.83	30.08
3.00	1.64	3.35	4.83	6.00	7.58	9.35	11.25	12.67	14.38	16.13	17.58	19.00	20.25	21.75	23.25	24.75	26.00	27.38	28.63	29.92
3.10	1.75	3.38	4.85	6.13	7.75	9.50	11.25	12.75	14.50	16.13	17.75	19.25	20.50	22.08	23.50	24.88	26.38	27.58	28.92	30.50
3.20	1.75	3.44	4.88	6.15	7.92	9.75	11.31	12.75	14.75	16.13	17.75	19.50	20.75	22.38	23.75	25.13	26.50	28.00	29.25	30.88
3.30	1.72	3.38	4.81	6.13	8.10	9.75	11.19	12.75	14.50	15.88	17.75	19.25	20.83	22.33	23.75	25.25	26.50	28.00	29.63	31.25
3.40	1.69	3.33	4.71	6.25	8.13	9.75	11.13	12.69	14.25	15.67	17.50	19.08	20.63	22.13	23.50	24.92	26.50	28.00	29.50	31.00
3.50	1.70	3.36	4.67	6.25	8.06	9.50	11.00	12.50	14.08	15.58	17.13	18.75	20.25	21.75	23.17	24.63	26.25	27.75	29.25	30.75
3.60	1.68	3.30	4.75	6.25	8.06	9.38	10.92	12.58	14.00	15.63	17.13	18.63	20.25	21.75	23.19	25.00	26.50	28.00	29.50	31.00
3.70	1.66	3.33	4.75	6.50	8.06	9.42	11.00	12.50	13.94	15.50	17.00	18.50	20.25	21.88	23.75	25.50	26.75	28.25	29.75	31.25
3.80	1.71	3.21	4.81	6.50	8.00	9.38	11.00	12.50	14.13	15.50	17.00	18.67	20.50	22.00	23.50	25.50	27.25	28.75	30.00	31.50
3.90	1.75	3.33	4.92	6.75	8.13	9.50	11.00	12.50	14.13	15.50	17.38	19.25	20.75	22.17	23.75	25.50	27.50	29.00	30.50	32.13
4.00	1.75	3.33	4.92	6.75	8.17	9.63	11.00	12.67	14.25	15.92	17.67	19.50	21.00	22.63	24.25	25.88	27.75	29.75	31.25	32.88

Table 4-29 Average reduction factors (*r*) for systems with α=0.8 and ξ=35%

Fundamental	μ_T																			
Period (T_{el})	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.30	1.75	2.04	2.19	2.42	2.65	2.92	3.17	3.30	3.50	3.60	3.81	4.00	4.25	4.38	4.50	4.67	4.83	4.88	5.08
0.20	1.89	3.05	3.92	4.67	5.17	5.75	6.08	6.50	7.00	7.63	8.25	9.00	9.33	9.75	10.25	10.75	11.00	11.25	11.75	12.00
0.30	2.13	3.80	4.75	5.67	6.25	7.25	8.00	9.00	9.50	10.25	10.83	11.42	12.00	12.58	13.08	13.69	14.25	14.92	15.63	16.25
0.40	2.35	4.06	5.15	6.33	7.58	8.63	9.63	10.75	11.50	12.50	13.13	13.75	14.25	15.00	15.67	16.50	17.50	18.50	19.50	20.00
0.50	2.65	4.69	5.94	7.17	8.75	10.00	11.38	12.75	13.50	14.50	15.50	16.75	17.75	19.00	20.13	21.50	22.75	23.75	24.50	25.58
0.60	2.45	4.56	5.75	7.00	8.38	9.63	11.33	12.25	13.75	14.75	16.25	17.75	18.50	19.50	20.38	21.25	22.25	23.25	24.25	25.25
0.70	2.38	4.50	5.92	7.50	9.25	10.75	12.25	13.63	14.75	16.08	17.13	18.00	19.00	20.38	22.08	23.38	24.42	25.50	26.75	27.75
0.80	2.19	4.00	5.83	7.38	9.00	10.38	11.75	12.88	13.75	15.00	16.25	17.75	18.75	20.00	21.00	22.00	23.25	24.13	25.00	26.25
0.90	2.16	4.05	5.67	7.35	8.80	10.25	11.56	12.67	13.88	15.25	16.50	17.50	18.50	19.75	20.75	21.92	23.00	24.00	25.13	26.25
1.00	2.15	3.90	5.50	7.13	8.63	9.88	11.50	12.50	13.42	14.42	15.44	16.75	17.88	18.88	19.83	20.75	21.92	23.00	24.33	25.63
1.10	2.15	3.88	5.50	7.00	8.50	9.75	11.25	12.25	13.50	14.67	15.75	16.75	17.88	18.92	20.25	22.00	23.50	25.00	26.50	27.75
1.20	2.25	4.13	5.67	7.25	8.70	10.25	11.67	13.25	14.38	15.50	16.50	18.00	18.88	21.00	22.75	24.00	25.25	26.50	28.00	29.25
1.30	2.25	4.20	5.75	7.25	8.95	10.50	11.75	13.13	14.50	16.00	17.25	18.00	20.50	21.50	22.75	24.38	25.75	27.50	29.00	30.25
1.40	2.31	4.21	5.92	7.61	9.00	10.50	12.00	12.94	14.75	16.75	17.63	19.25	20.63	22.25	23.75	25.33	26.88	28.25	29.50	30.88
1.50	2.22	4.19	5.94	7.50	8.75	10.50	11.75	13.25	15.00	16.38	18.00	19.44	21.00	22.50	24.25	25.75	27.50	29.25	30.75	32.00
1.60	2.14	3.91	5.58	7.30	8.75	10.00	11.38	12.75	14.75	16.00	17.50	18.75	20.00	21.75	23.50	24.75	26.08	27.38	28.75	30.25
1.70	2.00	3.82	5.38	6.81	8.25	9.75	11.25	12.67	14.50	15.75	17.08	18.50	19.88	21.25	23.00	24.25	25.50	26.75	28.00	29.63
1.80	1.92	3.65	5.30	6.50	8.00	9.50	11.17	12.75	14.00	15.50	16.75	18.00	19.38	21.00	22.75	24.00	25.50	26.75	28.00	29.50
1.90	1.86	3.55	5.31	6.75	8.50	9.58	10.75	12.08	13.50	15.00	16.75	18.25	19.58	21.00	22.75	24.25	25.75	27.00	28.13	29.75
2.00	1.84	3.68	5.38	6.63	8.25	9.67	10.75	12.00	13.44	15.50	17.25	18.75	20.33	22.00	23.50	24.50	26.00	28.00	29.75	30.00
2.10	1.86	3.75	5.50	6.67	8.38	9.67	11.13	12.50	13.88	16.25	17.75	19.38	21.06	22.38	23.88	25.75	27.50	29.25	30.75	32.00
2.20	1.80	3.75	5.38	6.88	8.35	9.88	11.50	13.25	14.50	16.17	18.25	19.88	21.50	22.88	24.50	26.00	27.75	29.50	31.25	32.00
2.30	1.75	3.68	5.20	6.88	8.38	9.92	11.50	13.00	14.50	16.50	18.25	19.75	21.88	23.50	24.75	26.50	28.00	29.75	31.50	33.00
2.40	1.79	3.64	5.25	6.88	8.25	10.00	11.75	13.25	15.00	16.75	18.38	20.00	22.25	23.75	25.00	26.50	28.25	30.13	32.00	33.25
2.50	1.73	3.71	5.33	6.83	8.50	10.25	12.00	13.75	15.50	17.13	18.75	20.50	22.25	24.00	25.50	27.13	28.75	30.50	32.00	33.25
2.60	1.75	3.71	5.42	6.75	8.25	10.50	12.50	14.25	15.63	17.75	19.50	21.25	22.75	24.50	26.00	27.50	29.13	30.75	32.25	34.00
2.70	1.75	3.71	5.25	6.69	8.00	10.50	12.25	13.88	15.75	17.75	19.25	21.00	22.50	24.00	25.50	27.00	28.50	30.13	31.58	33.00
2.80	1.75	3.66	5.15	6.44	7.92	9.88	11.63	13.75	15.50	17.50	19.00	20.25	21.75	23.50	24.75	26.25	27.75	29.25	30.50	31.92
2.90	1.75	3.58	5.00	6.33	7.88	9.63	11.75	13.75	15.75	17.25	18.50	20.00	21.38	23.00	24.50	25.75	27.25	28.50	29.75	31.25
3.00	1.75	3.54	5.08	6.38	7.90	9.75	11.75	13.75	15.50	17.00	18.25	19.75	21.25	23.00	24.25	25.75	27.00	28.25	29.67	31.25
3.10	1.82	3.57	5.13	6.50	8.08	10.00	12.00	13.50	15.50	17.00	18.25	19.75	21.75	23.00	24.50	26.00	27.25	28.63	30.25	31.75
3.20	1.81	3.58	5.10	6.45	8.19	10.17	11.92	13.50	15.25	16.75	18.42	20.00	21.75	23.25	24.75	26.00	27.38	29.00	30.50	32.00
3.30	1.79	3.58	5.00	6.50	8.35	10.31	11.88	13.38	15.25	16.75	18.38	20.25	21.75	23.25	24.75	26.00	27.75	29.00	30.50	32.00
3.40	1.81	3.55	4.96	6.50	8.50	10.30	11.67	13.25	15.00	16.63	18.25	20.00	21.50	23.25	24.25	26.00	27.50	29.00	30.25	31.92
3.50	1.75	3.46	4.90	6.50	8.33	10.06	11.63	13.25	14.83	16.50	18.00	19.75	21.25	22.75	24.13	25.75	27.25	28.75	30.38	32.00
3.60	1.73	3.46	4.92	6.50	8.42	9.94	11.58	13.30	14.75	16.33	18.00	19.50	21.00	22.50	24.25	25.75	27.25	28.83	30.38	31.92
3.70	1.75	3.43	5.08	6.75	8.38	9.94	11.50	13.19	14.67	16.38	18.00	19.50	21.13	22.58	24.00	25.75	27.50	29.00	30.50	32.13
3.80	1.79	3.43	5.00	6.75	8.42	9.92	11.50	13.19	14.75	16.25	18.00	19.50	21.13	22.67	24.25	26.00	27.75	29.25	30.88	32.50
3.90	1.75	3.54	5.17	6.75	8.50	10.00	11.50	13.25	14.75	16.50	18.00	19.75	21.38	23.00	25.00	26.75	28.25	29.75	31.38	33.25
4.00	1.75	3.63	5.25	7.00	8.58	10.00	11.75	13.33	15.00	16.75	18.25	20.06	21.75	23.50	25.25	27.25	28.75	30.25	32.00	34.00

Table 4-30 Average reduction factors (r) for systems with α=1.0 and ξ=5%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.10	1.48	1.63	1.76	1.85	1.98	2.05	2.13	2.20	2.31	2.41	2.47	2.55	2.66	2.75	2.81	2.89	2.94	3.03	3.07
0.20	1.21	1.97	2.47	2.92	3.26	3.56	3.78	4.05	4.27	4.47	4.71	4.94	5.20	5.42	5.75	6.10	6.36	6.86	7.10	7.18
0.30	1.23	2.19	2.86	3.35	3.74	4.33	4.91	5.23	5.55	5.84	6.32	6.65	7.08	7.50	7.87	8.41	8.87	9.34	9.73	10.09
0.40	1.24	2.23	3.11	3.69	4.27	4.73	5.34	5.85	6.51	7.29	7.84	8.46	8.85	9.30	9.81	10.46	11.02	11.61	12.11	12.61
0.50	1.30	2.55	3.44	4.12	4.74	5.71	6.45	7.37	8.03	8.80	9.67	10.35	10.97	11.61	12.28	12.87	13.56	14.12	14.72	15.49
0.60	1.24	2.41	3.33	4.20	5.12	5.97	6.57	7.11	7.96	8.66	9.38	10.15	10.74	11.28	11.93	13.00	13.72	14.27	15.01	15.94
0.70	1.30	2.39	3.45	4.40	5.32	6.32	7.04	7.90	8.78	9.39	10.43	11.04	11.63	12.44	13.10	13.87	14.56	15.30	16.09	16.55
0.80	1.24	2.36	3.18	4.13	5.22	6.30	7.08	7.90	8.64	9.51	10.23	10.84	11.66	12.33	12.94	13.42	13.96	14.91	15.58	16.32
0.90	1.27	2.33	3.40	4.37	5.26	6.40	7.39	8.21	9.08	9.77	10.57	11.17	12.12	12.75	13.41	14.04	14.70	15.43	16.23	17.35
1.00	1.24	2.34	3.42	4.20	5.34	6.39	7.24	8.06	8.91	9.71	10.31	11.03	11.98	12.74	13.33	14.47	15.41	16.11	16.99	17.63
1.10	1.23	2.38	3.44	4.30	5.44	6.34	7.18	8.42	9.00	9.82	10.64	11.58	12.61	13.33	14.03	14.84	15.69	16.58	17.52	18.42
1.20	1.25	2.50	3.61	4.37	5.56	6.59	7.53	8.74	9.59	10.57	11.58	12.41	13.31	14.12	15.23	16.50	17.55	18.32	18.95	19.70
1.30	1.26	2.49	3.55	4.54	5.69	6.71	7.76	8.82	9.72	10.62	11.62	12.53	13.41	14.81	16.12	16.96	17.77	18.58	19.42	20.33
1.40	1.28	2.62	3.65	4.67	5.95	6.85	8.09	9.06	10.01	11.14	11.97	12.91	13.99	14.97	16.31	17.14	18.10	19.03	20.33	21.63
1.50	1.25	2.64	3.67	4.84	5.98	6.90	8.08	9.07	10.30	11.07	11.78	13.18	14.29	15.31	16.33	17.51	18.48	19.76	20.71	21.66
1.60	1.26	2.60	3.69	4.73	5.69	6.59	8.02	8.86	9.71	10.65	11.54	12.65	13.90	14.70	15.98	16.90	17.85	19.06	20.28	21.56
1.70	1.28	2.53	3.70	4.42	5.33	6.08	7.57	8.59	9.42	10.12	10.88	12.08	13.25	14.27	15.28	16.57	17.94	19.11	20.28	21.06
1.80	1.25	2.45	3.51	4.40	5.16	6.20	7.25	8.06	8.69	9.67	10.72	11.86	13.15	14.36	15.41	16.39	17.24	18.22	19.11	20.53
1.90	1.22	2.40	3.44	4.33	5.27	6.05	6.78	7.71	8.66	9.87	11.03	12.22	13.33	14.10	15.00	15.93	16.90	18.32	19.29	20.55
2.00	1.20	2.41	3.57	4.72	5.50	6.08	6.94	8.23	9.30	10.44	11.61	12.61	13.38	14.19	15.36	17.07	18.08	19.10	20.31	21.68
2.10	1.20	2.55	3.66	4.75	5.46	6.26	7.41	8.49	9.48	10.63	11.74	12.68	13.61	14.96	16.04	17.21	18.37	19.86	21.10	22.19
2.20	1.19	2.54	3.64	4.61	5.55	6.58	7.68	8.59	9.69	10.88	11.96	12.94	14.06	15.04	16.17	17.57	18.88	20.22	21.77	23.19
2.30	1.19	2.52	3.58	4.51	5.68	6.72	7.52	8.67	9.86	10.84	11.82	13.14	14.25	15.66	17.18	18.31	19.68	21.11	22.63	24.08
2.40	1.16	2.48	3.50	4.62	5.63	6.55	7.47	8.66	9.72	10.75	11.97	13.20	14.46	15.93	17.42	18.79	20.47	22.05	23.32	24.77
2.50	1.16	2.44	3.44	4.61	5.59	6.49	7.51	8.75	9.90	10.99	12.33	13.41	15.09	16.97	18.18	19.48	20.90	22.39	23.71	25.43
2.60	1.19	2.47	3.54	4.60	5.65	6.50	7.74	9.07	10.41	11.54	12.55	14.51	15.78	16.99	18.42	19.67	21.19	22.65	23.94	25.60
2.70	1.19	2.49	3.51	4.63	5.58	6.52	7.95	9.24	10.76	11.71	13.02	14.29	15.76	17.38	18.58	19.92	21.78	23.21	24.81	26.12
2.80	1.18	2.56	3.47	4.49	5.45	6.37	7.81	9.65	10.74	11.90	13.02	14.60	15.90	17.14	18.43	20.54	21.81	23.28	24.67	25.91
2.90	1.19	2.59	3.44	4.49	5.37	6.53	8.20	9.54	11.14	12.10	13.39	14.65	15.94	17.08	18.65	20.14	21.38	22.61	24.00	25.94
3.00	1.20	2.61	3.45	4.46	5.42	6.85	8.31	9.80	11.21	12.46	13.63	14.93	15.99	17.33	18.60	19.81	21.23	23.43	24.69	25.76
3.10	1.23	2.60	3.56	4.58	5.62	7.12	8.63	9.99	11.23	12.55	13.90	14.91	16.30	17.45	18.66	20.89	22.66	23.65	24.95	26.44
3.20	1.26	2.62	3.59	4.72	5.90	7.35	8.71	10.04	11.40	12.61	13.87	15.19	16.48	17.80	20.23	21.43	22.68	24.02	25.29	26.65
3.30	1.27	2.63	3.63	4.83	6.07	7.55	8.77	10.11	11.38	12.44	14.26	15.62	17.07	18.86	20.05	21.38	22.65	23.90	25.26	26.68
3.40	1.26	2.62	3.58	4.77	6.12	7.56	8.73	10.02	11.12	12.35	14.30	16.32	17.46	18.62	20.01	21.26	22.36	23.68	25.12	26.25
3.50	1.24	2.58	3.58	4.64	5.98	7.51	8.68	9.78	10.89	12.43	14.84	16.21	17.32	18.59	19.92	21.00	22.23	23.61	24.76	25.84
3.60	1.24	2.57	3.65	4.54	6.30	7.38	8.63	9.66	11.16	13.29	14.87	16.18	17.39	18.54	19.80	21.04	22.40	23.49	24.64	25.78
3.70	1.25	2.56	3.67	4.58	6.20	7.43	8.61	9.69	11.96	13.56	15.00	16.32	17.66	18.69	19.88	21.16	22.41	23.49	24.63	25.90
3.80	1.25	2.56	3.76	4.73	6.21	7.39	8.68	10.02	12.40	13.89	15.01	16.51	18.04	19.08	20.24	21.40	22.57	23.75	24.99	26.34
3.90	1.25	2.55	3.88	4.94	6.33	7.45	8.81	11.08	12.60	14.06	15.36	16.82	17.94	19.43	20.55	21.64	22.76	24.18	25.50	27.00
4.00	1.24	2.53	4.03	5.08	6.42	7.72	9.03	11.30	12.67	14.08	15.59	16.97	18.06	19.66	20.74	21.91	23.25	24.69	26.40	28.01

Table 4-31 Average reduction factors (r) for systems with α=1.0 and ξ=10%

Fundamental	μ_T																			
Period (T_{el})	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.17	1.56	1.72	1.86	2.00	2.12	2.23	2.32	2.43	2.51	2.64	2.74	2.85	2.93	3.02	3.12	3.21	3.30	3.39	3.49
0.20	1.35	2.21	2.77	3.28	3.66	3.94	4.23	4.54	4.83	5.19	5.50	5.79	6.15	6.60	6.91	7.30	7.51	7.87	8.10	8.32
0.30	1.39	2.47	3.19	3.72	4.24	4.93	5.46	5.90	6.32	6.75	7.19	7.60	8.09	8.67	9.37	9.90	10.23	10.64	10.95	11.36
0.40	1.43	2.58	3.48	4.19	4.76	5.37	6.15	6.84	7.67	8.71	9.24	9.77	10.41	11.08	11.66	12.24	13.01	13.72	14.28	14.78
0.50	1.57	2.97	3.83	4.65	5.57	6.56	7.43	8.40	9.09	10.01	10.91	11.83	12.70	13.53	14.37	15.25	16.14	17.03	17.83	18.71
0.60	1.45	2.69	3.78	4.81	5.85	6.62	7.56	8.35	9.16	10.23	11.19	11.95	12.66	13.54	14.41	15.35	16.26	17.32	18.30	19.21
0.70	1.49	2.71	4.00	5.09	6.04	7.20	8.13	9.27	10.17	11.19	11.84	12.69	13.51	14.41	15.23	16.32	17.29	18.24	18.85	19.76
0.80	1.44	2.67	3.70	4.86	6.09	7.14	8.09	9.08	9.96	10.65	11.46	12.37	13.34	14.25	15.04	15.71	16.22	16.97	17.65	18.40
0.90	1.44	2.63	3.83	5.02	6.12	7.44	8.33	9.16	10.15	10.98	11.98	12.81	13.52	14.28	15.08	15.90	16.73	17.74	18.89	20.10
1.00	1.40	2.60	3.90	4.90	6.03	7.18	8.03	9.05	10.07	10.76	11.53	12.34	13.17	14.18	15.11	16.17	17.21	18.24	19.22	20.14
1.10	1.44	2.70	3.83	4.82	6.17	7.19	8.21	9.24	10.03	10.95	11.87	12.99	14.39	15.24	16.11	17.04	17.95	19.08	20.08	21.20
1.20	1.48	2.87	3.90	5.04	6.42	7.53	8.66	9.53	10.79	11.98	13.27	14.04	15.03	16.28	17.30	18.33	19.93	20.95	22.35	23.31
1.30	1.47	2.87	3.89	5.17	6.45	7.51	8.53	9.74	11.18	12.31	13.19	14.31	15.88	17.24	18.85	19.59	20.42	21.07	22.03	23.17
1.40	1.48	2.92	4.08	5.34	6.59	7.67	9.06	10.19	11.49	12.37	13.40	14.92	16.65	17.79	18.87	19.80	20.73	21.71	22.82	23.91
1.50	1.45	2.93	4.11	5.34	6.48	7.77	9.09	10.43	11.34	12.25	13.69	15.18	16.30	17.94	18.77	19.74	20.96	22.32	23.58	25.08
1.60	1.42	2.90	4.06	5.18	6.44	7.65	8.84	9.85	10.77	11.93	13.45	14.75	16.12	16.91	18.69	20.28	21.14	22.32	24.12	25.89
1.70	1.42	2.83	3.98	4.93	6.00	7.32	8.42	9.47	10.39	11.71	12.93	14.24	15.41	16.45	17.54	18.83	21.06	23.21	24.22	25.11
1.80	1.40	2.71	3.81	4.82	5.81	6.93	8.39	9.22	9.99	11.38	12.61	13.95	14.96	15.97	17.24	18.69	20.09	22.04	23.39	24.61
1.90	1.38	2.63	3.85	4.82	5.72	6.75	7.96	8.78	9.90	11.14	12.60	13.92	15.03	16.33	17.89	19.11	20.30	21.75	23.36	25.10
2.00	1.36	2.68	4.01	5.02	5.93	6.74	8.00	9.20	10.32	11.81	13.24	14.45	15.67	16.88	18.27	19.51	21.40	22.65	24.16	25.44
2.10	1.34	2.72	4.06	5.21	6.02	7.03	8.34	9.55	10.90	12.35	13.74	14.96	16.25	17.69	19.46	20.69	21.96	23.18	24.85	26.40
2.20	1.33	2.75	4.10	5.18	6.21	7.16	8.53	9.97	11.32	12.86	14.17	15.52	16.96	18.30	19.50	20.72	22.01	23.75	25.62	27.14
2.30	1.33	2.76	3.99	5.25	6.21	7.43	8.76	10.18	11.58	12.97	14.17	15.68	17.12	18.57	19.90	21.25	22.69	24.44	26.09	27.69
2.40	1.31	2.70	3.94	5.30	6.27	7.52	8.87	10.34	11.71	13.01	14.52	15.96	17.39	18.78	20.14	21.62	23.07	24.76	26.59	28.26
2.50	1.31	2.71	3.92	5.21	6.29	7.57	9.01	10.29	11.58	13.00	14.39	15.99	17.51	18.89	20.22	21.60	23.33	25.22	27.12	28.81
2.60	1.32	2.78	3.97	5.18	6.26	7.60	8.85	10.17	11.70	13.07	14.82	16.11	17.59	18.91	20.46	21.94	23.77	25.82	27.47	29.36
2.70	1.33	2.82	3.99	5.10	6.18	7.58	8.84	10.10	11.61	13.17	14.60	15.97	17.53	19.00	20.43	22.09	23.93	25.91	27.64	29.34
2.80	1.32	2.80	3.91	4.97	5.98	7.45	8.74	10.18	11.51	12.98	14.42	15.92	17.50	18.79	20.31	22.09	24.07	25.87	27.29	28.71
2.90	1.32	2.79	3.84	4.84	6.03	7.42	8.92	10.18	11.50	13.12	14.57	15.98	17.42	18.95	20.49	22.25	24.17	25.55	26.92	28.49
3.00	1.35	2.80	3.87	4.82	6.12	7.65	8.98	10.20	11.92	13.28	14.62	16.13	17.66	19.19	20.94	22.56	24.22	25.71	27.29	28.81
3.10	1.38	2.85	3.89	4.98	6.40	7.82	9.05	10.64	12.11	13.55	14.87	16.45	17.98	19.57	21.25	22.97	24.54	26.05	27.59	29.04
3.20	1.40	2.85	3.96	5.07	6.62	7.87	9.37	10.75	12.22	13.74	15.21	16.75	18.42	20.04	22.01	23.53	24.99	26.46	27.98	29.38
3.30	1.40	2.83	4.00	5.18	6.65	8.00	9.42	10.72	12.23	13.87	15.47	17.03	18.66	20.67	22.30	23.78	25.31	26.89	28.15	29.43
3.40	1.40	2.82	3.99	5.15	6.59	7.99	9.37	10.75	12.25	13.80	15.24	17.28	19.13	20.71	22.43	24.04	25.31	26.66	27.98	29.17
3.50	1.38	2.81	3.91	5.15	6.64	7.98	9.30	10.71	12.28	13.75	15.61	17.53	19.20	20.95	22.49	23.93	25.19	26.45	27.61	28.95
3.60	1.38	2.81	3.93	5.19	6.85	8.06	9.39	10.73	12.35	13.94	15.94	17.77	19.44	21.11	22.68	23.99	25.18	26.44	27.93	29.36
3.70	1.39	2.81	3.91	5.27	6.94	8.04	9.38	10.91	12.65	14.15	16.21	18.11	19.65	21.44	22.69	23.99	25.13	26.85	28.34	29.87
3.80	1.38	2.78	3.95	5.35	6.94	8.15	9.43	11.23	12.75	14.44	16.54	18.33	19.88	21.24	22.69	23.98	25.53	27.13	28.79	30.35
3.90	1.38	2.78	4.04	5.43	7.01	8.27	9.64	11.64	13.00	14.85	16.83	18.48	19.96	21.33	22.89	24.29	25.99	27.57	29.25	30.91
4.00	1.38	2.81	4.09	5.61	7.03	8.47	9.81	11.97	13.46	15.38	17.14	18.61	20.04	21.53	23.03	24.62	26.20	27.93	29.76	31.63

Table 4-32 Average reduction factors (r) for systems with α=1.0 and ξ=15%

Fundamental	μ_T																			
Period (T_{el})	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.22	1.63	1.82	1.98	2.16	2.27	2.41	2.49	2.64	2.73	2.84	2.98	3.07	3.22	3.33	3.46	3.58	3.69	3.82	3.95
0.20	1.49	2.40	3.00	3.58	3.95	4.36	4.73	5.12	5.53	5.83	6.20	6.69	7.15	7.52	7.83	8.24	8.57	8.91	9.15	9.37
0.30	1.53	2.75	3.50	4.09	4.77	5.56	5.99	6.56	7.17	7.54	8.08	8.66	9.43	10.01	10.50	10.90	11.31	11.80	12.34	12.78
0.40	1.61	2.95	3.90	4.68	5.35	6.17	6.95	7.92	8.96	9.75	10.35	10.98	11.66	12.45	13.24	14.01	14.76	15.71	16.47	17.20
0.50	1.82	3.36	4.31	5.31	6.38	7.59	8.44	9.39	10.41	11.47	12.46	13.48	14.37	15.30	16.49	17.53	18.38	19.17	19.95	20.85
0.60	1.65	2.99	4.27	5.48	6.56	7.50	8.44	9.64	10.76	11.62	12.70	13.92	15.08	16.25	17.25	18.38	19.64	20.36	21.47	22.35
0.70	1.67	3.02	4.47	5.67	6.85	8.04	9.38	10.75	11.83	12.80	13.65	14.51	15.68	16.89	17.84	18.92	20.03	20.80	21.57	22.47
0.80	1.61	3.02	4.21	5.55	6.94	7.97	9.05	10.21	11.15	11.95	12.95	14.05	15.52	16.25	17.14	17.99	18.87	19.69	20.48	21.47
0.90	1.60	3.03	4.30	5.54	6.90	8.24	9.31	10.28	11.31	12.35	13.46	14.46	15.34	16.23	17.09	17.99	18.92	19.80	21.02	22.35
1.00	1.56	3.04	4.24	5.55	6.81	7.95	8.91	10.01	11.02	11.92	12.79	13.67	14.72	15.99	17.11	18.40	19.59	20.56	21.50	22.46
1.10	1.58	3.08	4.23	5.54	6.74	7.97	9.26	10.22	11.12	12.06	13.11	14.48	15.53	16.68	17.73	19.07	20.54	21.79	22.96	24.14
1.20	1.66	3.20	4.27	5.75	7.06	8.42	9.69	10.76	12.09	13.17	14.37	15.60	17.05	18.24	20.01	21.04	22.09	23.31	24.78	26.44
1.30	1.64	3.18	4.32	5.79	7.22	8.36	9.44	10.96	12.25	13.50	14.81	16.19	17.48	18.81	19.83	21.51	23.37	24.64	25.63	26.85
1.40	1.66	3.28	4.56	6.00	7.28	8.45	9.89	11.52	12.66	13.58	15.28	17.14	18.75	20.07	21.06	22.14	23.30	24.67	26.44	28.01
1.50	1.65	3.29	4.54	5.83	7.13	8.57	10.22	11.45	12.43	13.79	15.61	17.22	18.64	19.82	20.95	22.07	23.13	24.47	25.86	27.40
1.60	1.56	3.24	4.41	5.69	7.07	8.48	9.63	10.82	11.95	13.42	14.97	16.47	17.90	19.11	20.10	21.27	22.54	23.96	25.40	26.96
1.70	1.56	3.10	4.37	5.39	6.86	8.21	9.29	10.30	11.71	13.18	14.49	16.25	17.62	18.89	20.23	21.49	22.86	24.38	25.96	27.17
1.80	1.56	2.97	4.24	5.20	6.40	7.88	9.14	10.09	11.37	12.65	14.14	15.74	17.44	18.43	19.76	21.38	23.28	25.05	26.28	27.47
1.90	1.50	2.87	4.16	5.22	6.24	7.88	8.87	10.01	11.33	12.68	14.28	15.44	16.95	18.87	20.64	22.36	23.86	25.02	26.30	27.54
2.00	1.49	2.93	4.31	5.37	6.39	7.84	8.93	10.23	11.64	13.20	14.87	16.04	17.49	19.39	21.16	22.63	24.02	25.49	27.16	28.95
2.10	1.47	2.94	4.33	5.51	6.55	7.85	9.23	10.57	12.20	13.86	15.24	16.73	18.25	19.87	21.62	23.11	24.86	26.49	28.14	29.74
2.20	1.46	2.98	4.46	5.62	6.71	7.92	9.46	11.01	12.69	14.33	15.69	17.24	18.73	20.55	22.50	24.00	25.49	27.23	28.99	30.71
2.30	1.44	2.99	4.44	5.75	6.73	8.14	9.79	11.41	13.12	14.67	16.24	17.72	19.58	21.01	22.77	24.19	26.00	27.57	29.38	31.09
2.40	1.43	2.97	4.38	5.72	6.82	8.32	10.03	11.68	13.29	14.88	16.95	18.19	19.56	20.95	22.58	24.44	26.40	27.93	29.89	31.38
2.50	1.42	2.97	4.38	5.66	6.89	8.47	10.19	11.95	13.41	15.02	16.62	18.15	19.52	21.15	22.79	25.02	26.85	28.34	30.00	31.38
2.60	1.44	3.01	4.46	5.67	6.89	8.59	10.32	11.86	13.52	15.09	16.63	18.19	19.70	21.43	23.24	25.62	27.16	28.81	30.13	31.38
2.70	1.45	3.01	4.37	5.56	6.86	8.51	10.00	11.75	13.29	14.86	16.42	17.96	19.65	21.32	23.34	25.46	27.00	28.34	29.62	31.01
2.80	1.45	3.02	4.25	5.41	6.79	8.30	9.70	11.45	12.97	14.58	16.12	17.75	19.40	20.94	23.34	25.01	26.24	27.55	29.02	30.41
2.90	1.44	2.99	4.24	5.36	6.73	8.24	9.79	11.16	12.72	14.43	16.10	17.69	19.24	20.96	23.03	24.36	25.76	27.29	28.59	30.37
3.00	1.45	2.97	4.21	5.31	6.84	8.46	9.90	11.20	12.73	14.36	16.18	17.79	19.32	21.23	22.89	24.21	25.83	27.38	29.03	30.67
3.10	1.51	3.00	4.23	5.38	6.98	8.49	9.85	11.28	13.12	14.76	16.40	17.88	19.61	21.34	23.07	24.72	26.44	28.06	29.83	31.43
3.20	1.50	3.04	4.22	5.48	7.22	8.51	9.99	11.71	13.39	15.05	16.51	18.02	20.01	21.70	23.44	24.93	26.52	28.63	30.43	31.78
3.30	1.52	3.00	4.26	5.66	7.23	8.54	10.15	11.66	13.41	14.92	16.48	18.32	20.34	22.06	23.40	24.73	26.46	28.26	30.16	31.49
3.40	1.51	2.97	4.21	5.65	7.11	8.56	9.98	11.61	13.27	14.82	16.53	18.74	20.46	21.85	23.19	24.52	26.15	27.93	29.60	31.13
3.50	1.49	2.95	4.21	5.67	7.04	8.44	9.88	11.55	13.17	14.95	16.98	18.80	20.24	21.62	22.95	24.43	25.97	27.62	29.37	31.18
3.60	1.50	2.96	4.25	5.76	7.17	8.44	9.89	11.62	13.27	15.35	17.21	18.91	20.34	21.67	23.23	24.73	26.24	27.81	29.54	31.21
3.70	1.51	2.97	4.20	5.78	7.26	8.56	10.09	11.78	13.65	15.43	17.39	19.04	20.50	22.03	23.48	25.02	26.63	28.29	29.92	31.53
3.80	1.51	2.99	4.19	5.80	7.37	8.72	10.19	12.15	13.75	15.35	17.53	19.16	20.89	22.40	23.82	25.39	27.00	28.81	30.49	32.25
3.90	1.52	2.98	4.22	5.86	7.46	8.91	10.52	12.36	13.74	15.66	17.89	19.63	21.11	22.63	24.17	25.77	27.93	29.54	31.38	33.02
4.00	1.52	3.02	4.32	6.06	7.61	9.21	10.86	12.53	13.98	16.05	18.11	19.91	21.51	23.23	24.70	26.61	28.66	30.43	32.18	33.77

Table 4-33 Average reduction factors (*r*) for systems with α=1.0 and ξ=20%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.28	1.70	1.92	2.11	2.26	2.42	2.56	2.69	2.81	2.96	3.08	3.25	3.39	3.55	3.67	3.82	3.94	4.13	4.21	4.37
0.20	1.61	2.61	3.22	3.86	4.31	4.80	5.20	5.64	6.06	6.52	7.14	7.57	8.01	8.43	8.77	9.04	9.31	9.58	9.91	10.24
0.30	1.67	3.03	3.92	4.46	5.38	6.00	6.67	7.37	7.89	8.43	9.00	9.93	10.52	10.97	11.54	12.10	13.00	13.78	14.53	15.16
0.40	1.81	3.29	4.28	5.15	5.98	6.94	7.91	9.01	9.95	10.70	11.45	12.24	13.14	13.96	14.94	15.80	16.80	17.83	18.91	19.73
0.50	2.05	3.73	4.77	5.80	7.29	8.42	9.40	10.67	11.63	12.78	14.37	15.38	16.46	17.49	18.46	19.38	20.42	21.50	22.58	23.58
0.60	1.86	3.50	4.69	6.06	7.40	8.30	9.64	11.02	12.04	13.41	14.68	15.84	16.98	18.24	19.30	20.26	21.53	22.82	23.97	24.85
0.70	1.84	3.40	4.93	6.27	7.61	9.18	10.75	11.86	13.05	14.05	15.29	16.57	18.30	19.84	20.75	21.71	22.95	24.08	25.12	26.08
0.80	1.77	3.31	4.70	6.22	7.66	8.82	10.25	11.26	12.44	13.81	14.94	16.06	17.24	18.19	19.25	20.35	21.38	22.49	23.72	24.98
0.90	1.76	3.41	4.72	6.19	7.73	9.19	10.20	11.46	12.44	13.63	14.85	15.97	16.93	17.89	19.11	20.60	21.70	22.88	24.19	25.36
1.00	1.71	3.34	4.65	6.05	7.45	8.82	9.84	11.05	12.33	13.30	14.26	15.62	16.71	17.77	18.92	20.30	21.63	22.68	23.85	25.13
1.10	1.73	3.35	4.67	6.22	7.37	8.68	9.97	11.12	12.20	13.19	14.62	15.92	17.37	18.57	19.82	21.21	22.64	24.04	25.20	26.33
1.20	1.83	3.49	4.70	6.36	7.78	9.38	10.78	11.77	13.06	14.39	15.83	17.46	19.00	20.43	21.80	22.94	24.25	25.64	27.14	28.70
1.30	1.82	3.51	4.84	6.49	7.92	9.34	10.51	11.83	13.48	14.76	16.35	17.98	19.24	20.68	22.58	24.08	25.62	27.13	28.68	30.24
1.40	1.83	3.54	5.01	6.67	8.00	9.27	10.80	12.36	13.62	15.17	16.96	18.44	19.66	21.11	23.31	25.11	26.52	28.06	29.46	30.78
1.50	1.81	3.57	4.99	6.38	7.81	9.39	10.93	12.36	13.69	15.36	17.31	18.93	20.47	21.91	23.27	24.74	26.50	28.04	29.63	31.38
1.60	1.74	3.49	4.82	6.19	7.70	9.15	10.57	11.74	13.28	15.08	16.72	18.21	19.48	20.84	22.20	23.47	24.88	26.57	28.68	30.00
1.70	1.71	3.37	4.73	6.04	7.44	8.91	10.13	11.49	13.13	14.55	16.16	17.46	18.86	20.28	21.64	23.06	24.49	25.92	27.47	29.08
1.80	1.66	3.19	4.59	5.65	7.13	8.60	9.90	11.32	12.67	14.20	16.11	17.46	18.80	20.20	21.53	22.90	24.39	25.94	27.29	28.84
1.90	1.61	3.14	4.51	5.63	6.95	8.48	9.75	11.02	12.52	14.32	16.12	17.52	18.83	20.25	21.76	23.38	24.84	26.49	28.27	30.62
2.00	1.61	3.19	4.65	5.77	7.10	8.74	10.13	11.45	13.00	14.43	16.04	17.61	19.32	20.99	22.86	24.41	26.20	28.42	30.05	31.64
2.10	1.58	3.21	4.69	5.87	7.30	8.79	10.20	11.70	13.28	14.95	16.87	18.34	19.92	21.81	23.60	25.49	27.49	29.29	31.60	33.41
2.20	1.57	3.19	4.70	6.17	7.41	8.85	10.45	12.13	13.75	15.44	17.11	18.93	20.65	22.48	24.32	26.38	28.34	30.20	31.79	33.54
2.30	1.56	3.20	4.80	6.20	7.41	8.95	10.78	12.47	14.29	15.85	17.49	19.56	21.30	23.19	25.03	26.78	28.56	30.43	32.21	33.77
2.40	1.55	3.24	4.82	6.07	7.41	9.22	11.10	12.96	14.68	16.36	18.03	20.10	21.92	23.63	25.41	27.14	28.76	30.60	32.32	33.85
2.50	1.53	3.27	4.84	6.05	7.50	9.40	11.30	13.20	14.95	16.70	18.48	20.26	21.95	23.91	25.53	27.03	28.73	30.50	32.25	33.68
2.60	1.55	3.27	4.86	6.11	7.56	9.56	11.56	13.20	15.19	16.89	18.52	20.41	22.04	24.08	25.66	27.28	28.96	30.51	32.25	33.78
2.70	1.55	3.24	4.78	5.98	7.55	9.45	11.38	13.20	14.94	16.73	18.39	20.23	21.86	23.90	25.36	26.87	28.47	30.11	31.60	33.29
2.80	1.54	3.20	4.61	5.86	7.43	9.23	11.16	12.94	14.56	16.34	18.13	19.75	21.47	23.30	24.78	26.26	27.92	29.33	30.93	32.78
2.90	1.55	3.16	4.52	5.80	7.39	9.14	11.00	12.66	14.28	16.17	17.90	19.51	21.35	22.86	24.38	25.92	27.45	29.04	30.80	32.63
3.00	1.57	3.18	4.48	5.82	7.43	9.20	10.88	12.64	14.21	16.16	17.84	19.48	21.28	22.77	24.29	25.92	27.53	29.19	31.01	32.80
3.10	1.59	3.20	4.52	5.85	7.56	9.46	10.92	12.67	14.39	16.21	17.97	19.60	21.42	22.99	24.62	26.31	28.10	29.74	31.54	33.43
3.20	1.61	3.22	4.56	5.94	7.78	9.41	10.97	12.84	14.55	16.35	18.00	19.72	21.48	23.25	24.91	26.60	28.51	30.27	32.05	33.87
3.30	1.62	3.21	4.49	6.04	7.78	9.35	11.00	12.87	14.60	16.24	17.85	19.81	21.51	23.19	24.84	26.87	28.81	30.62	32.30	34.08
3.40	1.61	3.18	4.46	6.03	7.66	9.33	10.94	12.80	14.45	15.94	17.75	19.83	21.38	23.01	24.71	26.54	28.68	30.33	32.14	34.01
3.50	1.60	3.13	4.46	6.01	7.57	9.22	10.91	12.60	14.23	15.83	17.90	19.74	21.29	22.79	24.38	26.24	28.18	30.15	32.28	33.80
3.60	1.59	3.14	4.52	6.11	7.67	9.23	10.91	12.54	14.24	16.18	18.18	19.86	21.39	22.75	24.45	26.28	28.26	30.12	32.28	34.20
3.70	1.60	3.14	4.50	6.16	7.70	9.19	10.93	12.58	14.45	16.44	18.42	20.09	21.53	22.89	24.56	26.57	28.35	30.10	32.21	34.31
3.80	1.61	3.14	4.52	6.26	7.77	9.35	10.91	12.74	14.67	16.59	18.60	20.15	21.62	23.05	24.95	26.73	28.37	30.10	32.21	34.31
3.90	1.62	3.18	4.54	6.32	7.92	9.47	11.13	13.04	14.96	16.88	18.78	20.32	21.91	23.51	25.46	26.99	28.60	30.37	32.31	34.50
4.00	1.63	3.21	4.65	6.54	8.11	9.57	11.55	13.36	15.15	17.15	19.00	20.64	22.35	24.13	25.79	27.42	29.04	30.68	32.81	34.91

Table 4-34 Average reduction factors (*r*) for systems with α=1.0 and ξ=25%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.31	1.77	2.03	2.22	2.38	2.56	2.71	2.85	3.04	3.18	3.35	3.54	3.68	3.84	3.99	4.11	4.30	4.48	4.67	4.81
0.20	1.73	2.76	3.54	4.15	4.65	5.19	5.65	6.10	6.70	7.34	7.92	8.36	8.76	9.16	9.44	9.75	10.19	10.50	10.96	11.45
0.30	1.81	3.31	4.20	4.90	5.87	6.62	7.44	8.05	8.69	9.39	10.25	10.97	11.56	12.05	13.10	13.97	14.97	15.59	16.37	17.24
0.40	2.00	3.55	4.72	5.62	6.74	7.77	8.80	9.92	10.75	11.64	12.38	13.25	14.24	15.32	16.46	17.59	18.68	19.84	20.99	21.94
0.50	2.22	4.12	5.21	6.39	7.95	9.18	10.63	11.69	12.72	14.35	16.17	17.27	18.42	19.40	20.50	21.73	22.98	24.35	25.51	26.82
0.60	2.04	3.89	5.15	6.51	7.98	9.06	10.73	12.26	13.46	14.97	16.07	17.34	18.38	19.49	20.78	22.22	23.40	24.66	26.13	27.29
0.70	2.02	3.84	5.36	6.83	8.43	10.41	11.84	13.18	14.38	15.36	16.97	18.50	20.03	21.49	22.70	24.24	25.56	27.00	28.10	29.33
0.80	1.92	3.61	5.25	6.85	8.45	9.94	11.15	12.36	13.93	15.50	16.52	17.63	19.08	20.02	21.31	22.96	24.29	25.69	27.05	28.41
0.90	1.94	3.75	5.18	6.90	8.55	9.91	11.26	12.60	13.73	15.10	16.46	17.74	18.97	20.21	21.75	23.33	24.71	25.96	27.28	28.66
1.00	1.86	3.62	5.10	6.62	8.20	9.57	10.88	11.96	13.46	14.72	16.01	17.25	18.53	19.89	21.11	22.45	24.00	25.58	27.05	28.42
1.10	1.89	3.63	5.15	6.72	7.99	9.41	10.84	12.15	13.48	14.91	16.11	17.45	18.93	20.42	21.81	23.32	24.79	26.53	28.03	29.39
1.20	1.97	3.77	5.17	6.87	8.54	10.12	11.54	12.78	14.14	16.18	17.58	19.19	20.72	22.11	23.92	25.45	26.73	28.28	29.71	31.41
1.30	2.00	3.82	5.31	7.05	8.66	10.38	11.57	13.21	14.72	16.52	18.10	19.67	21.16	22.56	24.05	25.56	27.21	29.18	30.80	32.59
1.40	2.01	3.87	5.50	7.27	8.68	10.14	11.85	13.43	15.09	16.87	18.41	19.71	21.20	23.28	25.46	27.38	29.25	31.08	32.74	34.25
1.50	1.98	3.85	5.47	7.01	8.49	10.31	11.98	13.35	15.00	16.82	18.42	20.04	21.68	24.08	25.99	28.00	29.83	31.41	33.03	34.55
1.60	1.90	3.71	5.22	6.67	8.32	9.85	11.40	12.91	14.70	16.47	18.06	19.68	21.25	23.04	24.81	26.40	28.22	29.79	31.38	33.03
1.70	1.86	3.59	5.06	6.60	8.13	9.54	11.00	12.64	14.43	16.13	17.63	19.07	20.53	22.06	23.60	25.23	27.16	29.25	30.99	32.38
1.80	1.77	3.45	4.99	6.33	7.82	9.36	10.77	12.53	14.00	15.79	17.28	18.56	19.95	21.44	22.99	24.50	26.11	28.00	29.85	31.53
1.90	1.72	3.39	4.92	6.19	7.79	9.24	10.55	12.14	13.85	15.85	17.24	18.63	20.05	21.58	23.08	24.64	26.40	28.04	29.68	31.69
2.00	1.73	3.47	5.02	6.27	7.88	9.36	10.88	12.41	14.49	16.38	18.06	19.40	20.92	22.41	24.04	25.73	27.42	29.43	31.79	34.07
2.10	1.71	3.48	5.08	6.47	7.98	9.74	11.27	12.92	14.68	16.29	18.41	20.01	21.54	23.17	24.78	26.64	28.90	31.49	33.92	36.06
2.20	1.69	3.46	5.10	6.57	8.13	9.89	11.42	13.08	14.91	16.75	18.33	20.28	22.00	23.71	25.59	28.23	30.74	32.95	35.09	36.76
2.30	1.68	3.45	5.15	6.64	8.14	9.95	11.68	13.42	15.17	17.24	18.78	20.65	22.45	24.68	27.06	29.54	31.79	33.54	35.13	36.86
2.40	1.66	3.46	5.15	6.58	8.15	10.14	11.95	13.87	15.68	17.54	19.35	21.38	23.61	25.51	27.61	29.63	31.78	33.46	35.09	36.89
2.50	1.66	3.49	5.10	6.49	8.09	10.34	12.31	14.36	16.15	18.17	20.17	22.17	23.99	25.99	27.73	29.64	31.63	33.30	35.03	36.99
2.60	1.67	3.52	5.16	6.51	8.23	10.55	12.62	14.58	16.63	18.65	20.54	22.26	24.28	26.05	27.95	29.66	31.78	33.46	35.21	37.34
2.70	1.67	3.48	5.10	6.36	8.17	10.43	12.37	14.41	16.49	18.36	20.21	21.94	23.93	25.65	27.47	29.33	31.12	32.91	34.86	37.01
2.80	1.65	3.41	4.92	6.27	8.02	10.08	12.02	14.14	16.11	18.04	19.74	21.37	23.34	25.11	26.90	28.66	30.21	32.07	34.16	36.23
2.90	1.64	3.36	4.85	6.23	7.95	9.92	11.98	13.74	15.88	17.69	19.36	21.22	22.96	24.67	26.40	27.98	29.71	31.59	33.56	35.51
3.00	1.66	3.34	4.84	6.28	7.98	9.99	11.85	13.69	15.75	17.65	19.31	21.10	22.81	24.59	26.11	27.80	29.67	31.54	33.54	35.24
3.10	1.68	3.37	4.86	6.37	8.17	10.14	11.96	13.77	15.93	17.70	19.36	21.36	23.06	24.79	26.34	28.22	30.12	31.93	33.85	35.63
3.20	1.72	3.39	4.88	6.47	8.41	10.25	12.04	14.00	15.94	17.73	19.51	21.41	23.14	24.94	26.64	28.55	30.43	32.27	34.21	36.01
3.30	1.72	3.41	4.85	6.53	8.40	10.26	12.11	13.97	15.81	17.57	19.45	21.33	23.21	24.91	26.68	28.65	30.60	32.39	34.25	36.09
3.40	1.69	3.36	4.80	6.52	8.32	10.18	11.96	13.92	15.61	17.32	19.26	21.17	22.89	24.56	26.47	28.49	30.48	32.27	34.12	36.03
3.50	1.67	3.32	4.84	6.48	8.26	10.12	11.94	13.70	15.33	17.10	19.16	20.95	22.62	24.34	26.36	28.43	30.32	32.02	33.76	35.59
3.60	1.69	3.29	4.85	6.60	8.33	10.24	11.93	13.68	15.28	17.10	19.16	20.92	22.54	24.42	26.60	28.71	30.50	32.21	33.85	35.64
3.70	1.69	3.30	4.90	6.64	8.32	10.16	11.81	13.66	15.47	17.28	19.39	21.00	22.74	24.68	26.90	28.72	30.67	32.38	34.20	35.94
3.80	1.69	3.29	4.94	6.69	8.34	10.13	11.79	13.78	15.50	17.61	19.48	21.14	22.90	24.84	26.81	28.74	30.78	32.81	34.41	36.23
3.90	1.72	3.36	5.00	6.81	8.39	10.18	11.92	13.87	15.71	17.89	19.73	21.48	23.26	25.33	26.98	28.84	30.80	33.13	35.09	36.75
4.00	1.73	3.39	5.09	6.94	8.57	10.26	12.17	14.14	15.98	18.21	20.02	21.71	23.73	25.55	27.28	29.17	31.14	33.49	35.51	37.34

Table 4-35 Average reduction factors (r) for systems with α=1.0 and ξ=30%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.34	1.83	2.12	2.32	2.51	2.70	2.86	3.05	3.26	3.43	3.60	3.81	3.95	4.11	4.27	4.43	4.64	4.85	5.02	5.28
0.20	1.84	2.98	3.78	4.39	5.03	5.59	6.02	6.57	7.35	8.03	8.53	9.01	9.49	9.89	10.41	10.70	11.19	11.67	12.20	12.82
0.30	1.96	3.54	4.48	5.40	6.37	7.32	8.06	8.74	9.58	10.36	11.17	11.79	12.45	13.84	14.84	15.56	16.37	17.42	18.08	18.54
0.40	2.17	3.86	5.10	6.20	7.42	8.51	9.64	10.72	11.55	12.50	13.44	14.41	15.61	16.80	18.01	19.27	20.52	21.80	23.04	24.36
0.50	2.39	4.44	5.69	7.20	8.62	10.18	11.67	12.65	14.14	15.90	17.54	18.92	19.97	21.30	22.71	24.31	25.96	27.34	28.96	30.35
0.60	2.24	4.20	5.57	7.06	8.54	10.40	11.63	13.36	15.03	16.20	17.63	19.05	20.19	21.56	23.21	24.46	26.03	27.31	28.69	30.05
0.70	2.19	4.18	5.81	7.48	9.52	11.40	12.94	14.38	15.54	17.33	18.86	20.41	21.99	23.35	24.80	26.36	28.00	29.46	31.06	32.58
0.80	2.06	4.00	5.79	7.42	9.40	10.83	12.02	13.55	15.13	16.50	17.97	19.65	21.18	22.58	23.96	25.48	27.06	28.64	30.20	31.62
0.90	2.10	4.00	5.68	7.59	9.25	10.99	12.43	13.70	14.99	16.54	17.97	19.65	21.04	22.38	23.84	25.45	27.20	28.91	30.35	31.80
1.00	2.01	3.90	5.58	7.24	8.89	10.46	11.87	13.12	14.92	16.28	17.61	18.91	20.43	21.99	23.60	25.14	26.72	28.26	29.78	31.47
1.10	2.03	3.85	5.58	7.17	8.73	10.34	11.99	13.45	14.79	16.30	17.75	19.21	20.77	22.37	24.09	25.76	27.42	29.13	30.92	32.54
1.20	2.13	4.09	5.75	7.43	9.21	10.72	12.31	14.01	15.67	17.49	19.22	20.78	22.38	24.13	25.94	27.58	29.30	30.97	32.66	34.42
1.30	2.14	4.12	5.77	7.63	9.28	10.90	12.52	14.11	16.25	17.85	19.52	21.36	23.09	24.80	26.57	28.16	29.88	31.65	33.58	35.49
1.40	2.18	4.17	6.00	7.78	9.41	11.22	12.78	14.62	16.57	18.28	19.97	21.64	23.34	25.01	26.83	28.79	31.00	33.12	35.09	37.06
1.50	2.16	4.15	5.97	7.69	9.34	11.19	12.93	14.62	16.60	18.22	19.75	21.41	23.34	25.63	28.01	30.13	32.28	34.35	36.28	38.04
1.60	2.06	3.99	5.60	7.27	8.95	10.65	12.23	14.11	15.90	17.51	19.17	21.07	23.34	25.56	27.57	29.83	31.60	33.43	35.28	37.13
1.70	1.98	3.83	5.44	6.97	8.74	10.26	11.80	13.77	15.63	17.08	18.81	20.58	22.35	24.30	26.23	28.02	29.89	31.63	33.54	35.24
1.80	1.91	3.67	5.39	7.08	8.51	10.08	11.79	13.66	15.41	17.11	18.75	20.32	22.07	23.64	25.37	27.38	29.46	31.14	33.02	34.73
1.90	1.83	3.61	5.27	6.88	8.48	9.95	11.57	13.27	15.45	17.04	18.50	19.98	21.62	23.30	24.91	26.57	28.68	30.75	32.87	34.89
2.00	1.84	3.72	5.43	6.82	8.56	10.07	11.71	13.60	15.92	17.44	18.94	20.46	22.07	23.77	25.55	27.40	29.47	31.55	33.57	35.74
2.10	1.85	3.74	5.49	7.00	8.66	10.35	12.06	14.35	16.32	17.94	19.40	20.92	22.55	24.44	26.39	28.54	30.54	32.78	35.17	37.62
2.20	1.80	3.71	5.47	7.09	8.71	10.75	12.55	14.48	16.38	18.19	19.84	21.37	23.20	25.20	27.29	29.55	31.99	34.45	36.62	38.48
2.30	1.79	3.68	5.48	7.13	8.95	10.83	12.67	14.60	16.53	18.29	20.05	21.84	23.81	25.99	28.17	30.75	33.25	35.35	37.27	39.19
2.40	1.78	3.68	5.49	7.09	9.07	10.94	12.86	14.84	16.82	18.65	20.41	22.43	24.59	26.85	29.24	31.72	33.78	35.92	37.65	39.32
2.50	1.77	3.76	5.49	7.01	9.00	11.19	13.21	15.17	17.24	19.00	21.01	23.11	25.20	27.59	29.96	32.10	34.13	35.98	37.63	39.42
2.60	1.79	3.77	5.50	7.02	8.91	11.39	13.51	15.44	17.61	19.83	21.73	23.74	25.96	28.35	30.47	32.39	34.26	36.01	37.74	39.60
2.70	1.77	3.73	5.40	6.91	8.91	11.30	13.32	15.37	17.84	19.95	21.78	23.79	25.95	27.96	29.95	31.78	33.63	35.34	37.18	39.11
2.80	1.75	3.61	5.25	6.74	8.55	11.03	13.09	15.18	17.56	19.48	21.30	23.42	25.24	27.15	29.10	30.93	32.68	34.39	36.09	38.02
2.90	1.74	3.54	5.15	6.64	8.42	10.77	13.06	14.97	17.31	19.15	20.91	22.98	24.82	26.61	28.45	30.45	32.21	33.83	35.59	37.31
3.00	1.76	3.55	5.12	6.66	8.54	10.78	12.89	15.03	17.21	19.02	20.95	22.87	24.73	26.47	28.28	30.24	31.93	33.65	35.35	37.02
3.10	1.81	3.55	5.17	6.79	8.75	11.03	12.87	15.12	17.24	19.17	21.16	23.03	24.80	26.57	28.53	30.48	32.13	33.90	35.59	37.33
3.20	1.83	3.61	5.19	6.97	8.93	10.98	12.86	15.23	17.23	19.20	21.27	23.08	24.91	26.77	28.83	30.67	32.42	34.17	35.85	37.71
3.30	1.81	3.55	5.18	7.05	8.97	10.97	12.96	15.05	17.09	18.96	21.13	23.03	24.83	26.75	28.77	30.56	32.38	34.12	35.86	37.75
3.40	1.80	3.53	5.15	7.04	8.95	10.91	12.92	14.87	16.84	18.75	20.83	22.70	24.54	26.55	28.47	30.26	32.11	33.86	35.85	37.78
3.50	1.78	3.48	5.20	6.98	8.89	10.80	12.82	14.68	16.51	18.58	20.58	22.45	24.28	26.31	28.18	30.07	31.83	33.75	35.67	37.65
3.60	1.76	3.45	5.20	6.99	8.98	10.80	12.78	14.59	16.48	18.63	20.55	22.38	24.29	26.36	28.16	29.94	31.97	33.91	35.97	37.91
3.70	1.79	3.45	5.28	7.08	8.99	10.85	12.78	14.63	16.47	18.85	20.60	22.36	24.30	26.44	28.12	30.09	32.12	34.01	36.03	38.05
3.80	1.82	3.47	5.26	7.14	9.00	10.84	12.77	14.69	16.51	18.87	20.63	22.46	24.34	26.45	28.34	30.47	32.36	34.20	36.21	38.14
3.90	1.80	3.57	5.36	7.22	9.11	10.87	12.88	14.78	16.70	19.00	20.77	22.62	24.53	26.68	28.75	30.80	32.81	34.58	36.47	38.46
4.00	1.81	3.61	5.44	7.29	9.20	11.03	13.02	15.00	17.07	19.11	21.11	22.99	25.00	27.07	29.23	31.41	33.21	35.17	37.01	38.93

Table 4-36 Average reduction factors (*r*) for systems with α=1.0 and ξ=35%

Fundamental Period (T_{el})	μ_T 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1.37	1.90	2.20	2.44	2.64	2.85	3.04	3.28	3.46	3.65	3.87	4.04	4.24	4.40	4.60	4.78	4.99	5.25	5.53	5.72
0.20	1.96	3.18	4.03	4.72	5.37	5.92	6.44	7.21	8.02	8.54	9.10	9.73	10.12	10.53	11.11	11.73	12.28	12.80	13.38	14.01
0.30	2.12	3.75	4.83	5.82	7.09	7.96	8.61	9.50	10.37	11.22	11.95	12.70	14.28	15.28	16.16	17.07	18.08	18.85	19.39	20.05
0.40	2.32	4.19	5.51	6.80	8.03	9.14	10.38	11.46	12.47	13.46	14.61	15.88	17.34	18.75	19.80	21.12	22.84	24.43	25.85	27.18
0.50	2.59	4.85	6.26	7.72	9.31	11.30	12.45	13.88	15.55	17.16	18.93	20.33	21.92	23.51	25.29	27.16	28.90	30.65	31.84	33.18
0.60	2.42	4.55	5.98	7.59	9.25	10.99	12.65	14.63	16.20	17.53	19.38	20.99	22.41	24.12	25.72	27.38	28.52	29.94	31.51	33.07
0.70	2.38	4.48	6.30	8.06	10.39	12.23	13.80	15.53	16.98	18.77	20.32	22.01	23.59	25.30	26.82	28.59	30.38	32.06	33.74	35.39
0.80	2.24	4.31	6.13	8.03	10.19	11.68	13.12	14.63	16.16	17.59	19.17	21.24	22.65	24.59	26.22	28.10	29.98	31.49	33.12	34.63
0.90	2.26	4.25	6.12	8.25	9.98	11.59	13.09	14.72	16.36	18.07	19.47	21.24	23.07	24.81	26.48	28.42	30.16	31.63	33.16	34.88
1.00	2.17	4.10	6.08	7.78	9.63	11.42	12.79	14.11	15.87	17.60	19.16	20.60	22.32	24.01	25.90	27.73	29.61	31.40	33.12	34.78
1.10	2.18	4.13	6.09	7.70	9.57	11.24	13.01	14.58	16.13	17.75	19.46	21.07	22.82	24.54	26.27	28.05	29.92	31.94	34.09	35.98
1.20	2.30	4.39	6.29	7.98	9.79	11.54	13.53	15.08	17.20	18.88	20.56	22.50	24.33	26.28	28.26	30.16	32.12	34.05	36.02	38.22
1.30	2.31	4.41	6.32	8.09	9.83	11.66	13.37	15.64	17.55	19.22	21.14	23.05	24.90	26.71	28.52	30.44	32.33	34.25	36.33	38.58
1.40	2.34	4.48	6.45	8.23	10.04	11.92	13.86	16.02	17.81	19.69	21.70	23.49	25.44	27.54	29.47	31.40	33.50	35.70	37.85	40.25
1.50	2.31	4.42	6.32	8.26	10.21	12.09	13.81	16.12	17.86	19.67	21.49	23.39	25.33	27.39	29.57	31.95	34.40	36.55	38.73	41.08
1.60	2.20	4.24	6.06	7.84	9.71	11.49	13.28	15.39	17.23	18.75	20.48	22.47	24.69	27.12	29.33	31.49	33.77	35.94	37.86	40.12
1.70	2.12	4.09	5.84	7.45	9.33	10.99	12.94	14.74	16.60	18.33	20.28	22.37	24.64	26.85	28.90	31.09	33.08	35.22	37.15	39.46
1.80	2.03	3.93	5.75	7.37	9.08	10.75	12.56	14.51	16.31	17.91	19.80	21.70	23.62	25.99	27.93	29.92	31.79	34.00	36.36	38.14
1.90	1.98	3.91	5.64	7.55	9.03	10.72	12.63	14.49	16.46	18.09	19.93	21.78	23.71	25.54	27.51	29.34	31.22	33.80	35.53	37.29
2.00	1.97	4.00	5.78	7.38	9.14	10.93	12.65	14.89	16.83	18.78	20.50	22.27	24.20	25.99	27.77	30.05	32.85	34.62	36.69	38.80
2.10	1.96	4.00	5.91	7.45	9.26	11.14	13.02	15.28	17.03	18.82	20.61	22.36	24.30	26.23	28.51	31.41	33.12	34.89	37.04	39.35
2.20	1.95	3.99	5.86	7.54	9.39	11.40	13.40	15.53	17.14	19.00	20.76	22.59	24.58	27.14	29.33	31.19	33.32	35.59	38.18	40.16
2.30	1.92	3.95	5.81	7.61	9.57	11.64	13.76	15.68	17.48	19.27	21.21	23.08	25.56	27.37	29.38	31.78	34.31	36.78	38.64	40.77
2.40	1.91	3.93	5.81	7.63	9.73	11.80	13.85	15.94	17.76	19.58	21.72	23.73	25.64	27.66	30.22	32.89	35.09	37.09	39.11	41.29
2.50	1.90	3.95	5.84	7.58	9.84	11.96	14.04	16.21	18.07	20.13	22.05	24.03	26.07	28.47	31.17	33.35	35.45	37.65	39.90	42.14
2.60	1.92	3.97	5.82	7.59	9.80	12.15	14.23	16.48	18.54	20.54	22.48	24.54	26.82	29.46	31.75	33.90	36.16	38.32	40.07	41.96
2.70	1.87	3.95	5.68	7.40	9.61	12.10	14.12	16.49	18.64	20.58	22.49	24.65	27.24	29.46	31.60	33.80	35.55	37.41	39.32	41.24
2.80	1.85	3.87	5.56	7.24	9.23	11.78	13.84	16.36	18.46	20.46	22.31	24.59	26.93	29.00	31.06	32.72	34.55	36.46	38.48	40.63
2.90	1.86	3.79	5.43	7.20	8.97	11.60	13.77	15.98	18.31	20.27	22.21	24.47	26.47	28.50	30.32	32.04	33.89	35.82	37.80	39.84
3.00	1.87	3.76	5.39	7.13	9.03	11.66	13.73	16.16	18.34	20.24	22.45	24.48	26.36	28.25	29.99	31.83	33.62	35.53	37.52	39.43
3.10	1.89	3.75	5.46	7.17	9.34	11.81	13.83	16.40	18.52	20.53	22.67	24.66	26.50	28.37	30.20	32.02	33.84	35.72	37.61	39.68
3.20	1.91	3.76	5.48	7.42	9.55	11.85	13.99	16.41	18.52	20.79	22.85	24.73	26.65	28.51	30.32	32.20	34.09	35.94	37.90	39.75
3.30	1.91	3.75	5.42	7.49	9.60	11.78	13.97	16.29	18.36	20.63	22.61	24.58	26.50	28.38	30.25	32.10	33.97	35.93	37.82	39.72
3.40	1.90	3.69	5.41	7.53	9.60	11.70	13.84	15.96	18.10	20.36	22.34	24.26	26.23	28.18	30.01	31.91	33.67	35.59	37.47	39.53
3.50	1.88	3.65	5.46	7.51	9.58	11.60	13.66	15.70	17.78	20.07	22.02	24.05	25.94	27.81	29.70	31.51	33.46	35.24	37.19	39.27
3.60	1.87	3.64	5.46	7.53	9.63	11.64	13.65	15.69	17.81	20.08	22.00	23.95	25.84	27.74	29.70	31.56	33.46	35.47	37.38	39.39
3.70	1.87	3.65	5.54	7.56	9.64	11.62	13.59	15.65	17.88	20.06	22.01	23.98	25.92	27.81	29.87	31.78	33.69	35.69	37.76	39.73
3.80	1.87	3.70	5.56	7.59	9.63	11.58	13.60	15.67	18.06	20.09	22.07	24.04	25.92	27.79	29.73	31.89	33.95	36.17	38.10	40.13
3.90	1.90	3.75	5.62	7.68	9.69	11.60	13.73	15.81	18.11	20.25	22.20	24.18	26.03	27.97	30.04	32.13	34.37	36.49	38.58	40.82
4.00	1.91	3.81	5.72	7.78	9.80	11.72	13.89	15.97	18.22	20.41	22.37	24.33	26.34	28.30	30.40	32.65	34.84	36.88	39.00	41.21

4.5 Extension of the analysis to multi-degrees-of-freedom systems

The main idea of the proposed simplified sidesway collapse analysis procedure is to simulate the response of MDOF building systems using nonlinear static (pushover) analysis. For this purpose, the building is subjected to a monotonic lateral force pattern that is proportional to the first elastic mode shape until a loss of 20% of the base shear capacity is observed. This approach is consistent with the FEMA P695 methodology. The period based ductility factor (μ_T) is then obtained from the bilinear curve fitted to the base shear-roof displacement pushover curve according to the FEMA P695 methodology (see Figure 4-21). The equivalent bilinear curve is easy to obtain since it depends only on the maximum strength of the building and the slope of the elastic segment of the pushover curve. The adequacy of using a simple bilinear fitting of the pushover curve was also suggested by De Luca *et al*. (2013), as discussed in Section 2.2. To evaluate the proposed procedure, the fundamental period of vibration of the building derived from an eigenvalue analysis is considered as the elastic period of the corresponding SDOF system. The static deformed shape of the building at the ultimate displacement (δ_u), which corresponds to the roof displacement at the point of 20% strength loss, is defined as the "inelastic mode shape" (ϕ_I). A simplifying assumption is made herein that considers the structure near collapse to vibrate according to its inelastic mode shape pattern. Therefore, an inelastic mode participation factor (Γ_I) can be defined as:

$$\Gamma_I = \frac{\phi_I^{\,T} M \,\{1\}}{\phi_I^{\,T} M \,\phi_I} \tag{4.27}$$

where {1} is the unity vector and M is the global mass matrix. The yield pseudo-acceleration for MDOF system can be obtained as follows:

$$A_y = \frac{4\pi^2 \delta_y}{T_{el}^{\,2} \Gamma_I \phi_{I,r}} = \frac{4\pi^2 \delta_u}{\mu_T T_{el}^{\,2} \Gamma_I \phi_{I,r}} \tag{4.28}$$

where $\phi_{I,r}$ is the roof component of the inelastic mode shape and δ_u is the ultimate roof displacement, as defined in Figure 4-21.

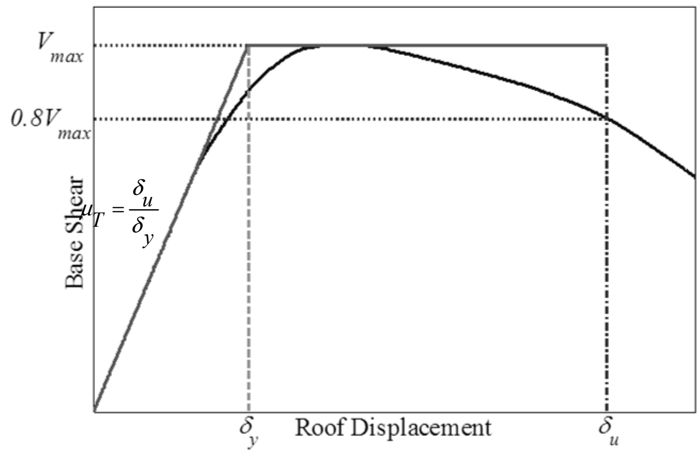


Figure 4-21 Idealized nonlinear static pushover curve (after (FEMA, 2009))

In addition, the elastic viscous damping ratio (ξ) is used for determination of the reduction factor (r). The supplemental damping ratio for the first mode of vibration (ξ_1)can be obtained by:

$$\xi_1 = \frac{\sum_{j=1}^{N_d} (2\pi)^{\alpha_j} T_{el}^{2-\alpha_j} \lambda_j C_j f_j^{1+\alpha_j} \delta_y^{\alpha_j - 1} (\phi_{1j} - \phi_{1(j-1)})^{1+\alpha_j}}{8\pi^3 \sum_{j=1}^{N_F} m_j \phi_{j1}^2} \tag{4.29}$$

The reduction factor (r) can be calculated from Equations 4.25 or 4.26 for given values of T_{el}, μ_T, ξ_1 and α, and is defined for MDOF systems as the ratio of the median value ($\hat{S}_{CT}$) of the 5-percent damped spectral acceleration at the fundamental period (T_{el}) of the building causing the roof of the building to exceed δ_u to the yield pseudo-acceleration (A_y). Thus, the reduction factor is defined as,

$$r = \frac{\hat{S}_{CT}}{A_y} \tag{4.30}$$

The FEMA P695 methodology also defines $\hat{S}_{CT}$ as the product of the *CMR* and the MCE level, 5-percent damped, spectral acceleration at the fundamental period of the building (T_{el}), for the assumed seismic design category (S_{MT}).

$$\hat{S}_{CT} = CMR \times S_{MT} \tag{4.31}$$

Using the MCE spectral shape of ASCE7-10 (see Figure 3-3) and substituting Equations 4.28 and 4.30 into Equation 4.31, the *CMR* can be expressed as:

$$\begin{cases} T_{el} < T_S \rightarrow CMR = \dfrac{4\pi^2 \delta_u r}{S_{MS} T_{el}{}^2 \mu_T \Gamma_I \phi_{I,r}} \\ T_{el} \geq T_S \rightarrow CMR = \dfrac{4\pi^2 \delta_u r}{S_{M1} T_{el} \mu_T \Gamma_I \phi_{I,r}} \end{cases} \tag{4.32}$$

where S_{M1} is the 5-percent damped MCE spectral acceleration at a period of one second and S_{MS} is the 5-percent damped MCE spectral acceleration at short periods, both for the assumed seismic design category. In addition, T_s is the short-period transition of the building in Equation 4.32, as defined in ASCE7-10 (see Figure 3-3). Figure 4-22 illustrates the schematic relationship between the governing parameters of the proposed procedure.

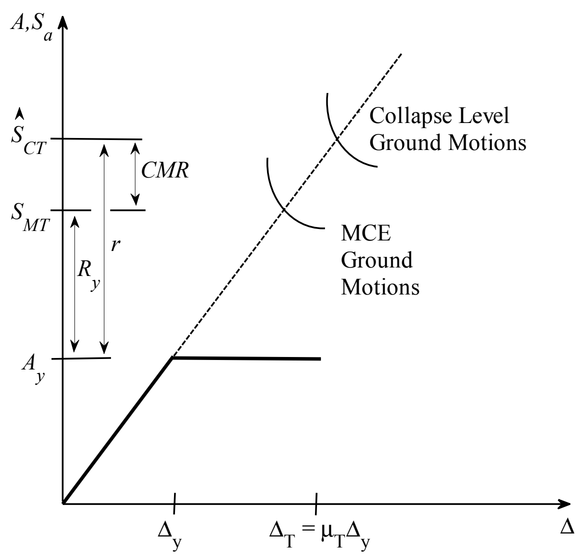


Figure 4-22 Schematic illustration of seismic performance factors as defined by the proposed procedure

4.6 Step-by-step procedure for estimating the *CMR*

The proposed simplified sidesway collapse assessment procedure described above can be succinctly summarized by the following steps:

1. Develop a numerical model of the building structure under evaluation, which incorporates monotonic nonlinear behavior and degradation characteristics of the structural components.
2. Obtain the elastic fundamental period of vibration (T_{el}) and the corresponding first mode shape through an eigenvalue analysis of the elastic building structure as discussed in Section 3.5.

3. Calculate the supplemental damping ratio for the first mode of vibration (ξ_I) provided by the viscous dampers from Equation 4.29.

4. Perform a nonlinear static (pushover) analysis of the building structure with a lateral load proportional to its first mode shape until the point of 20% strength loss.

5. Fit a bilinear curve to the base shear-roof displacement pushover curve according to the FEMA P695 methodology (see Figure 4-21), and obtain the ultimate roof displacement (δ_u) and target ductility ratio (μ_T).

6. From the pushover analysis, construct the inelastic mode shape (ϕ_I) using the displacement of each story when the ultimate roof displacement (δ_u) is reached.

7. Calculate the inelastic mode participation factor (Γ_I) using Equation 4.27.

8. Extract the reduction factor (r) from Equations 4.25or 4.26based on the elastic fundamental period of vibration (T_{el}), target ductility (μ_T), equivalent damping ratio for the first mode of vibration (ξ_I) and velocity exponent (α) computed above.

9. Calculate the *CMR* from Equation 4.32.

4.7 Assessment of proposed procedure

In order to assess the accuracy and robustness of the proposed procedure, the collapse capacities of three benchmark building models (three-, six- and nine-story) described in Section 3.7retrofitted with linear and nonlinear viscous dampers are evaluated. Figure 4-23 shows the geometry and dimensions of the considered frame buildings as well as the configuration of the viscous dampers. Detailed information about the selected buildings can be found in Hall (1995), Gupta and Krawinkler (1999), and FEMA (2005).

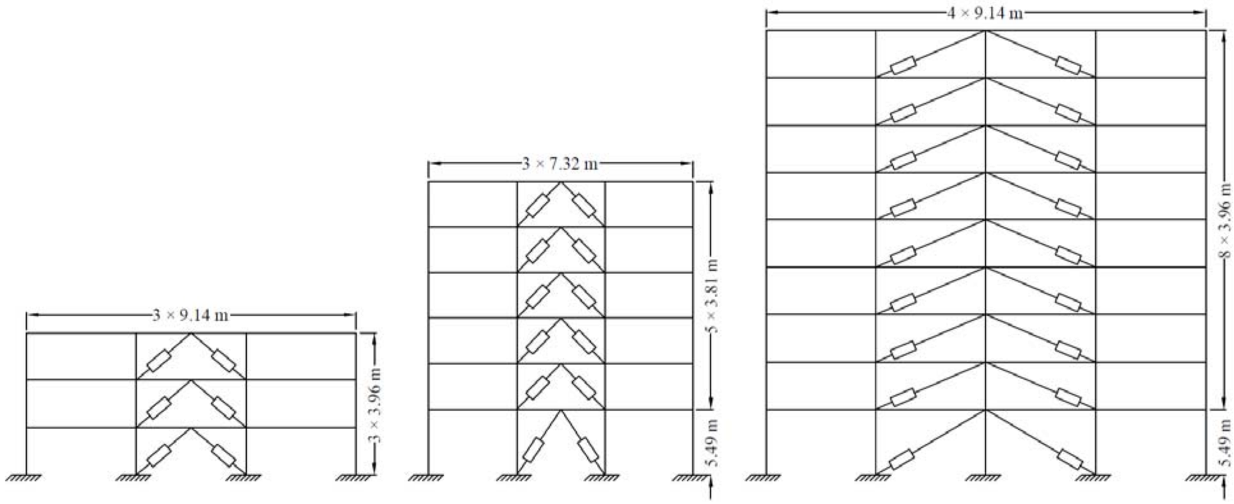


Figure 4-23 Elevation view of the selected steel frame buildings and configuration of viscous dampers

The Open Systems for Earthquake Engineering Simulation (OpenSees) (2013) software was used to carry out the structural analyses. The analytical model development process is described in Section 3.7. Inherent Rayleigh damping of 5% of critical was assigned based on the first and third elastic modes of vibration of the structure.

To assess the performance of the proposed procedure for a wide range of fundamental periods of the benchmark buildings and investigate the effect of the soft story phenomenon on the approximation of the inelastic mode shape, the first story height (h_1) was incrementally varied while all the other stories height (h_{typ}) remained unchanged. A ratio of h_1/h_{typ} from 1 to 1.5 at increments of 0.1 was considered and led to 18 different building models with elastic periods ranging from 0.94 s to 2.18 s. This range of geometries also avoids formation of extreme soft story irregularity as defined by the ASCE7-10 standard (2010). In addition, to avoid a bias of the results towards first floor sidesway collapse, another series of building models was considered and assessed by the proposed procedure. In this set, the heights of the three-story building were increased uniformly by a factor (h_{unif}/h_{typ}) ranging from 0.5 to 2.0 of their initial height at increments of 0.1. This led to a total of 16 more building models with elastic periods ranging from 0.41 s to 2.29 s.

Nonlinear viscous dampers with velocity exponents (α) ranging from 0.2 to 1.0 at 0.2 increments were incorporated as shown in Figure 4-23. The damper constants were chosen so that $C_j.f_j^{(1+\alpha)}$ were proportional to the interstory drift arising from the first mode shape along the height of the structure and provided 5% to 35% added critical damping ratios at 5% increments according to Equation 4.29.

Table 4-37 to Table 4-41 list the damping constants of the considered cases with ξ=5%. Damper constants associated with other damping ratios can be obtained by multiplication of the values by (ξ/0.05). In addition, the connecting bracing members were assumed to be rigid.

Table 4-37 Damper constants selected for three-story building with varying first story heights (ξ=5%)

h_1/h_{typ}	Velocity exponent (α)	Damper Constant, C_k [kN.(s/m)$^\alpha$]		
		1st Story	2nd Story	3rd Story
1.0	0.2	0.23	0.28	0.24
	0.4	0.35	0.43	0.37
	0.6	0.53	0.65	0.57
	0.8	0.80	0.99	0.87
	1.0	1.22	1.50	1.31
1.1	0.2	0.28	0.26	0.22
	0.4	0.43	0.41	0.34
	0.6	0.67	0.62	0.53
	0.8	1.03	0.95	0.81
	1.0	1.58	1.45	1.23
1.2	0.2	0.33	0.24	0.20
	0.4	0.51	0.37	0.31
	0.6	0.80	0.57	0.47
	0.8	1.24	0.87	0.71
	1.0	1.92	1.33	1.08
1.3	0.2	0.37	0.22	0.17
	0.4	0.58	0.33	0.26
	0.6	0.91	0.51	0.40
	0.8	1.41	0.76	0.60
	1.0	2.17	1.15	0.91
1.4	0.2	0.41	0.19	0.15
	0.4	0.64	0.29	0.22
	0.6	0.99	0.44	0.33
	0.8	1.54	0.65	0.50
	1.0	2.36	0.96	0.74
1.5	0.2	0.44	0.17	0.12
	0.4	0.69	0.25	0.19
	0.6	1.06	0.37	0.28
	0.8	1.63	0.54	0.41
	1.0	2.49	0.80	0.59

Table 4-38 Damper constants selected for six-story building with varying first story heights (ξ=5%)

h_1/h_{typ}	Velocity exponent (α)	Damper Constant, C_k [kN.(s/m)$^{\alpha}$]					
		1st Story	2nd Story	3rd Story	4th Story	5th Story	6th Story
1.0	0.2	0.18	0.26	0.29	0.26	0.24	0.15
	0.4	0.31	0.45	0.49	0.44	0.41	0.25
	0.6	0.52	0.76	0.83	0.75	0.69	0.43
	0.8	0.88	1.28	1.39	1.27	1.15	0.72
	1.0	1.47	2.14	2.34	2.13	1.94	1.20
1.1	0.2	0.23	0.26	0.28	0.25	0.23	0.14
	0.4	0.40	0.45	0.48	0.43	0.39	0.24
	0.6	0.70	0.77	0.82	0.74	0.67	0.42
	0.8	1.20	1.32	1.40	1.26	1.14	0.71
	1.0	2.06	2.23	2.37	2.14	1.94	1.20
1.2	0.2	0.29	0.26	0.27	0.24	0.22	0.13
	0.4	0.51	0.45	0.46	0.41	0.37	0.23
	0.6	0.90	0.77	0.80	0.72	0.65	0.40
	0.8	1.58	1.33	1.37	1.23	1.11	0.69
	1.0	2.76	2.27	2.35	2.10	1.89	1.17
1.3	0.2	0.35	0.25	0.25	0.22	0.20	0.12
	0.4	0.63	0.44	0.44	0.39	0.35	0.22
	0.6	1.13	0.76	0.76	0.68	0.61	0.37
	0.8	2.01	1.30	1.31	1.16	1.04	0.64
	1.0	3.54	2.23	2.25	1.99	1.79	1.11
1.4	0.2	0.42	0.24	0.23	0.21	0.18	0.11
	0.4	0.75	0.41	0.41	0.36	0.32	0.20
	0.6	1.35	0.72	0.70	0.62	0.55	0.34
	0.8	2.42	1.23	1.21	1.06	0.95	0.59
	1.0	4.31	2.10	2.06	1.81	1.62	1.00
1.5	0.2	0.48	0.22	0.21	0.18	0.16	0.10
	0.4	0.87	0.38	0.37	0.32	0.28	0.18
	0.6	1.56	0.66	0.63	0.55	0.49	0.30
	0.8	2.80	1.13	1.08	0.94	0.83	0.51
	1.0	4.99	1.91	1.82	1.59	1.41	0.87

Table 4-39 Damper constants selected for nine-story building with varying first story heights (ξ=5%)

h_1/h_{typ}	Velocity exponent (α)	Damper Constant, C_k [kN.(s/m)$^\alpha$]								
		1st Story	2nd Story	3rd Story	4th Story	5th Story	6th Story	7th Story	8th Story	9th Story
1.0	0.2	0.21	0.30	0.33	0.35	0.32	0.32	0.33	0.31	0.24
	0.4	0.38	0.53	0.58	0.62	0.57	0.56	0.58	0.56	0.43
	0.6	0.67	0.94	1.02	1.10	1.01	1.00	1.02	0.98	0.75
	0.8	1.18	1.66	1.80	1.93	1.78	1.75	1.79	1.72	1.32
	1.0	2.06	2.91	3.14	3.39	3.12	3.07	3.14	3.02	2.31
1.1	0.2	0.27	0.31	0.33	0.35	0.32	0.31	0.32	0.31	0.24
	0.4	0.48	0.55	0.58	0.62	0.57	0.56	0.57	0.55	0.42
	0.6	0.86	0.97	1.03	1.10	1.01	0.99	1.01	0.97	0.75
	0.8	1.53	1.72	1.82	1.94	1.79	1.75	1.79	1.72	1.32
	1.0	2.70	3.03	3.20	3.42	3.14	3.08	3.15	3.03	2.32
1.2	0.2	0.34	0.31	0.32	0.34	0.31	0.31	0.31	0.30	0.23
	0.4	0.60	0.56	0.57	0.61	0.56	0.55	0.56	0.53	0.41
	0.6	1.08	0.99	1.02	1.09	0.99	0.97	0.99	0.95	0.73
	0.8	1.93	1.75	1.81	1.92	1.76	1.72	1.76	1.69	1.29
	1.0	3.43	3.10	3.19	3.39	3.11	3.04	3.10	2.97	2.28
1.3	0.2	0.41	0.31	0.31	0.33	0.30	0.29	0.30	0.29	0.22
	0.4	0.73	0.55	0.56	0.59	0.54	0.53	0.53	0.51	0.39
	0.6	1.32	0.98	0.99	1.05	0.96	0.93	0.95	0.91	0.69
	0.8	2.36	1.74	1.75	1.85	1.69	1.65	1.68	1.61	1.23
	1.0	4.19	3.07	3.09	3.26	2.98	2.91	2.96	2.83	2.17
1.4	0.2	0.48	0.30	0.30	0.31	0.28	0.28	0.28	0.27	0.20
	0.4	0.86	0.54	0.53	0.55	0.50	0.49	0.50	0.48	0.36
	0.6	1.55	0.95	0.93	0.98	0.89	0.87	0.88	0.84	0.64
	0.8	2.75	1.67	1.64	1.72	1.57	1.52	1.55	1.48	1.13
	1.0	4.88	2.91	2.86	3.01	2.73	2.66	2.70	2.58	1.97
1.5	0.2	0.54	0.29	0.27	0.29	0.26	0.25	0.26	0.24	0.19
	0.4	0.97	0.50	0.48	0.50	0.46	0.44	0.45	0.43	0.33
	0.6	1.73	0.88	0.84	0.88	0.80	0.78	0.79	0.75	0.57
	0.8	3.05	1.52	1.46	1.53	1.38	1.34	1.36	1.30	0.99
	1.0	5.34	2.62	2.52	2.62	2.38	2.31	2.34	2.23	1.70

Table 4-40 Damper constants selected for three-story building with varying uniform story heights (ξ=5%)

h_{unif}/h_{typ}	Velocity exponent (α)	Damper Constant, C_k [kN.(s/m)$^{\alpha}$]		
		1st Story	2nd Story	3rd Story
0.5	0.2	0.32	0.44	0.43
	0.4	0.48	0.66	0.64
	0.6	0.71	0.98	0.96
	0.8	1.05	1.46	1.43
	1.0	1.56	2.15	2.11
0.6	0.2	0.28	0.38	0.36
	0.4	0.43	0.58	0.55
	0.6	0.64	0.86	0.82
	0.8	0.95	1.28	1.22
	1.0	1.41	1.90	1.82
0.7	0.2	0.26	0.34	0.32
	0.4	0.40	0.52	0.48
	0.6	0.59	0.78	0.73
	0.8	0.89	1.17	1.09
	1.0	1.32	1.74	1.62
0.8	0.2	0.25	0.32	0.29
	0.4	0.37	0.48	0.44
	0.6	0.56	0.72	0.66
	0.8	0.85	1.09	0.99
	1.0	1.27	1.63	1.48
0.9	0.2	0.24	0.30	0.26
	0.4	0.36	0.45	0.40
	0.6	0.54	0.68	0.61
	0.8	0.82	1.03	0.92
	1.0	1.24	1.55	1.38
1.0	0.2	0.23	0.28	0.24
	0.4	0.35	0.43	0.37
	0.6	0.53	0.65	0.57
	0.8	0.80	0.99	0.87
	1.0	1.22	1.50	1.31
1.1	0.2	0.22	0.27	0.23
	0.4	0.34	0.41	0.35
	0.6	0.52	0.63	0.54
	0.8	0.79	0.96	0.83
	1.0	1.21	1.46	1.26
1.2	0.2	0.22	0.26	0.22
	0.4	0.33	0.40	0.34
	0.6	0.52	0.61	0.52
	0.8	0.79	0.94	0.80
	1.0	1.21	1.44	1.22

Table 4-41 Damper constants selected for three-story building with varying uniform story heights (ξ=5%)

h_{unif}/h_{typ}	Velocity exponent (α)	Damper Constant, C_k [kN.(s/m)$^{\alpha}$]		
		1st Story	2nd Story	3rd Story
1.3	0.2	0.21	0.25	0.21
	0.4	0.33	0.39	0.32
	0.6	0.51	0.60	0.50
	0.8	0.79	0.93	0.77
	1.0	1.22	1.42	1.19
1.4	0.2	0.21	0.24	0.20
	0.4	0.33	0.38	0.31
	0.6	0.51	0.59	0.49
	0.8	0.79	0.92	0.75
	1.0	1.23	1.42	1.16
1.5	0.2	0.21	0.24	0.19
	0.4	0.33	0.37	0.30
	0.6	0.51	0.58	0.47
	0.8	0.80	0.91	0.74
	1.0	1.24	1.41	1.15
1.6	0.2	0.21	0.23	0.19
	0.4	0.33	0.37	0.29
	0.6	0.51	0.58	0.46
	0.8	0.80	0.91	0.73
	1.0	1.26	1.42	1.14
1.7	0.2	0.20	0.23	0.18
	0.4	0.33	0.36	0.29
	0.6	0.52	0.57	0.46
	0.8	0.81	0.90	0.72
	1.0	1.28	1.42	1.13
1.8	0.2	0.20	0.22	0.18
	0.4	0.33	0.36	0.28
	0.6	0.52	0.57	0.45
	0.8	0.82	0.90	0.71
	1.0	1.30	1.43	1.12
1.9	0.2	0.20	0.22	0.17
	0.4	0.33	0.36	0.28
	0.6	0.52	0.57	0.44
	0.8	0.83	0.91	0.71
	1.0	1.32	1.44	1.12
2.0	0.2	0.20	0.22	0.17
	0.4	0.33	0.35	0.27
	0.6	0.52	0.57	0.44
	0.8	0.84	0.91	0.70
	1.0	1.34	1.45	1.12

Results of the application of the proposed simplified collapse evaluation procedure to the ensemble of frame building structures considered above with selected values of ξ_1 (provided by the viscous dampers) and α are summarized in Table 4-42 to Table 4-55 for the buildings with varying first story heights and uniform story heights. Figure 4-24 compares the differences in *CMR* values predicted by the proposed simplified procedure with those obtained from nonlinear time history dynamic analyses as a function of the elastic period for the 1190 cases considered. Clearly, for the entire period range considered (0.41 s to 2.29 s) there is no discernible divergence in trend, which is indicative of the robustness of the proposed procedure. The averages of absolute differences between the simplified and nonlinear dynamic time history analyses are only 3.9, 4.8 and 5.0%, with standard deviations of 4.7, 5.8 and 5.9% for the three-, six- and nine-story building models with varying first story height, respectively. The corresponding values for the buildings with varying uniform story heights are 4.6% and 5.4% respectively. These results are acceptable considering that the proposed simplified procedure alleviates the need for nonlinear time history dynamic analyses and also uses regression analysis equations obtained from a large database.

Table 4-42 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for building with viscous dampers and varying first story heights (ξ=5%)

		Three-Story with Varying First Story Heights						Three-Story with Varying First Story Heights						Nine-Story with Varying First Story Heights					
		h_1/h_{typ}						h_1/h_{typ}						h_1/h_{typ}					
		1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5
	T_{el} (sec)	0.94	0.99	1.04	1.11	1.17	1.25	1.16	1.19	1.23	1.26	1.30	1.35	1.97	2.01	2.04	2.08	2.13	2.18
	δ_y (cm)	9.0	9.3	9.8	10.3	10.9	11.6	13.5	13.5	13.5	13.5	13.5	13.6	22.0	22.2	22.4	22.7	22.9	23.0
	δ_u (cm)	59.3	61.6	64.1	66.8	69.5	72.4	76.8	74.7	73.2	72.8	72.6	72.2	112.0	108.2	100.5	95.0	90.7	88.0
	Maximum Interstory Drift Ratio	5.4%	5.3%	5.3%	5.4%	5.4%	5.4%	5.7%	5.6%	5.6%	5.6%	5.7%	6.0%	5.8%	5.3%	5.1%	4.8%	4.9%	4.9%
	μ_T	6.59	6.62	6.54	6.49	6.38	6.24	5.69	5.53	5.42	5.39	5.38	5.31	5.09	4.87	4.49	4.19	3.96	3.83
	$\Gamma_1 \phi_{1,r}$	1.28	1.27	1.27	1.26	1.26	1.25	1.20	1.19	1.18	1.17	1.16	1.15	1.22	1.21	1.20	1.18	1.17	1.16
α=0.2, ξ=5%	r from Regression Equations	6.01	5.89	6.28	6.39	6.43	6.46	5.87	5.79	5.76	5.77	5.80	5.77	5.07	4.89	4.57	4.32	4.14	4.06
	CMR from Simplified Analyses	2.02	1.95	2.08	2.11	2.13	2.16	2.54	2.46	2.41	2.36	2.32	2.27	2.08	2.00	1.87	1.78	1.70	1.64
	CMR from IDA	1.87	1.87	1.90	1.89	2.02	1.94	2.30	2.19	2.12	2.16	2.10	2.11	1.92	1.75	1.65	1.56	1.55	1.45
	Differences (%)	8%	4%	9%	12%	5%	11%	10%	12%	14%	9%	10%	8%	8%	14%	13%	14%	10%	13%
α=0.4, ξ=5%	r from Regression Equations	6.12	6.00	6.35	6.46	6.50	6.53	5.93	5.84	5.81	5.82	5.85	5.82	5.12	4.93	4.60	4.35	4.17	4.09
	CMR from Simplified Analyses	2.06	1.99	2.10	2.13	2.15	2.18	2.56	2.49	2.43	2.38	2.34	2.29	2.10	2.02	1.88	1.79	1.71	1.66
	CMR from IDA	1.93	1.99	1.93	1.92	2.12	2.02	2.38	2.28	2.19	2.21	2.16	2.18	1.93	1.79	1.66	1.60	1.58	1.51
	Differences (%)	7%	0%	9%	11%	1%	8%	8%	9%	11%	8%	8%	5%	9%	13%	13%	12%	8%	10%
α=0.6, ξ=5%	r from Regression Equations	6.25	6.13	6.47	6.58	6.62	6.64	6.02	5.94	5.90	5.91	5.94	5.91	5.20	5.01	4.67	4.41	4.23	4.14
	CMR from Simplified Analyses	2.10	2.03	2.14	2.17	2.19	2.22	2.60	2.53	2.47	2.42	2.38	2.33	2.13	2.05	1.91	1.81	1.73	1.68
	CMR from IDA	2.08	2.05	1.99	1.96	2.18	2.08	2.56	2.37	2.22	2.31	2.26	2.25	2.02	1.82	1.73	1.63	1.64	1.67
	Differences (%)	1%	-1%	8%	11%	0%	7%	2%	7%	11%	5%	5%	4%	5%	13%	10%	11%	5%	1%
α=0.8, ξ=5%	r from Regression Equations	6.42	6.31	6.65	6.76	6.80	6.82	6.18	6.09	6.05	6.06	6.08	6.05	5.33	5.14	4.78	4.51	4.32	4.23
	CMR from Simplified Analyses	2.16	2.09	2.20	2.23	2.25	2.28	2.67	2.59	2.53	2.48	2.43	2.38	2.18	2.10	1.95	1.85	1.77	1.71
	CMR from IDA	2.13	2.14	2.21	2.20	2.35	2.12	2.63	2.56	2.42	2.43	2.31	2.39	2.11	1.86	1.80	1.69	1.78	1.80
	Differences (%)	1%	-2%	0%	1%	-4%	8%	2%	1%	5%	2%	5%	0%	3%	13%	8%	9%	-1%	-5%
α=1.0, ξ=5%	r from Regression Equations	6.54	6.57	6.58	6.71	6.75	6.78	6.05	5.95	5.91	5.93	5.97	5.94	5.19	5.01	4.67	4.43	4.27	4.22
	CMR from Simplified Analyses	2.20	2.17	2.18	2.21	2.24	2.26	2.61	2.53	2.47	2.43	2.39	2.34	2.13	2.05	1.91	1.82	1.75	1.71
	CMR from IDA	2.35	2.26	2.25	2.27	2.32	2.36	2.70	2.61	2.49	2.47	2.39	2.43	2.14	1.91	1.86	1.80	1.82	1.87
	Differences (%)	-6%	-4%	-3%	-2%	-4%	-4%	-3%	-3%	-1%	-2%	0%	-4%	-1%	7%	3%	1%	-4%	-9%

Table 4-43 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying uniform story heights (ξ=5%)

		h_{unif}/h_{typ}															
		0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	T_{el} (sec)	0.41	0.50	0.61	0.71	0.82	0.94	1.05	1.18	1.30	1.43	1.57	1.70	1.84	1.99	2.14	2.29
	δ_y (cm)	3.5	4.5	5.5	6.6	7.7	9.0	10.3	11.7	13.1	14.6	16.2	17.9	19.6	21.4	23.3	25.1
	δ_u (cm)	31.0	36.8	42.6	48.2	53.8	59.3	64.8	70.2	75.5	80.9	86.2	91.5	96.8	102.1	107.5	112.9
	Maximum Interstory Drift Ratio	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.5%	5.5%	5.5%	5.5%	5.6%	5.6%
	μ_T	8.86	8.18	7.75	7.30	6.99	6.59	6.29	6.00	5.76	5.54	5.32	5.11	4.94	4.77	4.61	4.50
	$\Gamma_I \phi_{I,r}$	1.27	1.27	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28
α=0.2, ξ=5%	r from Regression Equations	6.42	6.67	6.87	6.75	6.52	6.01	6.11	6.15	6.12	5.97	5.65	5.28	4.97	4.79	4.77	4.84
	CMR from Simplified Analyses	2.86	2.49	2.19	2.20	2.14	2.02	2.09	2.14	2.15	2.13	2.04	1.94	1.85	1.80	1.82	1.86
	CMR from IDA	2.71	2.25	2.01	2.18	1.92	1.87	2.00	2.13	1.94	2.04	1.97	1.95	1.76	1.61	1.62	1.81
	Differences (%)	6%	11%	9%	1%	11%	8%	4%	0%	11%	4%	4%	-1%	5%	12%	12%	3%
α=0.4, ξ=5%	r from Regression Equations	6.52	6.77	6.97	6.85	6.63	6.12	6.18	6.21	6.18	6.02	5.70	5.33	5.01	4.83	4.81	4.88
	CMR from Simplified Analyses	2.90	2.52	2.22	2.23	2.18	2.06	2.11	2.16	2.18	2.15	2.06	1.96	1.86	1.82	1.84	1.87
	CMR from IDA	2.80	2.32	2.08	2.26	2.00	1.93	2.04	2.15	2.00	2.10	2.03	2.01	1.78	1.66	1.68	1.88
	Differences (%)	4%	9%	7%	-1%	9%	7%	3%	0%	9%	2%	1%	-2%	4%	10%	10%	-1%
α=0.6, ξ=5%	r from Regression Equations	6.63	6.89	7.10	6.98	6.76	6.25	6.29	6.32	6.28	6.12	5.79	5.41	5.09	4.91	4.88	4.95
	CMR from Simplified Analyses	2.95	2.57	2.26	2.27	2.22	2.10	2.15	2.20	2.21	2.18	2.10	1.99	1.89	1.85	1.86	1.90
	CMR from IDA	2.87	2.41	2.11	2.33	2.05	2.08	2.14	2.33	2.11	2.17	2.07	2.11	1.88	1.69	1.70	1.98
	Differences (%)	3%	7%	7%	-3%	8%	1%	0%	-6%	5%	0%	1%	-6%	1%	9%	9%	-4%
α=0.8, ξ=5%	r from Regression Equations	6.79	7.04	7.26	7.14	6.93	6.42	6.47	6.48	6.44	6.27	5.93	5.55	5.22	5.03	5.00	5.06
	CMR from Simplified Analyses	3.02	2.62	2.31	2.32	2.27	2.16	2.21	2.26	2.27	2.24	2.15	2.04	1.94	1.89	1.91	1.94
	CMR from IDA	2.91	2.48	2.21	2.40	2.26	2.13	2.21	2.35	2.19	2.34	2.27	2.18	1.91	1.83	1.75	2.05
	Differences (%)	4%	6%	5%	-3%	0%	1%	0%	-4%	4%	-4%	-5%	-6%	2%	3%	9%	-5%
α=1.0, ξ=5%	r from Regression Equations	7.06	7.37	7.54	7.31	6.99	6.54	6.36	6.40	6.36	6.19	5.83	5.40	5.06	4.89	4.93	5.10
	CMR from Simplified Analyses	3.14	2.75	2.40	2.38	2.29	2.20	2.18	2.23	2.24	2.21	2.11	1.99	1.88	1.84	1.88	1.95
	CMR from IDA	3.00	2.89	2.34	2.44	2.49	2.35	2.32	2.41	2.49	2.52	2.36	2.23	2.13	2.11	2.00	2.12
	Differences (%)	5%	-5%	3%	-3%	-8%	-6%	-6%	-8%	-10%	-12%	-11%	-11%	-12%	-13%	-6%	-8%

Table 4-44 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying first story heights (ξ=10%)

		Three-Story with Varying First Story Heights						Three-Story with Varying First Story Heights						Nine-Story with Varying First Story Heights					
		h_1/h_{typ}						h_1/h_{typ}						h_1/h_{typ}					
		1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5
	T_{el} (sec)	0.94	0.99	1.04	1.11	1.17	1.25	1.16	1.19	1.23	1.26	1.30	1.35	1.97	2.01	2.04	2.08	2.13	2.18
	δ_y (cm)	9.0	9.3	9.8	10.3	10.9	11.6	13.5	13.5	13.5	13.5	13.5	13.6	22.0	22.2	22.4	22.7	22.9	23.0
	δ_u (cm)	59.3	61.6	64.1	66.8	69.5	72.4	76.8	74.7	73.2	72.8	72.6	72.2	112.0	108.2	100.5	95.0	90.7	88.0
	Maximum Interstory Drift Ratio	5.4%	5.3%	5.3%	5.4%	5.4%	5.4%	5.7%	5.6%	5.6%	5.6%	5.7%	6.0%	5.8%	5.3%	5.1%	4.8%	4.9%	4.9%
	μ_T	6.59	6.62	6.54	6.49	6.38	6.24	5.69	5.53	5.42	5.39	5.38	5.31	5.09	4.87	4.49	4.19	3.96	3.83
	$\Gamma_1 \phi_{1,r}$	1.28	1.27	1.27	1.26	1.26	1.25	1.20	1.19	1.18	1.17	1.16	1.15	1.22	1.21	1.20	1.18	1.17	1.16
α=0.2, ξ=10%	r from Regression Equations	6.36	6.24	6.53	6.64	6.68	6.71	6.11	6.03	6.00	6.01	6.04	6.01	5.30	5.12	4.79	4.54	4.37	4.28
	CMR from Simplified Analyses	2.14	2.07	2.16	2.19	2.21	2.24	2.64	2.57	2.51	2.46	2.42	2.37	2.17	2.09	1.96	1.87	1.79	1.73
	CMR from IDA	2.13	2.02	2.04	2.08	2.20	2.08	2.41	2.42	2.36	2.38	2.27	2.21	2.04	1.90	1.84	1.68	1.80	1.58
	Differences (%)	0%	2%	6%	5%	0%	8%	10%	6%	6%	3%	7%	7%	6%	10%	7%	11%	-1%	9%
α=0.4, ξ=10%	r from Regression Equations	6.58	6.46	6.67	6.79	6.82	6.85	6.23	6.14	6.10	6.11	6.14	6.11	5.40	5.21	4.87	4.61	4.42	4.33
	CMR from Simplified Analyses	2.22	2.14	2.21	2.24	2.26	2.29	2.69	2.61	2.55	2.50	2.46	2.41	2.21	2.13	1.99	1.90	1.81	1.75
	CMR from IDA	2.17	2.29	2.10	2.23	2.29	2.11	2.68	2.48	2.39	2.49	2.33	2.29	2.09	1.95	1.90	1.71	1.88	1.68
	Differences (%)	2%	-7%	5%	0%	-1%	9%	0%	5%	7%	0%	6%	5%	6%	9%	5%	11%	-4%	4%
α=0.6, ξ=10%	r from Regression Equations	6.84	6.73	6.91	7.02	7.06	7.07	6.42	6.33	6.29	6.30	6.32	6.29	5.57	5.37	5.00	4.73	4.53	4.44
	CMR from Simplified Analyses	2.30	2.23	2.29	2.32	2.34	2.36	2.78	2.69	2.63	2.58	2.53	2.48	2.28	2.20	2.04	1.95	1.86	1.80
	CMR from IDA	2.21	2.31	2.18	2.34	2.45	2.18	2.74	2.59	2.72	2.54	2.44	2.38	2.21	2.05	1.98	1.78	1.95	1.77
	Differences (%)	4%	-3%	5%	-1%	-4%	8%	1%	4%	-3%	2%	4%	4%	3%	7%	3%	10%	-5%	2%
α=0.8, ξ=10%	r from Regression Equations	7.18	7.08	7.28	7.39	7.41	7.42	6.73	6.63	6.58	6.58	6.61	6.57	5.83	5.62	5.23	4.93	4.72	4.62
	CMR from Simplified Analyses	2.42	2.34	2.41	2.44	2.45	2.48	2.91	2.82	2.75	2.69	2.64	2.59	2.39	2.30	2.14	2.03	1.93	1.87
	CMR from IDA	2.29	2.41	2.48	2.43	2.49	2.38	2.79	2.68	2.76	2.70	2.65	2.48	2.26	2.11	2.02	1.98	2.06	2.00
	Differences (%)	6%	-3%	-3%	0%	-2%	4%	4%	5%	0%	0%	0%	4%	6%	9%	6%	3%	-6%	-7%
α=1.0, ξ=10%	r from Regression Equations	7.50	7.50	7.49	7.61	7.64	7.65	6.84	6.72	6.67	6.68	6.72	6.68	5.90	5.68	5.30	5.01	4.83	4.75
	CMR from Simplified Analyses	2.53	2.48	2.48	2.51	2.53	2.56	2.96	2.86	2.79	2.74	2.69	2.63	2.42	2.33	2.16	2.06	1.98	1.93
	CMR from IDA	2.66	2.55	2.58	2.58	2.63	2.62	2.82	2.84	2.88	2.97	3.02	3.10	2.30	2.18	2.11	2.01	2.09	2.07
	Differences (%)	-5%	-3%	-4%	-3%	-4%	-2%	5%	1%	-3%	-8%	-11%	-15%	5%	7%	3%	3%	-5%	-7%

Table 4-45 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying uniform story heights (ξ=10%)

		h_{unif}/h_{typ}															
		0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	T_{el} (sec)	0.41	0.50	0.61	0.71	0.82	0.94	1.05	1.18	1.30	1.43	1.57	1.70	1.84	1.99	2.14	2.29
	δ_y (cm)	3.5	4.5	5.5	6.6	7.7	9.0	10.3	11.7	13.1	14.6	16.2	17.9	19.6	21.4	23.3	25.1
	δ_u (cm)	31.0	36.8	42.6	48.2	53.8	59.3	64.8	70.2	75.5	80.9	86.2	91.5	96.8	102.1	107.5	112.9
	Maximum Interstory Drift Ratio	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.5%	5.5%	5.5%	5.5%	5.6%	5.6%
	μ_T	8.86	8.18	7.75	7.30	6.99	6.59	6.29	6.00	5.76	5.54	5.32	5.11	4.94	4.77	4.61	4.50
	$\Gamma_I \phi_{I,r}$	1.27	1.27	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28
α=0.2, ξ=10%	*r* from Regression Equations	6.76	7.01	7.21	7.09	6.87	6.36	6.36	6.40	6.37	6.21	5.89	5.52	5.20	5.02	5.00	5.07
	CMR from Simplified Analyses	3.01	2.61	2.29	2.31	2.25	2.14	2.18	2.23	2.24	2.21	2.13	2.03	1.93	1.89	1.91	1.94
	CMR from IDA	2.99	2.38	2.08	2.32	2.11	2.13	2.03	2.24	2.14	2.15	2.14	2.03	1.82	1.74	1.71	1.91
	Differences (%)	1%	10%	10%	0%	7%	0%	7%	0%	5%	3%	0%	0%	6%	9%	12%	2%
α=0.4, ξ=10%	*r* from Regression Equations	6.95	7.21	7.42	7.30	7.08	6.58	6.50	6.52	6.48	6.32	5.99	5.61	5.29	5.11	5.08	5.15
	CMR from Simplified Analyses	3.09	2.69	2.36	2.37	2.32	2.22	2.22	2.27	2.28	2.25	2.17	2.06	1.97	1.92	1.94	1.97
	CMR from IDA	3.02	2.46	2.13	2.57	2.17	2.17	2.09	2.30	2.20	2.21	2.18	2.12	1.88	1.81	1.80	1.98
	Differences (%)	2%	9%	11%	-8%	7%	2%	6%	-1%	4%	2%	0%	-3%	5%	6%	8%	-1%
α=0.6, ξ=10%	*r* from Regression Equations	7.18	7.44	7.66	7.55	7.34	6.84	6.73	6.74	6.68	6.51	6.17	5.78	5.45	5.26	5.22	5.28
	CMR from Simplified Analyses	3.20	2.77	2.44	2.46	2.41	2.30	2.30	2.35	2.35	2.32	2.23	2.12	2.02	1.98	1.99	2.02
	CMR from IDA	3.11	2.55	2.31	2.68	2.26	2.21	2.18	2.41	2.31	2.28	2.27	2.21	1.96	1.88	1.86	2.03
	Differences (%)	3%	9%	6%	-8%	7%	4%	6%	-2%	2%	2%	-2%	-4%	3%	5%	7%	0%
α=0.8, ξ=10%	*r* from Regression Equations	7.49	7.75	7.99	7.88	7.68	7.18	7.07	7.06	7.00	6.81	6.45	6.05	5.71	5.50	5.45	5.51
	CMR from Simplified Analyses	3.33	2.89	2.54	2.56	2.52	2.42	2.42	2.46	2.46	2.43	2.33	2.22	2.12	2.07	2.08	2.11
	CMR from IDA	3.20	2.71	2.68	2.79	2.32	2.29	2.48	2.47	2.40	2.52	2.41	2.32	2.03	2.28	1.99	2.25
	Differences (%)	4%	7%	-5%	-8%	9%	6%	-2%	0%	3%	-4%	-3%	-4%	4%	-9%	5%	-6%
α=1.0, ξ=10%	*r* from Regression Equations	8.09	8.43	8.64	8.40	8.04	7.50	7.24	7.23	7.17	6.97	6.57	6.12	5.75	5.56	5.58	5.72
	CMR from Simplified Analyses	3.60	3.14	2.75	2.73	2.64	2.53	2.48	2.52	2.52	2.48	2.38	2.25	2.14	2.09	2.13	2.19
	CMR from IDA	3.42	3.28	2.70	2.86	2.77	2.66	2.57	2.74	2.81	2.76	2.64	2.45	2.36	2.42	2.30	2.36
	Differences (%)	5%	-4%	2%	-5%	-5%	-5%	-4%	-8%	-10%	-10%	-10%	-8%	-9%	-14%	-7%	-7%

Table 4-46 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying first story heights (ξ=15%)

		Three-Story with Varying First Story Heights						Three-Story with Varying First Story Heights						Nine-Story with Varying First Story Heights					
		h_1/h_{typ}						h_1/h_{typ}						h_1/h_{typ}					
		1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5
	T_{el} (sec)	0.94	0.99	1.04	1.11	1.17	1.25	1.16	1.19	1.23	1.26	1.30	1.35	1.97	2.01	2.04	2.08	2.13	2.18
	δ_y (cm)	9.0	9.3	9.8	10.3	10.9	11.6	13.5	13.5	13.5	13.5	13.5	13.6	22.0	22.2	22.4	22.7	22.9	23.0
	δ_u (cm)	59.3	61.6	64.1	66.8	69.5	72.4	76.8	74.7	73.2	72.8	72.6	72.2	112.0	108.2	100.5	95.0	90.7	88.0
	Maximum Interstory Drift Ratio	5.4%	5.3%	5.3%	5.4%	5.4%	5.4%	5.7%	5.6%	5.6%	5.6%	5.7%	6.0%	5.8%	5.3%	5.1%	4.8%	4.9%	4.9%
	μ_T	6.59	6.62	6.54	6.49	6.38	6.24	5.69	5.53	5.42	5.39	5.38	5.31	5.09	4.87	4.49	4.19	3.96	3.83
	$\Gamma_1 \phi_{1,r}$	1.28	1.27	1.27	1.26	1.26	1.25	1.20	1.19	1.18	1.17	1.16	1.15	1.22	1.21	1.20	1.18	1.17	1.16
α=0.2, ξ=15%	*r* from Regression Equations	6.71	6.59	6.78	6.90	6.94	6.96	6.35	6.27	6.24	6.25	6.28	6.24	5.54	5.36	5.02	4.77	4.59	4.50
	CMR from Simplified Analyses	2.26	2.18	2.25	2.28	2.30	2.32	2.75	2.67	2.61	2.56	2.51	2.46	2.27	2.19	2.05	1.96	1.88	1.82
	CMR from IDA	2.25	2.17	2.13	2.16	2.36	2.24	2.53	2.48	2.52	2.52	2.35	2.35	2.09	2.21	1.89	1.98	1.90	1.69
	Differences (%)	0%	0%	6%	6%	-3%	4%	9%	8%	4%	2%	7%	5%	9%	-1%	8%	-1%	-1%	8%
α=0.4, ξ=15%	*r* from Regression Equations	7.03	6.92	7.00	7.11	7.15	7.16	6.53	6.44	6.40	6.40	6.43	6.40	5.68	5.49	5.13	4.86	4.67	4.58
	CMR from Simplified Analyses	2.37	2.29	2.32	2.35	2.37	2.39	2.82	2.74	2.68	2.62	2.57	2.52	2.33	2.25	2.10	2.00	1.91	1.85
	CMR from IDA	2.27	2.35	2.24	2.43	2.43	2.40	2.75	2.59	2.64	2.66	2.43	2.47	2.19	2.28	1.94	2.07	1.93	1.77
	Differences (%)	4%	-3%	4%	-3%	-2%	0%	3%	6%	2%	-2%	6%	2%	6%	-1%	8%	-3%	-1%	5%
α=0.6, ξ=15%	*r* from Regression Equations	7.42	7.32	7.36	7.47	7.49	7.50	6.82	6.72	6.67	6.68	6.71	6.66	5.93	5.72	5.34	5.05	4.84	4.74
	CMR from Simplified Analyses	2.50	2.42	2.44	2.47	2.48	2.50	2.95	2.86	2.79	2.73	2.68	2.62	2.43	2.34	2.18	2.08	1.98	1.92
	CMR from IDA	2.44	2.39	2.45	2.50	2.58	2.46	2.91	2.90	2.84	2.74	2.54	2.55	2.34	2.33	2.01	2.11	1.99	1.82
	Differences (%)	2%	1%	0%	-1%	-4%	2%	1%	-1%	-2%	0%	6%	3%	4%	0%	8%	-1%	-1%	5%
α=0.8, ξ=15%	*r* from Regression Equations	7.93	7.85	7.91	8.01	8.03	8.02	7.28	7.16	7.10	7.11	7.13	7.09	6.33	6.10	5.67	5.35	5.12	5.00
	CMR from Simplified Analyses	2.67	2.60	2.62	2.65	2.66	2.68	3.15	3.05	2.97	2.91	2.85	2.79	2.59	2.49	2.32	2.20	2.10	2.02
	CMR from IDA	2.65	2.66	2.58	2.58	2.70	2.80	2.98	3.04	3.01	2.92	2.92	2.77	2.41	2.36	2.22	2.28	2.24	2.21
	Differences (%)	1%	-2%	2%	3%	-1%	-4%	6%	0%	-1%	0%	-2%	1%	7%	6%	5%	-4%	-6%	-9%
α=1.0, ξ=15%	*r* from Regression Equations	8.46	8.44	8.40	8.52	8.53	8.52	7.63	7.49	7.43	7.44	7.47	7.42	6.61	6.36	5.92	5.60	5.38	5.29
	CMR from Simplified Analyses	2.85	2.79	2.78	2.81	2.83	2.85	3.30	3.19	3.11	3.04	2.99	2.92	2.71	2.60	2.42	2.30	2.20	2.14
	CMR from IDA	2.84	2.84	2.87	2.89	2.91	2.87	3.06	3.13	3.17	3.41	3.40	3.41	2.48	2.41	2.30	2.30	2.29	2.27
	Differences (%)	0%	-2%	-3%	-3%	-3%	-1%	8%	2%	-2%	-11%	-12%	-14%	9%	8%	5%	0%	-4%	-6%

Table 4-47 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying uniform story heights (ξ=15%)

		h_{unif}/h_{typ}															
		0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	T_{el} (sec)	0.41	0.50	0.61	0.71	0.82	0.94	1.05	1.18	1.30	1.43	1.57	1.70	1.84	1.99	2.14	2.29
	δ_y (cm)	3.5	4.5	5.5	6.6	7.7	9.0	10.3	11.7	13.1	14.6	16.2	17.9	19.6	21.4	23.3	25.1
	δ_u (cm)	31.0	36.8	42.6	48.2	53.8	59.3	64.8	70.2	75.5	80.9	86.2	91.5	96.8	102.1	107.5	112.9
	Maximum Interstory Drift Ratio	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.5%	5.5%	5.5%	5.5%	5.6%	5.6%
	μ_T	8.86	8.18	7.75	7.30	6.99	6.59	6.29	6.00	5.76	5.54	5.32	5.11	4.94	4.77	4.61	4.50
	$\Gamma_I \phi_{I,r}$	1.27	1.27	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28
α=0.2, ξ=15%	*r* from Regression Equations	7.09	7.35	7.55	7.43	7.21	6.71	6.61	6.64	6.61	6.45	6.13	5.75	5.44	5.26	5.23	5.30
	CMR from Simplified Analyses	3.16	2.74	2.40	2.42	2.37	2.26	2.26	2.31	2.33	2.30	2.22	2.11	2.02	1.98	2.00	2.03
	CMR from IDA	3.23	2.68	2.39	2.53	2.25	2.25	2.16	2.32	2.24	2.20	2.28	2.12	1.97	1.92	1.84	1.98
	Differences (%)	-2%	2%	0%	-4%	5%	0%	5%	0%	4%	5%	-3%	0%	3%	3%	9%	3%
α=0.4, ξ=15%	*r* from Regression Equations	7.38	7.64	7.86	7.75	7.54	7.03	6.82	6.83	6.79	6.62	6.28	5.89	5.57	5.38	5.35	5.41
	CMR from Simplified Analyses	3.28	2.85	2.50	2.52	2.47	2.37	2.33	2.38	2.39	2.36	2.27	2.17	2.07	2.02	2.04	2.07
	CMR from IDA	3.41	2.78	2.78	2.61	2.38	2.27	2.23	2.38	2.27	2.39	2.37	2.19	2.04	1.97	1.91	2.04
	Differences (%)	-4%	3%	-10%	-3%	4%	4%	4%	0%	5%	-1%	-4%	-1%	1%	3%	7%	1%
α=0.6, ξ=15%	*r* from Regression Equations	7.73	8.00	8.23	8.12	7.92	7.42	7.16	7.15	7.09	6.90	6.55	6.15	5.81	5.61	5.56	5.62
	CMR from Simplified Analyses	3.44	2.98	2.62	2.64	2.60	2.50	2.45	2.49	2.50	2.46	2.37	2.26	2.16	2.11	2.12	2.15
	CMR from IDA	3.47	3.08	2.83	2.65	2.46	2.44	2.44	2.48	2.35	2.42	2.43	2.29	2.13	2.08	1.97	2.14
	Differences (%)	-1%	-3%	-7%	0%	6%	2%	0%	0%	6%	2%	-2%	-1%	1%	1%	8%	0%
α=0.8, ξ=15%	*r* from Regression Equations	8.18	8.46	8.72	8.62	8.43	7.93	7.68	7.64	7.55	7.35	6.97	6.55	6.19	5.97	5.91	5.96
	CMR from Simplified Analyses	3.64	3.15	2.78	2.80	2.77	2.67	2.63	2.66	2.66	2.62	2.52	2.41	2.30	2.25	2.26	2.28
	CMR from IDA	3.54	3.22	2.91	2.81	2.53	2.65	2.72	2.70	2.93	2.77	2.75	2.44	2.24	2.43	2.10	2.39
	Differences (%)	3%	-2%	-4%	0%	9%	1%	-3%	-1%	-9%	-5%	-8%	-1%	3%	-7%	8%	-5%
α=1.0, ξ=15%	*r* from Regression Equations	9.11	9.49	9.75	9.48	9.09	8.46	8.12	8.07	7.97	7.74	7.31	6.83	6.44	6.22	6.22	6.35
	CMR from Simplified Analyses	4.05	3.54	3.10	3.08	2.98	2.85	2.78	2.81	2.81	2.76	2.65	2.51	2.39	2.34	2.37	2.43
	CMR from IDA	3.80	3.69	2.98	3.17	3.08	2.84	2.80	2.90	3.01	3.07	2.80	2.72	2.59	2.62	2.58	2.58
	Differences (%)	7%	-4%	4%	-3%	-3%	0%	-1%	-3%	-7%	-10%	-5%	-8%	-8%	-11%	-8%	-6%

Table 4-48 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying first story heights (ξ=20%)

		Three-Story with Varying First Story Heights						Three-Story with Varying First Story Heights						Nine-Story with Varying First Story Heights					
		h_1/h_{typ}						h_1/h_{typ}						h_1/h_{typ}					
		1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5
	T_{el} (sec)	0.94	0.99	1.04	1.11	1.17	1.25	1.16	1.19	1.23	1.26	1.30	1.35	1.97	2.01	2.04	2.08	2.13	2.18
	δ_y (cm)	9.0	9.3	9.8	10.3	10.9	11.6	13.5	13.5	13.5	13.5	13.5	13.6	22.0	22.2	22.4	22.7	22.9	23.0
	δ_u (cm)	59.3	61.6	64.1	66.8	69.5	72.4	76.8	74.7	73.2	72.8	72.6	72.2	112.0	108.2	100.5	95.0	90.7	88.0
	Maximum Interstory Drift Ratio	5.4%	5.3%	5.3%	5.4%	5.4%	5.4%	5.7%	5.6%	5.6%	5.6%	5.7%	6.0%	5.8%	5.3%	5.1%	4.8%	4.9%	4.9%
	μ_T	6.59	6.62	6.54	6.49	6.38	6.24	5.69	5.53	5.42	5.39	5.38	5.31	5.09	4.87	4.49	4.19	3.96	3.83
	$\Gamma_1 \phi_{1,r}$	1.28	1.27	1.27	1.26	1.26	1.25	1.20	1.19	1.18	1.17	1.16	1.15	1.22	1.21	1.20	1.18	1.17	1.16
α=0.2, ξ=20%	*r* from Regression Equations	7.05	6.94	7.04	7.15	7.19	7.21	6.60	6.51	6.48	6.49	6.52	6.48	5.78	5.59	5.25	4.99	4.81	4.72
	CMR from Simplified Analyses	2.37	2.30	2.33	2.36	2.38	2.41	2.85	2.77	2.71	2.66	2.61	2.55	2.37	2.29	2.14	2.05	1.97	1.91
	CMR from IDA	2.41	2.29	2.24	2.52	2.41	2.44	2.89	2.78	2.63	2.56	2.62	2.49	2.18	2.41	2.11	2.10	1.94	1.93
	Differences (%)	-2%	0%	4%	-6%	-1%	-1%	-1%	0%	3%	4%	0%	2%	9%	-5%	1%	-2%	2%	-1%
α=0.4, ξ=20%	*r* from Regression Equations	7.49	7.38	7.33	7.44	7.47	7.48	6.83	6.73	6.69	6.70	6.72	6.69	5.96	5.76	5.40	5.12	4.92	4.82
	CMR from Simplified Analyses	2.52	2.44	2.43	2.46	2.47	2.50	2.95	2.87	2.80	2.74	2.69	2.63	2.44	2.36	2.21	2.11	2.02	1.95
	CMR from IDA	2.48	2.53	2.30	2.58	2.59	2.49	3.05	2.82	2.70	2.96	2.65	2.71	2.21	2.47	2.20	2.18	2.14	2.00
	Differences (%)	2%	-4%	6%	-5%	-5%	0%	-3%	2%	4%	-7%	2%	-3%	10%	-4%	0%	-3%	-6%	-3%
α=0.6, ξ=20%	*r* from Regression Equations	8.01	7.92	7.81	7.91	7.93	7.93	7.22	7.11	7.06	7.06	7.09	7.04	6.30	6.08	5.67	5.36	5.14	5.03
	CMR from Simplified Analyses	2.70	2.62	2.59	2.61	2.63	2.65	3.12	3.03	2.95	2.89	2.84	2.77	2.58	2.49	2.32	2.20	2.11	2.04
	CMR from IDA	2.71	2.68	2.68	2.70	2.69	2.63	3.15	3.07	3.15	3.01	2.90	2.81	2.39	2.50	2.28	2.21	2.25	2.25
	Differences (%)	0%	-2%	-3%	-3%	-2%	1%	-1%	-1%	-6%	-4%	-2%	-1%	8%	0%	2%	0%	-6%	-9%
α=0.8, ξ=20%	*r* from Regression Equations	8.69	8.62	8.54	8.63	8.64	8.62	7.84	7.70	7.63	7.63	7.66	7.60	6.83	6.58	6.12	5.77	5.52	5.39
	CMR from Simplified Analyses	2.93	2.85	2.83	2.85	2.86	2.88	3.39	3.28	3.19	3.12	3.06	2.99	2.80	2.69	2.50	2.37	2.26	2.18
	CMR from IDA	2.94	2.78	3.05	2.98	2.73	2.78	3.28	3.27	3.30	3.25	2.98	2.91	2.68	2.55	2.35	2.36	2.41	2.32
	Differences (%)	0%	3%	-7%	-4%	5%	4%	3%	0%	-3%	-4%	3%	3%	4%	5%	6%	0%	-6%	-6%
α=1.0, ξ=20%	*r* from Regression Equations	9.42	9.38	9.31	9.42	9.42	9.39	8.43	8.26	8.18	8.19	8.22	8.16	7.32	7.04	6.55	6.18	5.93	5.82
	CMR from Simplified Analyses	3.17	3.10	3.09	3.11	3.12	3.14	3.64	3.52	3.42	3.35	3.29	3.21	3.00	2.88	2.68	2.54	2.43	2.36
	CMR from IDA	3.07	3.06	3.13	3.21	3.15	3.03	3.34	3.37	3.46	3.71	3.69	3.70	2.73	2.62	2.53	2.47	2.50	2.46
	Differences (%)	3%	1%	-1%	-3%	-1%	4%	9%	4%	-1%	-10%	-11%	-13%	10%	10%	6%	3%	-3%	-4%

Table 4-49 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying uniform story heights (ξ=20%)

		h_{unif}/h_{typ}															
		0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	T_{el} (sec)	0.41	0.50	0.61	0.71	0.82	0.94	1.05	1.18	1.30	1.43	1.57	1.70	1.84	1.99	2.14	2.29
	δ_y (cm)	3.5	4.5	5.5	6.6	7.7	9.0	10.3	11.7	13.1	14.6	16.2	17.9	19.6	21.4	23.3	25.1
	δ_u (cm)	31.0	36.8	42.6	48.2	53.8	59.3	64.8	70.2	75.5	80.9	86.2	91.5	96.8	102.1	107.5	112.9
	Maximum Interstory Drift Ratio	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.5%	5.5%	5.5%	5.5%	5.6%	5.6%
	μ_T	8.86	8.18	7.75	7.30	6.99	6.59	6.29	6.00	5.76	5.54	5.32	5.11	4.94	4.77	4.61	4.50
	$\Gamma_I \phi_{I,r}$	1.27	1.27	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28
α=0.2, ξ=20%	*r* from Regression Equations	7.43	7.69	7.90	7.78	7.56	7.05	6.86	6.89	6.85	6.69	6.37	5.99	5.67	5.49	5.46	5.53
	CMR from Simplified Analyses	3.31	2.87	2.51	2.53	2.48	2.37	2.35	2.40	2.41	2.39	2.31	2.20	2.11	2.07	2.08	2.12
	CMR from IDA	3.32	2.97	2.76	2.73	2.39	2.41	2.29	2.53	2.29	2.42	2.39	2.17	2.02	2.10	2.01	2.08
	Differences (%)	0%	-3%	-9%	-7%	4%	-2%	3%	-5%	5%	-1%	-3%	1%	4%	-1%	3%	2%
α=0.4, ξ=20%	*r* from Regression Equations	7.82	8.08	8.31	8.20	7.99	7.49	7.14	7.14	7.09	6.91	6.57	6.18	5.85	5.65	5.61	5.68
	CMR from Simplified Analyses	3.48	3.01	2.64	2.67	2.62	2.52	2.44	2.49	2.50	2.46	2.38	2.27	2.17	2.13	2.14	2.18
	CMR from IDA	3.46	3.28	2.90	2.94	2.61	2.48	2.37	2.58	2.37	2.46	2.41	2.38	2.34	2.25	2.05	2.16
	Differences (%)	1%	-8%	-9%	-9%	0%	2%	3%	-3%	5%	0%	-1%	-5%	-7%	-5%	4%	1%
α=0.6, ξ=20%	*r* from Regression Equations	8.28	8.55	8.80	8.70	8.50	8.01	7.59	7.57	7.49	7.29	6.93	6.52	6.17	5.96	5.90	5.95
	CMR from Simplified Analyses	3.69	3.19	2.80	2.83	2.79	2.70	2.60	2.63	2.64	2.60	2.51	2.40	2.29	2.24	2.25	2.28
	CMR from IDA	3.71	3.40	2.95	3.09	2.69	2.71	2.50	2.72	2.50	2.54	2.47	2.43	2.58	2.30	2.34	2.25
	Differences (%)	-1%	-6%	-5%	-8%	4%	0%	4%	-3%	6%	2%	2%	-1%	-11%	-3%	-4%	1%
α=0.8, ξ=20%	*r* from Regression Equations	8.88	9.17	9.45	9.35	9.18	8.69	8.29	8.22	8.11	7.89	7.49	7.05	6.68	6.44	6.37	6.40
	CMR from Simplified Analyses	3.95	3.42	3.01	3.04	3.01	2.93	2.84	2.86	2.86	2.81	2.71	2.59	2.48	2.42	2.43	2.45
	CMR from IDA	3.77	3.43	3.10	3.18	2.88	2.94	2.98	2.91	3.04	2.82	2.86	2.60	2.65	2.65	2.46	2.44
	Differences (%)	5%	0%	-3%	-4%	5%	0%	-5%	-2%	-6%	0%	-5%	0%	-6%	-9%	-1%	0%
α=1.0, ξ=20%	*r* from Regression Equations	10.13	10.55	10.85	10.57	10.14	9.42	9.00	8.91	8.78	8.51	8.05	7.54	7.13	6.89	6.86	6.98
	CMR from Simplified Analyses	4.51	3.93	3.45	3.44	3.33	3.17	3.08	3.10	3.09	3.04	2.91	2.77	2.65	2.59	2.62	2.68
	CMR from IDA	4.09	3.87	3.15	3.37	3.28	3.07	3.05	3.07	3.27	3.29	3.12	3.02	2.82	2.86	2.81	2.81
	Differences (%)	10%	2%	10%	2%	1%	3%	1%	1%	-6%	-8%	-7%	-8%	-6%	-9%	-7%	-5%

Table 4-50 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying first story heights (ξ=25%)

		Three-Story with Varying First Story Heights						Three-Story with Varying First Story Heights						Nine-Story with Varying First Story Heights					
		h_1/h_{typ}						h_1/h_{typ}						h_1/h_{typ}					
		1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5
	T_{el} (sec)	0.94	0.99	1.04	1.11	1.17	1.25	1.16	1.19	1.23	1.26	1.30	1.35	1.97	2.01	2.04	2.08	2.13	2.18
	δ_y (cm)	9.0	9.3	9.8	10.3	10.9	11.6	13.5	13.5	13.5	13.5	13.5	13.6	22.0	22.2	22.4	22.7	22.9	23.0
	δ_u (cm)	59.3	61.6	64.1	66.8	69.5	72.4	76.8	74.7	73.2	72.8	72.6	72.2	112.0	108.2	100.5	95.0	90.7	88.0
	Maximum Interstory Drift Ratio	5.4%	5.3%	5.3%	5.4%	5.4%	5.4%	5.7%	5.6%	5.6%	5.6%	5.7%	6.0%	5.8%	5.3%	5.1%	4.8%	4.9%	4.9%
	μ_T	6.59	6.62	6.54	6.49	6.38	6.24	5.69	5.53	5.42	5.39	5.38	5.31	5.09	4.87	4.49	4.19	3.96	3.83
	$\Gamma_1\phi_{1,r}$	1.28	1.27	1.27	1.26	1.26	1.25	1.20	1.19	1.18	1.17	1.16	1.15	1.22	1.21	1.20	1.18	1.17	1.16
α=0.2, ξ=25%	*r* from Regression Equations	7.40	7.29	7.29	7.40	7.44	7.46	6.84	6.75	6.72	6.73	6.76	6.72	6.01	5.82	5.48	5.22	5.03	4.94
	CMR from Simplified Analyses	2.49	2.41	2.42	2.44	2.46	2.49	2.96	2.87	2.81	2.75	2.70	2.65	2.46	2.38	2.24	2.15	2.06	2.00
	CMR from IDA	2.59	2.42	2.28	2.53	2.60	2.55	2.94	2.81	2.94	3.03	2.67	2.56	2.35	2.48	2.23	2.20	2.03	2.01
	Differences (%)	-4%	0%	6%	-4%	-5%	-2%	1%	2%	-4%	-9%	1%	4%	5%	-4%	0%	-2%	1%	0%
α=0.4, ξ=25%	*r* from Regression Equations	7.94	7.85	7.65	7.76	7.79	7.80	7.13	7.03	6.98	6.99	7.02	6.98	6.25	6.04	5.66	5.37	5.17	5.07
	CMR from Simplified Analyses	2.67	2.60	2.54	2.56	2.58	2.60	3.08	2.99	2.92	2.86	2.81	2.75	2.56	2.47	2.31	2.21	2.12	2.05
	CMR from IDA	2.79	2.72	2.34	2.63	2.69	2.66	3.03	2.89	3.22	3.09	2.71	2.94	2.38	2.51	2.34	2.31	2.18	2.08
	Differences (%)	-4%	-4%	9%	-3%	-4%	-2%	2%	3%	-9%	-7%	4%	-6%	8%	-2%	-1%	-4%	-3%	-1%
α=0.6, ξ=25%	*r* from Regression Equations	8.59	8.51	8.25	8.35	8.37	8.36	7.62	7.50	7.44	7.44	7.47	7.42	6.67	6.43	6.01	5.68	5.45	5.33
	CMR from Simplified Analyses	2.89	2.82	2.73	2.76	2.77	2.79	3.29	3.19	3.11	3.05	2.99	2.92	2.73	2.63	2.46	2.34	2.23	2.16
	CMR from IDA	2.99	2.78	2.73	2.86	2.78	2.79	3.30	3.14	3.48	3.31	3.11	3.02	2.45	2.59	2.57	2.37	2.31	2.33
	Differences (%)	-3%	1%	0%	-3%	0%	0%	0%	2%	-11%	-8%	-4%	-3%	11%	2%	-4%	-1%	-3%	-7%
α=0.8, ξ=25%	*r* from Regression Equations	9.45	9.40	9.17	9.26	9.25	9.22	8.39	8.24	8.16	8.16	8.19	8.12	7.33	7.06	6.57	6.19	5.91	5.78
	CMR from Simplified Analyses	3.18	3.11	3.04	3.06	3.06	3.08	3.63	3.51	3.41	3.34	3.28	3.20	3.00	2.89	2.68	2.55	2.42	2.34
	CMR from IDA	3.39	2.92	3.02	2.92	2.82	2.97	3.57	3.68	3.65	3.45	3.19	3.42	2.91	2.69	2.64	2.54	2.65	2.42
	Differences (%)	-6%	7%	1%	5%	9%	4%	2%	-5%	-7%	-3%	3%	-6%	3%	7%	2%	0%	-9%	-3%
α=1.0, ξ=25%	*r* from Regression Equations	10.38	10.31	10.22	10.33	10.31	10.26	9.22	9.04	8.94	8.94	8.97	8.91	8.03	7.72	7.18	6.77	6.48	6.36
	CMR from Simplified Analyses	3.49	3.41	3.39	3.41	3.41	3.43	3.99	3.85	3.74	3.66	3.59	3.51	3.29	3.16	2.93	2.78	2.66	2.57
	CMR from IDA	3.29	3.25	3.41	3.42	3.38	3.26	3.68	3.75	3.80	3.88	3.92	3.90	2.97	2.86	2.72	2.74	2.72	2.65
	Differences (%)	6%	5%	-1%	0%	1%	5%	8%	3%	-2%	-6%	-8%	-10%	11%	10%	8%	2%	-2%	-3%

Table 4-51 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying uniform story heights (ξ=25%)

		h_{unif}/h_{typ}															
		0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	T_{el} (sec)	0.41	0.50	0.61	0.71	0.82	0.94	1.05	1.18	1.30	1.43	1.57	1.70	1.84	1.99	2.14	2.29
	δ_y (cm)	3.5	4.5	5.5	6.6	7.7	9.0	10.3	11.7	13.1	14.6	16.2	17.9	19.6	21.4	23.3	25.1
	δ_u (cm)	31.0	36.8	42.6	48.2	53.8	59.3	64.8	70.2	75.5	80.9	86.2	91.5	96.8	102.1	107.5	112.9
	Maximum Interstory Drift Ratio	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.5%	5.5%	5.5%	5.5%	5.6%	5.6%
	μ_T	8.86	8.18	7.75	7.30	6.99	6.59	6.29	6.00	5.76	5.54	5.32	5.11	4.94	4.77	4.61	4.50
	$\Gamma_I \phi_{I,r}$	1.27	1.27	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28
α=0.2, ξ=25%	*r* from Regression Equations	7.77	8.02	8.24	8.12	7.91	7.40	7.11	7.14	7.10	6.93	6.61	6.22	5.91	5.72	5.69	5.76
	CMR from Simplified Analyses	3.46	2.99	2.62	2.64	2.60	2.49	2.43	2.49	2.50	2.47	2.39	2.29	2.20	2.15	2.17	2.21
	CMR from IDA	3.38	3.08	2.89	2.89	2.64	2.59	2.38	2.69	2.39	2.51	2.58	2.21	2.13	2.17	2.14	2.14
	Differences (%)	2%	-3%	-9%	-9%	-2%	-4%	2%	-7%	5%	-2%	-7%	4%	3%	-1%	1%	3%
α=0.4, ξ=25%	*r* from Regression Equations	8.25	8.52	8.75	8.64	8.44	7.94	7.45	7.45	7.39	7.21	6.86	6.46	6.13	5.93	5.88	5.94
	CMR from Simplified Analyses	3.67	3.18	2.78	2.81	2.77	2.67	2.55	2.59	2.60	2.57	2.48	2.37	2.28	2.23	2.24	2.28
	CMR from IDA	3.75	3.54	3.03	3.08	2.70	2.79	2.48	2.76	2.51	2.55	2.67	2.41	2.49	2.29	2.36	2.29
	Differences (%)	-2%	-10%	-8%	-9%	3%	-4%	3%	-6%	4%	1%	-7%	-2%	-8%	-3%	-5%	0%
α=0.6, ξ=25%	*r* from Regression Equations	8.82	9.10	9.37	9.27	9.09	8.59	8.02	7.98	7.89	7.68	7.31	6.88	6.53	6.30	6.24	6.29
	CMR from Simplified Analyses	3.93	3.39	2.98	3.01	2.98	2.89	2.74	2.78	2.78	2.74	2.65	2.53	2.43	2.37	2.38	2.41
	CMR from IDA	4.07	3.69	3.12	3.19	2.88	2.99	2.55	2.85	2.79	2.68	2.74	2.51	2.63	2.38	2.51	2.60
	Differences (%)	-3%	-8%	-4%	-6%	3%	-3%	7%	-2%	0%	2%	-3%	1%	-8%	0%	-5%	-7%
α=0.8, ξ=25%	*r* from Regression Equations	9.58	9.88	10.18	10.09	9.93	9.45	8.89	8.81	8.67	8.43	8.01	7.55	7.17	6.91	6.82	6.85
	CMR from Simplified Analyses	4.26	3.68	3.24	3.28	3.26	3.18	3.04	3.07	3.05	3.01	2.90	2.78	2.66	2.60	2.60	2.63
	CMR from IDA	4.19	3.80	3.20	3.43	2.91	3.39	3.02	3.05	3.19	3.02	2.96	2.79	2.72	2.90	2.58	2.78
	Differences (%)	2%	-3%	1%	-4%	12%	-6%	1%	1%	-4%	0%	-2%	0%	-2%	-10%	1%	-5%
α=1.0, ξ=25%	*r* from Regression Equations	11.15	11.61	11.95	11.66	11.20	10.38	9.87	9.74	9.58	9.28	8.80	8.25	7.82	7.56	7.51	7.61
	CMR from Simplified Analyses	4.96	4.33	3.80	3.79	3.67	3.49	3.38	3.39	3.37	3.31	3.18	3.03	2.90	2.84	2.86	2.92
	CMR from IDA	4.59	4.25	3.54	3.53	3.42	3.29	3.27	3.32	3.42	3.48	3.38	3.23	3.12	3.06	3.00	3.06
	Differences (%)	8%	2%	7%	7%	7%	6%	3%	2%	-1%	-5%	-6%	-6%	-7%	-7%	-5%	-5%

Table 4-52 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying first story heights (ξ=30%)

		Three-Story with Varying First Story Heights						Three-Story with Varying First Story Heights						Nine-Story with Varying First Story Heights					
		h_1/h_{typ}						h_1/h_{typ}						h_1/h_{typ}					
		1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5
	T_{el} (sec)	0.94	0.99	1.04	1.11	1.17	1.25	1.16	1.19	1.23	1.26	1.30	1.35	1.97	2.01	2.04	2.08	2.13	2.18
	δ_y (cm)	9.0	9.3	9.8	10.3	10.9	11.6	13.5	13.5	13.5	13.5	13.5	13.6	22.0	22.2	22.4	22.7	22.9	23.0
	δ_u (cm)	59.3	61.6	64.1	66.8	69.5	72.4	76.8	74.7	73.2	72.8	72.6	72.2	112.0	108.2	100.5	95.0	90.7	88.0
	Maximum Interstory Drift Ratio	5.4%	5.3%	5.3%	5.4%	5.4%	5.4%	5.7%	5.6%	5.6%	5.6%	5.7%	6.0%	5.8%	5.3%	5.1%	4.8%	4.9%	4.9%
	μ_T	6.59	6.62	6.54	6.49	6.38	6.24	5.69	5.53	5.42	5.39	5.38	5.31	5.09	4.87	4.49	4.19	3.96	3.83
	$\Gamma_1 \phi_{1,r}$	1.28	1.27	1.27	1.26	1.26	1.25	1.20	1.19	1.18	1.17	1.16	1.15	1.22	1.21	1.20	1.18	1.17	1.16
α=0.2, ξ=30%	r from Regression Equations	7.75	7.64	7.54	7.65	7.69	7.71	7.08	7.00	6.96	6.97	7.00	6.96	6.25	6.06	5.71	5.44	5.25	5.17
	CMR from Simplified Analyses	2.61	2.53	2.50	2.53	2.55	2.57	3.06	2.98	2.91	2.85	2.80	2.74	2.56	2.48	2.33	2.24	2.15	2.09
	CMR from IDA	2.72	2.67	2.51	2.57	2.74	2.66	3.02	2.84	3.12	3.13	2.78	2.64	2.40	2.52	2.36	2.34	2.09	2.21
	Differences (%)	-4%	-5%	0%	-2%	-7%	-3%	1%	5%	-7%	-9%	1%	4%	7%	-2%	-1%	-4%	3%	-5%
α=0.4, ξ=30%	r from Regression Equations	8.40	8.31	7.98	8.08	8.11	8.12	7.43	7.32	7.27	7.28	7.31	7.26	6.53	6.31	5.93	5.63	5.42	5.31
	CMR from Simplified Analyses	2.83	2.75	2.64	2.67	2.69	2.71	3.21	3.12	3.04	2.98	2.92	2.86	2.68	2.58	2.42	2.32	2.22	2.15
	CMR from IDA	2.90	2.85	2.68	2.81	2.86	2.93	3.15	2.93	3.37	3.19	2.84	3.05	2.68	2.56	2.46	2.51	2.28	2.30
	Differences (%)	-2%	-4%	-1%	-5%	-6%	-8%	2%	6%	-10%	-7%	3%	-6%	0%	1%	-2%	-8%	-3%	-7%
α=0.6, ξ=30%	r from Regression Equations	9.18	9.11	8.70	8.80	8.80	8.79	8.02	7.89	7.83	7.83	7.85	7.80	7.03	6.78	6.34	6.00	5.75	5.63
	CMR from Simplified Analyses	3.09	3.02	2.88	2.91	2.91	2.94	3.47	3.36	3.28	3.20	3.14	3.07	2.88	2.77	2.59	2.47	2.36	2.28
	CMR from IDA	3.25	3.16	2.83	2.89	3.18	3.09	3.59	3.58	3.64	3.44	3.42	3.14	2.75	2.65	2.64	2.59	2.48	2.42
	Differences (%)	-5%	-4%	2%	1%	-8%	-5%	-3%	-6%	-10%	-7%	-8%	-2%	5%	5%	-2%	-5%	-5%	-6%
α=0.8, ξ=30%	r from Regression Equations	10.20	10.17	9.79	9.88	9.87	9.83	8.94	8.78	8.69	8.69	8.71	8.64	7.83	7.53	7.01	6.61	6.31	6.16
	CMR from Simplified Analyses	3.43	3.37	3.24	3.26	3.27	3.28	3.87	3.74	3.64	3.56	3.48	3.40	3.21	3.08	2.86	2.72	2.59	2.49
	CMR from IDA	3.31	3.37	3.07	2.95	3.28	3.38	3.86	3.71	3.77	3.75	3.51	3.56	3.21	3.09	2.76	2.68	2.75	2.45
	Differences (%)	4%	0%	6%	11%	0%	-3%	0%	1%	-3%	-5%	-1%	-4%	0%	0%	4%	1%	-6%	2%
α=1.0, ξ=30%	r from Regression Equations	11.34	11.25	11.14	11.23	11.20	11.13	10.02	9.81	9.70	9.69	9.73	9.65	8.74	8.40	7.80	7.35	7.04	6.89
	CMR from Simplified Analyses	3.82	3.72	3.69	3.71	3.71	3.72	4.33	4.18	4.06	3.97	3.89	3.80	3.58	3.44	3.19	3.02	2.88	2.79
	CMR from IDA	3.53	3.60	3.71	3.69	3.62	3.49	4.02	4.09	4.14	4.16	4.11	4.09	3.31	3.19	2.95	2.91	2.88	2.84
	Differences (%)	8%	3%	-1%	1%	2%	7%	8%	2%	-2%	-5%	-5%	-7%	8%	8%	8%	4%	0%	-2%

Table 4-53 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying uniform story heights (ξ=30%)

		h_{unif}/h_{typ}															
		0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	T_{el} (sec)	0.41	0.50	0.61	0.71	0.82	0.94	1.05	1.18	1.30	1.43	1.57	1.70	1.84	1.99	2.14	2.29
	δ_y (cm)	3.5	4.5	5.5	6.6	7.7	9.0	10.3	11.7	13.1	14.6	16.2	17.9	19.6	21.4	23.3	25.1
	δ_u (cm)	31.0	36.8	42.6	48.2	53.8	59.3	64.8	70.2	75.5	80.9	86.2	91.5	96.8	102.1	107.5	112.9
	Maximum Interstory Drift Ratio	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.5%	5.5%	5.5%	5.5%	5.6%	5.6%
	μ_T	8.86	8.18	7.75	7.30	6.99	6.59	6.29	6.00	5.76	5.54	5.32	5.11	4.94	4.77	4.61	4.50
	$\Gamma_I \phi_{I,r}$	1.27	1.27	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28
α=0.2, ξ=30%	*r* from Regression Equations	8.10	8.36	8.58	8.46	8.25	7.75	7.36	7.38	7.34	7.18	6.85	6.46	6.14	5.95	5.92	5.99
	CMR from Simplified Analyses	3.61	3.12	2.73	2.75	2.71	2.61	2.52	2.57	2.58	2.56	2.48	2.37	2.28	2.24	2.26	2.30
	CMR from IDA	3.71	3.27	2.93	3.08	2.72	2.72	2.49	2.73	2.53	2.67	2.63	2.36	2.29	2.27	2.36	2.16
	Differences (%)	-3%	-5%	-7%	-11%	0%	-4%	1%	-6%	2%	-4%	-6%	0%	0%	-1%	-4%	6%
α=0.4, ξ=30%	*r* from Regression Equations	8.68	8.95	9.20	9.09	8.90	8.40	7.77	7.76	7.69	7.50	7.15	6.74	6.41	6.20	6.15	6.21
	CMR from Simplified Analyses	3.86	3.34	2.93	2.96	2.92	2.83	2.66	2.70	2.71	2.67	2.59	2.48	2.38	2.33	2.35	2.38
	CMR from IDA	3.86	3.67	3.14	3.18	2.87	2.90	2.57	2.84	2.62	2.77	2.79	2.49	2.68	2.38	2.46	2.38
	Differences (%)	0%	-9%	-7%	-7%	2%	-2%	4%	-5%	3%	-4%	-7%	0%	-11%	-2%	-4%	0%
α=0.6, ξ=30%	*r* from Regression Equations	9.37	9.66	9.93	9.84	9.67	9.18	8.46	8.40	8.30	8.08	7.69	7.25	6.89	6.65	6.58	6.62
	CMR from Simplified Analyses	4.17	3.60	3.16	3.20	3.17	3.09	2.89	2.92	2.92	2.88	2.78	2.67	2.56	2.50	2.51	2.54
	CMR from IDA	4.14	3.72	3.25	3.45	3.03	3.25	2.67	2.93	3.12	2.86	2.88	2.63	2.72	2.46	2.69	2.74
	Differences (%)	1%	-3%	-3%	-7%	5%	-5%	8%	0%	-6%	1%	-3%	2%	-6%	2%	-7%	-7%
α=0.8, ξ=30%	*r* from Regression Equations	10.28	10.58	10.91	10.83	10.68	10.20	9.50	9.39	9.23	8.96	8.53	8.05	7.65	7.38	7.28	7.29
	CMR from Simplified Analyses	4.58	3.94	3.47	3.52	3.50	3.43	3.25	3.27	3.25	3.20	3.09	2.96	2.84	2.78	2.78	2.79
	CMR from IDA	4.54	4.20	3.44	3.81	3.69	3.31	3.05	3.15	3.31	3.12	3.30	3.30	2.98	3.13	2.74	3.11
	Differences (%)	1%	-6%	1%	-8%	-5%	4%	7%	4%	-2%	3%	-6%	-10%	-5%	-11%	1%	-10%
α=1.0, ξ=30%	*r* from Regression Equations	12.17	12.67	13.06	12.74	12.25	11.34	10.75	10.58	10.38	10.06	9.54	8.97	8.51	8.22	8.15	8.23
	CMR from Simplified Analyses	5.42	4.72	4.15	4.14	4.02	3.82	3.68	3.68	3.66	3.59	3.45	3.30	3.16	3.09	3.11	3.16
	CMR from IDA	4.94	4.58	3.85	3.89	3.81	3.53	3.46	3.60	3.64	3.81	3.64	3.49	3.37	3.37	3.21	3.31
	Differences (%)	10%	3%	8%	7%	5%	8%	6%	2%	0%	-6%	-5%	-6%	-6%	-8%	-3%	-5%

Table 4-54 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying first story heights (ξ=35%)

		Three-Story with Varying First Story Heights						Three-Story with Varying First Story Heights						Nine-Story with Varying First Story Heights					
		h_1/h_{typ}						h_1/h_{typ}						h_1/h_{typ}					
		1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5	1.0	1.1	1.2	1.3	1.4	1.5
	T_{el} (sec)	0.94	0.99	1.04	1.11	1.17	1.25	1.16	1.19	1.23	1.26	1.30	1.35	1.97	2.01	2.04	2.08	2.13	2.18
	δ_y (cm)	9.0	9.3	9.8	10.3	10.9	11.6	13.5	13.5	13.5	13.5	13.5	13.6	22.0	22.2	22.4	22.7	22.9	23.0
	δ_u (cm)	59.3	61.6	64.1	66.8	69.5	72.4	76.8	74.7	73.2	72.8	72.6	72.2	112.0	108.2	100.5	95.0	90.7	88.0
	Maximum Interstory Drift Ratio	5.4%	5.3%	5.3%	5.4%	5.4%	5.4%	5.7%	5.6%	5.6%	5.6%	5.7%	6.0%	5.8%	5.3%	5.1%	4.8%	4.9%	4.9%
	μ_T	6.59	6.62	6.54	6.49	6.38	6.24	5.69	5.53	5.42	5.39	5.38	5.31	5.09	4.87	4.49	4.19	3.96	3.83
	$\Gamma_1 \phi_{1,r}$	1.28	1.27	1.27	1.26	1.26	1.25	1.20	1.19	1.18	1.17	1.16	1.15	1.22	1.21	1.20	1.18	1.17	1.16
α=0.2, ξ=35%	*r* from Regression Equations	8.09	7.99	7.80	7.91	7.94	7.96	7.33	7.24	7.20	7.21	7.23	7.20	6.48	6.29	5.94	5.67	5.48	5.39
	CMR from Simplified Analyses	2.72	2.65	2.59	2.61	2.63	2.66	3.17	3.08	3.01	2.95	2.89	2.83	2.66	2.57	2.43	2.33	2.25	2.18
	CMR from IDA	2.98	2.92	2.68	2.62	2.87	2.83	3.12	3.01	3.38	3.19	3.02	3.05	2.59	2.66	2.52	2.51	2.20	2.28
	Differences (%)	-9%	-9%	-3%	0%	-8%	-6%	2%	2%	-11%	-8%	-4%	-7%	3%	-3%	-4%	-7%	2%	-4%
α=0.4, ξ=35%	*r* from Regression Equations	8.86	8.77	8.30	8.41	8.43	8.43	7.73	7.62	7.57	7.57	7.60	7.55	6.81	6.59	6.19	5.89	5.66	5.56
	CMR from Simplified Analyses	2.98	2.90	2.75	2.78	2.79	2.82	3.34	3.24	3.17	3.10	3.04	2.97	2.79	2.70	2.53	2.42	2.32	2.25
	CMR from IDA	3.21	3.23	2.87	2.98	3.01	3.02	3.38	3.17	3.49	3.41	3.35	3.20	2.98	2.68	2.55	2.62	2.40	2.35
	Differences (%)	-7%	-10%	-4%	-7%	-7%	-7%	-1%	2%	-9%	-9%	-9%	-7%	-6%	1%	-1%	-8%	-3%	-4%
α=0.6, ξ=35%	*r* from Regression Equations	9.76	9.71	9.14	9.24	9.24	9.22	8.42	8.28	8.21	8.21	8.24	8.18	7.40	7.14	6.67	6.31	6.05	5.93
	CMR from Simplified Analyses	3.29	3.21	3.03	3.05	3.06	3.08	3.64	3.52	3.43	3.36	3.30	3.22	3.03	2.92	2.72	2.60	2.48	2.40
	CMR from IDA	3.47	3.66	2.97	3.19	3.38	3.29	3.78	3.72	3.81	3.60	3.52	3.49	3.04	2.84	2.90	2.80	2.65	2.46
	Differences (%)	-5%	-12%	2%	-4%	-9%	-6%	-4%	-5%	-10%	-7%	-6%	-8%	0%	3%	-6%	-7%	-6%	-2%
α=0.8, ξ=35%	*r* from Regression Equations	10.96	10.94	10.42	10.51	10.48	10.43	9.49	9.32	9.22	9.21	9.24	9.16	8.33	8.01	7.46	7.03	6.71	6.55
	CMR from Simplified Analyses	3.69	3.62	3.45	3.47	3.47	3.48	4.10	3.97	3.86	3.77	3.70	3.61	3.41	3.28	3.05	2.89	2.75	2.65
	CMR from IDA	3.59	3.78	3.68	3.41	3.61	3.52	4.08	4.13	4.01	4.04	3.92	3.86	3.60	3.48	3.29	3.12	2.85	2.61
	Differences (%)	3%	-4%	-6%	2%	-4%	-1%	0%	-4%	-4%	-7%	-6%	-6%	-5%	-6%	-7%	-7%	-4%	2%
α=1.0, ξ=35%	*r* from Regression Equations	12.29	12.19	12.05	12.14	12.09	12.01	10.81	10.58	10.45	10.44	10.48	10.39	9.45	9.08	8.43	7.94	7.59	7.43
	CMR from Simplified Analyses	4.14	4.03	3.99	4.01	4.00	4.01	4.67	4.50	4.37	4.27	4.19	4.09	3.87	3.71	3.44	3.26	3.11	3.01
	CMR from IDA	3.78	3.86	4.11	3.97	3.87	3.66	4.38	4.44	4.52	4.58	4.63	4.52	3.68	3.55	3.38	3.26	3.19	3.04
	Differences (%)	9%	5%	-3%	1%	3%	10%	7%	1%	-3%	-7%	-9%	-10%	5%	5%	2%	0%	-3%	-1%

Table 4-55 Simplified analyses results and comparison with *CMR* obtained from incremental dynamic analyses for buildings with viscous dampers and varying uniform story heights (ξ=35%)

		h_{unif}/h_{typ}															
		0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	T_{el} (sec)	0.41	0.50	0.61	0.71	0.82	0.94	1.05	1.18	1.30	1.43	1.57	1.70	1.84	1.99	2.14	2.29
	δ_y (cm)	3.5	4.5	5.5	6.6	7.7	9.0	10.3	11.7	13.1	14.6	16.2	17.9	19.6	21.4	23.3	25.1
	δ_u (cm)	31.0	36.8	42.6	48.2	53.8	59.3	64.8	70.2	75.5	80.9	86.2	91.5	96.8	102.1	107.5	112.9
	Maximum Interstory Drift Ratio	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.4%	5.5%	5.5%	5.5%	5.5%	5.6%	5.6%
	μ_T	8.86	8.18	7.75	7.30	6.99	6.59	6.29	6.00	5.76	5.54	5.32	5.11	4.94	4.77	4.61	4.50
	$\Gamma_I \phi_{I,r}$	1.27	1.27	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28
α=0.2, ξ=35%	*r* from Regression Equations	8.44	8.70	8.92	8.81	8.60	8.09	7.61	7.63	7.58	7.42	7.08	6.70	6.37	6.18	6.15	6.22
	CMR from Simplified Analyses	3.76	3.24	2.84	2.87	2.82	2.72	2.60	2.66	2.67	2.65	2.56	2.46	2.37	2.33	2.35	2.38
	CMR from IDA	4.05	3.42	3.08	3.11	2.86	2.98	2.78	2.98	2.73	2.84	2.74	2.51	2.38	2.41	2.48	2.29
	Differences (%)	-7%	-5%	-8%	-8%	-1%	-9%	-6%	-11%	-2%	-7%	-7%	-2%	0%	-3%	-5%	4%
α=0.4, ξ=35%	*r* from Regression Equations	9.11	9.39	9.64	9.54	9.35	8.86	8.09	8.07	8.00	7.80	7.44	7.03	6.68	6.47	6.42	6.47
	CMR from Simplified Analyses	4.05	3.50	3.07	3.10	3.07	2.98	2.77	2.81	2.82	2.78	2.69	2.58	2.48	2.43	2.45	2.48
	CMR from IDA	4.14	3.73	3.29	3.28	2.99	3.21	2.91	3.03	3.01	2.91	2.89	2.62	2.72	2.49	2.53	2.68
	Differences (%)	-2%	-6%	-7%	-5%	3%	-7%	-5%	-7%	-6%	-4%	-7%	-2%	-9%	-2%	-3%	-7%
α=0.6, ξ=35%	*r* from Regression Equations	9.92	10.21	10.50	10.41	10.25	9.76	8.89	8.82	8.70	8.47	8.07	7.62	7.24	7.00	6.92	6.95
	CMR from Simplified Analyses	4.42	3.81	3.34	3.39	3.36	3.29	3.04	3.07	3.06	3.02	2.92	2.80	2.69	2.63	2.64	2.66
	CMR from IDA	4.57	3.87	3.38	3.58	3.19	3.47	2.96	3.11	3.24	3.03	3.00	2.78	2.81	2.69	2.77	2.88
	Differences (%)	-3%	-2%	-1%	-5%	5%	-5%	3%	-1%	-6%	0%	-3%	1%	-4%	-2%	-5%	-8%
α=0.8, ξ=35%	*r* from Regression Equations	10.98	11.29	11.64	11.57	11.44	10.96	10.11	9.97	9.79	9.50	9.05	8.55	8.14	7.85	7.73	7.74
	CMR from Simplified Analyses	4.89	4.21	3.70	3.76	3.75	3.69	3.46	3.47	3.45	3.39	3.28	3.14	3.02	2.95	2.95	2.97
	CMR from IDA	4.76	4.40	3.57	3.90	3.76	3.59	3.31	3.48	3.50	3.51	3.58	3.42	3.12	3.21	2.85	3.31
	Differences (%)	3%	-4%	4%	-4%	0%	3%	5%	0%	-1%	-3%	-8%	-8%	-3%	-8%	4%	-10%
α=1.0, ξ=35%	*r* from Regression Equations	13.19	13.74	14.16	13.83	13.30	12.29	11.63	11.42	11.19	10.83	10.28	9.68	9.19	8.89	8.79	8.86
	CMR from Simplified Analyses	5.87	5.12	4.51	4.50	4.36	4.14	3.98	3.97	3.94	3.86	3.72	3.56	3.42	3.34	3.35	3.40
	CMR from IDA	5.43	5.06	4.27	4.13	4.04	3.78	3.65	3.79	3.87	3.92	3.88	3.64	3.62	3.60	3.53	3.61
	Differences (%)	8%	1%	6%	9%	8%	10%	9%	5%	2%	-1%	-4%	-2%	-6%	-7%	-5%	-6%

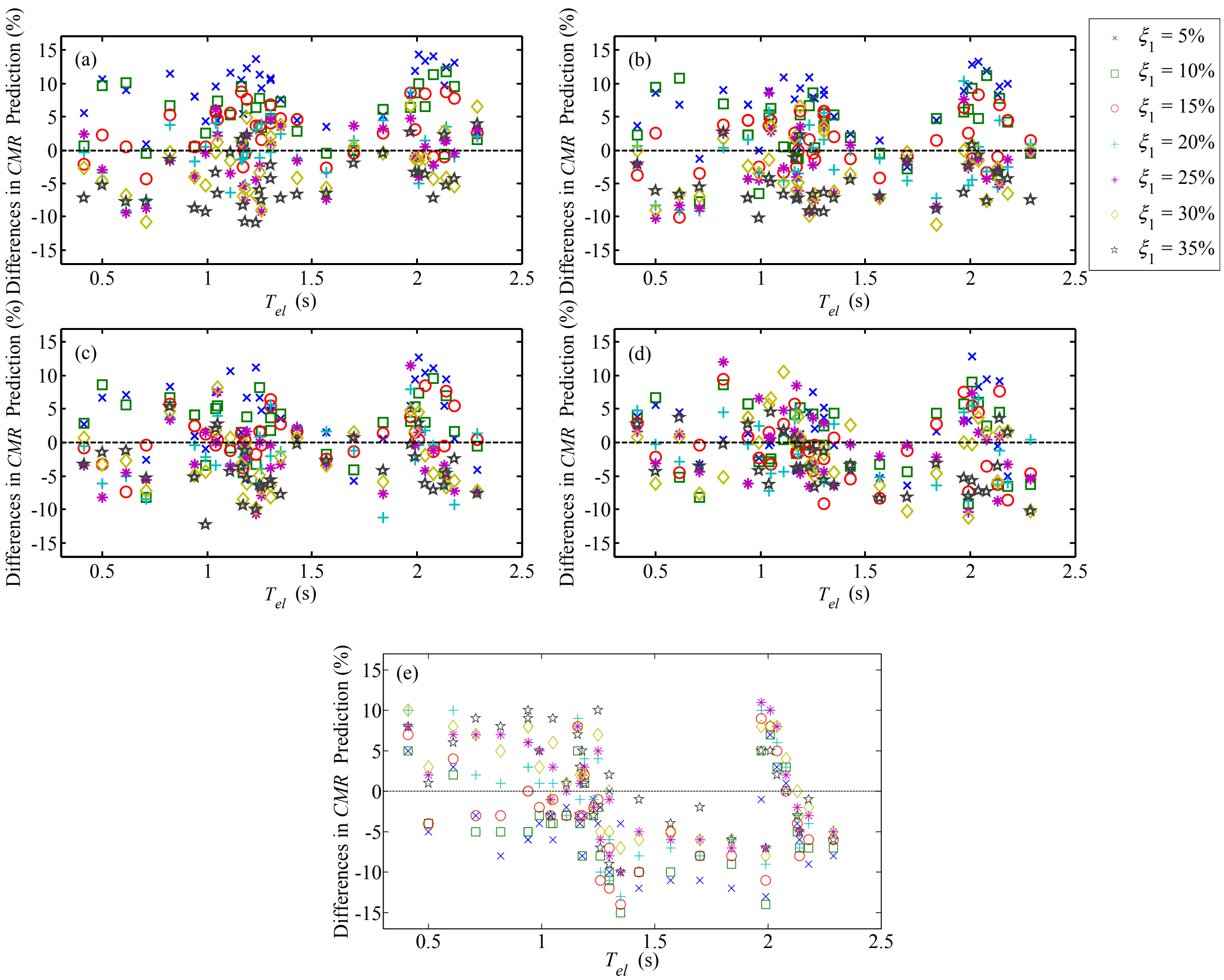


Figure 4-24 Errors in estimating *CMR* for various fundamental periods and critical damping ratios with proposed procedure a) *α*=0.2 b) *α*=0.4 c) *α*=0.6 d) *α*=0.8 e) *α*=1.0

4.8 Development of simplified design procedure

The FEMA P695 methodology defines acceptable values of the adjusted collapse margin ratio *(ACMR)* in terms of an acceptably low probability of collapse (i.e. 10% for average of a given performance group) under MCE ground motions, given a total system collapse uncertainty *(β_{TOT})*. Smaller target probability of collapse should be adopted for structures incorporating supplemental damping systems since the objective of these systems is to enhance the seismic performance of the retrofitted building. A design approach for frame building structures incorporating viscous dampers can be developed using the proposed approximate procedure by conducting a back calculation for the desired *ACMR*. Figure 4-25 shows the minimum required *ACMRs* with respect to the total system collapse uncertainties for different values of collapse probability under MCE ground motions. According to the FEMA P695 methodology, the total system collapse uncertainty *(β_{TOT})* is given by:

$$\beta_{TOT} = \sqrt{\beta_{RTR}^2 + \beta_{DR}^2 + \beta_{TD}^2 + \beta_{MDL}^2} \tag{4.33}$$

where β_{RTR} is the record-to-record collapse uncertainty, β_{DR} is the design requirements-related collapse uncertainty, β_{TD} is the test data-related collapse uncertainty and β_{MDL} is the modeling-related collapse uncertainty. The *CMR* can then be back calculated by:

$$CMR = \frac{ACMR_{\chi\%}}{SSF} \tag{4.34}$$

where *SSF* is the simplified spectral shape factor that accounts for the unique spectral shape of rare ground motions and depends on the elastic fundamental building period, the building target ductility ratio, and the properties of nearby faults, approximately addressed by the seismic design categories. Values of *SSF* are summarized in Table 4-56 for SDC D_{max}. Further information on the derivation of spectral shape factors along with corresponding values for various SDCs can be found in FEMA P695 (2009).

Table 4-56 Spectral Shape Factor (*SSF*) for buildings evaluated by SDC D_{max}, after FEMA P695 (2009)

	Target Ductility Ratio (μ_T)							
T_{el}	1.00	1.10	1.50	2.00	3.00	4.00	6.00	≥ 8.00
≤0.5	1.00	1.05	1.10	1.13	1.18	1.22	1.28	1.33
0.60	1.00	1.05	1.11	1.14	1.20	1.24	1.30	1.36
0.70	1.00	1.06	1.11	1.15	1.21	1.25	1.32	1.38
0.80	1.00	1.06	1.12	1.16	1.22	1.27	1.35	1.41
0.90	1.00	1.06	1.13	1.17	1.24	1.29	1.37	1.44
1.00	1.00	1.07	1.13	1.18	1.25	1.31	1.39	1.46
1.10	1.00	1.07	1.14	1.19	1.27	1.32	1.41	1.49
1.20	1.00	1.07	1.15	1.20	1.28	1.34	1.44	1.52
1.30	1.00	1.08	1.16	1.21	1.29	1.36	1.46	1.55
1.40	1.00	1.08	1.16	1.22	1.31	1.38	1.49	1.58
≥1.5	1.00	1.08	1.17	1.23	1.32	1.40	1.51	1.61

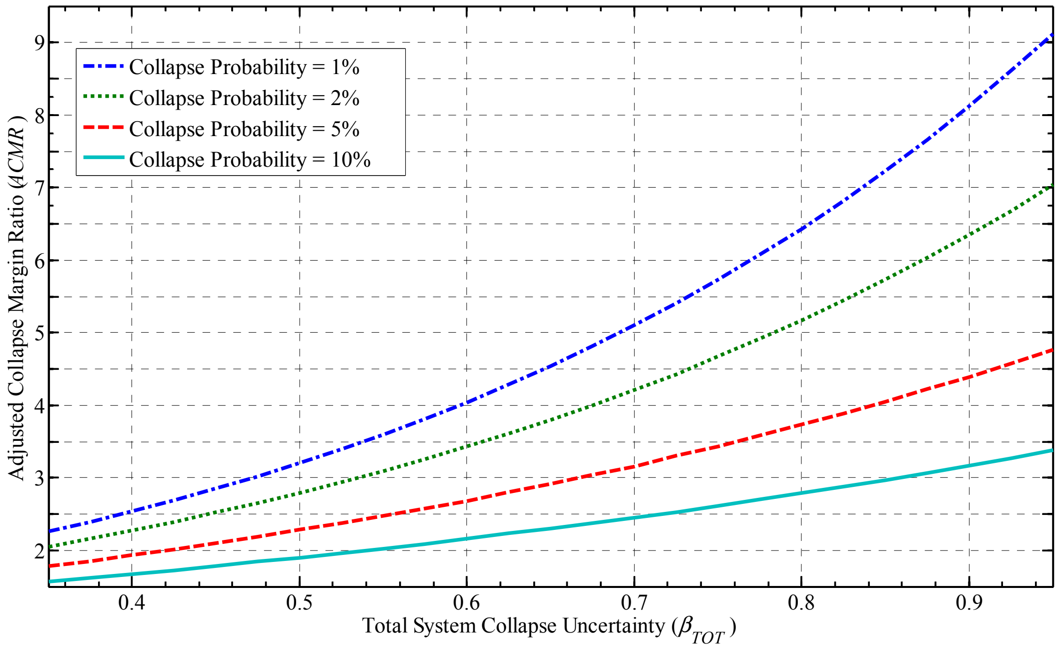


Figure 4-25 Minimum required values of *ACMR* with respect to desired collapse probability under MCE

The velocity exponent (α)is usually pre-selected for a design project and is assumed to be known. The simplified collapsed capacity-based design procedure for frame buildings incorporating viscous dampers can be summarized by the following steps:

1. Design the seismic force resisting system of the building without dampers for the minimum design base shear per Chapter 18 ASCE7-10(2010).
2. Choose an *ACMR* value corresponding to the desired probability of collapse under MCE ground motions from Figure 4-25.
3. Calculate the *CMR* from Equation 4.34.
4. Develop a numerical model of the building structure under evaluation, which incorporates monotonic nonlinear behavior and degradation characteristics of the structural components.
5. Obtain the elastic fundamental period of vibration (T_{el}) and the corresponding first mode shape through an eigenvalue analysis of the elastic building structure.
6. Perform nonlinear static (pushover) analysis of the building structure with the lateral force distribution that is proportional to its first mode shape until the point of 20% strength loss.

7. Fit a bilinear curve to the base shear-roof displacement pushover curve according to the FEMA P695 methodology and obtain the ultimate roof displacement (δ_u) and target ductility ratio (μ_T).

8. Construct the inelastic mode shape (ϕ_I) from pushover data, which represents the displacement of each story when the initial ultimate roof displacement (δ_u) is reached.

9. Calculate the inelastic mode participation factor (Γ_I) using Equation 4.27.

10. Extract the reduction factor (r) from the following equation:

$$\begin{cases} T_{el} < T_S \rightarrow r \geq \dfrac{S_{MS} T_{el}{}^2 \mu_T \Gamma_I \phi_{I,r} CMR}{4\pi^2 \delta_u} \\ T_{el} > T_S \rightarrow r \geq \dfrac{S_{M1} T_{el} \mu_T \Gamma_I \phi_{I,r} CMR}{4\pi^2 \delta_u} \end{cases} \tag{4.35}$$

11. Given the elastic fundamental period of vibration (T_{el}), target ductility (μ_T) and velocity exponent (α) find the appropriate first mode damping ratio (ξ_I) from Equations 4.25 or 4.26. This may be achieved by iterating on damping ratio provided by supplemental viscous dampers (ξ_I) in these equations until the target reduction factor (r) is obtained within a tolerance.

12. Find the nonlinear damper constant at the k^{th} story *(C_k)* from the following equation, given $C_j . f_j^{(1+\alpha)}$ is distributed along the height of the building proportional to the first mode shape.

$$C_k = \frac{8\pi^3 (\xi_1)(\phi_{1k} - \phi_{1(k-1)}) \sum_{j=1}^{N_F} m_j \phi_{1j}{}^2}{\sum_{j=1}^{N_d} (2\pi)^{\alpha_j} T_{el}^{2-\alpha_j} \lambda_j f_j^{1+\alpha_j} \delta_y^{\alpha_j - 1} (\phi_{1j} - \phi_{1(j-1)})^{2+\alpha_j}} \tag{4.36}$$

4.9 Design example with nonlinear viscous dampers

Design calculations are presented herein for the original three-story building described in Section 3.7 for which the height of each floor is 3.96 m. The seismic masses of the first, second and roof level are equal to 478.15, 478.15 and 517.3 kN.sec^2/m, respectively. The fundamental period of the building obtained by eigenvalue analysis is 0.94 s. The total system collapse uncertainty (β_{TOT}) for this example is considered to be 0.60, which corresponds to a good model quality due to the detailed deterioration and degradation models considered, a good quality of test data obtained from over 300 specimens, and a fair quality of the design requirements due to the application of the older Uniform Building Code (UBC 1994). Assuming a desired 2% probability of collapse under MCE ground motions, the minimum required *ACMR*=3.43 from Figure 4-25. The *SSF* is 1.40 from Table B-8 FEMA P695 (2009) and *CMR*=2.45 from Equation 4.34.

The target ductility ratio μ_T = 6.59 from nonlinear static (pushover) analysis and the inelastic mode shape is defined as:

$$\phi_I = \begin{Bmatrix} 59.3 \\ 37.9 \\ 17.1 \end{Bmatrix} \tag{4.37}$$

which results in $\Gamma_I \phi_{I,r}$ = 1.28 from Equation 4.27. Given the spectral acceleration at a period of 1 s (S_{M1}=0.9g for SDC D_{max}), the reduction factor $r \geq 7.32$ from Equation 4.35. Given a velocity exponent (α=0.4), the minimum required supplemental critical damping ratio provided by the nonlinear viscous dampers in the first mode of vibration ξ_I=19% from Equations 4.25. Here nonlinear viscous dampers are incorporated to provide ξ_I=20%, which results in a conservative r = 7.49 and corresponding *CMR* = 2.52 from Equation 4.32. The corresponding values for the nonlinear damper constants are equal to: 1.4, 1.7, and 1.5 kN. $(\text{sec/mm})^{0.4}$ from Equation 4.36 for the first, second and third story, respectively. The *CMR* obtained from nonlinear time history analysis is 2.48 as given in Table 4-48. This represents a difference of 2% with the approximate proposed procedure, which is quite acceptable.

4.10 Design example with linear viscous dampers

Design calculations are presented herein for the original three-story building. The desired reduction factor is $r \geq 7.32$as calculated in Section 4.9. The minimum required supplemental damping ratio isξ_I=10% from Equations 4.26, which results in a *CMR*=2.53 from Equation 4.32. The corresponding values for the linear damper constants are equal to: 2.4, 3.0 and 2.6kN.sec/mm from Equation 4.36 for the first, second and third story, respectively. The *CMR* obtained from nonlinear time history analysis is 2.66 as given in Table 4-44. This represents a difference of 5% with the approximate proposed procedure, which is quite acceptable.

4.11 Limitations of the proposed procedure

In addition to the limitations addressed in Section 3.10, the proposed procedure is not applicable to buildings with extreme soft story irregularities and weak columns – strong beams per ASCE 7. The evaluation and design procedure has been assessed for a distribution of viscous dampers proportional to the interstory drifts arising from first mode shape and has not been validated for other distributions.

4.12 Summary

The simplified procedure to evaluate the seismic sidesway collapse capacity of retrofitted building structures incorporating viscous dampers presented in this section is based on the development of a robust database of nonlinear seismic responses of single-degree-of-freedom systems for various seismic intensities and uses nonlinear static (pushover) analysis without the need for nonlinear time history dynamic analyses. The procedure was assessed using a total of 1,190 different retrofitted building structure models with a wide range of fundamental periods (0.94 s to 2.29 s) and added damping ratios (5% to 35%). The collapse capacities predicted by the proposed simplified procedure are in very good agreement with those obtained by nonlinear time history dynamic analyses. For the series of three-, six- and nine-story benchmark building models with varying first story height considered in this study, the average values of absolute differences between the simplified and nonlinear dynamic time history analyses are only 3.9, 4.8 and 5.0%, with standard deviations of 4.7, 5.8 and 5.9%, respectively. The corresponding values for a series of benchmark building models with varying uniform story heights are 4.6% and 5.4% respectively. Finally, using back calculation on the *CMR*, a simplified design procedure was introduced for frame building incorporating viscous dampers. The procedure is consistent with the ASCE7-10 seismic provisions and is aimed at achieving a pre-determined probability of collapse under MCE events. Again, it is important to note that the proposed procedure is only an approximate approach for the quick estimation of the collapse capacity of frame buildings incorporating linear and nonlinear viscous dampers that cannot replace more rigorous nonlinear time history dynamic analyses.

SECTION 5

COLLAPSE CAPACITY BASED EVALUATION AND DESIGN OF FRAME BUILDINGS WITH HYSTERETIC DAMPERS

5.1 Introduction

This section describes the assessment of the proposed simplified procedure described in Section 3.4 for estimating the seismic sidesway collapse margin ratio (*CMR*) of frame building structures equipped with hysteretic dampers. The proposed simplified procedure is assessed by comparing its collapse capacity predictions on 210 different building models with those obtained by incremental nonlinear dynamic analyses. A straightforward collapse capacity based design procedure is also introduced for hysteretically damped structures without extreme soft story irregularities.

5.2 Characteristics of hysteretic dampers

Metallic and friction dampers belong to the category of displacement-activated supplemental damping systems known as hysteretic dampers. Metallic dampers take advantage of the hysteretic behavior of metals when deformed into the post-elastic range to dissipate energy. Friction dampers, on the other hand, dissipate the seismic energy by friction that develops at the interface between two solid bodies sliding relative to each other. Both types of dampers when used in a bracing configuration exhibit hysteretic behavior that can be idealized by an elastic-perfectly plastic load-displacement relationship. Figure 5-1 shows load-displacement characteristics of hysteretic dampers where F_{hd} stands for the force provided by the damping system. For a metallic damper, the load F_a, which activates the damper, corresponds to the yield load of the damper. For a friction damper, F_a corresponds to the slip load of the damper. The elastic stiffness (K_b) can be associated with the stiffness of the connecting bracing elements (Christopoulos and Filiatrault, 2006).

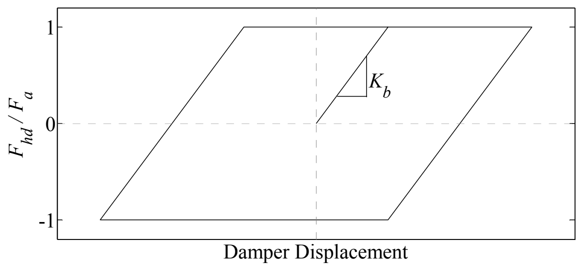


Figure 5-1 Idealized load-displacement characteristics of hysteretic dampers in a bracing configuration

5.3 Step-by step procedure for estimating the *CMR*

The proposed simplified collapse assessment procedure described above can be summarized by the following steps:

1. Develop a numerical model of the building structure under evaluation with hysteretic dampers, which incorporates monotonic nonlinear behavior and degradation characteristics of the structural components.
2. Obtain the elastic fundamental period of vibration (T_{el}) and the corresponding first mode shape using eigenvalue analysis of the elastic building structure as discussed in Section 3.5. This elastic period of vibration corresponds to the condition before activation of the hysteretic dampers.
3. Perform a nonlinear static (pushover) analysis of the building structure incorporating nonlinear response of entire damping system with the lateral force distribution that is proportional to its first mode shape until the point of 20% strength loss.
4. Fit a bilinear curve to the base shear-roof displacement pushover curve according to the FEMA P695 methodology (see Figure 3-9) and obtain the ultimate roof displacement (δ_u) and target ductility ratio (μ_T).
5. Construct the inelastic mode shape (ϕ_I) from pushover data, which represents the relative displacement of each story when the ultimate roof displacement (δ_u) is reached.
6. Calculate the inelastic mode participation factor (Γ_I) using Equation 3.7.
7. Extract the reduction factor (r) by linear interpolation from Table 3-2 to Table 3-5 based on the given elastic fundamental period of vibration (T_{el}) and target ductility (μ_T).
8. Calculate the *CMR* from Equation 3.11.

5.4 Assessment of proposed procedure

5.4.1 Original buildings without hysteretic dampers

In order to assess the accuracy and robustness of the proposed procedure, the collapse capacities of the same three steel unbraced (original) benchmark frame building models (three-, six- and nine-story) described in Section 3.7 were evaluated. The OpenSees software (2013) was used to carry out the structural analyses. Analytical modeling development assumptions are described in Section 3.7.

The results of the application of the proposed simplified collapse evaluation procedure to the three frame building structures considered above are summarized in Table 5-1. The predicted *CMR* values based on the proposed simplified procedure are in very good agreement with those obtained from nonlinear time history dynamic analyses, but with much reduced computational demand.

Table 5-1 Simplified analyses results and comparison with *CMR* obtained from time history analyses for unbraced buildings

Type	T_u (sec)	δ_y (cm)	δ_u (cm)	μ_T	V_{max} (kN)	r	$\Gamma_I \phi_{I,r}$	*CMR* fom Equation 11	*CMR* from Nonlinear Time History Analyses	Absolute Differences
3 Story	0.94	9.0	59.3	6.59	4082	5.87	1.28	1.96	1.93	2%
6 Story	1.33	13.5	72.5	5.37	2762	5.39	1.16	2.11	1.97	7%
9 Story	2.12	22.8	91.4	4.01	5489	4.17	1.17	1.71	1.65	4%

5.4.2 Building incorporating hysteretic dampers

Hysteretic dampers were incorporated to the considered frame buildings as shown in Figure 5-2. It was assumed that braces would be installed after the building would be completed, and thus would not carry gravity loads. The braced cross-sectional areas were chosen so that the lateral stiffness of the bracing members were proportional to the interstory drift arising from first mode shape of the unbraced buildings. This assumption is a simple way to increase the lateral stiffness to the softer stories of the building. The braces were added to obtain a ratio of fundamental period of the braced building (T_b) to that of an unbraced building (T_u) ranging from 0.3 to 0.9 at 0.1 increments. Table 5-2 to Table 5-4 list the cross-sectional area of the braces in each story for the considered cases.

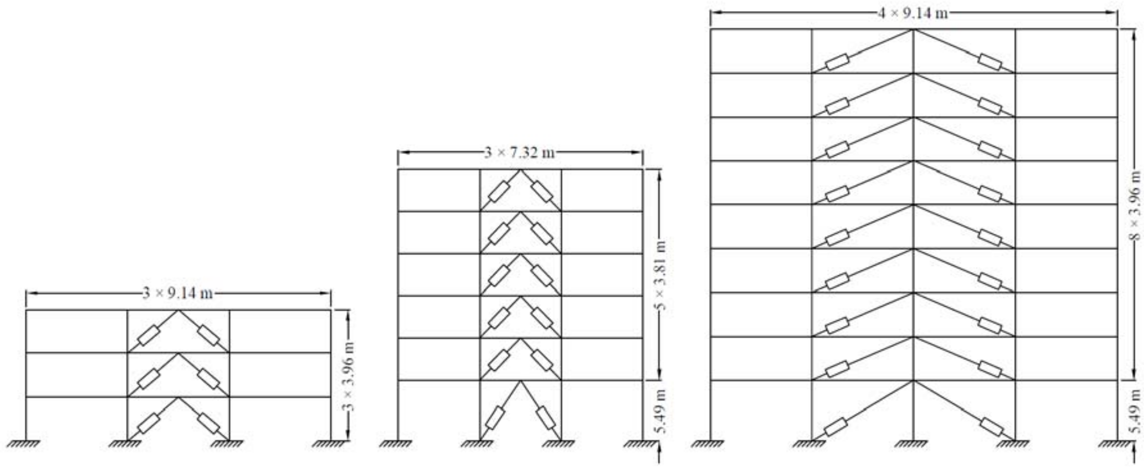


Figure 5-2 Elevation view of the selected steel frame buildings and configuration of hysteretic dampers

Table 5-2 Brace cross-sectional areas of three-story building

T_b/T_u	Brace Cross-Sectional Area (m^2)		
	1st Story	2nd Story	3rd Story
0.3	0.0331	0.0408	0.0356
0.4	0.0152	0.0188	0.0164
0.5	0.0084	0.0104	0.0091
0.6	0.0049	0.0061	0.0053
0.7	0.0027	0.0034	0.0029
0.8	0.0015	0.0018	0.0016
0.9	0.0006	0.0007	0.0006

Table 5-3 Brace cross-sectional areas of six-story building

T_b/T_u	Brace Cross-Sectional Area (m^2)					
	1st Story	2nd Story	3rd Story	4th Story	5th Story	6th Story
0.3	0.1400	0.0494	0.0478	0.0416	0.0369	0.0227
0.4	0.0622	0.0219	0.0212	0.0185	0.0164	0.0101
0.5	0.0303	0.0107	0.0103	0.0090	0.0080	0.0049
0.6	0.0142	0.0050	0.0049	0.0042	0.0038	0.0023
0.7	0.0072	0.0026	0.0025	0.0022	0.0019	0.0012
0.8	0.0036	0.0013	0.0012	0.0011	0.0009	0.0006
0.9	0.0013	0.0005	0.0005	0.0004	0.0003	0.0002

Table 5-4 Brace cross-sectional areas of nine-story building

T_b/T_u	Brace Cross-Sectional Area (m^2)								
	1st Story	2nd Story	3rd Story	4th Story	5th Story	6th Story	7th Story	8th Story	9th Story
0.3	0.2031	0.1179	0.1166	0.1226	0.1116	0.1087	0.1103	0.1055	0.0806
0.4	0.0601	0.0349	0.0345	0.0363	0.0330	0.0322	0.0327	0.0312	0.0239
0.5	0.0285	0.0166	0.0164	0.0172	0.0157	0.0153	0.0155	0.0148	0.0113
0.6	0.0155	0.0090	0.0089	0.0094	0.0085	0.0083	0.0084	0.0081	0.0062
0.7	0.0086	0.0050	0.0049	0.0052	0.0047	0.0046	0.0047	0.0045	0.0034
0.8	0.0044	0.0025	0.0025	0.0026	0.0024	0.0023	0.0024	0.0023	0.0017
0.9	0.0018	0.0010	0.0010	0.0011	0.0010	0.0010	0.0010	0.0009	0.0007

The activation loads of the dampers were set according to the procedure developed by Filiatrault and Cherry (1988). The distribution of shear forces required to activate all hysteretic dampers is proportional to the first mode shape of the braced structure. The ratio of the total shear force required to activate all dampers in the structure (V_0) to the maximum lateral force of the fully-yielded unbraced buildings (D_{max}) (see Figure 3-9) was chosen from 0.2 to 2 at 0.2 increments. Table 5-5 to Table 5-7 list the activation load of the hysteretic dampers in each story of the considered cases where $V_0=V_{max}$. The activation load for other cases can be found by multiplying the values by V_0/V_{max} ratio.

Table 5-5 Activation load of the hysteretic dampers of three-story building ($V_0=V_{max}$)

T_b/T_u	Activation Load (kN)		
	1st Story	2nd Story	3rd Story
0.3	1176	965	561
0.4	1184	964	553
0.5	1183	965	553
0.6	1178	967	556
0.7	1171	969	562
0.8	1163	970	568
0.9	1155	971	574

Table 5-6 Activation load of the hysteretic dampers of three-story building ($V_0=V_{max}$)

T_b/T_u	Activation Load (kN)					
	1st Story	2nd Story	3rd Story	4th Story	5th Story	6th Story
0.3	562	435	397	334	245	132
0.4	562	435	397	334	245	132
0.5	562	435	397	334	245	132
0.6	575	440	397	330	239	127
0.7	584	443	397	327	235	124
0.8	590	446	396	324	232	122
0.9	595	447	396	323	230	120

Table 5-7 Activation load of the hysteretic dampers of three-story building (V_0=V_{max})

T_b/T_u	Activation Load (kN)								
	1st Story	2nd Story	3rd Story	4th Story	5th Story	6th Story	7th Story	8th Story	9th Story
0.3	492	453	437	410	372	321	259	184	98
0.4	505	463	443	412	370	316	251	176	92
0.5	511	467	446	413	368	313	247	172	89
0.6	514	469	447	413	368	312	245	171	88
0.7	516	470	447	413	367	311	245	170	88
0.8	517	470	447	412	366	311	245	171	88
0.9	518	470	446	411	366	310	245	171	89

Table 5-8 to Table 5-10 list the increases in *CMR* values obtained through the incorporation of hysteretic dampers for the three-, six- and nine-story buildings, respectively. These results were obtained from nonlinear time history dynamic analyses of 210 considered models and clearly show that the seismic performance of the studied systems has been improved significantly. Figure shows the variation of the increase in *CMR* for various T_b/T_u ratios. The *CMR* generally increases with a reduction of T_b/T_u and an increase of V_0/V_{max}. In addition, as it is clearly observed in the case of the six-story building, a rapid increase in *CMR* occurs for T_b/T_u=0.4.

Table 5-8 Increase in *CMR* due to incorporation of hysteretic dampers in three-story building obtained from nonlinear time history dynamic analyses

T_b/T_u	V_0/V_{max}									
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.3	148%	155%	162%	170%	176%	179%	185%	193%	197%	202%
0.4	111%	115%	120%	123%	131%	137%	139%	142%	147%	150%
0.5	82%	90%	92%	96%	102%	106%	117%	124%	132%	135%
0.6	40%	48%	51%	53%	60%	64%	67%	74%	77%	88%
0.7	22%	25%	32%	35%	44%	47%	49%	52%	56%	59%
0.8	10%	18%	24%	31%	40%	46%	52%	64%	72%	83%
0.9	7%	11%	22%	28%	36%	49%	58%	64%	71%	77%

Table 5-9 Increase in *CMR* due to incorporation of hysteretic dampers in six-story building obtained from nonlinear time history dynamic analyses

T_b/T_u	V_0/V_{max}									
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.3	178%	189%	190%	193%	193%	195%	196%	197%	197%	200%
0.4	114%	117%	119%	120%	122%	124%	124%	127%	130%	132%
0.5	56%	61%	63%	63%	66%	68%	69%	71%	73%	76%
0.6	48%	54%	56%	58%	61%	61%	63%	63%	65%	66%
0.7	21%	23%	26%	29%	31%	38%	43%	48%	50%	54%
0.8	15%	19%	25%	29%	35%	39%	43%	46%	48%	50%
0.9	7%	9%	13%	20%	22%	25%	29%	36%	39%	43%

Table 5-10 Increase in *CMR* due to incorporation of hysteretic dampers in nine-story building obtained from nonlinear time history dynamic analyses

T_b/T_u	V_0/V_{max}									
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.3	117%	118%	119%	121%	122%	122%	124%	126%	126%	127%
0.4	92%	93%	95%	97%	98%	98%	101%	102%	103%	104%
0.5	70%	72%	72%	74%	76%	78%	78%	79%	80%	80%
0.6	52%	52%	54%	54%	58%	59%	59%	62%	62%	62%
0.7	38%	44%	46%	50%	54%	59%	63%	65%	67%	72%
0.8	16%	18%	18%	22%	22%	25%	25%	27%	27%	28%
0.9	4%	6%	6%	8%	9%	13%	19%	24%	26%	28%

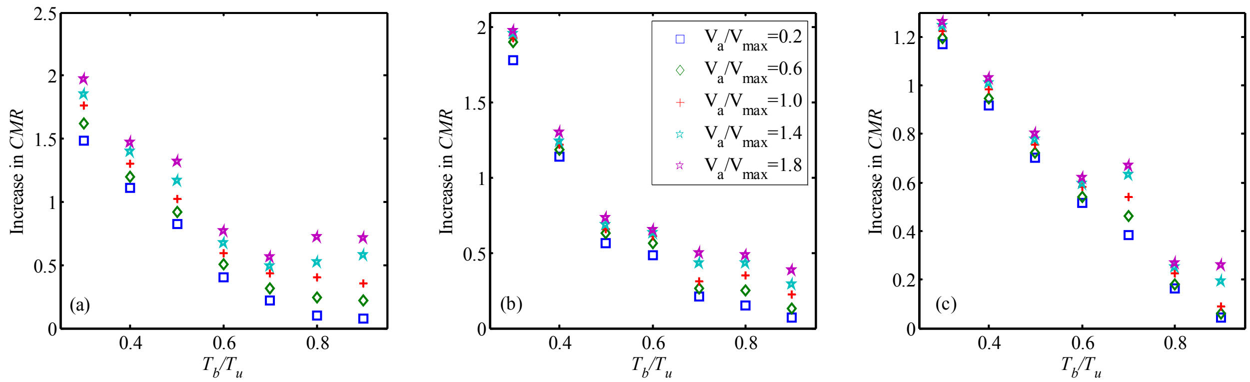


Figure 5-3 Increase in *CMR* due to incorporation of hysteretic dampers in a) three-story b) six-story c) nine-story building obtained from nonlinear time history dynamic analyses

Results of the application of the proposed simplified collapse evaluation procedure to the ensemble of frame building structures incorporating hysteretic dampers considered above are summarized in Table 5-11 to Table 5-20. Figure 5-4 to Figure 5-6 compare the differences in *CMR* values predicted by the proposed simplified procedure with those obtained from nonlinear time history dynamic analyses as a function of the T_b/T_u for the three-, six- and nine-story buildings, respectively. Clearly, for the entire period range considered (0.28 s to 1.9 s), there is no discernible divergence in trend, which is indicative of the robustness of the proposed procedure. The averages of absolute differences between the simplified and nonlinear dynamic time history analyses are only 3.8, 4.4 and 2.8%, with standard deviations of 4.7, 5.1 and 2.7% for the three-, six- and nine-story buildings respectively. These results are acceptable considering that the proposed simplified procedure alleviates the need for nonlinear time history dynamic analyses.

Table 5-11 Simplified analyses results and comparison with *CMR* values obtained from time history analyses for selected buildings with hysteretic dampers and V_0/V_{max}=0.2

Type	T_b/T_u	δ_y (cm)	δ_u (cm)	μ_T	r	$\Gamma_I \phi_{I,r}$	*CMR* fom Simplified Analysis	*CMR* from Nonlinear Time History Analyses	Absolute Differences
3 Story	0.30	0.83	61.78	74.2	18.81	1.28	4.20	4.79	12%
	0.40	1.50	61.90	41.3	16.94	1.28	3.70	4.08	9%
	0.50	2.33	61.88	26.6	15.18	1.28	3.37	3.52	4%
	0.60	3.33	61.84	18.6	12.02	1.28	2.68	2.71	1%
	0.70	4.64	61.74	13.3	9.70	1.28	2.39	2.35	2%
	0.80	6.07	61.65	10.1	8.24	1.28	2.34	2.12	10%
	0.90	7.89	61.55	7.8	6.79	1.28	2.21	2.07	7%
6 Story	0.30	2.62	76.32	29.1	13.37	1.17	5.04	5.48	8%
	0.40	3.19	76.32	23.9	15.16	1.17	3.82	4.22	10%
	0.50	4.12	76.30	18.5	12.65	1.17	2.99	3.08	3%
	0.60	5.62	75.43	13.4	10.40	1.16	2.82	2.92	3%
	0.70	7.43	74.87	10.1	8.49	1.16	2.59	2.38	9%
	0.80	9.46	74.47	7.9	6.98	1.16	2.39	2.27	5%
	0.90	11.78	74.13	6.3	5.91	1.15	2.23	2.11	6%
9 Story	0.30	2.32	98.58	42.4	24.92	1.20	3.39	3.58	5%
	0.40	3.83	95.14	24.8	17.31	1.18	2.95	3.16	7%
	0.50	5.78	93.88	16.2	12.78	1.18	2.65	2.81	6%
	0.60	8.18	93.30	11.4	9.98	1.18	2.44	2.50	2%
	0.70	11.12	92.96	8.4	8.44	1.18	2.41	2.28	6%
	0.80	14.71	92.80	6.3	5.75	1.18	1.89	1.92	2%
	0.90	18.70	92.61	5.0	4.63	1.18	1.72	1.72	0%

Table 5-12 Simplified analyses results and comparison with *CMR* values obtained from time history analyses for selected buildings with hysteretic dampers and V_0/V_{max}=0.4

Type	T_b/T_u	δ_y (cm)	δ_u (cm)	μ_T	r	$\Gamma_I \phi_{I,r}$	*CMR* fom Simplified Analysis	*CMR* from Nonlinear Time History Analyses	Absolute Differences
3 Story	0.30	0.90	64.07	70.9	18.27	1.28	4.43	4.93	10%
	0.40	1.62	64.19	39.7	16.46	1.28	3.87	4.15	7%
	0.50	2.51	64.18	25.6	14.76	1.28	3.52	3.66	4%
	0.60	3.59	64.12	17.9	11.59	1.28	2.79	2.85	2%
	0.70	5.00	64.04	12.8	9.39	1.28	2.50	2.42	3%
	0.80	6.55	63.94	9.8	7.97	1.28	2.44	2.27	8%
	0.90	8.51	63.85	7.5	6.53	1.28	2.29	2.14	7%
6 Story	0.30	2.74	77.72	28.4	13.15	1.17	5.18	5.69	9%
	0.40	3.33	77.69	23.4	14.79	1.17	3.88	4.28	9%
	0.50	4.30	77.64	18.1	12.36	1.17	3.04	3.17	4%
	0.60	5.87	76.72	13.1	10.18	1.16	2.88	3.03	5%
	0.70	7.76	76.14	9.8	8.31	1.16	2.65	2.42	10%
	0.80	9.88	75.71	7.7	6.80	1.16	2.43	2.35	3%
	0.90	12.31	75.34	6.1	5.75	1.15	2.27	2.15	6%
9 Story	0.30	2.40	99.92	41.7	24.56	1.20	3.44	3.60	4%
	0.40	3.94	96.46	24.5	17.12	1.19	3.00	3.18	6%
	0.50	5.97	95.18	15.9	12.60	1.18	2.69	2.83	5%
	0.60	8.44	94.58	11.2	9.83	1.18	2.48	2.51	1%
	0.70	11.47	94.17	8.2	8.31	1.18	2.45	2.37	3%
	0.80	15.17	93.84	6.2	5.64	1.18	1.91	1.95	2%
	0.90	19.30	93.56	4.8	4.56	1.18	1.76	1.75	1%

Table 5-13 Simplified analyses results and comparison with *CMR* values obtained from time history analyses for selected buildings with hysteretic dampers and V_0/V_{max}=0.6

Type	T_b/T_u	δ_y (cm)	δ_u (cm)	μ_T	r	$\Gamma_I \phi_{I,r}$	*CMR* fom Simplified Analysis	*CMR* from Nonlinear Time History Analyses	Absolute Differences
3 Story	0.30	0.96	66.37	68.9	17.88	1.28	4.61	5.05	9%
	0.40	1.74	66.47	38.3	15.89	1.28	4.01	4.24	5%
	0.50	2.69	66.46	24.7	14.38	1.28	3.68	3.71	1%
	0.60	3.85	66.41	17.2	11.29	1.28	2.91	2.91	0%
	0.70	5.37	66.32	12.3	9.13	1.28	2.60	2.54	2%
	0.80	7.04	66.24	9.4	7.70	1.28	2.53	2.40	5%
	0.90	9.13	66.14	7.2	6.31	1.28	2.37	2.36	0%
6 Story	0.30	2.86	79.04	27.7	12.92	1.17	5.31	5.72	7%
	0.40	3.46	79.01	22.8	14.43	1.17	3.95	4.31	8%
	0.50	4.49	78.94	17.6	12.13	1.17	3.12	3.21	3%
	0.60	6.11	78.02	12.8	9.97	1.16	2.94	3.08	5%
	0.70	8.09	77.44	9.6	8.17	1.16	2.72	2.49	9%
	0.80	10.31	76.98	7.5	6.64	1.15	2.48	2.46	1%
	0.90	12.83	76.58	6.0	5.62	1.15	2.32	2.23	4%
9 Story	0.30	2.45	101.16	41.2	24.35	1.20	3.49	3.62	4%
	0.40	4.07	97.65	24.0	16.88	1.19	3.05	3.21	5%
	0.50	6.15	96.29	15.6	12.41	1.18	2.73	2.84	4%
	0.60	8.70	95.59	11.0	9.66	1.18	2.51	2.54	1%
	0.70	11.81	95.14	8.1	8.17	1.18	2.48	2.41	3%
	0.80	15.64	94.79	6.1	5.53	1.18	1.94	1.95	1%
	0.90	19.88	94.49	4.8	4.50	1.17	1.78	1.75	2%

Table 5-14 Simplified analyses results and comparison with *CMR* values obtained from time history analyses for selected buildings with hysteretic dampers and V_0/V_{max}=0.8

Type	T_b/T_u	δ_y (cm)	δ_u (cm)	μ_T	r	$\Gamma_I \phi_{I,r}$	*CMR* fom Simplified Analysis	*CMR* from Nonlinear Time History Analyses	Absolute Differences
3 Story	0.30	1.03	68.65	66.4	17.30	1.28	4.80	5.21	8%
	0.40	1.85	68.76	37.1	15.46	1.28	4.17	4.31	3%
	0.50	2.88	68.74	23.9	14.00	1.28	3.83	3.78	1%
	0.60	4.11	68.70	16.7	11.01	1.28	3.04	2.96	3%
	0.70	5.74	68.60	11.9	8.89	1.28	2.71	2.61	4%
	0.80	7.51	68.52	9.1	7.48	1.28	2.62	2.53	4%
	0.90	9.76	68.42	7.0	6.11	1.28	2.46	2.48	1%
6 Story	0.30	2.96	80.31	27.1	12.71	1.17	5.42	5.77	6%
	0.40	3.60	80.26	22.3	14.09	1.17	4.01	4.34	8%
	0.50	4.66	80.18	17.2	11.95	1.17	3.19	3.21	1%
	0.60	6.36	79.26	12.5	9.78	1.16	3.00	3.12	4%
	0.70	8.41	78.66	9.4	8.02	1.16	2.78	2.55	9%
	0.80	10.72	78.18	7.3	6.49	1.15	2.52	2.54	1%
	0.90	13.35	77.82	5.8	5.51	1.15	2.36	2.36	0%
9 Story	0.30	2.53	102.15	40.4	23.95	1.20	3.54	3.65	3%
	0.40	4.18	98.64	23.6	16.60	1.19	3.08	3.25	5%
	0.50	6.34	97.26	15.3	12.22	1.18	2.77	2.87	4%
	0.60	8.96	96.55	10.8	9.50	1.18	2.55	2.54	0%
	0.70	12.16	96.09	7.9	8.02	1.18	2.51	2.48	1%
	0.80	16.08	95.72	6.0	5.45	1.17	1.96	2.02	3%
	0.90	20.45	95.40	4.7	4.44	1.17	1.81	1.78	2%

Table 5-15 Simplified analyses results and comparison with *CMR* values obtained from time history analyses for selected buildings with hysteretic dampers and V_0/V_{max}=1.0

Type	$T_{b/}T_u$	δ_y (cm)	δ_u (cm)	μ_T	r	$\Gamma_1 \phi_{1,r}$	*CMR* fom Equation 11	*CMR* from Nonlinear Time History Analyses	Absolute Differences
3 Story	0.30	1.09	70.93	64.9	16.97	1.28	4.97	5.33	7%
	0.40	1.97	71.04	36.0	15.10	1.28	4.34	4.45	2%
	0.50	3.06	71.03	23.2	13.64	1.28	3.96	3.90	2%
	0.60	4.37	70.98	16.2	10.77	1.28	3.16	3.08	3%
	0.70	6.10	70.90	11.6	8.70	1.28	2.81	2.77	1%
	0.80	7.99	70.81	8.9	7.29	1.28	2.72	2.71	0%
	0.90	10.38	70.72	6.8	5.94	1.28	2.54	2.62	3%
6 Story	0.30	3.07	81.61	26.6	12.52	1.17	5.53	5.77	4%
	0.40	3.74	81.55	21.8	13.80	1.17	4.07	4.37	7%
	0.50	4.84	81.46	16.8	11.79	1.17	3.27	3.27	0%
	0.60	6.60	80.53	12.2	9.62	1.16	3.06	3.17	3%
	0.70	8.73	79.90	9.1	7.89	1.16	2.84	2.58	10%
	0.80	11.14	79.39	7.1	6.35	1.15	2.57	2.66	3%
	0.90	13.87	79.27	5.7	5.42	1.15	2.41	2.41	0%
9 Story	0.30	2.60	103.14	39.6	23.44	1.20	3.57	3.67	3%
	0.40	4.31	99.60	23.1	16.26	1.18	3.12	3.27	5%
	0.50	6.51	98.21	15.1	12.06	1.18	2.81	2.90	3%
	0.60	9.22	97.48	10.6	9.35	1.18	2.58	2.61	1%
	0.70	12.51	97.02	7.8	7.87	1.18	2.53	2.54	0%
	0.80	16.55	96.63	5.8	5.37	1.17	1.99	2.02	1%
	0.90	21.05	96.31	4.6	4.38	1.17	1.84	1.80	2%

Table 5-16 Simplified analyses results and comparison with *CMR* values obtained from time history analyses for selected buildings with hysteretic dampers and V_0/V_{max}=1.2

Type	$T_{b/}T_u$	δ_y (cm)	δ_u (cm)	μ_T	r	$\Gamma_1 \phi_{1,r}$	*CMR* fom Simplified Analysis	*CMR* from Nonlinear Time History Analyses	Absolute Differences
3 Story	0.30	1.16	73.20	62.8	16.62	1.28	5.18	5.38	4%
	0.40	2.09	73.31	35.0	14.73	1.28	4.48	4.57	2%
	0.50	3.25	73.30	22.6	13.28	1.28	4.10	3.97	3%
	0.60	4.64	73.25	15.8	10.55	1.28	3.27	3.17	3%
	0.70	6.47	73.17	11.3	8.51	1.28	2.92	2.83	3%
	0.80	8.46	73.08	8.6	7.12	1.28	2.81	2.82	0%
	0.90	10.89	74.58	6.8	5.97	1.28	2.68	2.88	7%
6 Story	0.30	3.19	82.85	26.0	12.31	1.17	5.65	5.81	3%
	0.40	3.88	82.78	21.4	13.54	1.17	4.14	4.42	6%
	0.50	5.01	82.67	16.5	11.64	1.17	3.34	3.30	1%
	0.60	6.84	81.73	11.9	9.45	1.16	3.12	3.17	2%
	0.70	9.06	81.10	9.0	7.77	1.16	2.90	2.71	7%
	0.80	11.55	80.58	7.0	6.23	1.15	2.61	2.73	4%
	0.90	14.36	80.91	5.6	5.36	1.15	2.47	2.47	0%
9 Story	0.30	2.68	104.12	38.9	22.82	1.19	3.58	3.67	3%
	0.40	4.42	100.55	22.7	16.00	1.18	3.15	3.27	4%
	0.50	6.69	99.16	14.8	11.88	1.18	2.85	2.93	3%
	0.60	9.46	98.41	10.4	9.22	1.18	2.61	2.63	1%
	0.70	12.85	97.93	7.6	7.73	1.17	2.56	2.62	2%
	0.80	17.01	97.54	5.7	5.29	1.17	2.02	2.06	2%
	0.90	21.62	97.20	4.5	4.33	1.17	1.87	1.86	1%

Table 5-17 Simplified analyses results and comparison with *CMR* values obtained from time history analyses for selected buildings with hysteretic dampers and V_0/V_{max}=1.4

Type	T_b/T_u	δ_y (cm)	δ_u (cm)	μ_T	r	$\Gamma_1 \phi_{1,r}$	*CMR* fom Simplified Analysis	*CMR* from Nonlinear Time History Analyses	Absolute Differences
3 Story	0.30	1.22	75.48	61.7	16.41	1.28	5.38	5.50	2%
	0.40	2.21	75.59	34.2	14.41	1.28	4.63	4.62	0%
	0.50	3.42	75.58	22.1	13.00	1.28	4.23	4.18	1%
	0.60	4.90	75.53	15.4	10.33	1.28	3.39	3.23	5%
	0.70	6.84	75.45	11.0	8.34	1.28	3.02	2.88	5%
	0.80	8.95	75.36	8.4	6.95	1.28	2.90	2.94	1%
	0.90	11.32	79.82	7.1	6.15	1.28	2.86	3.04	6%
6 Story	0.30	3.30	84.11	25.5	12.17	1.17	5.77	5.83	1%
	0.40	4.01	84.02	20.9	13.31	1.17	4.21	4.42	5%
	0.50	5.18	83.88	16.2	11.49	1.17	3.41	3.32	3%
	0.60	7.08	82.95	11.7	9.30	1.16	3.17	3.22	2%
	0.70	9.38	82.28	8.8	7.65	1.16	2.96	2.82	5%
	0.80	11.95	81.60	6.8	6.10	1.15	2.65	2.82	6%
	0.90	14.85	82.46	5.6	5.29	1.16	2.52	2.54	1%
9 Story	0.30	2.75	105.07	38.2	22.23	1.19	3.58	3.70	3%
	0.40	4.54	101.50	22.4	15.76	1.18	3.18	3.31	4%
	0.50	6.88	100.09	14.5	11.72	1.18	2.89	2.93	1%
	0.60	9.72	99.32	10.2	9.07	1.18	2.65	2.63	1%
	0.70	13.20	98.84	7.5	7.59	1.17	2.58	2.69	4%
	0.80	17.48	98.43	5.6	5.22	1.17	2.05	2.06	1%
	0.90	22.22	98.08	4.4	4.27	1.17	1.90	1.97	4%

Table 5-18 Simplified analyses results and comparison with *CMR* values obtained from time history analyses for selected buildings with hysteretic dampers and V_0/V_{max}=1.6

Type	T_b/T_u	δ_y (cm)	δ_u (cm)	μ_T	r	$\Gamma_1 \phi_{1,r}$	*CMR* fom Simplified Analysis	*CMR* from Nonlinear Time History Analyses	Absolute Differences
3 Story	0.30	1.30	77.77	60.0	16.13	1.28	5.59	5.66	1%
	0.40	2.33	77.86	33.4	14.13	1.28	4.78	4.68	2%
	0.50	3.60	77.86	21.6	12.75	1.28	4.36	4.32	1%
	0.60	5.16	77.80	15.1	10.13	1.28	3.50	3.35	5%
	0.70	7.19	77.73	10.8	8.20	1.28	3.13	2.94	7%
	0.80	9.43	77.65	8.2	6.81	1.28	3.00	3.17	5%
	0.90	11.74	85.78	7.3	6.36	1.28	3.07	3.17	3%
6 Story	0.30	3.41	85.36	25.1	12.03	1.17	5.89	5.86	1%
	0.40	4.14	85.27	20.6	13.12	1.17	4.29	4.47	4%
	0.50	5.36	85.11	15.9	11.34	1.17	3.48	3.37	3%
	0.60	7.32	84.18	11.5	9.16	1.16	3.23	3.22	0%
	0.70	9.70	83.19	8.6	7.53	1.16	3.01	2.91	3%
	0.80	12.37	82.11	6.6	5.94	1.15	2.67	2.88	7%
	0.90	15.34	83.88	5.5	5.22	1.16	2.56	2.68	5%
9 Story	0.30	2.83	106.02	37.5	21.86	1.19	3.62	3.73	3%
	0.40	4.67	102.43	21.9	15.46	1.18	3.22	3.33	3%
	0.50	7.07	101.00	14.3	11.56	1.18	2.93	2.95	1%
	0.60	9.98	100.24	10.0	8.94	1.17	2.68	2.67	0%
	0.70	13.55	99.71	7.4	7.46	1.17	2.61	2.72	4%
	0.80	17.92	99.31	5.5	5.15	1.17	2.08	2.09	1%
	0.90	22.79	98.93	4.3	4.23	1.17	1.93	2.04	5%

Table 5-19 Simplified analyses results and comparison with *CMR* values obtained from time history analyses for selected buildings with hysteretic dampers and V_0/V_{max}=1.8

Type	$T_{b/}T_u$	δ_y (cm)	δ_u (cm)	μ_T	r	$\Gamma_I \phi_{I,r}$	*CMR* fom Simplified Analysis	*CMR* from Nonlinear Time History Analyses	Absolute Differences
3 Story	0.30	1.36	80.17	59.2	15.95	1.28	5.79	5.74	1%
	0.40	2.45	80.26	32.8	13.89	1.28	4.95	4.77	4%
	0.50	3.79	80.25	21.2	12.50	1.28	4.50	4.47	1%
	0.60	5.42	80.20	14.8	9.97	1.28	3.62	3.42	6%
	0.70	7.56	80.14	10.6	8.08	1.28	3.24	3.01	8%
	0.80	9.90	80.07	8.1	6.70	1.28	3.09	3.32	7%
	0.90	12.17	92.20	7.6	6.59	1.28	3.30	3.30	0%
6 Story	0.30	3.51	86.59	24.6	11.90	1.17	6.00	5.86	2%
	0.40	4.28	86.47	20.2	12.90	1.17	4.35	4.53	4%
	0.50	5.53	86.29	15.6	11.18	1.17	3.54	3.41	4%
	0.60	7.56	85.03	11.3	9.00	1.16	3.28	3.25	1%
	0.70	10.01	83.57	8.3	7.38	1.16	3.05	2.95	3%
	0.80	12.78	82.56	6.5	5.79	1.15	2.69	2.92	8%
	0.90	15.80	85.51	5.4	5.18	1.16	2.61	2.73	4%
9 Story	0.30	2.90	106.95	36.9	21.57	1.19	3.67	3.73	2%
	0.40	4.78	103.34	21.6	15.27	1.18	3.25	3.35	3%
	0.50	7.23	101.89	14.1	11.43	1.18	2.97	2.97	0%
	0.60	10.24	101.13	9.9	8.82	1.17	2.71	2.67	2%
	0.70	13.89	100.61	7.2	7.34	1.17	2.63	2.75	4%
	0.80	18.39	100.16	5.4	5.09	1.17	2.11	2.09	1%
	0.90	23.39	99.77	4.3	4.18	1.17	1.96	2.08	6%

Table 5-20 Results of simplified analyses and comparison with *CMR* values obtained from time history analyses for selected buildings with hysteretic dampers and V_0/V_{max}=2.0

Type	$T_{b/}T_u$	δ_y (cm)	δ_u (cm)	μ_T	r	$\Gamma_I \phi_{I,r}$	*CMR* fom Simplified Analysis	*CMR* from Nonlinear Time History Analyses	Absolute Differences
3 Story	0.30	1.43	82.71	58.0	15.71	1.28	6.00	5.82	3%
	0.40	2.57	82.79	32.2	13.70	1.28	5.11	4.83	6%
	0.50	3.97	82.78	20.9	12.32	1.28	4.65	4.53	3%
	0.60	5.68	82.75	14.6	9.84	1.28	3.74	3.62	3%
	0.70	7.93	82.69	10.4	7.97	1.28	3.35	3.07	9%
	0.80	10.38	82.63	8.0	6.61	1.28	3.20	3.53	9%
	0.90	12.59	98.41	7.8	6.80	1.28	3.52	3.42	3%
6 Story	0.30	3.62	87.83	24.3	11.78	1.17	6.12	5.91	4%
	0.40	4.41	87.69	19.9	12.73	1.17	4.42	4.58	4%
	0.50	5.70	87.48	15.3	11.01	1.17	3.60	3.47	4%
	0.60	7.79	85.57	11.0	8.83	1.16	3.32	3.28	1%
	0.70	10.32	84.03	8.1	7.25	1.16	3.09	3.04	2%
	0.80	13.17	83.06	6.3	5.66	1.15	2.71	2.95	8%
	0.90	16.24	87.24	5.4	5.15	1.16	2.66	2.81	5%
9 Story	0.30	2.96	107.88	36.5	21.39	1.19	3.71	3.75	1%
	0.40	4.89	104.23	21.3	15.08	1.18	3.29	3.37	2%
	0.50	7.42	102.78	13.9	11.28	1.18	3.00	2.97	1%
	0.60	10.49	102.00	9.7	8.72	1.17	2.75	2.67	3%
	0.70	14.24	101.46	7.1	7.22	1.17	2.66	2.83	6%
	0.80	18.85	101.02	5.4	5.02	1.17	2.13	2.12	1%
	0.90	23.97	100.61	4.2	4.13	1.17	1.99	2.12	6%

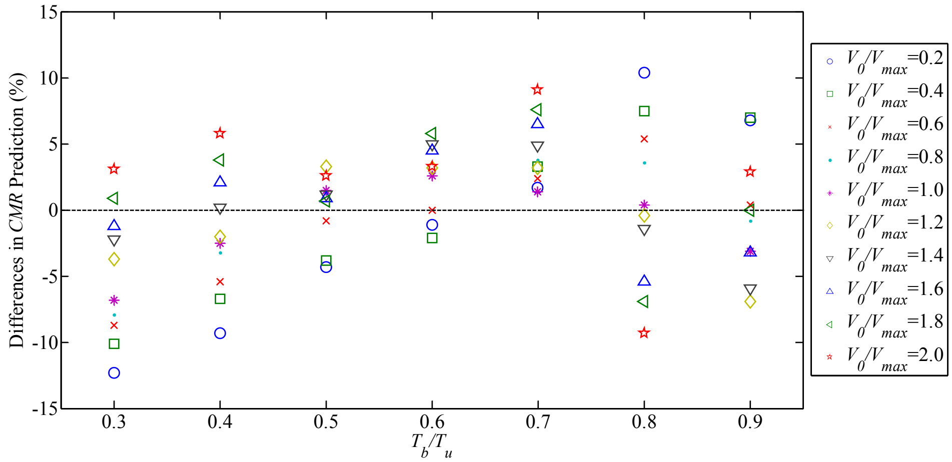


Figure 5-4 Error in estimating collapse margin ratio (*CMR*) of three-story building incorporating hysteretic dampers

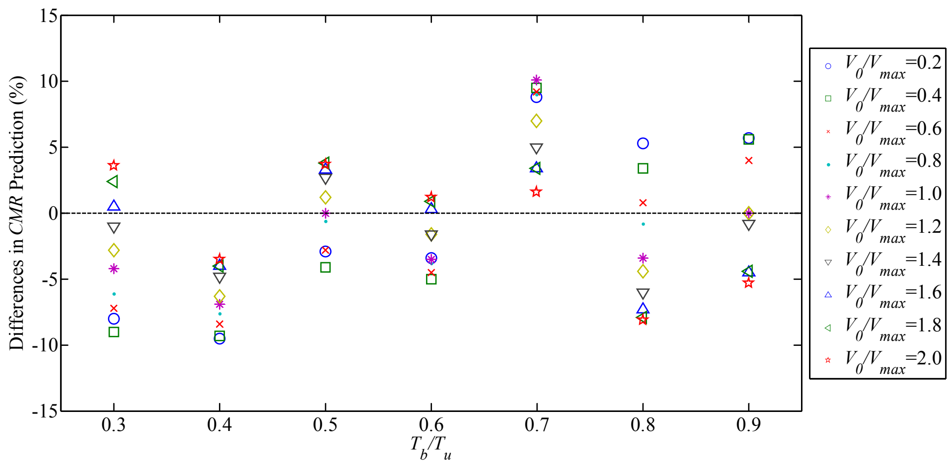


Figure 5-5 Error in estimating collapse margin ratio (*CMR*) of six-story building incorporating hysteretic dampers

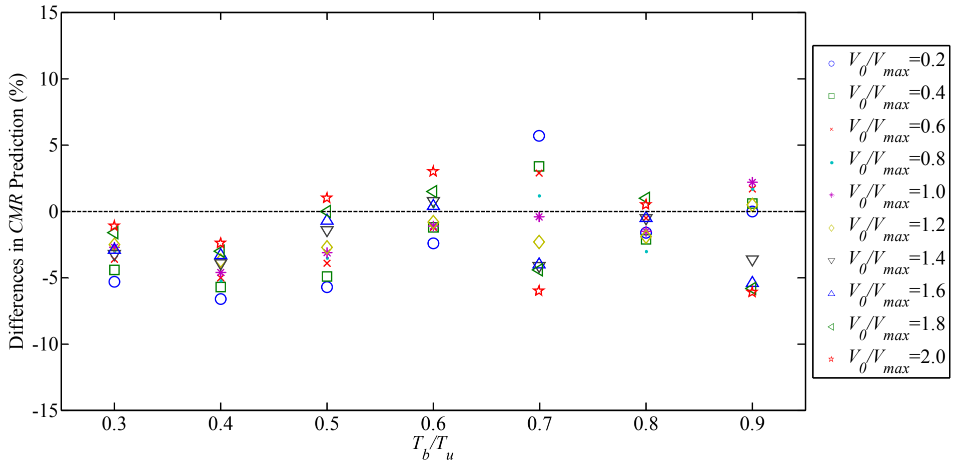


Figure 5-6 Error in estimating collapse margin ratio (*CMR*) of nine-story building incorporating hysteretic dampers

5.5 Development of simplified design procedure for buildings with hysteretic dampers

A design approach for frame building structures equipped with hysteretic dampers can be developed using the proposed approximate procedure by conducting a back calculation for the desired *ACMR* aiming at achieving a pre-determined probability of collapse under MCE events (see Section 4.8).

The design parameters for buildings incorporating hysteretic dampers include T_b/T_aunt V_0/V_{max}. The authors recommend selection of brace sections such that T_b/T_u be less or equal than 0.4, consistent with the recommendation by Filiatrault and Cherry (1988), and iteration on V_0 to achieve desired *CMR* values.

The simplified collapsed capacity-based iterative design procedure for frame buildings incorporating hysteretic dampers can be summarized by the following steps:

1. Design the seismic force resisting system of the building without dampers for minimum design base shear as addressed by Chapter 18 of ASCE7-10 (2010).
2. Choose an *ACMR* value corresponding to the desired probability of collapse under MCE ground motions from Figure 4-25.
3. Calculate the desired *CMR* from Equation 4.34.
4. Develop a numerical model of the building structure under evaluation without dampers, which incorporates monotonic nonlinear behavior and degradation characteristics of the structural components.

5. Obtain the elastic fundamental period of vibration (T_u) and the corresponding first mode shape through an eigenvalue analysis of the elastic building structure without hysteretic dampers.
6. Perform nonlinear static (pushover) analysis of the building structure with the lateral force distribution that is proportional to its first mode shape and obtain V_{max} as shown in Figure 3-9.
7. Incorporate elastic braces into the numerical model of the building. Choose the brace cross-sectional areas so that the lateral stiffness of the bracing members becomes proportional to the interstory drift arising from first mode shape of the unbraced buildings. Iterate through eigenvalue analyses on the bracing section properties in order to achieve an elastic fundamental period of vibration $T_b \leq 0.4T_u$ (if economically possible).
8. Incorporate hysteretic dampers into the numerical model of the building with a small value ofV_0/V_{max} (e.g. 0.2), given the shear forces required to activate all hysteretic dampers is distributed proportionally to the first mode shape of the braced structure.
9. Perform nonlinear static (pushover) analysis of the braced building structure incorporating nonlinear response of entire damping system with the lateral force distribution that is proportional to its first mode shape until the point of 20% strength loss.
10. Fit a bilinear curve to the base shear-roof displacement pushover curve according to the FEMA P695 methodology (see Figure 3-9) and obtain the ultimate roof displacement (δ_u) and target ductility ratio (μ_T).
11. Construct the inelastic mode shape (ϕ_I) from pushover data, which represents the displacement of each story when the initial ultimate roof displacement (δ_u) is reached.
12. Calculate the inelastic mode participation factor (Γ_I) using Equation 3.7.
13. Extract the reduction factor (r) by linear interpolation from Table 3-2 to Table 3-5 based on the given elastic fundamental period of vibration (T_b) and target ductility (μ_T).
14. Calculate the *CMR* from Equation 3.11.
15. Repeat from step 8 with increased V_0/V_{max} ratio if calculated *CMR* in step 14 is less than desired *CMR* calculated in step 3.

5.6 Design example

Design calculations are presented herein for the original three-story building described in Section 3.7 for which the height of each floor is 3.96 m. It is assumed that the building is located on site class D in downtown Los Angeles (Latitude: 34.05°, Longitude: -118.25°) characterized with S_{MS}=2.402g, S_{M1}=1.264g and T_s=0.527 s. The total system collapse uncertainty (β_{TOT}) for this example is considered to be 0.60 according to the P-695 methodology, which corresponds to a good model quality due to the detailed deterioration and degradation models considered, a good quality of test data obtained from over 300 specimens, and a fair quality of the design requirements due to the application of the older Uniform Building Code (UBC, 1994). Assuming a desired 2% probability of collapse under MCE ground motions, the minimum required *ACMR*=3.43 from Figure 4-25. The spectral shape factor (*SSF)* is 1.28 from Table B-9 FEMA P695 (FEMA, 2009) and *CMR*=2.68 from Equation 4.34. The seismic masses of the first, second and roof floor are equal to 478.15, 478.15 and 517.3 kN.sec^2/m, respectively. The fundamental period of the unbraced building (T_u) obtained by eigenvalue analysis is 0.94 s and V_{max}=4085 kN from pushover analysis. The brace cross-sectional areas were incorporated so that T_b=0.4T_u=0.38 s from eigenvalue analysis of the braced building (see Table 5-2). A ratio of V_0/V_{max}=0.6 was chosen for the first iteration and the hysteretic dampers were added to the braced building model. The corresponding activation load of the hysteretic dampers of the first, second and roof floor are equal to 710, 580 and 332 kN, respectively. The V_0/V_{max} will increase if the *CMR* does not meet the minimum required *CMR*=2.68. The target ductility ratio μ_T = 38.3 from pushover analysis of the building with hysteretic dampers and the inelastic mode shape is defined as:

$$\phi_I = \begin{Bmatrix} 66.5 \\ 42.4 \\ 19.2 \end{Bmatrix} \tag{5.1}$$

which results in $\Gamma_I\phi_{I,r}$ = 1.28 from Equation 3.7. By linear interpolation in Table 3-3, the reduction factor *r*=15.89.Given the MCE spectral acceleration at a period of 1.0 s, S_{M1}=2.402g, the *CMR* predicted by the proposed simplified seismic collapse analysis procedure is then obtained as:

$$CMR = \frac{4\pi^2 \delta_u r}{S_{Ms} T_1^2 \mu_T \Gamma_I \phi_{I,r}} = \frac{4\pi^2 \times 66.5 \times 15.89}{2.402 \times 981 \times (0.38)^2 \times 38.3 \times 1.28} = 2.50 \tag{5.2}$$

The calculated *CMR* is less than the minimum required *CMR* = 2.68; therefore, the V_0/V_{max} ratio is increased. A ratio of V_0/V_{max}=1.0 was chosen and the hysteretic dampers were added to the braced building model. The corresponding activation load of the hysteretic dampers of the first, second and roof floor are equal to 1184, 964 and 553 kN, respectively. The target ductility ratio μ_T = 36.0 from pushover analysis of the building with hysteretic dampers and the inelastic mode shape is defined as:

$$\phi_I = \begin{Bmatrix} 71.0 \\ 45.2 \\ 20.4 \end{Bmatrix} \tag{5.3}$$

which results in $\Gamma_I\phi_{I,r}$ = 1.28 from Equation 3.7. By linear interpolation in Table 3-3, the reduction factor r=15.10.Given the MCE spectral acceleration at a period of 1.0 s, S_{M1}=2.402g, the *CMR* predicted by the proposed simplified seismic collapse analysis procedure is then obtained as:

$$CMR = \frac{4\pi^2\delta_u r}{S_{Ms}T_1^2\mu_T\Gamma_I\phi_{I,r}} = \frac{4\pi^2 \times 71.04 \times 15.10}{2.402 \times 981 \times (0.38)^2 \times 36.0 \times 1.28} = 2.70 \tag{5.4}$$

The *CMR* obtained from nonlinear time history analysis for SDC D_{max} is 4.45 as given in Table 5-15. The *CMR* value corresponding to the considered site is then: 4.45×1.5g/2.402g=2.77. This represents a difference of 3% with the approximate proposed procedure, which is quite acceptable.

5.7 Limitations of the proposed procedure

In addition to the limitations addressed in Section 3.10, the evaluation and design procedure has been assessed for a distribution of hysteretic dampers proportional to the first mode shape and has not been validated for other distributions.

5.8 Summary

The simplified procedure to evaluate the seismic sidesway collapse capacity of building structures incorporating hysteretic dampers presented in this section is based on the development of a robust database of nonlinear seismic responses of single-degree-of-freedom systems for various seismic intensities and uses nonlinear static (pushover) analysis without the need for nonlinear time history dynamic analyses. The procedure was assessed using a total of 210 different building structure models with a wide range of fundamental periods (from 0.28 s to 2.12 s) and damper activation loads. The collapse capacities predicted by the proposed simplified procedure are in very good agreement with those obtained by nonlinear time history dynamic analyses. For the series of three-, six- and nine-story hysteretically damped benchmark building models considered in this study, the average values of absolute differences between the simplified and nonlinear dynamic time history analyses are only 3.8, 4.4 and 2.8%, with standard deviations of 4.7, 5.1 and 2.7%, respectively. Finally, using back calculation on the *CMR*, a simplified design procedure was introduced for frame building incorporating hysteretic dampers. The procedure is consistent with the ASCE7-10 seismic provisions and is aimed at achieving a pre-determined probability of collapse under MCE events. Again, it is important to note that the proposed procedure is only an approximate approach for the quick estimation of the collapse capacity of frame buildings incorporating hysteretic dampers that cannot replace more rigorous nonlinear time history dynamic analyses.

SECTION 6

DESIGN EXAMPLE OF A NEW BUILDING WITH SUPPLEMENTAL DAMPING SYSTEMS

6.1 Introduction

Design calculations are presented in this section applying equivalent lateral force (ELF) procedure according to ASCE7-10 Chapter 18 for a five-story office building incorporating linear viscous dampers, nonlinear viscous dampers and hysteretic dampers. The validity of the assumptions for only using the ELF procedure is verified in Section 6.3.4. The collapse safety of the building with and without each of the three considered damping systems is then quantified in Section 6.4. The damping systems are re-designed in Section 6.5 for a pre-determined 2% probability of collapse under MCE ground motions.

6.2 Building information

Figure 6-1shows the geometry and dimensions of the considered frame building as well as the configuration of the dampers. It is assumed that the building is located on stiff soil (site class D) in downtown Los Angeles (Latitude: 34.1°, Longitude: -118.2°) characterized with design response spectrum as shown in Figure 6-2. Uniform floor dead load of 100 psf and live load of 80 psf were considered. The seismic mass includes dead load in combination with 25% of the live load. The seismic force-resisting system (SFRS) is composed of four steel special moment frames in the periphery of the building with response modification factor R = 8.0, overstrength factor Ω_0=3 and deflection amplification factor C_d=5.5. The importance factor I_e = 1.0 for an office building with risk category II. It is further assumed that the building does not possess vertical and horizontal structural irregularities and the inherent damping is considered as β_I=5%.Table 6-1 summarizes the initial assumed sections of the steel special moment frame and Table 6-2 lists the elastic periods of the building obtained from eigenvalue analysis.

Table 6-1 Assumed beam and column sections

Floor Number	Beam Section	Column Section
5	W16×57	W14×145
4	W16×57	W14×145
3	W18×97	W14×176
2	W18×97	W14×176
1	W18×97	W14×176

Table 6-2 Elastic periods of the considered frame

Mode Number	Period (s)
1	1.54
2	0.53
3	0.26
4	0.16
5	0.11

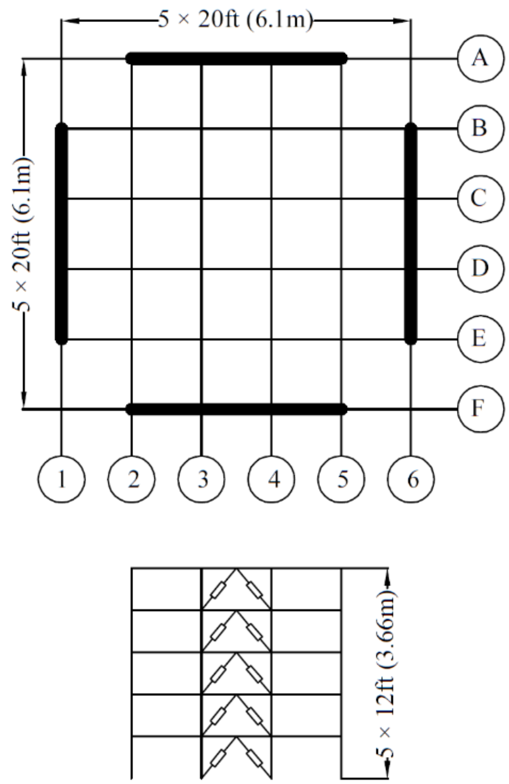


Figure 6-1 Plan and elevation view of the selected steel frame building and configuration of dampers

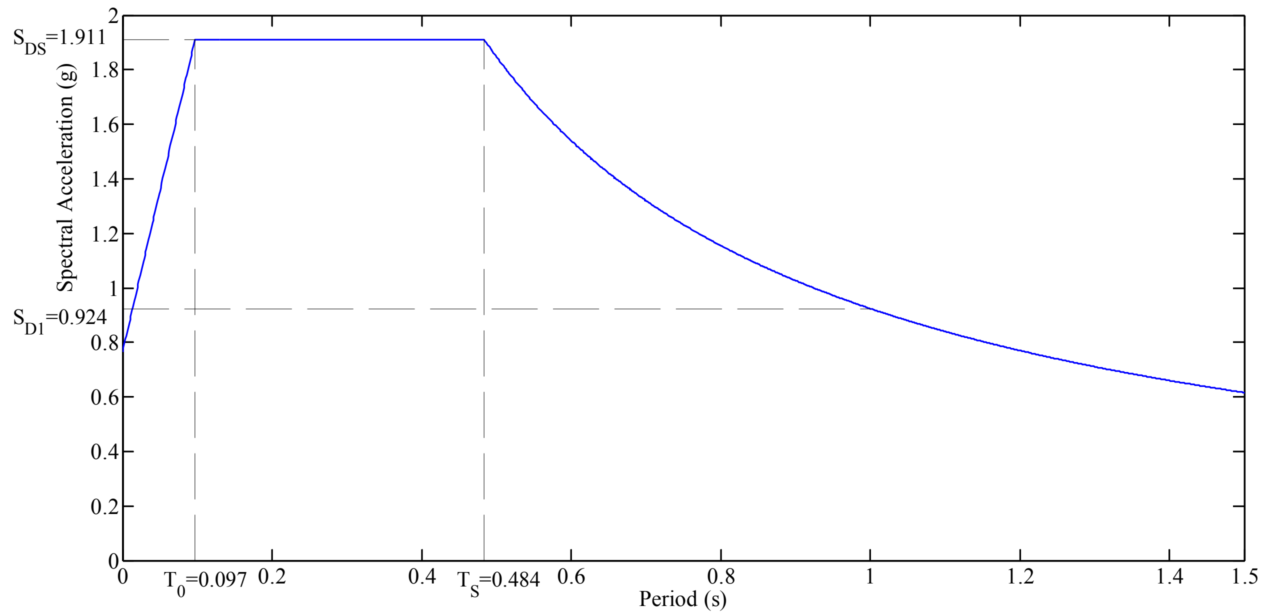


Figure 6-2 Design response spectral acceleration for the considered site

6.3 Design of supplemental damping systems per ASCE7-10.

6.3.1 Building with linear viscous dampers

Equivalent lateral load procedure requires an iterative procedure to satisfy the requirements of ASCE7-10 Chapter 18. The following calculations are according to Section 18.5 ASCE7-10. (The values in [] represent the corresponding equation caption in ASCE7-10).

The fundamental mode shape (ϕ_1), effective seismic weight ($\bar{W}_1$) and participation factor (Γ_1) is determined as follows:

[18.5-3] $$\phi_{i1} = \frac{h_i}{h_r} = \begin{Bmatrix} 1.0 \\ 0.8 \\ 0.6 \\ 0.4 \\ 0.2 \end{Bmatrix} \tag{6.1}$$

[18.4-2b] $$\bar{W}_1 = \frac{\left(\sum_{i=1}^{N_f} w_i \phi_{i1}\right)^2}{\sum_{i=1}^{N_f} w_i \phi_{i1}^2} = \frac{\left(\frac{W}{5} \times (0.2+0.4+0.6+0.8+1.0)\right)^2}{\frac{W}{5} \times (0.2^2+0.4^2+0.6^2+0.8^2+1.0^2)} = \frac{9}{11} W = 2455 \text{ kips} \tag{6.2}$$

[18.5-4] $$\Gamma_1 = \frac{\bar{W}_1}{\sum_{i=1}^{N_f} w_i \phi_{i1}} = \frac{\frac{9}{11}W}{\frac{W}{5} \times (0.2+0.4+0.6+0.8+1.0)} = \frac{15}{11} \quad (6.3)$$

where h_i is the height above the base level to floor i, h_r is the height of the structure above the base to the roof level, w_i is the seismic weight at floor i, W=3000 kips is the total seismic weight of the structure and N_f is the total number of floors (N_f = 5).

Assuming the effective ductility demand on the seismic force resisting system due to the design earthquake ground motion μ_D=2.0, the effective fundamental period at the design earthquake ground motion is:

[18.4-6] $$T_{1D} = T_1\sqrt{\mu_D} = 1.54\sqrt{2} = 2.18 \text{ s} \quad (6.4)$$

where T_1 is the fundamental elastic period of the building. The hysteretic loop adjustment factor for the SFRS is obtained as follows:

[18.6-5] $$q_H = 0.67\frac{T_S}{T_1} = 0.67\left(\frac{0.484}{1.54}\right) = 0.21 \quad (6.5)$$

where T_s is the short-period transition in design response spectrum. The value of q_H shall not be taken as greater than 1.0 and as less than 0.5. Therefore q_H=0.5 is used. The effective hysteretic damping of the seismic force-resisting system is determined as:

[18.6-3] $$\beta_{HD} = q_H(0.64-\beta_I)\left(1-\frac{1}{\mu_D}\right) = 0.5(0.64-0.05)\left(1-\frac{1}{2}\right) = 14.7\% \quad (6.6)$$

The effective damping ratio for the first mode of vibration due to incorporation of linear viscous dampers can be obtained by:

$$\beta_{V1} = \frac{T_1\sum_{j=1}^{N_F} C_j f_j^{\,2}(\phi_{j1}-\phi_{(j-1)1})^2}{4\pi\sum_{j=1}^{N_F} m_j \phi_{j1}^{\,2}} \quad (6.7)$$

where C_j is the sum of all the damping constants for all linear viscous dampers at floor level j, f_j is a displacement magnification factor that depends on the geometrical arrangement of the dampers at floor level j and m_j is the lumped mass at floor level j. The factor f_j = cos(θ_j) for the diagonal configuration, where θ_j is the angle of inclination of the device j.

As an initial assumption, the effective damping ratio is assumed to be β_{V1}=10%. The value will be increased later if the design requirements are not satisfied. Linear viscous damper constants are distributed along the height of the structure proportional to the interstory drifts arising from the first mode shape. This approach results in uniform distribution of the damper constants. Linear viscous damper constants are determined from the following equation:

$$C_j = \frac{4\pi\beta_{V1}\sum_{j=1}^{N_F} m_j \phi_{j1}^{\ 2}}{T_1 \sum_{j=1}^{N_F} f_j^{\ 2}(\phi_{j1}-\phi_{(j-1)1})^2} = \frac{4\pi\times 0.1\times 0.2\times 3000\times\left(0.2^2+0.4^2+0.6^2+0.8^2+1.0^2\right)}{386\times 1.54\times \cos(50.2^\circ)^2\times 5\times 0.2^2} = 34.1 \text{ kip.s/in} \quad (6.8)$$

The effective damping at the design displacement of the fundamental mode of vibration is:

[18.6-1] $$\beta_{1D} = \beta_I + \beta_{V1}\sqrt{\mu_D} + \beta_{HD} = 0.05 + 0.10\sqrt{2} + 0.147 = 34\% \quad (6.9)$$

The corresponding numerical coefficient for β_{1D} is determined as B_{1D}=1.92 from Table 18.6-1 of ASCE7-10. The fundamental mode seismic response coefficient (C_{S1}) and corresponding base shear (V_1) are calculated as follows:

[18.5-7] $$C_{S1} = \left(\frac{R}{C_d}\right)\frac{S_{D1}}{T_{1D}\left(\Omega_0 B_{1D}\right)} = \frac{8}{5.5}\times\frac{0.924}{2.18(3\times 1.92)} = 0.107 \quad (6.10)$$

[18.5-2] $$V_1 = C_{S1}\overline{W}_1 = 0.107\times 2455 = 263 \text{ kips} \quad (6.11)$$

Residual mode shape (ϕ_R), participation factor (Γ_R), effective seismic weight ($\overline{W}_R$) and effective period (T_R) are determined as:

[18.5-11] $$\phi_{iR} = \frac{1-\Gamma_1\phi_{i1}}{1-\Gamma_1} = \begin{Bmatrix} 1.0 \\ 0.25 \\ -0.5 \\ -1.25 \\ -2 \end{Bmatrix} \quad (6.12)$$

[18.5-12] $$\Gamma_R = 1-\Gamma_1 = 1-\frac{15}{11} = \frac{-4}{11} \quad (6.13)$$

[18.5-13] $$\overline{W}_R = W - \overline{W}_1 = \frac{2}{11}W = 545 \text{ kips} \quad (6.14)$$

[18.5-14] $$T_R = 0.4T_1 = 0.4\times 1.54 = 0.62 \text{ s} \quad (6.15)$$

Thus, total effective damping in the residual mode of vibration is:

$$\beta_R = \frac{T_1 \sum_{j=1}^{N_F} C_j f_j^{\,2} (\phi_{jR} - \phi_{(j-1)R})^2}{4\pi \sum_{j=1}^{N_F} m_j \phi_{jR}^{\,2}} + \beta_I = \frac{0.62 \times 386 \times 34.1 \times \cos(50.2^\circ)^2 \times \left(4 \times 0.75^2 + 2^2\right)}{4\pi \times 0.2 \times 3000 \times \left(2^2 + 1.25^2 + 0.5^2 + 0.25^2 + 1.0^2\right)} + 5\% = 45\% \quad (6.16)$$

The corresponding numerical coefficient for β_R is determined as B_R=2.25 from Table 18.6-1 of ASCE 7-10. The residual mode seismic response coefficient (C_{SR}) and base shear (V_R) are calculated as follows:

[18.5-15] $$C_{SR} = \left(\frac{R}{C_d}\right)\frac{S_{DS}}{\Omega_0 B_R} = \left(\frac{8}{5.5}\right) \times \frac{1.911}{3 \times 2.25} = 0.412 \quad (6.17)$$

[18.5-10] $$V_R = C_{SR} \overline{W}_R = 0.412 \times 545 = 225 \text{ kips} \quad (6.18)$$

The significant contribution from residual mode is due to the independency of C_{SR} with T_R and also ignoring the hysteretic damping in the definition of β_R. The total design seismic base shear of the seismic force-resisting system is defined as:

[18.5-1] $$V_d = \sqrt{V_1^2 + V_R^2} = \sqrt{263^2 + 225^2} = 346 \text{ kips} \quad (6.19)$$

ASCE 7-10 defines minimum allowable value of the base shear permitted for design of the seismic force-resisting system as the greater of the following values:

[18.2-2] $$V_{\min} = 0.75V \quad (6.20)$$

[18.2-1] $$V_{\min} = \frac{V}{B_{V+I}} \quad (6.21)$$

where V is the seismic base shear determined in accordance with Section 12.8 for the seismic force-resisting system without damping systems. The calculation of V is according to the following procedure.

The approximate fundamental period is defined as:

[12.8-7] $$T = C_u C_t h_r^x = 1.4 \times 0.028 (60 \text{ ft})^{0.8} = 1.04 \text{ s} \quad (6.22)$$

where C_t=0.028, x=0.8 from Table 12.8-2 ASCE7-10 and C_u=1.4 from Table 12.8-1 ASCE7-10. The seismic response coefficient (C_S) and seismic base shear (V) are calculated accordingly as:

[12.8-3] $$C_s = \frac{S_{D1}}{T\left(\frac{R}{I_e}\right)} = \frac{0.924}{1.04\left(\frac{8}{1.0}\right)} = 0.111 \quad (6.23)$$

[12.8-1] $$V = C_S W = 0.111 \times 3000 = 333 \text{ kips} \quad (6.24)$$

substituting V from Equation 6.24 into Equations 6.20 and 6.21 results in:

[18.2-2] $$V_{\min} = 0.75V = 0.75\times 333 = 250 \text{ kips} < \frac{V}{B_{V+I}} = \frac{333}{1.35} = 247 \text{ kips} \quad (6.25)$$

which is less than the calculated total design base shear (V_d). Thus, the seismic force-resisting system is controlled against the base shear V_d = 346 kips. The seismic force-resisting system has the required strength to meet the forces induced by V_d. In addition, the special moment frame passes the criteria defined by AISC seismic provisions for structural steel buildings (AISC 341-10). The maximum design interstory drift shall not exceed form 0.02×floor height/C_d=0.02×144in/5.5=0.52 in for conventional five-story office buildings without damping systems. As listed in Table 6-3, the maximum design interstory drift obtained from elastic analysis of the frame building exceeds the mentioned value.

Table 6-3 Interstory drift obtained from elastic analysis of the frame building with base shear V_d

Floor Number	Interstory Drift (in)
5	1.15
4	1.41
3	1.48
2	1.57
1	1.05

However, the seismic force-resisting system in the building incorporating damping system is allowed to exceed this limit and the combination of the seismic force-resisting system and the damping system is permitted to be used to meet the drift requirements.

The fundamental mode displacement due to the design earthquake ground motions at roof level of the building is determined as:

[18.5-20b] $$D_{1D} = \left(\frac{g}{4\pi^2}\right)\Gamma_1 \frac{S_{D1}T_{1D}}{B_{1D}} = \frac{g}{4\pi^2}\times\frac{15}{11}\times\frac{0.924\times 2.18}{1.92} = 13.98 \text{ in} \quad (6.26)$$

and cannot be less than:

[18.5-20b] $$\left(\frac{g}{4\pi^2}\right)\Gamma_1 \frac{S_{D1}T_1}{B_{1E}} = \frac{g}{4\pi^2}\times\frac{15}{11}\times\frac{0.924\times 1.54}{1.35} = 14.06 \text{ in} \quad (6.27)$$

Thus, D_{1D}=14.06 in. The displacement at roof level of the building at the effective yield point of the seismic force-resisting system is defined as:

[18.6-10] $$D_Y = \left(\frac{g}{4\pi^2}\right)\left(\frac{\Omega_0 C_d}{R}\right)\Gamma_1 C_{S1}T_1^2 = \frac{g}{4\pi^2}\left(\frac{3\times 5.5}{8}\right)\times\frac{15}{11}\times 0.107\times 1.54^2 = 6.98 \text{ in} \quad (6.28)$$

The effective ductility demand on the seismic force resisting system due to the design earthquake ground motion will be:

[18.6-8] $$\mu_D = \frac{D_{1D}}{D_Y} = \frac{14.06}{6.98} = 2.01 > 1.0 \quad (6.29)$$

Comparing to the assumed value of μ_D=2.0, the difference is less than 1%. In addition it is less than the maximum allowable value of effective ductility demand determined as:

[18.6-12] $$\mu_{\max} = \frac{R}{\Omega_0 I_e} = \frac{8}{3\times 1.0} = 2.67 \quad (6.30)$$

Therefore, the assumed effective ductility demand is acceptable and no iteration is needed. The residual mode displacement due to the design earthquake ground motions at roof level of the building is determined as:

[18.5-21] $$D_{RD} = \left(\frac{g}{4\pi^2}\right)\Gamma_R \frac{S_{D1}T_R}{B_R} = \frac{g}{4\pi^2}\times\frac{-4}{11}\times\frac{0.924\times 0.62}{2.25} = -0.91 \text{ in} \quad (6.31)$$

and cannot be larger than:

[18.5-21] $$\left(\frac{g}{4\pi^2}\right)\Gamma_R \frac{S_{DS}T_R^2}{B_R} = \frac{g}{4\pi^2}\times\frac{-4}{11}\times\frac{1.911\times 0.62^2}{2.25} = -1.16 \text{ in} \quad (6.32)$$

Thus D_{RD}=-1.16. Consequently, the fundamental and residual mode deflections due to the design earthquake ground motions at level *i* are defined as:

[18.5-18] $$\delta_{i1D} = D_{1D}\phi_{i1} = 14.06 \text{ in}\times\begin{Bmatrix}1.0\\0.8\\0.6\\0.4\\0.2\end{Bmatrix} = \begin{Bmatrix}14.06\\11.25\\8.44\\5.62\\2.81\end{Bmatrix} \text{ in} \quad (6.33)$$

[18.5-19] $$\delta_{iRD} = D_{RD}\phi_{iR} = -1.16 \text{ in}\times\begin{Bmatrix}1.0\\0.25\\-0.5\\-1.25\\-2\end{Bmatrix} = \begin{Bmatrix}-1.16\\-0.29\\0.58\\1.45\\2.32\end{Bmatrix} \text{ in} \quad (6.34)$$

The design story drift is then calculated as:

[18.5-22] $$\Delta_D = \sqrt{\Delta_{1D}^2 + \Delta_{RD}^2} = \begin{Bmatrix} 2.94 \\ 2.94 \\ 2.95 \\ 2.94 \\ 3.64 \end{Bmatrix} \text{ in} \tag{6.35}$$

The calculated design story drift shall not exceed the following value:

[18.5-22] $$0.02 \frac{R}{C_d} h_f = 0.02 \times \frac{8}{5.5} \times 144 = 4.19 \text{ in} \tag{6.36}$$

where h_f is the floor height. The design story drift is close to and below the allowable drift. Therefore, seismic force-resisting system design is acceptable and also optimized.

6.3.2 Building with nonlinear viscous dampers

The building described in Section 6.2 is designed herein incorporating nonlinear viscous dampers with velocity exponent α=0.5. Several equations derived in previous sections are repeated in order to make the section self-contained. The fundamental mode shape (ϕ_1), effective seismic weight ($\bar{W}_1$) and participation factor (Γ_1) is determined as follows:

[18.5-3] $$\phi_{i1} = \frac{h_i}{h_r} = \begin{Bmatrix} 1.0 \\ 0.8 \\ 0.6 \\ 0.4 \\ 0.2 \end{Bmatrix} \tag{6.37}$$

[18.4-2b] $$\bar{W}_1 = \frac{\left(\sum_{i=1}^{N_f} w_i \phi_{i1}\right)^2}{\sum_{i=1}^{N_f} w_i \phi_{i1}^2} = \frac{\left(\frac{W}{5} \times (0.2+0.4+0.6+0.8+1.0)\right)^2}{\frac{W}{5} \times (0.2^2+0.4^2+0.6^2+0.8^2+1.0^2)} = \frac{9}{11} W = 2455 \text{ kips} \tag{6.38}$$

[18.5-4] $$\Gamma_1 = \frac{\bar{W}_1}{\sum_{i=1}^{N_f} w_i \phi_{i1}} = \frac{\frac{9}{11} W}{\frac{W}{5} \times (0.2+0.4+0.6+0.8+1.0)} = \frac{15}{11} \tag{6.39}$$

Assuming the effective ductility demand on the seismic force resisting system due to the design earthquake ground motion μ_D=2.0, the effective fundamental period at the design earthquake ground motion is:

[18.4-6] $$T_{1D} = T_1\sqrt{\mu_D} = 1.54\sqrt{2} = 2.18 \text{ s} \quad (6.40)$$

The hysteretic loop adjustment factor is obtained as follows:

[18.6-5] $$q_H = 0.67\frac{T_S}{T_1} = 0.67\left(\frac{0.484}{1.54}\right) = 0.21 \quad (6.41)$$

The value of q_H shall not be taken as greater than 1.0 and as less than 0.5. Therefore, $q_H = 0.5$ is used. The effective hysteretic damping of the seismic force-resisting system is determined as:

[18.6-3] $$\beta_{HD} = q_H(0.64-\beta_I)\left(1-\frac{1}{\mu_D}\right) = 0.5(0.64-0.05)\left(1-\frac{1}{2}\right) = 14.7\% \quad (6.42)$$

As an initial assumption, the effective damping ratio is assumed to be β_{VI}=10%. The value will be increased later if the design requirements are not satisfied. The effective damping at the design displacement of the fundamental mode of vibration is:

[18.6-1] $$\beta_{1D} = \beta_I + \beta_{V1}\sqrt{\mu_D} + \beta_{HD} = 0.05 + 0.1\sqrt{2} + 0.147 = 34\% \quad (6.43)$$

The corresponding numerical coefficient for β_{ID} is determined as B_{ID}=1.92 from Table 18.6-1 of ASCE7-10. The fundamental mode seismic response coefficient (C_{SI}) and corresponding base shear (V_I) are calculated as follows:

[18.5-7] $$C_{S1} = \left(\frac{R}{C_d}\right)\frac{S_{D1}}{T_{1D}\left(\Omega_0 B_{1D}\right)} = \frac{8}{5.5}\times\frac{0.924}{2.18(3\times1.92)} = 0.107 \quad (6.44)$$

[18.5-2] $$V_1 = C_{S1}\overline{W}_1 = 0.107\times2455 = 263 \text{ kips} \quad (6.45)$$

The fundamental mode displacement due to the design earthquake ground motions at roof level of the building is determined as:

[18.5-20b] $$D_{1D} = \left(\frac{g}{4\pi^2}\right)\Gamma_1\frac{S_{D1}T_{1D}}{B_{1D}} = \frac{g}{4\pi^2}\times\frac{15}{11}\times\frac{0.924\times2.18}{1.92} = 13.98 \text{ in} \quad (6.46)$$

and cannot be less than:

[18.5-20b] $$\left(\frac{g}{4\pi^2}\right)\Gamma_1\frac{S_{D1}T_1}{B_{1E}} = \frac{g}{4\pi^2}\times\frac{15}{11}\times\frac{0.924\times1.54}{1.35} = 14.06 \text{ in} \quad (6.47)$$

Thus, D_{ID}=14.06 in. The displacement at roof level of the building at the effective yield point of the seismic force-resisting system is defined as:

[18.6-10] $$D_Y = \left(\frac{g}{4\pi^2}\right)\left(\frac{\Omega_0 C_d}{R}\right)\Gamma_1 C_{S1} T_1^2 = \frac{g}{4\pi^2}\left(\frac{3\times 5.5}{8}\right)\times\frac{15}{11}\times 0.107\times 1.54^2 = 6.98 \text{ in} \tag{6.48}$$

The effective ductility demand on the seismic force-resisting system due to the design earthquake ground motion will be:

[18.6-8] $$\mu_D = \frac{D_{1D}}{D_Y} = \frac{14.06}{6.98} = 2.01 > 1.0 \tag{6.49}$$

Comparing to the assumed value of μ_D=2.0, the difference is less than 1%. In addition, it is less than the maximum allowable value of effective ductility demand determined as:

[18.6-12] $$\mu_{\max} = \frac{R}{\Omega_0 I_e} = \frac{8}{3\times 1.0} = 2.67 \tag{6.50}$$

Therefore, the assumed effective ductility demand is acceptable and no iteration is needed. The effective damping ratio for the first mode of vibration due to incorporation of nonlinear viscous dampers can be obtained by:

$$\beta_{V1} = \frac{(2\pi)^{\alpha} T_1^{2-\alpha} \lambda D_{1D}^{\alpha-1} \sum_{j=1}^{N_F} f_j^{1+\alpha} C_{NLj} (\phi_{j1} - \phi_{(j-1)1})^{1+\alpha}}{8\pi^3 \sum_{j=1}^{N_F} m_j \phi_{j1}^2} \tag{6.51}$$

where C_{NLj} is the sum of all the damping constants for all nonlinear viscous dampers at floor level j and constant λ is equal to:

$$\lambda = \frac{2^{2+\alpha}\Gamma^2(1+\alpha/2)}{\Gamma(2+\alpha)} \tag{6.52}$$

where Γ is the Euler Gamma function. Table 6-4 lists values of λ for various values of the exponent α.

Table 6-4 Values of *λ* for various values of exponent *α*

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
λ	3.88	3.77	3.67	3.58	3.50	3.42	3.34	3.27	3.20	3.14

Nonlinear viscous damper constants are distributed along the height of the structure proportional to the interstory drifts arising from the first mode shape. This approach results in uniform distribution of the damper constants. Nonlinear viscous damper constants are determined from the following equation:

$$C_{NLj} = \frac{8\pi^3 \beta_{V1} \sum_{j=1}^{N_F} m_j \phi_{j1}^{\ 2}}{(2\pi)^{\alpha} T_1^{2-\alpha} \lambda D_Y^{\alpha-1} \sum_{j=1}^{N_F} f_j^{1+\alpha} (\phi_{1j} - \phi_{1(j-1)})^{1+\alpha}} = \frac{8\pi^3 \times 0.1 \times 0.2 \times 3000 \times \left(0.2^2 + 0.4^2 + 0.6^2 + 0.8^2 + 1.0^2\right)}{386 \times \sqrt{2\pi} \times 1.54^{1.5} \times 3.50 \times 14.06^{-0.5} \times \cos(50.2^{\circ})^{1.5} \times 5 \times 0.2^{1.5}} = 82.8 \text{ kip.}\left(\text{s/in}\right)^{0.5} \quad (6.53)$$

The residual mode shape (ϕ_R), participation factor (Γ_R), effective seismic weight ($\bar{W}_R$) and effective period (T_R) are determined as:

[18.5-11]
$$\phi_{iR} = \frac{1 - \Gamma_1 \phi_{i1}}{1 - \Gamma_1} = \begin{Bmatrix} 1.0 \\ 0.25 \\ -0.5 \\ -1.25 \\ -2 \end{Bmatrix} \quad (6.54)$$

[18.5-12]
$$\Gamma_R = 1 - \Gamma_1 = 1 - \frac{15}{11} = \frac{-4}{11} \quad (6.55)$$

[18.5-13]
$$\bar{W}_R = W - \bar{W}_1 = \frac{2}{11} W = 545 \text{ kips} \quad (6.56)$$

[18.5-14]
$$T_R = 0.4 T_1 = 0.4 \times 1.54 = 0.62 \text{ s} \quad (6.57)$$

Assuming a total effective damping in the residual mode of vibration β_R=38%, the corresponding numerical coefficient for β_R is determined as B_R=2.04 from Table 18.6-1 of ASCE 7-10. The residual mode displacement due to the design earthquake ground motions at roof level of the building is determined as:

[18.5-21]
$$D_{RD} = \left(\frac{g}{4\pi^2}\right) \Gamma_R \frac{S_{D1} T_R}{B_R} = \frac{g}{4\pi^2} \times \frac{-4}{11} \times \frac{0.924 \times 0.62}{2.04} = -1.0 \text{ in} \quad (6.58)$$

and cannot be larger than:

[18.5-21]
$$\left(\frac{g}{4\pi^2}\right) \Gamma_R \frac{S_{DS} T_R^2}{B_R} = \frac{g}{4\pi^2} \times \frac{-4}{11} \times \frac{1.911 \times 0.62^2}{2.04} = -1.28 \text{ in} \quad (6.59)$$

Thus D_{RD}=-1.28.Thus, total effective damping in the residual mode of vibration is:

$$\beta_R = \beta_{VR} + \beta_I = \frac{(2\pi)^\alpha T_R^{2-\alpha} \lambda \left| D_{RD} \right|^{\alpha-1} \sum_{j=1}^{N_F} f_j^{1+\alpha} C_{NLj} (\phi_{jR} - \phi_{(j-1)R})^{1+\alpha}}{8\pi^3 \sum_{j=1}^{N_F} m_j \phi_{jR}^2} + \beta_I$$

$$= \frac{386 \times \sqrt{2\pi} \times 0.62^{1.5} \times 3.50 \times 1.28^{-0.5} \times \cos(50.2^\circ)^{1.5} \times 82.8 \times \left(4 \times 0.75^{1.5} + 2^{1.5}\right)}{8\pi^3 \times 0.2 \times 3000 \times \left(2^2 + 1.25^2 + 0.5^2 + 0.25^2 + 1.0^2\right)} + 5\% = 37.8\% \tag{6.60}$$

Comparing to the assumed value of β_R=38%, the difference is less than 1%. Therefore, the assumed total effective damping in the residual mode of vibration is acceptable and no iteration is needed. The residual mode seismic response coefficient (C_{SR}) and base shear (V_R) are calculated as follows:

[18.5-15] $$C_{SR} = \left(\frac{R}{C_d}\right)\frac{S_{DS}}{\Omega_0 B_R} = \left(\frac{8}{5.5}\right) \times \frac{1.911}{3 \times 2.04} = 0.454 \tag{6.61}$$

[18.5-10] $$V_R = C_{SR}\overline{W}_R = 0.454 \times 545 = 247 \text{ kips} \tag{6.62}$$

The total design seismic base shear of the seismic force-resisting system is defined as:

[18.5-1] $$V_d = \sqrt{V_1^2 + V_R^2} = \sqrt{263^2 + 247^2} = 361 \text{ kips} \tag{6.63}$$

ASCE 7-10 defines minimum allowable value of the base shear permitted for design of the seismic force-resisting system as the greater of the following values:

[18.2-2] $$V_{\min} = 0.75V \tag{6.64}$$

[18.2-1] $$V_{\min} = \frac{V}{B_{V+I}} \tag{6.65}$$

where V is the seismic base shear determined in accordance with Section 12.8 for the seismic force-resisting system without damping systems. The calculation of V is according to the following procedure.

The approximate fundamental period is defined as:

[12.8-7] $$T = C_u C_t h_r^x = 1.4 \times 0.028(60 \text{ ft})^{0.8} = 1.04 \text{ s} \tag{6.66}$$

where C_t=0.028, x=0.8 from Table 12.8-2 ASCE7-10 and C_u=1.4 from Table 12.8-1 ASCE7-10. The seismic response coefficient (C_S) and seismic base shear (V) are calculated accordingly as:

[12.8-3] $$C_s = \frac{S_{D1}}{T\left(\frac{R}{I_e}\right)} = \frac{0.924}{1.04\left(\frac{8}{1.0}\right)} = 0.111 \tag{6.67}$$

[12.8-1] $$V = C_S W = 0.111 \times 3000 = 333 \text{ kips} \quad (6.68)$$

substituting V from Equation 6.68 into Equations 6.64 and 6.65 results in:

[18.2-2] $$V_{\min} = 0.75V = 0.75 \times 333 = 250 \text{ kips} < \frac{V}{B_{V+I}} = \frac{333}{1.35} = 247 \text{ kips} \quad (6.69)$$

which is less than the calculated total design base shear (V_d). Thus, the seismic force-resisting system is controlled against the base shear V_d=361 kips. The seismic force-resisting system has the required strength to meet the forces induced by V_d. In addition, the special moment frame passes the criteria defined by AISC seismic provisions for structural steel buildings (AISC 341-10). The maximum design interstory drift shall not exceed form 0.02×floor height/C_d=0.02×144/5.5=0.52 in. for conventional five-story office buildings without damping systems. As listed in Table 6-5, the maximum design interstory drift obtained from elastic analysis of the frame building exceeds the mentioned value.

Table 6-5 Interstory drift obtained from elastic analysis of the frame building with base shear V_d

Floor Number	Interstory Drift (in)
5	1.2
4	1.47
3	1.54
2	1.64
1	1.1

However, the seismic force-resisting system in a building incorporating damping system is allowed to exceed this limit and the combination of the seismic force-resisting system and the damping system is permitted to be used to meet the drift requirements.

The fundamental and residual mode deflections due to the design earthquake ground motions at level i are defined as:

[18.5-18] $$\delta_{i1D} = D_{1D}\phi_{i1} = 14.06 \text{ in} \times \begin{Bmatrix} 1.0 \\ 0.8 \\ 0.6 \\ 0.4 \\ 0.2 \end{Bmatrix} = \begin{Bmatrix} 14.06 \\ 11.25 \\ 8.44 \\ 5.62 \\ 2.81 \end{Bmatrix} \text{ in} \quad (6.70)$$

[18.5-19] $$\delta_{iRD} = D_{RD}\phi_{iR} = -1.28 \text{ in} \times \begin{Bmatrix} 1.0 \\ 0.25 \\ -0.5 \\ -1.25 \\ -2 \end{Bmatrix} = \begin{Bmatrix} -1.28 \\ -0.32 \\ 0.64 \\ 1.60 \\ 2.56 \end{Bmatrix} \text{ in} \quad (6.71)$$

The design story drift is then calculated as:

[18.5-22] $$\Delta_D = \sqrt{\Delta_{1D}^2 + \Delta_{RD}^2} = \begin{Bmatrix} 2.97 \\ 2.97 \\ 2.98 \\ 2.97 \\ 3.80 \end{Bmatrix} \text{ in} \quad (6.72)$$

The calculated design story drift shall not exceed the following value:

[18.5-22] $$0.02\frac{R}{C_d}h_f = 0.02 \times \frac{8}{5.5} \times 144 = 4.19 \text{ in} \quad (6.73)$$

The design story drift is close to and below the allowable drift. Therefore, seismic force-resisting system design is acceptable and also optimized.

6.3.3 Building with hysteretic dampers

The building described in Section 6.2 is designed herein incorporating hysteretic dampers. The design parameters for buildings incorporating hysteretic dampers include T_b/T_u and V_0/V_{max}. The authors recommend selection of brace sections such that T_b/T_u be less or equal than 0.4, consistent with the recommendation by Filiatrault and Cherry (1988), and iteration on V_0 to fulfill ASCE7-10 Chapter 18 requirements. The unbraced fundamental period of the building is T_u=1.54 s. The brace cross-sectional areas were chosen so that lateral stiffness of the bracing members becomes proportional to the interstory drift arising from simplified first mode shape of the unbraced buildings. This approach results in identical sections for this example. Incorporating braces with HSS 12×10×½ sections, the fundamental period of the braced building T_b=0.4T_u=0.62 s. It was assumed the ratio V_0/V_{max}=1.0thus V_0= 448 kips and the corresponding activation load of the hysteretic dampers for the first, second, third, fourth and roof floor are equal to 35, 56, 77, 87.5, 94.5 kips, respectively. The design presented here follows the guidelines developed by Ramirez et al. (2001).

Figure 6-3 shows the pushover curve of the building with hysteretic dampers and the corresponding parameters used in design procedure.

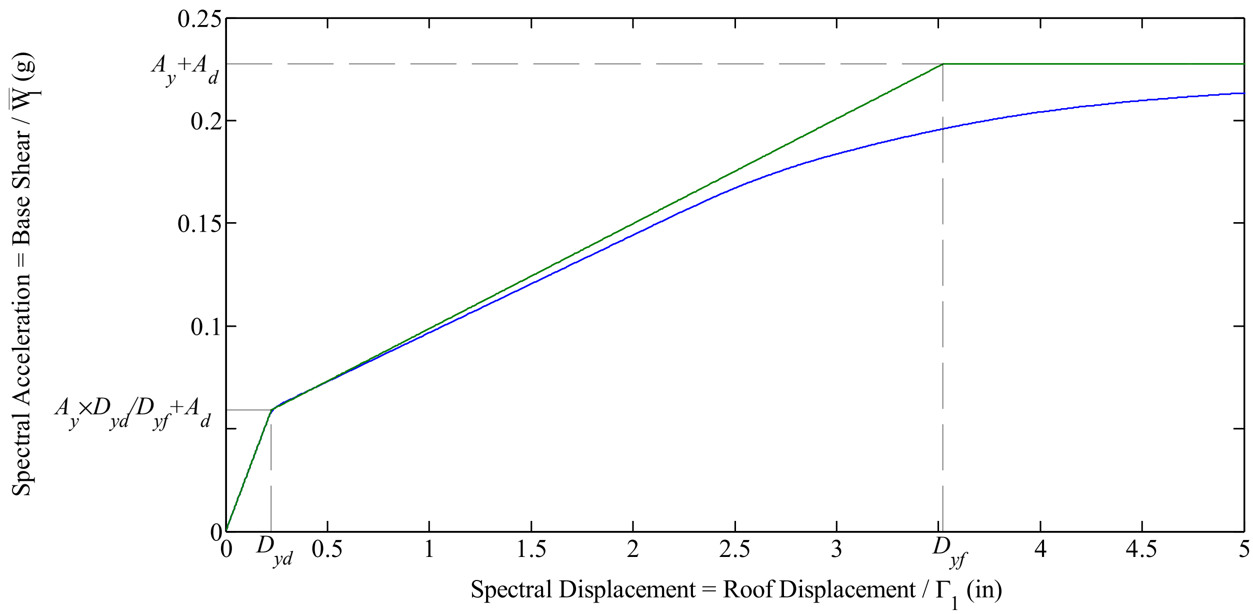


Figure 6-3 Pushover curve of the considered building with hysteretic dampers and corresponding trilinear fit

The yield spectral displacement of the frame D_{yf}=3.522in and the yield spectral displacement of the damping system D_{yd} = 0.222in, as shown in the Figure 6-3. Solving the following equations results in a yield spectral acceleration of the frame A_y=0.180g and a yield spectral acceleration of the damping system A_d=0.048g.

$$\begin{cases} A_y + A_d = 0.228g \\ A_y \dfrac{D_{yd}}{D_{yf}} + A_d = 0.059g \end{cases} \tag{6.74}$$

The fundamental mode shape (ϕ_1), effective seismic weight ($\overline{W}_1$) and participation factor (Γ_1) are determined as follows:

[18.5-3]

$$\phi_{i1} = \frac{h_i}{h_r} = \begin{Bmatrix} 1.0 \\ 0.8 \\ 0.6 \\ 0.4 \\ 0.2 \end{Bmatrix} \tag{6.75}$$

[18.4-2b] $$\bar{W}_1 = \frac{\left(\sum_{i=1}^{N_f} w_i \phi_{i1}\right)^2}{\sum_{i=1}^{N_f} w_i \phi_{i1}^2} = \frac{\left(\frac{W}{5} \times (0.2+0.4+0.6+0.8+1.0)\right)^2}{\frac{W}{5} \times (0.2^2+0.4^2+0.6^2+0.8^2+1.0^2)} = \frac{9}{11} W = 2455 \text{ kips} \quad (6.76)$$

[18.5-4] $$\Gamma_1 = \frac{\bar{W}_1}{\sum_{i=1}^{N_f} w_i \phi_{i1}} = \frac{\frac{9}{11} W}{\frac{W}{5} \times (0.2+0.4+0.6+0.8+1.0)} = \frac{15}{11} \quad (6.77)$$

Assuming D_{1D}=14in, the effective fundamental period of the system at the design earthquake ground motion is:

$$T_{1D} = 2\pi \sqrt{\frac{D_{1D}}{\Gamma_1 (A_y + A_d)}} = 2\pi \sqrt{\frac{11\times 14}{15\times(0.180+0.048)\times 386}} = 2.15 \text{ s} \quad (6.78)$$

The ductility ratio for the frame can be obtained as:

$$\mu_f = \frac{D_{1D}}{\Gamma_1 D_{yf}} = \frac{11\times 14}{15\times 3.522} = 2.92 \quad (6.79)$$

And the ductility ratio for the damping system is as follows:

$$\mu_d = \frac{D_{1D}}{\Gamma_1 D_{yd}} = \frac{11\times 14}{15\times 0.222} = 46.2 \quad (6.80)$$

The hysteretic loop adjustment factor for the SFRS is obtained as follows:

[18.6-5] $$q_H = 0.67 \frac{T_S}{T_1} = 0.67 \left(\frac{0.484}{1.54}\right) = 0.21 \quad (6.81)$$

The value of q_H shall not be taken as greater than 1.0 and as less than 0.5. Therefore q_H=0.5 is used. The effective hysteretic damping of the system is determined as:

$$\beta_{HD} = \frac{2q_H\left(1-\frac{1}{\mu_f}\right) + 2\left(\frac{A_d}{A_y}\right)\left(1-\frac{1}{\mu_d}\right)}{\pi\left(1+\frac{A_d}{A_y}\right)} = \frac{2\times 0.5\times\left(1-\frac{1}{2.92}\right) + 2\times\left(\frac{0.048\text{g}}{0.180\text{g}}\right)\left(1-\frac{1}{46.2}\right)}{\pi\left(1+\frac{0.048\text{g}}{0.180\text{g}}\right)} = 29\% \quad (6.82)$$

Given β_{V1}=0, the effective damping at the design displacement of the fundamental mode of vibration is:

[18.6-1] $$\beta_{1D} = \beta_I \sqrt{\frac{1}{1+\frac{A_d}{A_y}}} + \beta_{V1}\sqrt{\mu_D} + \beta_{HD} = 0.05\sqrt{\frac{1}{1+\frac{0.048\text{g}}{0.180\text{g}}}} + 0 + 0.29 = 33\% \tag{6.83}$$

The corresponding numerical coefficient for β_{1D} is determined as B_{1D}=1.89 from Table 18.6-1 of ASCE7-10. The fundamental mode seismic response coefficient (C_{S1}) and corresponding base shear (V_1) are calculated as follows:

[18.5-7] $$C_{S1} = \left(\frac{R}{C_d}\right)\frac{S_{D1}}{T_{1D}\left(\Omega_0 B_{1D}\right)} = \frac{8}{5.5}\times\frac{0.924}{2.15(3\times1.89)} = 0.110 \tag{6.84}$$

[18.5-2] $$V_1 = C_{S1}\bar{W}_1 = 0.110\times2455 = 270 \text{ kips} \tag{6.85}$$

The residual mode shape (ϕ_R), participation factor (Γ_R), effective seismic weight ($\bar{W}_R$) and effective period (T_R) are determined as:

[18.5-11] $$\phi_{iR} = \frac{1-\Gamma_1\phi_{i1}}{1-\Gamma_1} = \begin{Bmatrix} 1.0 \\ 0.25 \\ -0.5 \\ -1.25 \\ -2 \end{Bmatrix} \tag{6.86}$$

[18.5-12] $$\Gamma_R = 1-\Gamma_1 = 1-\frac{15}{11} = \frac{-4}{11} \tag{6.87}$$

[18.5-13] $$\bar{W}_R = W - \bar{W}_1 = \frac{2}{11}W = 545 \text{ kips} \tag{6.88}$$

[18.5-14] $$T_R = 0.4T_1 = 0.4\times0.62 = 0.25 \text{ s} \tag{6.89}$$

Given β_{VR}=0, the corresponding numerical coefficient for β_R is determined as B_R=1.0 from Table 18.6-1 of ASCE 7-10. The residual mode seismic response coefficient (C_{SR}) and base shear (V_R) are calculated as follows:

[18.5-15] $$C_{SR} = \left(\frac{R}{C_d}\right)\frac{S_{DS}}{\Omega_0 B_R} = \left(\frac{8}{5.5}\right)\times\frac{1.911}{3\times1.0} = 0.926 \tag{6.90}$$

[18.5-10] $$V_R = C_{SR}\bar{W}_R = 0.926\times545 = 505 \text{ kips} \tag{6.91}$$

The important contribution of the residual mode is also addressed in Ramirez et al. (2002b) and is due to the following reasons:

- The effective hysteretic damping of the system is not considered in definition of β_R.
- No effective viscous damping ratio is included in the calculation of β_R. (β_{VR}=0)

Considering the issues addressed in design of structures with other supplemental damping systems the concept of residual mode in ASCE Chapter 18 may need to be reconsidered.

The total design seismic base shear of the seismic force-resisting system is defined as:

[18.5-1] $$V_d = \sqrt{V_1^2 + V_R^2} = \sqrt{270^2 + 505^2} = 573 \text{ kips} \tag{6.92}$$

ASCE 7-10 defines minimum allowable value of the base shear permitted for design of the seismic force-resisting system as the greater of the following values:

[18.2-2] $$V_{\min} = 0.75V \tag{6.93}$$

[18.2-1] $$V_{\min} = \frac{V}{B_{V+I}} \tag{6.94}$$

where V is the seismic base shear determined in accordance with Section 12.8 for the seismic force-resisting system without damping systems. The calculation of V is according to the following procedure.

The approximate fundamental period is defined as:

[12.8-7] $$T = C_u C_t h_r^x = 1.4 \times 0.02(60 \text{ ft})^{0.75} = 0.60 \text{ s} \tag{6.95}$$

where C_t=0.02, x=0.75 from Table 12.8-2 ASCE7-10 and C_u=1.4 from Table 12.8-1 ASCE7-10. The seismic response coefficient (C_S) and seismic base shear (V) are calculated accordingly as:

[12.8-3] $$C_s = \frac{S_{D1}}{T\left(\frac{R}{I_e}\right)} = \frac{0.924}{0.60\left(\frac{8}{1.0}\right)} = 0.193 \tag{6.96}$$

[12.8-1] $$V = C_S W = 0.193 \times 3000 = 579 \text{ kips} \tag{6.97}$$

substituting V from Equation 6.97 into Equations 6.93 and 6.94 results in:

[18.2-2] $$V_{\min} = 0.75V = 0.75 \times 579 = 434 \text{ kips} < \frac{V}{B_{V+I}} = \frac{579}{1.00} = 579 \text{ kips} \tag{6.98}$$

which is larger than the calculated total design base shear (V_d). Thus, seismic force-resisting system is controlled against the base shear V_d=579 kips. The seismic force-resisting system has the required strength to meet the forces induced by V_d. In addition, the special moment frame passes the criteria defined by AISC seismic provisions for structural steel buildings (AISC 341-10).

The fundamental mode displacement due to the design earthquake ground motions at roof level of the building is determined as:

[18.5-20b] $$D_{1D} = \left(\frac{g}{4\pi^2}\right)\Gamma_1 \frac{S_{D1}T_{1D}}{B_{1D}} = \frac{g}{4\pi^2}\times\frac{15}{11}\times\frac{0.924\times 2.15}{1.89} = 14 \text{ in} \quad (6.99)$$

and cannot be less than:

[18.5-20b] $$\left(\frac{g}{4\pi^2}\right)\Gamma_1 \frac{S_{D1}T_1}{B_{1E}} = \frac{g}{4\pi^2}\times\frac{15}{11}\times\frac{0.924\times 0.62}{1.0} = 7.64 \text{ in} \quad (6.100)$$

Thus, D_{1D}=14in as assumed before. The displacement at the roof level of the building at the effective yield point of the seismic force-resisting system is defined as:

[18.6-10] $$D_Y = \left(\frac{g}{4\pi^2}\right)\left(\frac{\Omega_0 C_d}{R}\right)\Gamma_1 C_{S1} T_u^2 = \frac{g}{4\pi^2}\left(\frac{3\times 5.5}{8}\right)\times\frac{15}{11}\times 0.111\times 1.54^2 = 7.23 \text{ in} \quad (6.101)$$

The effective ductility demand on the seismic force-resisting system due to the design earthquake ground motion will be:

[18.6-8] $$\mu_D = \frac{D_{1D}}{D_Y} = \frac{14.0}{7.23} = 1.94 > 1.0 \quad (6.102)$$

This value is less than the maximum allowable value of effective ductility demand determined as:

[18.6-12] $$\mu_{\max} = \frac{R}{\Omega_0 I_e} = \frac{8}{3\times 1.0} = 2.67 \quad (6.103)$$

The residual mode displacement due to the design earthquake ground motions at roof level of the building is determined as:

[18.5-21] $$D_{RD} = \left(\frac{g}{4\pi^2}\right)\Gamma_R \frac{S_{D1}T_R}{B_R} = \frac{g}{4\pi^2}\times\frac{-4}{11}\times\frac{0.924\times 0.25}{1.0} = -0.82 \text{ in} \quad (6.104)$$

and cannot be larger than:

[18.5-21] $$\left(\frac{g}{4\pi^2}\right)\Gamma_R \frac{S_{DS}T_R^2}{B_R} = \frac{g}{4\pi^2}\times\frac{-4}{11}\times\frac{1.911\times 0.25^2}{1.0} = -0.42 \text{ in} \quad (6.105)$$

Thus D_{RD}=-0.82. Consequently, the fundamental and residual mode deflections due to the design earthquake ground motions at level i are defined as:

[18.5-18]
$$\delta_{i1D} = D_{1D}\phi_{i1} = 14.0 \text{ in} \times \begin{Bmatrix} 1.0 \\ 0.8 \\ 0.6 \\ 0.4 \\ 0.2 \end{Bmatrix} = \begin{Bmatrix} 14.0 \\ 11.2 \\ 8.4 \\ 5.6 \\ 2.8 \end{Bmatrix} \text{ in} \quad (6.106)$$

[18.5-19]
$$\delta_{iRD} = D_{RD}\phi_{iR} = -0.82 \text{ in} \times \begin{Bmatrix} 1.0 \\ 0.25 \\ -0.5 \\ -1.25 \\ -2 \end{Bmatrix} = \begin{Bmatrix} -0.82 \\ -0.21 \\ 0.41 \\ 1.03 \\ 1.64 \end{Bmatrix} \text{ in} \quad (6.107)$$

The design story drift is then calculated as:

[18.5-22]
$$\Delta_D = \sqrt{\Delta_{1D}^2 + \Delta_{RD}^2} = \begin{Bmatrix} 2.87 \\ 2.87 \\ 2.87 \\ 2.87 \\ 3.25 \end{Bmatrix} \text{ in} \quad (6.108)$$

The calculated design story drift shall not exceed the following value:

[18.5-22]
$$0.02\frac{R}{C_d}h_f = 0.02 \times \frac{8}{5.5} \times 144 = 4.19 \text{ in} \quad (6.109)$$

The design story drift is close to and below the allowable drift. Therefore, seismic force-resisting system design is acceptable and also optimized.

6.3.4 Verification of validity of using ELF method

The ELF procedure is permitted to be used as the only design procedure for the design of the considered building with each of the three damping systems according to requirements of Section 18.2.4.3 as follows:

1. In each direction four damping devices are installed at each story configured to resist torsion.
2. The total effective damping of the fundamental mode, β_{1D}, is less than 35% of critical for all three considered cases.

3. The SFRS does not have horizontal or vertical irregularities.

4. Floor diaphragms are assumed to be rigid.

5. The height of the structure measured from the base is 60 ft and does not exceed 100 ft.

6.4 Evaluation of seismic collapse capacity

6.4.1 Original building without damper

In this section, the seismic sidesway collapse capacity of the original structure identified in Section 6.2 is evaluated. The *CMR* of the structure without damper is calculated by the procedure proposed in this report and is compared to the value obtained from incremental dynamic analysis. Figure 6-4 shows the result of nonlinear static (pushover) analysis. The target ductility ratio is defined as:

$$\mu_T = \frac{\delta_u}{\delta_y} = \frac{39.45 \text{ in}}{5.10 \text{ in}} = 7.74 \tag{6.110}$$

where δ_u is the ultimate roof displacement and δ_y is the roof yield displacement. By linear interpolation in Table 3-2, the reduction factor r=7.68. The inelastic mode shape is obtained from pushover analysis as follows:

$$\phi_I = \begin{Bmatrix} 39.45 \\ 33.45 \\ 26.01 \\ 17.41 \\ 8.23 \end{Bmatrix} \tag{6.111}$$

Given the uniform mass distribution, the inelastic mode shape participation factor can be defined as:

$$\Gamma_I \phi_{I,r} = \frac{\phi_I^T M \{1\}}{\phi_I^T M \phi_I} \phi_{I,r} = 1.32 \tag{6.112}$$

Given the MCE spectral acceleration at a period of 1.0 s, S_{M1}=1.5×S_{D1}=1.386, the *CMR* predicted by the proposed simplified seismic collapse analysis procedure is then obtained as:

$$CMR = \frac{4\pi^2 \delta_u r}{S_{M1} T_1 \mu_T \Gamma_I \phi_{I,r}} = \frac{4\pi^2 \times 39.45 \times 7.68}{1.386 \times 386 \times 1.54 \times 7.74 \times 1.32} = 1.42 \tag{6.113}$$

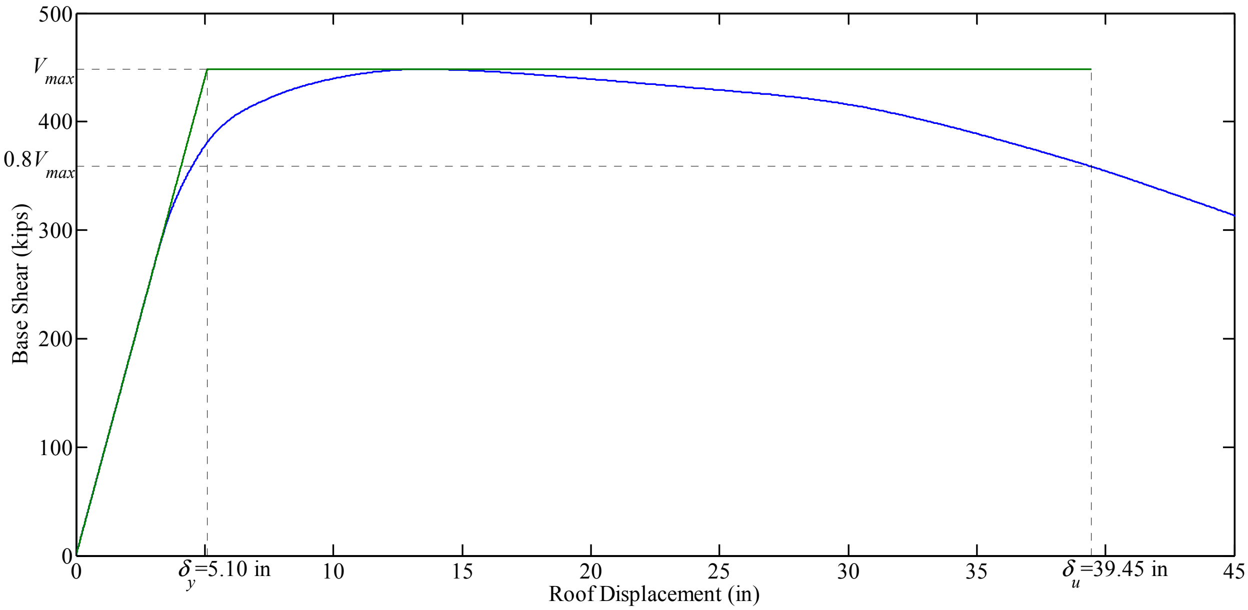
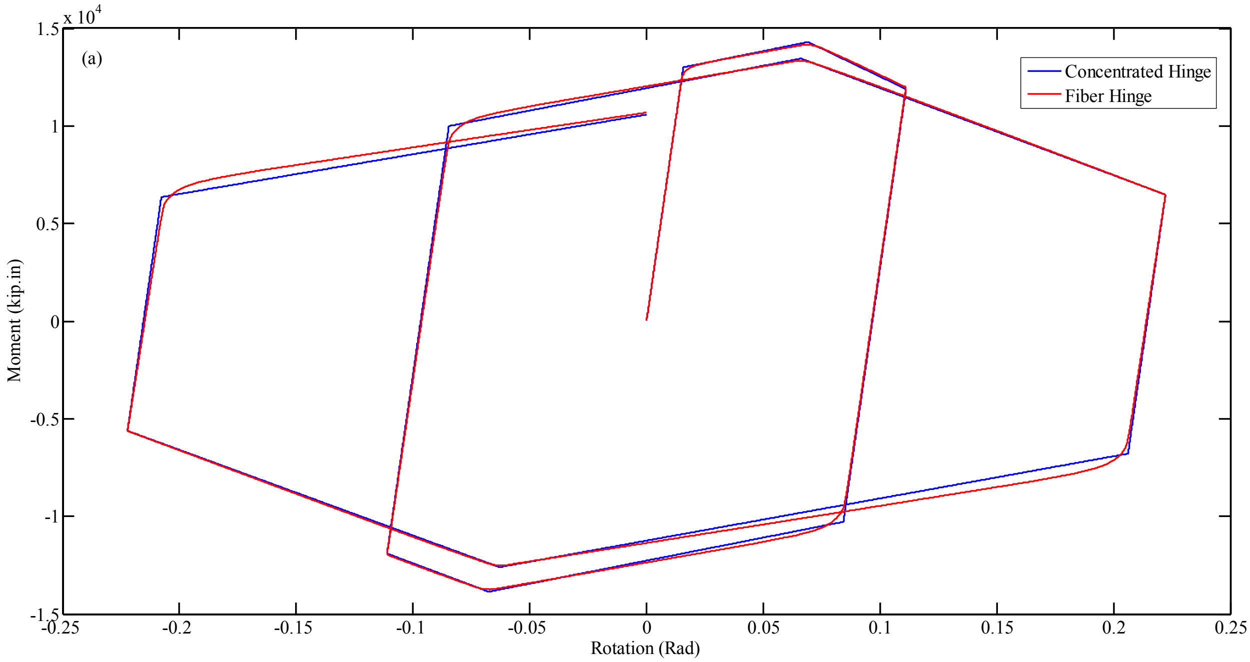


Figure 6-4 Nonlinear static (pushover) curve of the original building with corresponding bilinear fit

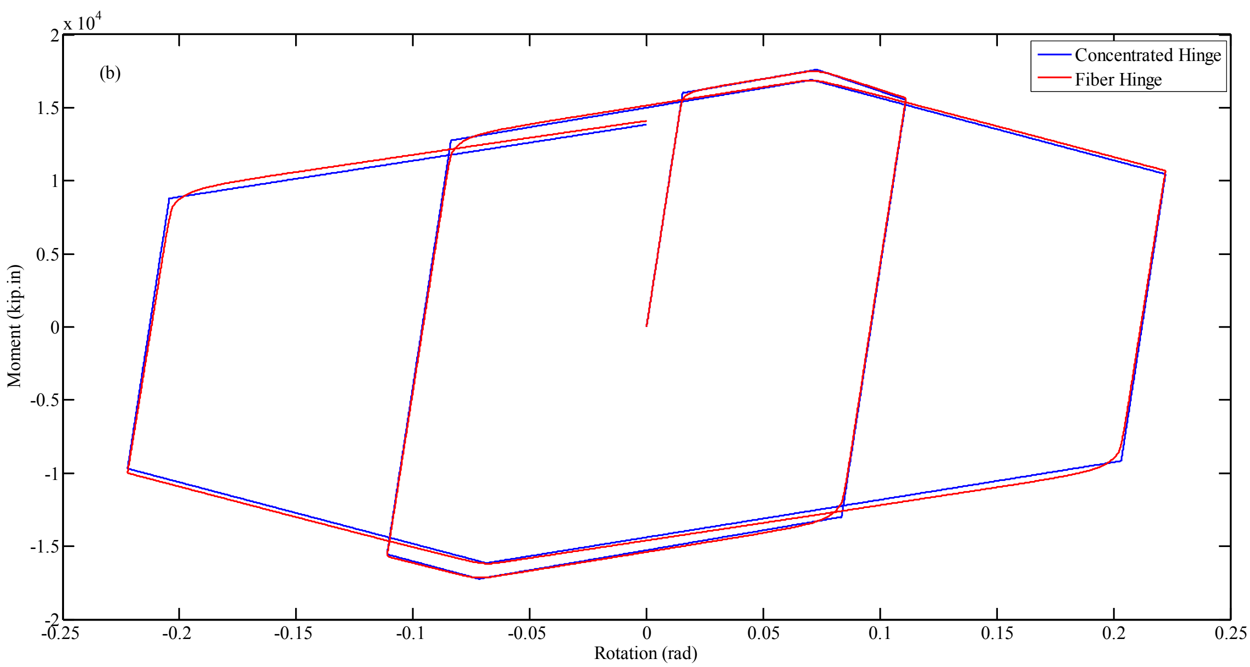
The *CMR* obtained from incremental dynamic analysis using OpenSees is 1.36. This represents a difference of 4% with the *CMR* predicted by the simplified analysis. Modeling assumption for the model created by OpenSees is described in Section 3.7. Table 6-6 lists the parameters calculated based on IK model and Figure 6-5 shows the calibration of fiber hinges as described in Section 3.7.

Table 6-6 IK model corresponding parameters

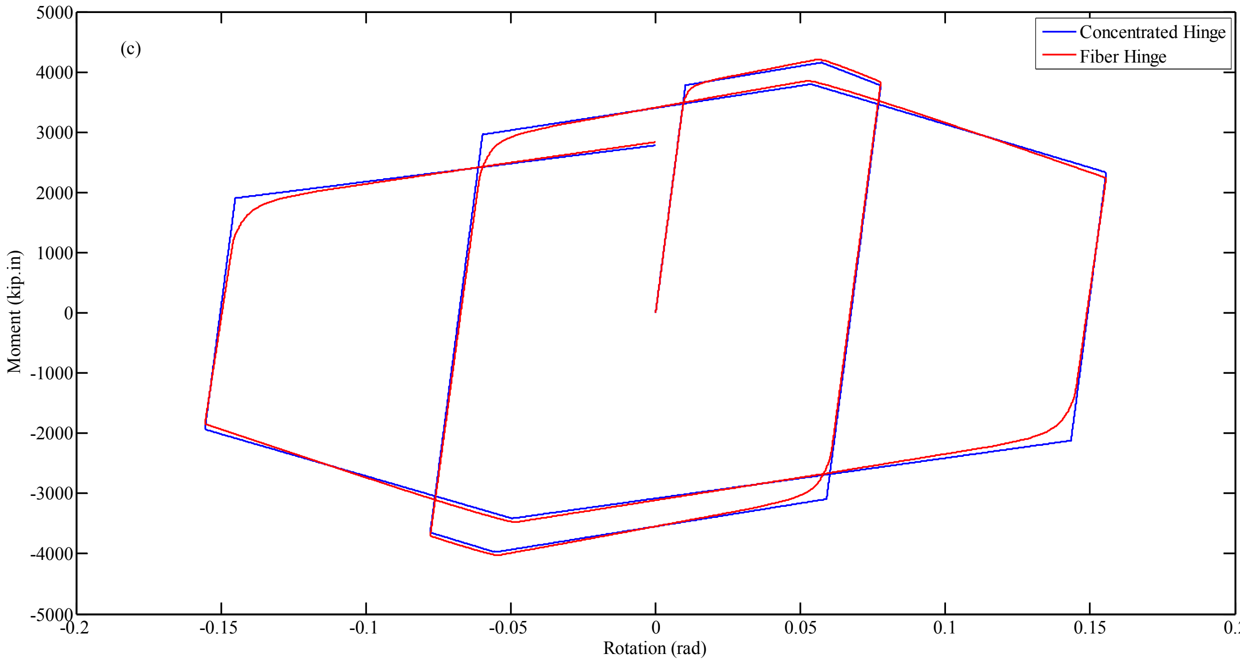
Section	θ_P	θ_{PC}	Λ
W14X145	0.05	0.27	3.55
W14X176	0.06	0.34	5.18
W16X057	0.05	0.27	2.00
W18X097	0.04	0.22	1.95



x 10^4
(a)
Concentrated Hinge
Fiber Hinge
Moment (kip.in)
Rotation (Rad)
1.5
1
0.5
0
-0.5
-1
-1.5
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
0.25



x 10^4
(b)
Concentrated Hinge
Fiber Hinge
Moment (kip.in)
Rotation (rad)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
0.25



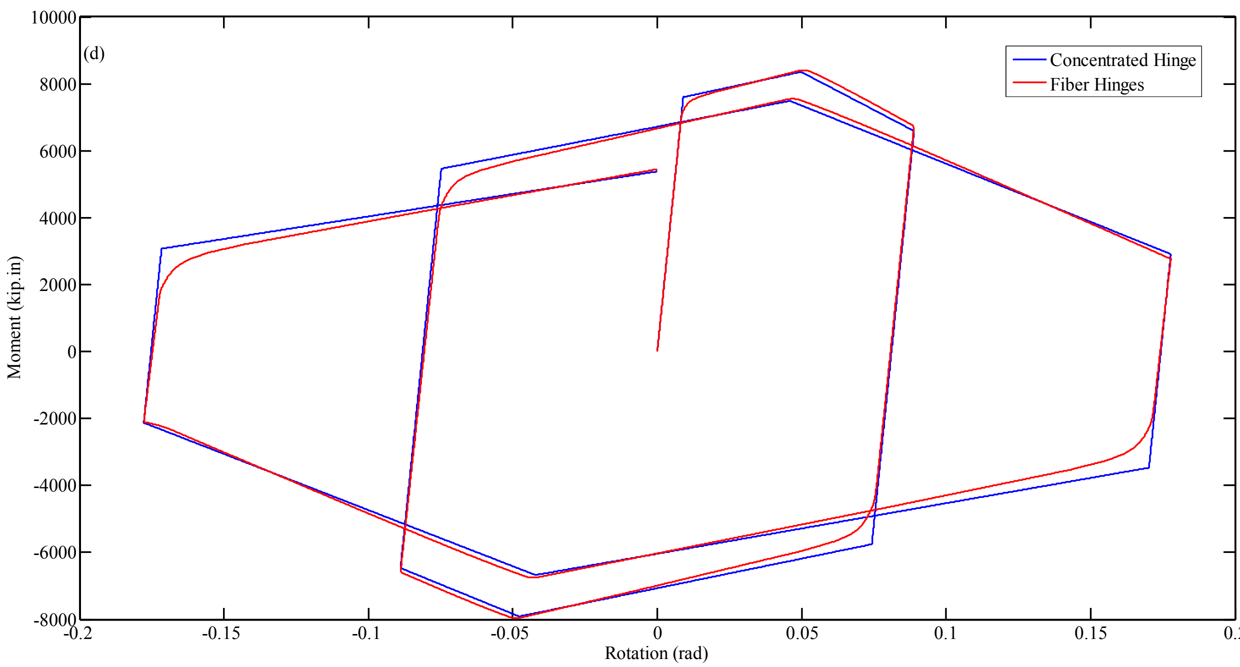


Figure 6-5 Calibrated fiber hinges used in OpenSees Models a) W14X145 b) W14X176 c) W16X57 d) W18X97

In order to compare the *CMR*, RUAUMOKO was used to model the structure. The modeling concept follows the principles discussed in Section 3.7. RUAUMOKO does not include the phenomenological IK deterioration model. The flexural strength degradation model shown in Figure 6-6 was introduced at the ends of the beam and column elements. The *CMR* obtained from incremental dynamic analysis using RUAUMOKO is 1.50. This represents a difference of 11% with the *CMR* obtained by OpenSees model.

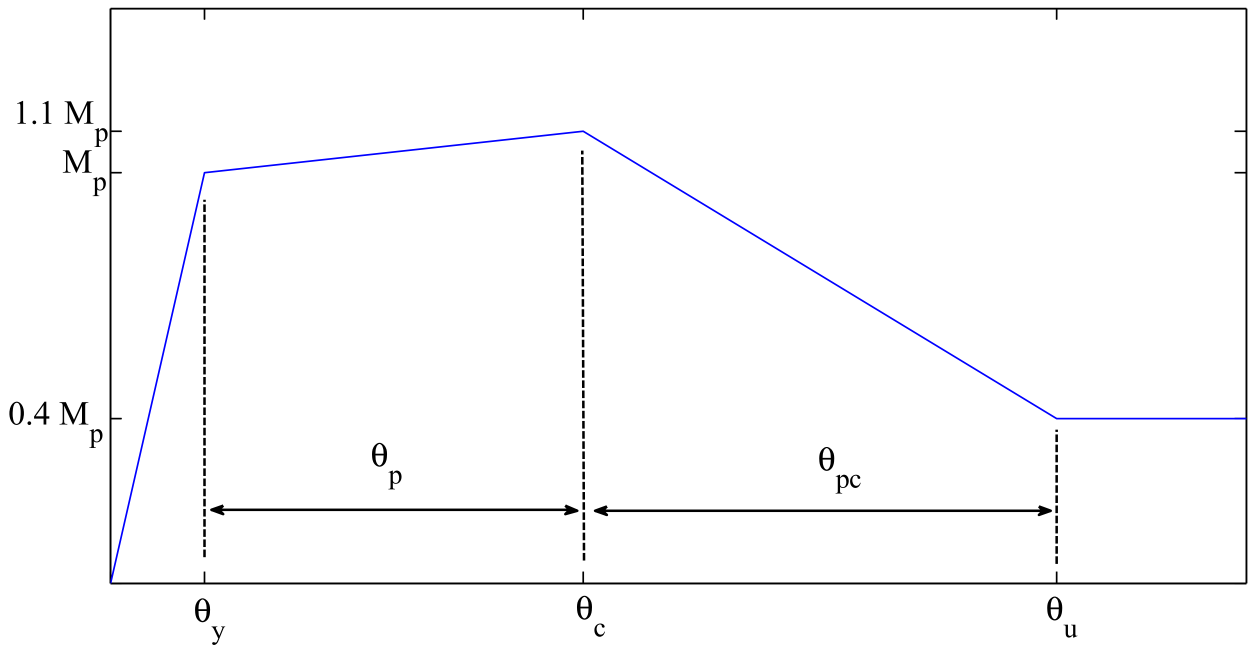


Figure 6-6 Moment-rotation behavior of the hinges used in RUAUMOKO model

ASCE7-10 assigns seismic design category (SDC) E to risk category II structures located where the mapped spectral response acceleration parameter at 1-s period, S_1, is greater than 0.75. Given spectral shape factor, *SSF*=1.45 from Table 6-7 for seismic design category E, the adjusted collapse margin ratio is defined as:

$$ACMR = CMR \times SSF = 1.36 \times 1.45 = 1.97 \quad (6.114)$$

The total system collapse uncertainty (β_{TOT}) for this example is considered to be 0.525 according to the P-695 methodology, which corresponds to a good model quality due to the detailed deterioration and degradation models considered, a good quality of test data obtained from over 300 specimens, and a good quality of the design requirements due to the application of the ASCE7-10. Given β_{TOT}=0.525, the probability of collapse under MCE ground motions is defined as 9.8% from Figure 6-7.

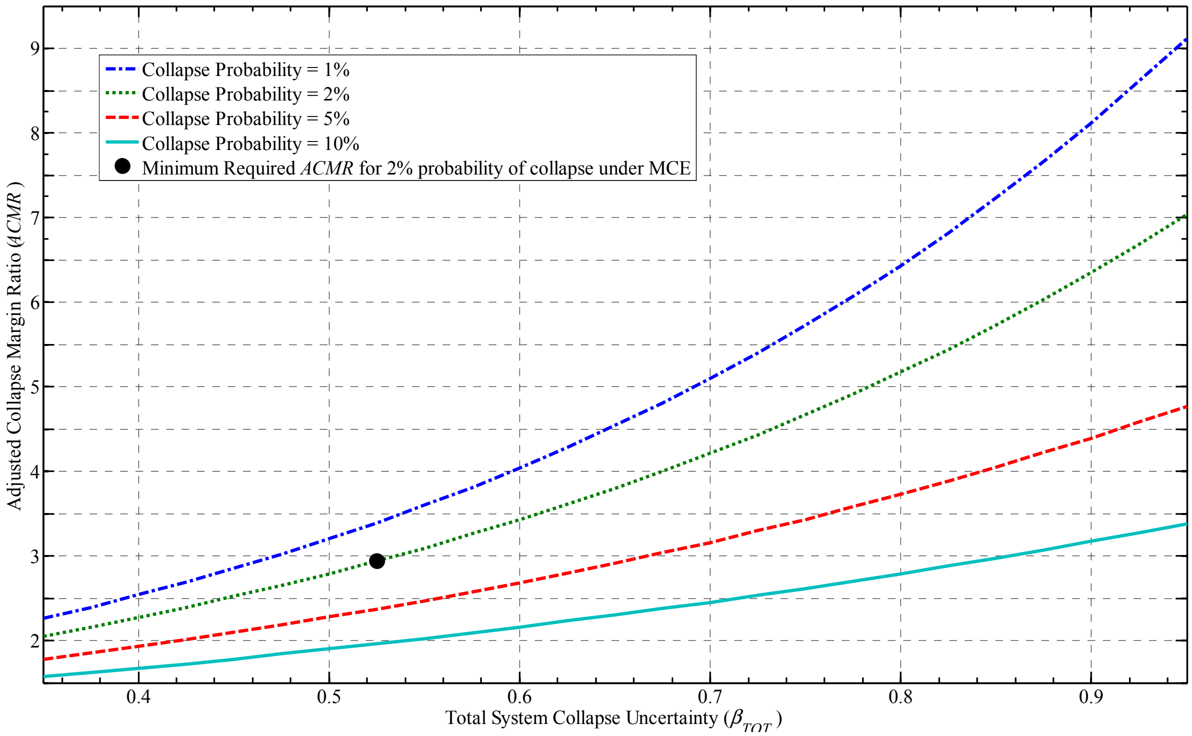


Figure 6-7 Values of *ACMR* with respect to collapse probability under MCE

Table 6-7 Spectral Shape Factor (*SSF)* for buildings evaluated by SDF E, after FEMA P695

	Target Ductility Ratio (μ_T)							
T_{el}	1.00	1.10	1.50	2.00	3.00	4.00	6.00	≥ 8.00
≤0.5	1.00	1.03	1.06	1.09	1.12	1.14	1.18	1.21
0.60	1.00	1.04	1.07	1.10	1.13	1.16	1.20	1.23
0.70	1.00	1.04	1.08	1.11	1.14	1.17	1.22	1.26
0.80	1.00	1.04	1.09	1.12	1.16	1.19	1.24	1.28
0.90	1.00	1.05	1.09	1.12	1.17	1.21	1.26	1.31
1.00	1.00	1.05	1.10	1.13	1.18	1.22	1.28	1.33
1.10	1.00	1.05	1.11	1.14	1.20	1.24	1.30	1.36
1.20	1.00	1.06	1.11	1.50	1.21	1.25	1.32	1.38
1.30	1.00	1.06	1.12	1.16	1.22	1.27	1.35	1.41
1.40	1.00	1.06	1.13	1.17	1.24	1.29	1.37	1.44
≥1.5	1.00	1.07	1.13	1.18	1.25	1.31	1.39	1.46

6.4.2 Building with linear viscous dampers

In this section, the seismic sidesway collapse capacity of the structure with linear viscous dampers designed in Section 6.3.1 is evaluated. The fundamental mode shape of the building is obtained as follows from eigenvalue analysis:

$$\phi_1 = \begin{Bmatrix} 1.00 \\ 0.84 \\ 0.65 \\ 0.14 \\ 0.19 \end{Bmatrix} \tag{6.115}$$

The elastic viscous damping ratio (ξ_L) used for determination of the reduction factor (r) can be defined as follows:

$$\xi_1 = \frac{T_1 \sum_{j=1}^{N_F} C_j f_j^{\,2} (\phi_{j1} - \phi_{(j-1)1})^2}{4\pi \sum_{j=1}^{N_F} m_j \phi_{j1}^{\,2}} = \frac{1.54 \times 386 \times 34.1 \times \cos(50.2^\circ)^2 \times \left(0.19^2 + 0.25^2 + 0.21^2 + 0.19^2 + 0.16^2\right)}{4\pi \times 0.2 \times 3000 \times \left(0.19^2 + 0.44^2 + 0.65^2 + 0.84^2 + 1.0^2\right)} = 9.5\% \tag{6.116}$$

The reduction factor is obtained from the following regression analysis equation (See Equation 4.26) as:

$$r = 0.55 + 0.79\mu_T + 2.79\mu_T\xi + 0.0023T_{el}^2\mu_T^2 + 0.065\mu_T \sin(5.67T_{el}) = 9.36 \tag{6.117}$$

The corresponding *CMR* predicted by the proposed simplified seismic collapse analysis procedure is then obtained as:

$$CMR = \frac{4\pi^2 \delta_u r}{S_{M1} T_1 \mu_T \Gamma_I \phi_{I,r}} = \frac{4\pi^2 \times 39.45 \times 9.36}{1.386 \times 386 \times 1.54 \times 7.74 \times 1.32} = 1.73 \tag{6.118}$$

The *CMR* obtained from incremental dynamic analysis is 1.81. This represents a difference of 5% with the *CMR* predicted by the simplified analysis. The adjusted collapse margin ratio is defined as:

$$ACMR = CMR \times SSF = 1.81 \times 1.45 = 2.62 \tag{6.119}$$

Given β_{TOT}=0.525, the probability of collapse under MCE ground motions is defined as 3.3% from Figure 6-7. In addition, The *CMR* obtained from incremental dynamic analysis using RUAUMOKO is 1.90. This represents a difference of 5% with the *CMR* obtained by OpenSees model.

6.4.3 Building with nonlinear viscous dampers

In this section, the seismic sidesway collapse capacity of the structure with nonlinear viscous dampers designed in section 6.3.2 is evaluated. The elastic viscous damping ratio (ξ_L) used for determination of the reduction factor (r) can be defined as follows:

$$\xi_1 = \frac{(2\pi)^{\alpha} T_1^{2-\alpha} \lambda \delta_y^{\alpha-1} \sum_{j=1}^{N_F} f_j^{1+\alpha} C_{NLj} (\phi_{j1} - \phi_{(j-1)1})^{1+\alpha}}{8\pi^3 \sum_{j=1}^{N_F} m_j \phi_{j1}^2} \tag{6.120}$$

$$= \frac{386 \times \sqrt{2\pi} \times 1.54^{1.5} \times 3.50 \times 5.1^{-0.5} \times \cos(50.2^{\circ})^{1.5} \times 82.8 \times \left(0.19^{1.5} + 0.25^{1.5} + 0.21^{1.5} + 0.19^{1.5} + 0.16^{1.5}\right)}{8\pi^3 \times 0.2 \times 3000 \times \left(0.19^2 + 0.44^2 + 0.65^2 + 0.84^2 + 1.0^2\right)} = 15.6\%$$

The reduction factor is obtained from the following regression analysis equation (see Equation 4.25) as:

$$r = 2.36 + 4.33\xi_1 + 0.53\mu_T + 0.2T_1\mu_T + 2.73^{\alpha}\mu_T\xi_1\alpha - T_1 - 4.3\xi_1\alpha + 0.052\mu_T \sin(5.68T_1) = 8.89 \tag{6.121}$$

The corresponding *CMR* predicted by the proposed simplified seismic collapse analysis procedure is then obtained as:

$$CMR = \frac{4\pi^2 \delta_u r}{S_{M1} T_1 \mu_T \Gamma_I \phi_{I,r}} = \frac{4\pi^2 \times 39.45 \times 8.89}{1.386 \times 386 \times 1.54 \times 7.74 \times 1.32} = 1.64 \tag{6.122}$$

The *CMR* obtained from incremental dynamic analysis is 1.71. This represents a difference of 4% with the CMR predicted by simplified analysis. The adjusted collapse margin ratio is defined as:

$$ACMR = CMR \times SSF = 1.71 \times 1.45 = 2.48 \tag{6.123}$$

Given $\beta_{TOT} = 0.525$, the probability of collapse under MCE ground motions is defined as 4.2% from Figure 6-7. In addition, the *CMR* obtained from incremental dynamic analysis using RUAUMOKO is 1.85. This represents a difference of 8% with the *CMR* obtained by OpenSees model.

6.4.4 Building with hysteretic dampers

In this section, the seismic sidesway collapse capacity of the structure with hysteretic dampers designed in section 6.3.3 is evaluated. The collapse margin ratio (*CMR*) of the hysteretically damped structure is calculated by the procedure proposed in this report and is compared to the value obtained from incremental dynamic analysis. Figure 6-8 shows the result of nonlinear static (pushover) analysis. The target ductility ratio is defined as:

$$\mu_T = \frac{\delta_u}{\delta_y} = \frac{46.3 \text{ in}}{1.18 \text{ in}} = 39.2 \tag{6.124}$$

By linear interpolation in Table 3-3, the reduction factor r=22.05. The inelastic mode shape is obtained from pushover analysis as follows:

$$\phi_I = \begin{Bmatrix} 46.30 \\ 39.38 \\ 30.81 \\ 20.83 \\ 10.03 \end{Bmatrix} \tag{6.125}$$

The inelastic mode shape participation factor can be defined as:

$$\Gamma_I = \frac{\phi_I^{\,T} M\{1\}}{\phi_I^{\,T} M \phi_I} = 1.32 \tag{6.126}$$

The *CMR* predicted by the proposed simplified seismic collapse analysis procedure is then obtained as:

$$CMR = \frac{4\pi^2 \delta_u r}{S_{M1} T_b \mu_T \Gamma_I \phi_{I,r}} = \frac{4\pi^2 \times 46.3 \times 22.05}{1.386 \times 386 \times 0.62 \times 39.2 \times 1.32} = 2.35 \tag{6.127}$$

The *CMR* obtained from incremental dynamic analysis is 2.46. This represents a difference of 5% with the CMR predicted by the simplified analysis. Given spectral shape factor, *SSF*=1.23 from Table 6-7 for seismic design category E, the adjusted collapse margin ratio is defined as:

$$ACMR = CMR \times SSF = 2.46 \times 1.23 = 3.03 \tag{6.128}$$

Given $\beta_{TOT} = 0.525$, the probability of collapse under MCE ground motions is defined as 1.8% from Figure 6-7. In addition, the *CMR* obtained from incremental dynamic analysis using RUAUMOKO is 2.6. This represents a difference of 6% with the *CMR* obtained by OpenSees model.

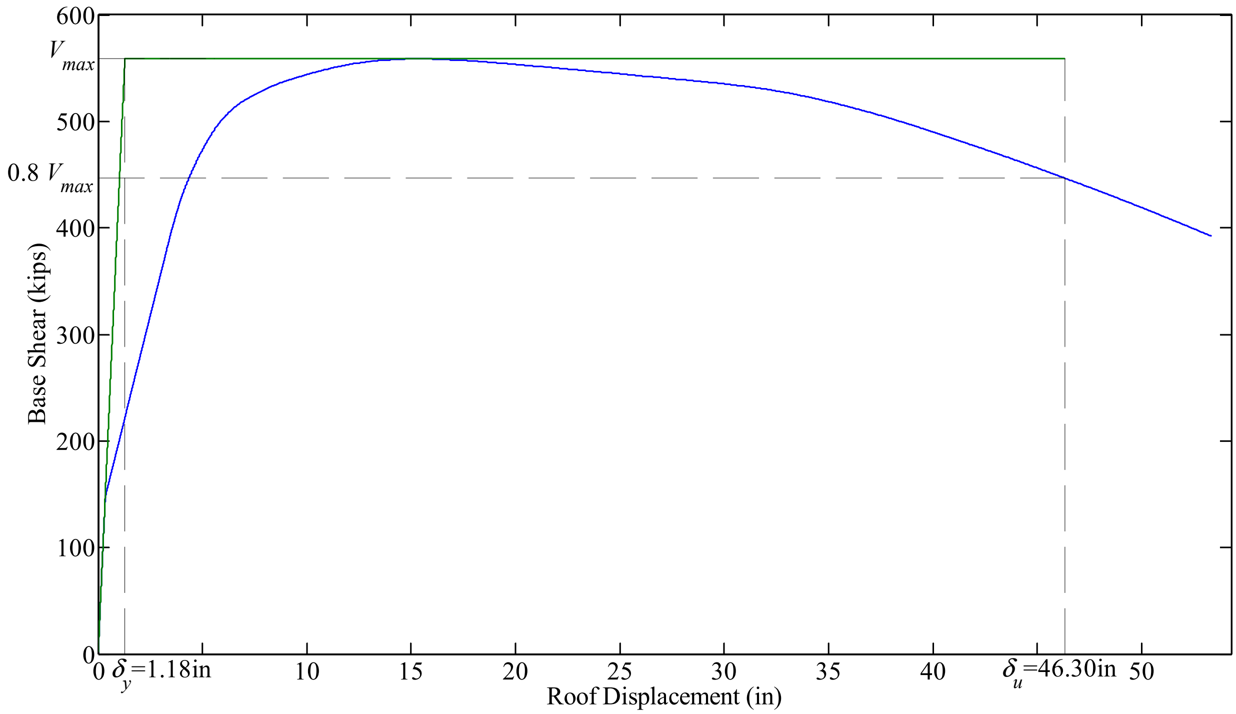


Figure 6-8 Nonlinear static (pushover) curve of the building with hysteretic dampers and corresponding bilinear fit

Table 6-8 compares the collapse capacities of the structure incorporating various damping systems obtained from the proposed simplified procedure and from nonlinear time history dynamic analyses. The addition of supplemental damping systems improves the collapse margin ratios and reduces the MCE probability of collapse of the building structure. Also, the introduction of linear viscous dampers results in a smaller probability of collapse under MCE ground motions in comparison with nonlinear ones. Moreover, although the application of hysteretic dampers has the largest effect on collapse margin ratio, heavy sections and high slip loads are required to meet the ASCE7-10 criteria, which may become uneconomical compared to the supplemental viscous damping solutions.

Table 6-8 Collapse capacity of the structure with various damping systems obtained from simplified analyses and nonlinear time history dynamic analyses

Type	*CMR* from proposed simplified procedure	*ACMR* from proposed simplified procedure	Probability of collapse obtained from prposed simplified procedure	*CMR* from incremental dynamic analyses	*ACMR* from incremental dynamic analyses	Probability of Collapse obtained from incremental dynamic analyses	Differences in *CMR* prediction
Original Building	1.42	2.06	8.40%	1.36	1.97	9.80%	4%
Building with Linear Viscous Dampers	1.73	2.51	3.90%	1.81	2.62	3.30%	5%
Building with Nonlinear Viscous Dampers	1.64	2.38	4.90%	1.71	2.48	4.20%	4%
Building with Hysteretic Dampers	2.35	2.89	2.10%	2.46	3.03	1.80%	7%

6.5 Design of the building with supplemental damping systems for a pre-determined 2% probability of collapse

The building with hysteretic dampers designed per ASCE7-10 had approximately 2% probability of collapse and a re-design was not conducted.

6.5.1 Building with linear viscous dampers

Design calculations are presented herein for the building identified in Section 6.2. The design objective is to achieve a pre-determined probability of collapse, as addressed in Section 4.8 for the building incorporating linear viscous dampers. Assuming a desired 2% probability of collapse under MCE ground motions, the minimum required *ACMR*=2.94 as highlighted in Figure 6-7. The *SSF* is 1.45 from Table 6-7 and *CMR*=2.03 from Equation 4.34. Given μ_T = 7.74 and $\Gamma_I\phi_{I,r}$ = 1.32 from Section 3.4, the minimum required reduction factor can be obtained as:

$$r \geq \frac{S_{M1}T_1\mu_T\Gamma_I\phi_{I,r}CMR}{4\pi^2\delta_u} = \frac{1.386\times386\times1.54\times7.74\times1.32\times2.03}{4\pi^2\times39.45} = 10.97 \quad (6.129)$$

The minimum required total equivalent viscous damping ratio ξ=17% from Equation 4.26. The corresponding values for the linear damper constants are equal to: 61 kip.s/in from Equation 4.36. Therefore, design for a pre-determined 2% probability of collapse results in a 78% increase in the damping constant of all linear viscous dampers. The *CMR* obtained from nonlinear time history analysis is 2.11. This represents a difference of 4% with the approximate proposed procedure.

6.5.2 Building with nonlinear viscous dampers

Design calculations are presented herein for the building identified in Section 6.2. The design objective is to achieve a pre-determined probability of collapse, as addressed in Section 4.8 for the building incorporating nonlinear viscous dampers with the same velocity exponent (α=0.5). Assuming a desired 2% probability of collapse under MCE ground motions, the minimum required *ACMR* = 2.94 as highlighted in Figure 6-7. The *SSF* is 1.45 from Table 6-7 and *CMR* = 2.03 from Equation 4.34. Given μ_T = 7.74 and $\Gamma_I\phi_{I,r}$=1.32 from Section 3.4, the minimum required reduction factor can be obtained as:

$$r \geq \frac{S_{M1}T_1\mu_T\Gamma_I\phi_{I,r}CMR}{4\pi^2\delta_u} = \frac{1.386\times386\times1.54\times7.74\times1.32\times2.03}{4\pi^2\times39.45} = 10.97 \quad (6.130)$$

The minimum required equivalent viscous damping ratio ξ=34% from Equation 4.25. The corresponding values for the nonlinear damper constant is equal to: 180 kip.(s/in)$^{0.5}$ from Equation 4.36. Therefore, design for a pre-determined 2% probability of collapse results in 117% increase in the damping constant of all nonlinear viscous dampers. The *CMR* obtained from nonlinear time history analysis is 2.08. This represents a difference of 2% with the approximate proposed procedure.

6.6 Summary

A practical example application of ASCE7-10 design procedure for a five-story building equipped with linear viscous dampers, nonlinear viscous dampers and hysteretic dampers was presented in this section. The collapse capacity of the building with various supplemental damping systems under MCE ground motions was evaluated. The results reveal that, although collapse safety of the building increases significantly with the introduction of supplemental damping systems, different damping systems result in different probability of collapse when designed per ASCE7-10 requirements. The building was then redesigned with linear and nonlinear viscous dampers for a pre-determined 2% probability of collapse. The building with hysteretic dampers designed per ASCE7-10had approximately 2% probability of collapse and re-design was not conducted. Design for a pre-determined probability of collapse resulted in 78% and 117% increases in linear and nonlinear viscous damper constants, respectively in comparison with design per ASCE7.

SECTION 7
SUMMARY AND CONCLUSION

7.1 Summary

The simplified procedure to evaluate the seismic sidesway collapse capacity of frame building structures with and without supplemental damping systems presented in this report is based on the development of a robust database of nonlinear seismic responses of single-degree-of-freedom systems for various seismic intensities and uses nonlinear static (pushover) analysis without the need for nonlinear time history dynamic analyses.

The proposed assessment and design procedure for frame buildings developed in this study is in accordance with the FEMA P695 requirements. The proposed procedure can effectively estimate the *CMR* of archetype buildings incorporating viscous and hysteretic dampers and governed by sidesway inelastic response without the need of nonlinear time history dynamic analyses. The procedure can be used by design professionals without the need for ancillary software. The simplified procedure requires parameters only from a simple bilinear fit of a pushover curve. Thus, it is not sensitive to post-elastic parameters that depend on the way a pushover curve is idealized. The derivation uses a reduction factor of the spectral acceleration that is independent of the yield reduction factor, R_y, and is only based on the target ductility (μ_T) and fundamental elastic period (T_1) of the building. The inelastic mode shape introduced in the proposed procedure is a novel concept to create a bridge between SDOF and MDOF systems with degrading components and subjected to P-Δ effects. Based on the sidesway collapse assessment of 1470 different frame building models incorporating hysteretic, linear and nonlinear viscous dampers, the proposed procedure predicts *CMR* with acceptable accuracies considering its simplicity. In addition, the design procedure introduced herein is a step toward unifying collapse capacity based design of buildings incorporating supplemental damping systems for pre-determined probability of collapse under MCE events.

The collapse capacities predicted by the proposed simplified procedure are in very good agreement with those obtained by nonlinear response history dynamic analyses. Finally, using back calculation on the *CMR*, a simplified design procedure was introduced for frame buildings incorporating viscous and hysteretic dampers. The procedure is consistent with the ASCE7-10 seismic provisions and is aimed at achieving a pre-determined probability of collapse under MCE events.

7.2 Original contributions

- The proposed evaluation procedure is simpler than the available approaches and requires only the application of a standard pushover analysis of the building under evaluation without the need of specialized software, thereby making the application of the FEMA P695 methodology less computationally demanding and more user-friendly.
- Development of a reduction factor for the spectral acceleration in the seismic collapse context, independent of the strength reduction factor, and only based on the target ductility and fundamental elastic period of the building.
- Introduction of a novel inelastic mode shape concept to create a bridge between SDOF and MDOF systems.
- Development of a new simplified probabilistic design procedure for frame buildings incorporating viscous and hysteretic dampers. The procedure is consistent with the ASCE7-10 seismic provisions and is aimed at achieving a pre-determined probability of collapse under MCE events.

7.3 Future work

The proposed procedure can be refined with complementary research in the following area:

- The estimation of the seismic response parameters through a SDOF model in combination with nonlinear static (pushover) analysis has some inherent limitations. For example, the effect of cyclic deterioration in strength and stiffness of structural components is not considered. The SDOF models can be modified to account for these effects.
- The proposed procedure is aimed at estimating the *CMR* due to sidesway seismic collapse of frame buildings dominated by a single mode of deformation and is unable to predict collapse modes dominated by forces and overturning moments along the height of a frame building.
- The procedure can be verified for building types other than frame buildings.
- The proposed procedure is strictly valid for the P695 far-field ground motions ensemble adopted in this study. The procedure can be verified by other ground motions sets.
- The design procedure is verified for structures without sever soft stories. The procedure can be modified to obtain robust inelastic mode shape of structures with soft story irregularities.

- The procedure can be modified to be applied for other seismic protection systems such as self-centering systems, tuned mass dampers and base isolation systems.

7.4 Conclusions

Given the inherent simplicity and paralleled consistency with current methodologies used in seismic design practice, the proposed procedure represents a useful and efficient tool for practicing engineers and researchers interested in evaluating the safety of frame structures against seismic sidesway collapse and for designing new buildings with minimal computational overhead.

SECTION 8

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APPENDIX A

OPENSEES SCRIPTS

A.1 Script used for SDOF systems

```
# ------------------------------------------------------------
# SDOF
#                    Written By:
#          Mohammad Javad Hamidia, 2012
# ------------------------------------------------------------
#

# SET UP ----------------------------------------------------------------------
wipe;                                  # clear memory of all past model definitions
model BasicBuilder -ndm 1 -ndf 1;         # Define the model builder, ndm=#dimension, ndf=#dofs
source LibUnits.tcl;                      # define units
node 1 [expr 0*$mm]
node 2 [expr 1000*$mm]
fix 1 1
set InelasticMat 1
set Damper 2
set DamperNL 3
set Fy [expr $Fy*$N]
set K0 [expr $K0*($N/$m)]
set C [expr $C_Inherent*$N*$sec]
set CNL [expr $C_NLDamper*$N*$sec]
set b 0
set alphaC 1
set alphaCNL $alpha
recorder Node -file Disp.out -time -node 2  -dof 1 disp;
uniaxialMaterial Steel01 $InelasticMat $Fy $K0 $b
uniaxialMaterial Viscous $Damper $C $alphaC
uniaxialMaterial Viscous $DamperNL $CNL $alphaCNL
element truss 1 1 2 [expr 1*pow($m,2)] $InelasticMat
element truss 2 1 2 [expr 1*pow($m,2)] $Damper
element truss 3 1 2 [expr 1*pow($m,2)] $DamperNL
mass 2 [expr 9.806*$N/$g]
puts "Model Built"
# --------------------------------------------------------------------------
#
# Dynamic Analysis Procedure, Under Uniform Ground Acceleration
#
# --------------------------------------------------------------------------
#
# Uniform Earthquake ground motion (uniform acceleration input at all support nodes)
```

```
set GMdirection 1;                                    # ground-motion direction
set GMfile "1.AT2" ;                          # ground-motion filenames
set GMfact $Factor;                                   # ground-motion scaling factor
set DtAnalysis   [expr 0.0025*$sec];        # time-step Dt for lateral analysis
set TmaxAnalysis [expr 60*$sec];            # maximum duration of ground-motion analysis

# ----------- set up analysis parameters
source LibAnalysisDynamicParameters.tcl;        # constraintsHandler,DOFnumberer,system-
ofequations,convergenceTest,solutionAlgorithm,integrator

#  ---------------------------------    perform Dynamic Ground-Motion Analysis
# the following commands are unique to the Uniform Earthquake excitation
set IDloadTag 400;        # for uniformSupport excitation
# Uniform EXCITATION: acceleration input
set outFile $GMfile.g3; # set variable holding new filename (PEER files have .at2/dt2 extension)
set GMfatt [expr $g*$GMfact];              # data in input file is in g Unifts -- ACCELERATION TH
set AccelSeries "Series -dt 0.02 -filePath $outFile -factor  $GMfatt";       # time series information
pattern UniformExcitation  $IDloadTag  $GMdirection -accel  $AccelSeries  ;                 # create
Unifform excitation
set dt_anal_Step $DtAnalysis
set GMtime $TmaxAnalysis
set numStories 1
set FloorNodes {1 2}
set h1 1
set htyp 1
set DriftLimit 3
global CollapseFlag;                                  # global variable to monitor collapse
global Diverge;                               # global variable to monitor collapse
global ok
set k 1;
source MaxDriftTester.tcl;                            # For Collapse Studies
set CollapseFlag "NO"
set Diverge "NO"
wipeAnalysis
constraints Plain
numberer RCM
system UmfPack
test EnergyIncr 1.0e-4 100
algorithm Newton
integrator Newmark 0.50 0.25
analysis Transient
set dt_analysis $dt_anal_Step;                                                  # timestep of analysis
set NumSteps [expr round(($GMtime + 0.0)/$dt_analysis)];         # number of steps in analysis
set ok [analyze $NumSteps $dt_analysis];
# Check Max Drifts for Collapse by Monitoring the CollapseFlag Variable
MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
if  {$CollapseFlag == "YES"} {
          set ok 0
}
```

```
#set dt_analysis [expr $dt_analysis/10];                                   # timestep of analysis
 if {$ok != 0} {
	puts "Analysis did not converge..."
	set TmaxAnalysis $GMtime;
	# The analysis will be time-controlled and is done for the remaining time
	set ok 0;
	set controlTime [getTime];
		while {$controlTime < [expr round(1.0*$TmaxAnalysis)] || $ok !=0 } {
		MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
		if  {$CollapseFlag == "YES"} {
			set ok 0; break;
		} else {
			set ok 1
		}
	   # Get Control Time inside the loop
		set controlTime [getTime];
		set startTime [getTime];
		if {$ok != 0} {
			puts "!!!!!Run Newton with 1/2 of step.."
			test EnergyIncr 1.0e-3 100   0
			set controlTime [getTime]
			set remainTime [expr $TmaxAnalysis - $controlTime]
		     puts $remainTime
			algorithm Newton
			integrator Newmark 0.50 0.25
			set ok [analyze 100 [expr $dt_analysis/2.0]]
			MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
			if  {$CollapseFlag == "YES"} {
				set ok 0
			}
		}
			if {$ok != 0} {
			puts "!!!!!Run Newton with 1/10 of step.."
			test EnergyIncr 1.0e-3 100   0
			set controlTime [getTime]
			set remainTime [expr $TmaxAnalysis - $controlTime]
		     puts $remainTime
			algorithm Newton
			integrator Newmark 0.50 0.25
			set ok [analyze 100 [expr $dt_analysis/10.0]]
			MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
			if  {$CollapseFlag == "YES"} {
				set ok 0
			}
		}
			if {$ok != 0} {
			puts "!!!!!Run Newton with 1/100 of step.."
			test EnergyIncr 1.0e-3 100   0
			set controlTime [getTime]
```

```
			set remainTime [expr $TmaxAnalysis - $controlTime]
		puts $remainTime
			algorithm Newton
			integrator Newmark 0.50 0.25
			set ok [analyze 100 [expr $dt_analysis/100.0]]
			MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
			if  {$CollapseFlag == "YES"} {
				set ok 0
			}
		}

		if {$ok != 0 } {
			puts "!!!!!Run Newton with Initial Tangent with 1/10 of original step.."
			test EnergyIncr 1.0e-2 100 0
			set controlTime [getTime]
			set remainTime [expr $TmaxAnalysis - $controlTime]
			algorithm Newton -initial
			set ok [analyze 100 [expr $dt_analysis/10.0]]
			MaxDriftTester $numStories $DriftLimit $FloorNodes  $h1 $htyp
			if  {$CollapseFlag == "YES"} {
				set ok 0
			}
		}

		if {$ok != 0 } {
			puts "!!!!!Newton with line Search and 1/10 of the original step .."
			test EnergyIncr 1.0e-2 100 0
			algorithm NewtonLineSearch 0.50
			set ok [analyze 100 [expr $dt_analysis/10.0]]
			MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
			if  {$CollapseFlag == "YES"} {
				set ok 0
			}
		}

		if {$ok != 0 } {
			puts "!!!!!Newton Initial with 1/10 of step and Displacement Control 
Convergence.."
			test NormDispIncr 1.0e-2 100  0
			algorithm Newton -initial
			set ok [analyze 100 [expr $dt_analysis/10.0]]
			MaxDriftTester $numStories $DriftLimit $FloorNodes  $h1 $htyp
			if  {$CollapseFlag == "YES"} {
				set ok 0
			}
		}

		if {$ok != 0 } {
```

```
                puts "!!!!!Newton Initial with 1/10 of step energy Control Convergence and
HTTP Inegrator.."
                test EnergyIncr 1.0e-2 100  0
                algorithm Newton -initial
                integrator HHTHSIncrReduct 0.5 0.95
                set ok [analyze 100 [expr $dt_analysis/10.0]]
                MaxDriftTester $numStories $DriftLimit $FloorNodes  $h1 $htyp
                if  {$CollapseFlag == "YES"} {
                        set ok 0
                }
        }
        if {$ok != 0 } {
                puts "Go Back to Newton with tangent Tangent and original step.."
                test EnergyIncr 1.0e-2 50   0
                set controlTime [getTime]
                set remainTime [expr $TmaxAnalysis - $controlTime]
                set NewRemainSteps [expr round(($remainTime)/($dt_analysis))]
                algorithm Newton
                integrator Newmark 0.50 0.25
                set ok [analyze $NewRemainSteps [expr $dt_analysis]]
                MaxDriftTester $numStories $DriftLimit $FloorNodes  $h1 $htyp
                if  {$CollapseFlag == "YES"} {
                        set ok 0
                }
        }

        if {$ok != 0 } {
                puts "Newton with 1/1000 of step and do only 1 step.."
                test NormDispIncr 1.0e-1 100  0
                set controlTime [getTime]
                set remainTime [expr $TmaxAnalysis - $controlTime]
                set NewRemainSteps [expr round(($remainTime)/($dt_analysis/1000))]
                algorithm Newton -initial
                set ok [analyze 10 [expr $dt_analysis/1000.0]]
                MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
                if  {$CollapseFlag == "YES"} {
                        set ok 0
                }
        }
        set controlTime [getTime]
        set endTime [getTime];
        set passTime [expr $endTime-$startTime];
        puts "PassTime is equal to $passTime"
        if { $passTime < 0.0000001 } {
        set k [expr $k+1];
        puts " K is equal to $k"
        } else {
        set k 1;
        }
```

```
                    if {$k > 20} {
                    set CollapseFlag "YES"
                    set Diverge "YES"
                    set ok 0; break;
                    }

          }
 }
##}
#
#
```

A.2 Script used for design example of Section 6

```
# --------------------------------------------------------------
# Example Building
#                   Written By:
#          Mohammad Javad Hamidia, 2012
# --------------------------------------------------------------
#

# SET UP ----------------------------------------------------------------------
wipe;                                  # clear memory of all past model definitions
model BasicBuilder -ndm 2 -ndf 3;         # Define the model builder, ndm=#dimension, ndf=#dofs
set DynamicDataDir DynamicData;                         # set up name of data directory (can remove this)
source LibUnits.tcl;                          # define units
source DisplayPlane.tcl;             # procedure for displaying a plane in model
source DisplayModel2D.tcl;                  # procedure for displaying 2D perspective of model
source Wsection.tcl;                          # procedure to define fiber W section

# define GEOMETRY --------------------------------------------------------
# define structure-geometry paramters
# define NODAL COORDINATES

set d14145 [expr 16.4*$in];   # depth of W section
set d14176 [expr 16.7*$in];       # depth of W section
set d16057 [expr 18.4*$in];       # depth of W section
set d18097 [expr 21.8*$in];       # depth of W section

set h2 [expr 144*$in]
set L1 [expr 240*$in]

node 1 [expr 0] [expr 5*$h2];
node 2 [expr $d14145/2] [expr 5*$h2];
node 3 [expr $L1-$d14145/2] [expr 5*$h2];
node 4 [expr $L1] [expr 5*$h2];
node 5 [expr $L1+$d14145/2] [expr 5*$h2];
node 6 [expr 2*$L1-$d14145/2] [expr 5*$h2];
node 7 [expr 2*$L1] [expr 5*$h2];
node 8 [expr 2*$L1+$d14145/2] [expr 5*$h2];
node 9 [expr 3*$L1-$d14145/2] [expr 5*$h2];
node 10 [expr 3*$L1] [expr 5*$h2];
node 11 [expr 4*$L1] [expr 5*$h2];
node 12 [expr 0] [expr 5*$h2-$d16057/2];
node 13 [expr $L1] [expr 5*$h2-$d16057/2];
node 14 [expr 2*$L1] [expr 5*$h2-$d16057/2];
node 15 [expr 3*$L1] [expr 5*$h2-$d16057/2];

node 16 [expr 0] [expr 4*$h2+$d16057/2];
node 17 [expr $L1] [expr 4*$h2+$d16057/2];
```

```
node 18 [expr 2*$L1] [expr 4*$h2+$d16057/2];
node 19 [expr 3*$L1] [expr 4*$h2+$d16057/2];
node 20 [expr 0] [expr 4*$h2];
node 21 [expr $d14145/2] [expr 4*$h2];
node 22 [expr $L1-$d14145/2] [expr 4*$h2];
node 23 [expr $L1] [expr 4*$h2];
node 24 [expr $L1+$d14145/2] [expr 4*$h2];
node 25 [expr 2*$L1-$d14145/2] [expr 4*$h2];
node 26 [expr 2*$L1] [expr 4*$h2];
node 27 [expr 2*$L1+$d14145/2] [expr 4*$h2];
node 28 [expr 3*$L1-$d14145/2] [expr 4*$h2];
node 29 [expr 3*$L1] [expr 4*$h2];
node 30 [expr 4*$L1] [expr 4*$h2];
node 31 [expr 0] [expr 4*$h2-$d16057/2];
node 32 [expr $L1] [expr 4*$h2-$d16057/2];
node 33 [expr 2*$L1] [expr 4*$h2-$d16057/2];
node 34 [expr 3*$L1] [expr 4*$h2-$d16057/2];


node 35 [expr 0] [expr 3*$h2+$d18097/2];
node 36 [expr $L1] [expr 3*$h2+$d18097/2];
node 37 [expr 2*$L1] [expr 3*$h2+$d18097/2];
node 38 [expr 3*$L1] [expr 3*$h2+$d18097/2];
node 39 [expr 0] [expr 3*$h2];
node 40 [expr $d14176/2] [expr 3*$h2];
node 41 [expr $L1-$d14176/2] [expr 3*$h2];
node 42 [expr $L1] [expr 3*$h2];
node 43 [expr $L1+$d14176/2] [expr 3*$h2];
node 44 [expr 2*$L1-$d14176/2] [expr 3*$h2];
node 45 [expr 2*$L1] [expr 3*$h2];
node 46 [expr 2*$L1+$d14176/2] [expr 3*$h2];
node 47 [expr 3*$L1-$d14176/2] [expr 3*$h2];
node 48 [expr 3*$L1] [expr 3*$h2];
node 49 [expr 4*$L1] [expr 3*$h2];
node 50 [expr 0] [expr 3*$h2-$d18097/2];
node 51 [expr $L1] [expr 3*$h2-$d18097/2];
node 52 [expr 2*$L1] [expr 3*$h2-$d18097/2];
node 53 [expr 3*$L1] [expr 3*$h2-$d18097/2];


node 54 [expr 0] [expr 2*$h2+$d18097/2];
node 55 [expr $L1] [expr 2*$h2+$d18097/2];
node 56 [expr 2*$L1] [expr 2*$h2+$d18097/2];
node 57 [expr 3*$L1] [expr 2*$h2+$d18097/2];
node 58 [expr 0] [expr 2*$h2];
node 59 [expr $d14176/2] [expr 2*$h2];
node 60 [expr $L1-$d14176/2] [expr 2*$h2];
node 61 [expr $L1] [expr 2*$h2];
node 62 [expr $L1+$d14176/2] [expr 2*$h2];
```

```
node 63 [expr 2*$L1-$d14176/2] [expr 2*$h2];
node 64 [expr 2*$L1] [expr 2*$h2];
node 65 [expr 2*$L1+$d14176/2] [expr 2*$h2];
node 66 [expr 3*$L1-$d14176/2] [expr 2*$h2];
node 67 [expr 3*$L1] [expr 2*$h2];
node 68 [expr 4*$L1] [expr 2*$h2];
node 69 [expr 0] [expr 2*$h2-$d18097/2];
node 70 [expr $L1] [expr 2*$h2-$d18097/2];
node 71 [expr 2*$L1] [expr 2*$h2-$d18097/2];
node 72 [expr 3*$L1] [expr 2*$h2-$d18097/2];


node 73 [expr 0] [expr 1*$h2+$d18097/2];
node 74 [expr $L1] [expr 1*$h2+$d18097/2];
node 75 [expr 2*$L1] [expr 1*$h2+$d18097/2];
node 76 [expr 3*$L1] [expr 1*$h2+$d18097/2];
node 77 [expr 0] [expr 1*$h2];
node 78 [expr $d14176/2] [expr 1*$h2];
node 79 [expr $L1-$d14176/2] [expr 1*$h2];
node 80 [expr $L1] [expr 1*$h2];
node 81 [expr $L1+$d14176/2] [expr 1*$h2];
node 82 [expr 2*$L1-$d14176/2] [expr 1*$h2];
node 83 [expr 2*$L1] [expr 1*$h2];
node 84 [expr 2*$L1+$d14176/2] [expr 1*$h2];
node 85 [expr 3*$L1-$d14176/2] [expr 1*$h2];
node 86 [expr 3*$L1] [expr 1*$h2];
node 87 [expr 4*$L1] [expr 1*$h2];
node 88 [expr 0] [expr 1*$h2-$d18097/2];
node 89 [expr $L1] [expr 1*$h2-$d18097/2];
node 90 [expr 2*$L1] [expr 1*$h2-$d18097/2];
node 91 [expr 3*$L1] [expr 1*$h2-$d18097/2];

node 92 [expr 0] [expr 0*$h2];
node 93 [expr $L1] [expr 0*$h2];
node 94 [expr 2*$L1] [expr 0*$h2];
node 95 [expr 3*$L1] [expr 0*$h2];
node 96 [expr 4*$L1] [expr 0*$h2];

# For PDelta

node 97 [expr 4*$L1] [expr 4*$h2];
node 98 [expr 4*$L1] [expr 3*$h2];
node 99 [expr 4*$L1] [expr 2*$h2];
node 100 [expr 4*$L1] [expr 1*$h2];

# Half of Beam

node 101 [expr 0.5*$L1] [expr 5*$h2];
node 102 [expr 1.5*$L1] [expr 5*$h2];
```

```
node 103 [expr 2.5*$L1] [expr 5*$h2];

node 104 [expr 0.5*$L1] [expr 4*$h2];
node 105 [expr 1.5*$L1] [expr 4*$h2];
node 106 [expr 2.5*$L1] [expr 4*$h2];

node 107 [expr 0.5*$L1] [expr 3*$h2];
node 108 [expr 1.5*$L1] [expr 3*$h2];
node 109 [expr 2.5*$L1] [expr 3*$h2];

node 110 [expr 0.5*$L1] [expr 2*$h2];
node 111 [expr 1.5*$L1] [expr 2*$h2];
node 112 [expr 2.5*$L1] [expr 2*$h2];

node 113 [expr 0.5*$L1] [expr $h2];
node 114 [expr 1.5*$L1] [expr $h2];
node 115 [expr 2.5*$L1] [expr $h2];

# BOUNDARY CONDITIONS

fix 92 1 1 1;     # Fix base nodes
fix 93 1 1 1;     # Fix base nodes
fix 94 1 1 1;     # Fix base nodes
fix 95 1 1 1;     # Fix base nodes
fix 96 1 1 0;     # Fix gravity columns base node except rotational DOF

# Define Diaphragms
equalDOF 11 1 1
equalDOF 11 2 1
equalDOF 11 3 1
equalDOF 11 4 1
equalDOF 11 5 1
equalDOF 11 6 1
equalDOF 11 7 1
equalDOF 11 8 1
equalDOF 11 9 1
equalDOF 11 10 1
equalDOF 11 101 1
equalDOF 11 102 1
equalDOF 11 103 1

equalDOF 30 20 1
equalDOF 30 21 1
equalDOF 30 22 1
equalDOF 30 23 1
equalDOF 30 24 1
equalDOF 30 25 1
equalDOF 30 26 1
equalDOF 30 27 1
```

```
equalDOF 30 28 1
equalDOF 30 29 1
equalDOF 30 104 1
equalDOF 30 105 1
equalDOF 30 106 1

equalDOF 49 39 1
equalDOF 49 40 1
equalDOF 49 41 1
equalDOF 49 42 1
equalDOF 49 43 1
equalDOF 49 44 1
equalDOF 49 45 1
equalDOF 49 46 1
equalDOF 49 47 1
equalDOF 49 48 1
equalDOF 49 107 1
equalDOF 49 108 1
equalDOF 49 109 1


equalDOF 68 58 1
equalDOF 68 59 1
equalDOF 68 60 1
equalDOF 68 61 1
equalDOF 68 62 1
equalDOF 68 63 1
equalDOF 68 64 1
equalDOF 68 65 1
equalDOF 68 66 1
equalDOF 68 67 1
equalDOF 68 110 1
equalDOF 68 111 1
equalDOF 68 112 1


equalDOF 87 77 1
equalDOF 87 78 1
equalDOF 87 79 1
equalDOF 87 80 1
equalDOF 87 81 1
equalDOF 87 82 1
equalDOF 87 83 1
equalDOF 87 84 1
equalDOF 87 85 1
equalDOF 87 86 1
equalDOF 87 113 1
equalDOF 87 114 1
equalDOF 87 115 1
```

```
# Gravity Constrain
equalDOF 30 97 1 2
equalDOF 49 98 1 2
equalDOF 68 99 1 2
equalDOF 87 100 1 2

# Calculated MODEL PARAMETERS,

# Define MATERIAL properties ----------------------------------------
set FyBeam [expr 2.48*pow(10,8)*$N/pow($m,2)]
set FyCol [expr 3.45*pow(10,8)*$N/pow($m,2)]
set Es [expr 2.0*pow(10,11)*$N/pow($m,2)];              # Steel Young's Modulus
set epyBeam [expr $FyBeam/$Es]
set epyCol [expr $FyCol/$Es]


set mat14145 1
uniaxialMaterial Bilin $mat14145 $Es 0.0065 0.0065 [expr 1.01*$FyCol] [expr -1.01*$FyCol] 1.6 1.6
1.6 1.6 1 1 1 1 0.025 0.025 0.115 0.115 0.40 0.40 10 10 1 1
# uniaxialMaterial Bilin $matIDhard $Es $as_Plus $as_Neg $My_Plus $My_Neg $Lamda_S $Lamda_C
$Lamda_A $Lamda_K $c_S $c_C $c_A $c_K $theta_p_Plus $theta_p_Neg $theta_pc_Plus
$theta_pc_Neg $Res_Pos $Res_Neg $theta_u_Plus $theta_u_Neg $D_Plus $D_Neg

set mat14176 2
uniaxialMaterial Bilin $mat14176 $Es 0.0065 0.0065 [expr 1.01*$FyCol] [expr -1.01*$FyCol] 2.5 2.5
2.5 2.5 1 1 1 1 0.026 0.026 0.15 0.15 0.4 0.4 10 10 1 1

set mat16057 3
uniaxialMaterial Bilin $mat16057 $Es 0.009 0.009 [expr 1.01*$FyBeam] [expr -1.01*$FyBeam] 0.8 0.8
0.8 0.8 1 1 1 1 0.022 0.022 0.09 0.09 0.4 0.4 10 10 1 1

set mat18097 4
uniaxialMaterial Bilin $mat18097 $Es 0.009 0.009 [expr 1.01*$FyBeam] [expr -1.01*$FyBeam] 0.8 0.8
0.8 0.8 1 1 1 1 0.02 0.02 0.08 0.08 0.4 0.4 10 10 1 1


set GravcolumnsmatTag 10
set Egrav [expr 5.5*pow(10,10)*$N/pow($mm,2)]
uniaxialMaterial Elastic $GravcolumnsmatTag $Es


# ELEMENT properties
# Structural-Steel W-section properties


set W14145 1
set W14176 2
set W16057 3
```

```
set W18097 4


# column sections: W14x145
set d [expr 14.8*$in];     # depth
set bf [expr 15.5*$in];    # flange width
set tf [expr 1.09*$in];    # flange thickness
set tw [expr 0.68*$in];    # web thickness
set nfdw 20;               # number of fibers along dw
set nftw 1;                # number of fibers along tw
set nfbf 1;                # number of fibers along bf
set nftf 6;                        # number of fibers along tf
set lp14145 [expr $d]; # length of plastic hinge
set A14145 [expr 42.7*pow($in,2)]; # area of the section
set Iz14145 [expr 1710*pow($in,4)]; # moment of inertia of the section
Wsection  $W14145 $mat14145 $d $bf $tf $tw $nfdw $nftw $nfbf $nftf

# column sections: W14x176
set d [expr 15.2*$in];     # depth
set bf [expr 15.7*$in];    # flange width
set tf [expr 1.31*$in];    # flange thickness
set tw [expr 0.83*$in];    # web thickness
#set nfdw 16;              # number of fibers along dw
#set nftw 2;               # number of fibers along tw
#set nfbf 16;              # number of fibers along bf
#set nftf 4;                       # number of fibers along tf
set lp14176 [expr $d]; # length of plastic hinge
set A14176 [expr 51.8*pow($in,2)]; # area of the section
set Iz14176 [expr 2140*pow($in,4)]; # moment of inertia of the section
Wsection  $W14176 $mat14176 $d $bf $tf $tw $nfdw $nftw $nfbf $nftf

#column sections: W16x057
set d [expr 16.4*$in];     # depth
set bf [expr 7.12*$in];    # flange width
set tf [expr 0.715*$in];   # flange thickness
set tw [expr 0.43*$in];    # web thickness
#set nfdw 16;              # number of fibers along dw
#set nftw 2;               # number of fibers along tw
#set nfbf 16;              # number of fibers along bf
#set nftf 4;                       # number of fibers along tf
set lp16057 [expr $d]; # length of plastic hinge
set A16057 [expr 16.8*pow($in,2)]; # area of the section
set Iz16057 [expr 758*pow($in,4)]; # moment of inertia of the section
Wsection  $W16057 $mat16057 $d $bf $tf $tw $nfdw $nftw $nfbf $nftf

#column sections: W18x097
set d [expr 18.6*$in];     # depth
set bf [expr 11.1*$in];    # flange width
set tf [expr 0.87*$in];    # flange thickness
```

```
set tw [expr 0.535*$in]; # web thickness
#set nfdw 16;                # number of fibers along dw
#set nftw 2;                 # number of fibers along tw
#set nfbf 16;                # number of fibers along bf
#set nftf 4;                 # number of fibers along tf
set lp18097 [expr $d]; # length of plastic hinge
set A18097 [expr 28.5*pow($in,2)]; # area of the section
set Iz18097 [expr 1750*pow($in,4)]; # moment of inertia of the section
Wsection  $W18097 $mat18097 $d $bf $tf $tw $nfdw $nftw $nfbf $nftf




# define ELEMENTS
# set up geometric transformations of element
# separate columns and beams, in case of P-Delta analysis for columns
set IDColTransf 1; # all columns
set IDBeamTransf 2; # all beams
set ColTransfType PDelta ;                          # options, Linear PDelta Corotational
geomTransf $ColTransfType $IDColTransf  ;      # only columns can have PDelta effects (gravity effects)
geomTransf Linear $IDBeamTransf

# Define Beam-Column Elements
set np 5;          # number of Gauss integration points for nonlinear curvature distribution-- np=2 for
linear distribution ok

# columns


element beamWithHinges 1 16 12   $W14145    $lp14145   $W14145 $lp14145 $Es $A14145 $Iz14145
$IDColTransf
element beamWithHinges 2 17 13   $W14145    $lp14145   $W14145 $lp14145 $Es $A14145 $Iz14145
$IDColTransf
element beamWithHinges 3 18 14   $W14145    $lp14145   $W14145 $lp14145 $Es $A14145 $Iz14145
$IDColTransf
element beamWithHinges 4 19 15   $W14145    $lp14145   $W14145 $lp14145 $Es $A14145 $Iz14145
$IDColTransf
element beamWithHinges 5 35 31   $W14145    $lp14145   $W14145 $lp14145 $Es $A14145 $Iz14145
$IDColTransf
element beamWithHinges 6 36 32   $W14145    $lp14145   $W14145 $lp14145 $Es $A14145 $Iz14145
$IDColTransf
element beamWithHinges 7 37 33   $W14145    $lp14145   $W14145 $lp14145 $Es $A14145 $Iz14145
$IDColTransf
element beamWithHinges 8 38 34   $W14145    $lp14145   $W14145 $lp14145 $Es $A14145 $Iz14145
$IDColTransf

element beamWithHinges 9 54 50   $W14176    $lp14176   $W14176 $lp14176 $Es $A14176 $Iz14176
$IDColTransf
```

```
element beamWithHinges 10 55 51  $W14176   $lp14176   $W14176 $lp14176 $Es $A14176 $Iz14176
$IDColTransf
element beamWithHinges 11 56 52  $W14176   $lp14176   $W14176 $lp14176 $Es $A14176 $Iz14176
$IDColTransf
element beamWithHinges 12 57 53  $W14176   $lp14176   $W14176 $lp14176 $Es $A14176 $Iz14176
$IDColTransf
element beamWithHinges 13 73 69  $W14176   $lp14176   $W14176 $lp14176 $Es $A14176 $Iz14176
$IDColTransf
element beamWithHinges 14 74 70  $W14176   $lp14176   $W14176 $lp14176 $Es $A14176 $Iz14176
$IDColTransf
element beamWithHinges 15 75 71  $W14176   $lp14176   $W14176 $lp14176 $Es $A14176 $Iz14176
$IDColTransf
element beamWithHinges 16 76 72  $W14176   $lp14176   $W14176 $lp14176 $Es $A14176 $Iz14176
$IDColTransf
element beamWithHinges 17 92 88  $W14176   $lp14176   $W14176 $lp14176 $Es $A14176 $Iz14176
$IDColTransf
element beamWithHinges 18 93 89  $W14176   $lp14176   $W14176 $lp14176 $Es $A14176 $Iz14176
$IDColTransf
element beamWithHinges 19 94 90  $W14176   $lp14176   $W14176 $lp14176 $Es $A14176 $Iz14176
$IDColTransf
element beamWithHinges 20 95 91  $W14176   $lp14176   $W14176 $lp14176 $Es $A14176 $Iz14176
$IDColTransf


## beams

element beamWithHinges 21 2 101  $W16057 $lp16057 $W16057 $lp16057 $Es $A16057 $Iz16057
$IDBeamTransf
element beamWithHinges 22 101 3  $W16057 $lp16057 $W16057 $lp16057 $Es $A16057 $Iz16057
$IDBeamTransf
element beamWithHinges 23 5 102  $W16057 $lp16057 $W16057 $lp16057 $Es $A16057 $Iz16057
$IDBeamTransf
element beamWithHinges 24 102 6  $W16057 $lp16057 $W16057 $lp16057 $Es $A16057 $Iz16057
$IDBeamTransf
element beamWithHinges 25 8 103  $W16057 $lp16057 $W16057 $lp16057 $Es $A16057 $Iz16057
$IDBeamTransf
element beamWithHinges 26 103 9  $W16057 $lp16057 $W16057 $lp16057 $Es $A16057 $Iz16057
$IDBeamTransf
element beamWithHinges 27 21 104 $W16057 $lp16057 $W16057 $lp16057 $Es $A16057 $Iz16057
$IDBeamTransf
element beamWithHinges 28 104 22 $W16057 $lp16057 $W16057 $lp16057 $Es $A16057 $Iz16057
$IDBeamTransf
element beamWithHinges 29 24 105 $W16057 $lp16057 $W16057 $lp16057 $Es $A16057 $Iz16057
$IDBeamTransf
element beamWithHinges 30 105 25 $W16057 $lp16057 $W16057 $lp16057 $Es $A16057 $Iz16057
$IDBeamTransf
element beamWithHinges 31 27 106 $W16057 $lp16057 $W16057 $lp16057 $Es $A16057 $Iz16057
$IDBeamTransf
```

```
element beamWithHinges 32 106 28 $W16057 $lp16057 $W16057 $lp16057 $Es $A16057 $Iz16057
$IDBeamTransf


element beamWithHinges 33 40 107 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 34 107 41 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 35 43 108 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 36 108 44 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 37 46 109 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 38 109 47 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 301 59 110 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 302 110 60 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 303 62 111 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 304 111 63 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 305 65 112 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 306 112 66 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 307 78 113 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 308 113 79 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 309 81 114 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 310 114 82 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 311 84 115 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf
element beamWithHinges 312 115 85 $W18097 $lp18097 $W18097 $lp18097 $Es $A18097 $Iz18097
$IDBeamTransf


# Rigid End Zone


set RGDBeamTransf 4
set RGDColumnTransf 3
geomTransf Linear $RGDBeamTransf
```

```
geomTransf PDelta $RGDColumnTransf
set ARGD [expr 9000*pow($in,2)]; # area of inertia of the assumed rigid section
set IzRGD [expr 82300000*pow($in,4)]; # moment of inertia of the assumed rigid section


element elasticBeamColumn 39 1 2 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 40 3 4 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 41 4 5 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 42 6 7 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 43 7 8 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 44 9 10 $ARGD $Es $IzRGD $RGDBeamTransf

element elasticBeamColumn 45 1 12 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 46 4 13 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 47 7 14 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 48 10 15 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 49 16 20 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 50 17 23 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 51 18 26 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 52 19 29 $ARGD $Es $IzRGD $RGDColumnTransf

element elasticBeamColumn 53 20 21 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 54 22 23 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 55 23 24 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 56 25 26 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 57 26 27 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 58 28 29 $ARGD $Es $IzRGD $RGDBeamTransf

element elasticBeamColumn 59 20 31 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 60 23 32 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 61 26 33 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 62 29 34 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 63 35 39 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 64 36 42 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 65 37 45 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 66 38 48 $ARGD $Es $IzRGD $RGDColumnTransf

element elasticBeamColumn 67 39 40 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 68 41 42 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 69 42 43 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 70 44 45 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 71 45 46 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 72 47 48 $ARGD $Es $IzRGD $RGDBeamTransf

element elasticBeamColumn 73 39 50 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 74 42 51 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 75 45 52 $ARGD $Es $IzRGD $RGDColumnTransf
element elasticBeamColumn 76 48 53 $ARGD $Es $IzRGD $RGDColumnTransf
```

```
element elasticBeamColumn 77 54 58 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 78 55 61 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 79 56 64 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 80 57 67 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 81 58 59 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 82 60 61 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 83 61 62 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 84 63 64 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 85 64 65 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 86 66 67 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 87 58 69 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 88 61 70 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 89 64 71 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 90 67 72 $ARGD $Es $IzRGD $RGDBeamTransf


element elasticBeamColumn 91 73 77 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 92 74 80 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 93 75 83 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 94 76 86 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 95 77 78 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 96 79 80 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 97 80 81 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 98 82 83 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 99 83 84 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 100 85 86 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 101 77 88 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 102 80 89 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 103 83 90 $ARGD $Es $IzRGD $RGDBeamTransf
element elasticBeamColumn 104 86 91 $ARGD $Es $IzRGD $RGDBeamTransf



## Gravity Columns


set AGCol [expr 5100*pow($in,2)]
set IzGCol [expr 100*pow($in,4)]
set GravColumnTransfTag 5
geomTransf PDelta $GravColumnTransfTag

element elasticBeamColumn 105 96 100 $AGCol $Es $IzGCol $GravColumnTransfTag
element elasticBeamColumn 106 87 99 $AGCol $Es $IzGCol $GravColumnTransfTag
element elasticBeamColumn 107 68 98 $AGCol $Es $IzGCol $GravColumnTransfTag
element elasticBeamColumn 108 49 97 $AGCol $Es $IzGCol $GravColumnTransfTag
element elasticBeamColumn 109 30 11 $AGCol $Es $IzGCol $GravColumnTransfTag
```

```
# element elasticBeamColumn $eleTag $iNode $jNode $A $E $Iz $transfTag


## Links to gravity columns

element truss 401 10 11 [expr 5*pow($m,2)] $GravcolumnsmatTag
element truss 402 29 30 [expr 5*pow($m,2)] $GravcolumnsmatTag
element truss 403 48 49 [expr 5*pow($m,2)] $GravcolumnsmatTag
element truss 404 67 68 [expr 5*pow($m,2)] $GravcolumnsmatTag
element truss 405 86 87 [expr 5*pow($m,2)] $GravcolumnsmatTag
# element truss $eleTag $iNode $jNode $A $matTag


## Define GRAVITY LOADS, weight and masses

mass 102  [expr 2669*$kN/$m/$g] 1 0.0

mass 105  [expr 2669*$kN/$m/$g] 1 0.0

mass 108  [expr 2669*$kN/$m/$g] 1 0.0

mass 111  [expr 2669*$kN/$m/$g] 1 0.0

mass 114  [expr 2669*$kN/$m/$g] 1 0.0



# Define RECORDERS -------------------------------------------------------------

set FreeNodeID 1;                                    # ID: free node
#set SupportNodeFirst 129;                  # ID: first support node
#set SupportNodeLast 133;                   # ID: last support node
#set FirstColumn 21

# General Recorders
#recorder Node -file $DynamicDataDir/DFree.out -time -node $FreeNodeID  -dof 1 2 3 disp;
                        # displacements of free node
#recorder Node -file $DynamicDataDir/DBase.out -time -nodeRange $SupportNodeFirst
$SupportNodeLast -dof 1 2 3 disp;         # displacements of support nodes
#recorder Node -file $DynamicDataDir/RBase.out -time -nodeRange $SupportNodeFirst
$SupportNodeLast -dof 1 2 3 reaction;   # support reaction
#recorder Drift -file $DynamicDataDir/DrNode.out -time -iNode $SupportNodeFirst  -jNode
$FreeNodeID  -dof 1 -perpDirn 2;          # lateral drift
#recorder Element -file $DynamicDataDir/Fel1.out -time -ele $FirstColumn localForce;
                        # element forces in local coordinates
#recorder Element -file $DynamicDataDir/ForceEle1sec1.out -time -ele $FirstColumn section 1 force;
                        # section forces, axial and moment, node i
```

#recorder Element -file $DynamicDataDir/DefoEle21sec1.out -time -ele $FirstColumn section 1 deformation; # section deformations, axial and curvature, node i
#recorder Element -file $DynamicDataDir/ForceEle21sec$np.out -time -ele $FirstColumn section $np force; # section forces, axial and moment, node j
#recorder Element -file $DynamicDataDir/DefoEle21sec$np.out -time -ele $FirstColumn section $np deformation; # section deformations, axial and curvature, node j
#recorder Element -file $DynamicDataDir/SSEle1sec21.out -time -ele $FirstColumn section $np fiber 0 0 $matIDhard stressStrain; # steel fiber stress-strain, node i
#
#recorder Element -file $DynamicDataDir/ForceEle23sec1.out -time -ele 23 section 1 force; # section forces, axial and moment, node i
#recorder Element -file $DynamicDataDir/DefoEle23sec1.out -time -ele 23 section 1 deformation; # section deformations, axial and curvature, node i
#
#recorder Node -file $DynamicDataDir/RN48.out -time -node 48 -dof 1 2 3 reaction; # support reaction
#
#recorder Element -file $DynamicDataDir/ForceEle57sec1.out -time -ele 57 section 1 force; # section forces, axial and moment, node i
#recorder Element -file $DynamicDataDir/DefoEle57sec1.out -time -ele 57 section 1 deformation; # section deformations, axial and curvature, node i

Recorder of Interstory Drifts

Recorder for Calculation of Energy and Plastic Hinges

#recorder Node -file DispStories.out -time -node $6thstorynode $5thstorynode $4thstorynode $3rdstorynode $2ndstorynode $1ststorynode $basenode -dof 1 disp;
#recorder Node -file VelStories.out -time -node $6thstorynode $5thstorynode $4thstorynode $3rdstorynode $2ndstorynode $1ststorynode $basenode -dof 1 vel;
#recorder Node -file RelACCStories.out -time -node $6thstorynode $5thstorynode $4thstorynode $3rdstorynode $2ndstorynode $1ststorynode $basenode -dof 1 accel;

Recorder Needed for Pushover
#recorder Node -file $DynamicDataDir/ReactionBase.out -time -nodeRange $SupportNodeFirst $SupportNodeLast -dof 1 reaction;

define GRAVITY ---
GRAVITY LOADS

pattern Plain 101 Linear {

  load 101 0 [expr -160*$kN] 0
  load 102 0 [expr -160*$kN] 0

```
   load 103  0 [expr -160*$kN] 0
   load 104  0 [expr -160*$kN] 0
   load 105  0 [expr -160*$kN] 0
   load 106  0 [expr -160*$kN] 0
   load 107  0 [expr -160*$kN] 0
   load 108  0 [expr -160*$kN] 0
   load 109  0 [expr -160*$kN] 0
   load 110  0 [expr -160*$kN] 0
   load 111 0 [expr -160*$kN] 0
   load 112  0 [expr -160*$kN] 0
   load 113  0 [expr -160*$kN] 0
   load 114  0 [expr -160*$kN] 0
   load 115  0 [expr -160*$kN] 0


   load 11 0 [expr -2669*$kN] 0
   load 30  0 [expr -2669*$kN] 0
   load 49  0 [expr -2669*$kN] 0
   load 68  0 [expr -2669*$kN] 0
   load 87  0 [expr -2669*$kN] 0


}

# Gravity-analysis parameters -- load-controlled static analysis
set Tol 1.0e-8;                        # convergence tolerance for test
variable constraintsTypeGravity Plain;             # default;
if {  [info exists RigidDiaphragm] == 1} {
        if {$RigidDiaphragm=="ON"} {
                variable constraintsTypeGravity Lagrange;        #  large model: try Transformation
        };        # if rigid diaphragm is on
};        # if rigid diaphragm exists
constraints $constraintsTypeGravity ;              # how it handles boundary conditions
numberer RCM;                              # renumber dof's to minimize band-width (optimization), if you
want to
system BandGeneral ;              # how to store and solve the system of equations in the analysis (large
model: try UmfPack)
test NormDispIncr $Tol 6 ;                 # determine if convergence has been achieved at the end of an
iteration step
algorithm Newton;                          # use Newton's solution algorithm: updates tangent stiffness at
every iteration
set NstepGravity 10;              # apply gravity in 10 steps
set DGravity [expr 1./$NstepGravity];    # first load increment;
integrator LoadControl $DGravity;        # determine the next time step for an analysis
analysis Static;                  # define type of analysis static or transient
analyze $NstepGravity;            # apply gravity

# ------------------------------------------------ maintain constant gravity loads and reset time to zero
loadConst -time 0.0
```

```
# Activate this section if viscous dampers are used (linear and nonlinear)

set FifthStory 12
set FourthStory 13
set ThirdStory 14
set SecondStory 15
set FirstStory 16

set A5 [expr $Area5*pow($in,2)]
set A4 [expr $Area4*pow($in,2)]
set A3 [expr $Area3*pow($in,2)]
set A2 [expr $Area2*pow($in,2)]
set A1 [expr $Area1*pow($in,2)]


set C5 [expr $C_NL5*$kN*$sec*5281.4/$A5]
set C4 [expr $C_NL4*$kN*$sec*5281.4/$A4]
set C3 [expr $C_NL3*$kN*$sec*5281.4/$A3]
set C2 [expr $C_NL2*$kN*$sec*5281.4/$A2]
set C1 [expr $C_NL1*$kN*$sec*3791.7418/$A1]
set alphaC $alphaC_NL

uniaxialMaterial Viscous $FifthStory $C5 $alphaC
uniaxialMaterial Viscous $FourthStory $C4 $alphaC
uniaxialMaterial Viscous $ThirdStory $C3 $alphaC
uniaxialMaterial Viscous $SecondStory $C2 $alphaC
uniaxialMaterial Viscous $FirstStory $C1 $alphaC
#uniaxialMaterial Viscous $matTag $C $alpha

element truss  701 93 114  $A1 $ FirstStory
element truss  702 94 114  $A1 $ FirstStory
element truss  703 80 111  $A2 $ SecondStory
element truss  704 83 111  $A2 $ SecondStory
element truss  705 61 108  $A3 $ ThirdStory
element truss  706 64 108  $A3 $ ThirdStory
element truss  707 42 105  $A4 $ FourthStory
element truss  708 45 105  $A4 $ FourthStory
element truss  709 23 102  $A5 $ FifthStory
element truss  710 26 102  $A5 $ FifthStory

# Activate this section if hysteretic dampers are used

set Costet 0.64
set Atot [expr 0.0613*pow($m,2)];
set Fytot [expr 1994*$kN/$Costet/2]

set A1 [expr 0.2*$Atot]
set A2 [expr 0.2*$Atot]
set A3 [expr 0.2*$Atot]
```

```
set A4 [expr 0.2*$Atot]
set A5 [expr 0.2*$Atot]

set Fy1 [expr 0.27*$Fytot/$A1]
set Fy2 [expr 0.25*$Fytot/$A2]
set Fy3 [expr 0.22*$Fytot/$A3]
set Fy4 [expr 0.16*$Fytot/$A4]
set Fy5 [expr 0.10*$Fytot/$A5]

set A1 [expr 0.2*$Atot]
set A2 [expr 0.2*$Atot]
set A3 [expr 0.2*$Atot]
set A4 [expr 0.2*$Atot]
set A5 [expr 0.2*$Atot]

set HD1 301
set HD2 302
set HD3 303
set HD4 304
set HD5 305

uniaxialMaterial Steel01 $HD1 $Fy1 $Es 0
uniaxialMaterial Steel01 $HD2 $Fy2 $Es 0
uniaxialMaterial Steel01 $HD3 $Fy3 $Es 0
uniaxialMaterial Steel01 $HD4 $Fy4 $Es 0
uniaxialMaterial Steel01 $HD5 $Fy5 $Es 0

element truss  701 93 114  $A1 $HD1
element truss  702 94 114  $A1 $HD1
element truss  703 80 111  $A2 $HD2
element truss  704 83 111  $A2 $HD2
element truss  705 61 108  $A3 $HD3
element truss  706 64 108  $A3 $HD3
element truss  707 42 105  $A4 $HD4
element truss  708 45 105  $A4 $HD4
element truss  709 23 102  $A5 $HD5
element truss  710 26 102  $A5 $HD5


# Activate this section for Pushover Analysis

## LATERAL-LOAD distribution for static pushover analysis
## create node and load vectors for lateral-load distribution in static analysis
set iFPush ""
set iNodePush ""
lappend iNodePush 77
lappend iNodePush 58
lappend iNodePush 39
lappend iNodePush 20
```

```
lappend iNodePush 1

#
## Mode 1

lappend iFPush [expr 25*$kN*0.06]
lappend iFPush [expr 25*$kN*0.14]
lappend iFPush [expr 25*$kN*0.21]
lappend iFPush [expr 25*$kN*0.27]
lappend iFPush [expr 25*$kN*0.32]
recorder Node -file ReactionBase_1.out -time -nodeRange 92 96 -dof 1 reaction;
recorder Node -file Pushdisp_1.out -time -node 77 58 39 20 1 -dof 1 disp;
        # displacements of free node
set HBuilding [expr (5*$h2)];    # total building height

## Recorder Needed for Pushover


# calculated MODEL PARAMETERS,
set IDctrlNode 1;                # node where displacement is read for displacement control
set IDctrlDOF 1;                 # degree of freedom of displacement read for displacement control



source PushoverAnalysis.tcl

recorder Drift -file DriftStories_5_10.out -time -iNode 1 20 39 58 77 -jNode 20 39 58 77 92 -dof 1 -
perpDirn 2;      # lateral drift


puts "Model Built"


# Activate this section for nonlinear response history dynamic analysis


# ------------------------------------------------------------------------------
#
# Dynamic Analysis Procedure, Under Uniform Ground Acceleration
#
# ------------------------------------------------------------------------------
#

# source in procedures
source ReadSMDFile.tcl;                   # procedure for reading GM file and converting it to proper
format

# Uniform Earthquake ground motion (uniform acceleration input at all support nodes)
set GMdirection 1;                                # ground-motion direction
```

```
set GMfile "1.AT2" ;                          # ground-motion filenames
set GMfact $Fact;                                         # ground-motion scaling factor

# set up ground-motion-analysis parameters

set DtAnalysis   [expr 0.005*$sec];        # time-step Dt for lateral analysis
set TmaxAnalysis          [expr 120*$sec];           # maximum duration of ground-motion analysis

# ----------- set up analysis parameters
#source LibAnalysisDynamicParameters.tcl;       # constraintsHandler,DOFnumberer,system-
ofequations,convergenceTest,solutionAlgorithm,integrator

# ------------ define & apply damping
# RAYLEIGH damping parameters, Where to put M/K-prop damping, switches
(http://opensees.berkeley.edu/OpenSees/manuals/usermanual/1099.htm)
#         D=$alphaM*M + $betaKcurr*Kcurrent + $betaKcomm*KlastCommit + $beatKinit*$Kinitial
set xDamp 0.05;                                                 # damping ratio
set MpropSwitch 1.0;
set KcurrSwitch 1.0;
set KcommSwitch 0.0;
set KinitSwitch 1.0;
set nEigenI 1;              # mode 1
set nEigenJ 3;              # mode 3
set lambdaN [eigen [expr $nEigenJ]];                      # eigenvalue analysis for nEigenJ modes
set lambdaI [lindex $lambdaN [expr $nEigenI-1]];                  # eigenvalue mode i
set lambdaJ [lindex $lambdaN [expr $nEigenJ-1]];         # eigenvalue mode j
set omegaI [expr pow($lambdaI,0.5)];
set omegaJ [expr pow($lambdaJ,0.5)];
set alphaM [expr $MpropSwitch*$xDamp*(2*$omegaI*$omegaJ)/($omegaI+$omegaJ)];# M-prop.
damping; D = alphaM*M
set betaKcurr [expr $KcurrSwitch*2.*$xDamp/($omegaI+$omegaJ)];                    # current-K;
+beatKcurr*KCurrent
set betaKcomm [expr $KcommSwitch*2.*$xDamp/($omegaI+$omegaJ)];                # last-
committed K;   +betaKcomm*KlastCommitt
set betaKinit [expr $KinitSwitch*2.*$xDamp/($omegaI+$omegaJ)];                          #
initial-K;     +beatKinit*Kini

# define damping
rayleigh $alphaM $betaKcurr $betaKinit $betaKcomm;                              # RAYLEIGH damping

#  ---------------------------------    perform Dynamic Ground-Motion Analysis
# the following commands are unique to the Uniform Earthquake excitation
set IDloadTag 400;        # for uniformSupport excitation
# Uniform EXCITATION: acceleration input
set outFile $GMfile.g3;  # set variable holding new filename (PEER files have .at2/dt2 extension)
set GMfatt [expr $g*$GMfact];              # data in input file is in g Unifts -- ACCELERATION TH

set AccelSeries "Series -dt 0.01 -filePath $outFile -factor  $GMfatt";       # time series information
```

```
pattern UniformExcitation  $IDloadTag  $GMdirection -accel  $AccelSeries  ;                 # create
Unifform excitation


set dt_anal_Step $DtAnalysis
set GMtime $TmaxAnalysis
set numStories 6


set FloorNodes {77 58 39 20 1}
set h1 [expr 3657*$mm]

set htyp [expr 3657*$mm]
set DriftLimit 0.15

global CollapseFlag;                                   # global variable to monitor collapse
global Diverge;                                  # global variable to monitor collapse
global ok
set k 1;
source MaxDriftTester.tcl;                              # For Collapse Studies
set CollapseFlag "NO"
set Diverge "NO"
wipeAnalysis
constraints Plain
numberer RCM
system UmfPack
test EnergyIncr 1.0e-2 100
algorithm Newton
integrator Newmark 0.50 0.25
analysis Transient
set dt_analysis $dt_anal_Step;                                            # timestep of analysis
set NumSteps [expr round(($GMtime + 0.0)/$dt_analysis)];         # number of steps in analysis
set ok [analyze $NumSteps $dt_analysis];
# Check Max Drifts for Collapse by Monitoring the CollapseFlag Variable
MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
if  {$CollapseFlag == "YES"} {
	set ok 0
}
#set dt_analysis [expr $dt_analysis/10];                                         # timestep of analysis

 if {$ok != 0} {
	puts "Analysis did not converge..."
	set TmaxAnalysis $GMtime;
	# The analysis will be time-controlled and is done for the remaining time
	set ok 0;
	set controlTime [getTime];

	while {$controlTime < [expr round(1.0*$TmaxAnalysis)] || $ok !=0 } {
		MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
```

```
        if  {$CollapseFlag == "YES"} {
                set ok 0; break;
        } else {
                set ok 1
        }
# Get Control Time inside the loop
        set controlTime [getTime];
        set startTime [getTime];
        if {$ok != 0} {
                puts "!!!!!Run Newton with 1/2 of step.."
                test EnergyIncr 1.0e-3 100   0
                set controlTime [getTime]
                set remainTime [expr $TmaxAnalysis - $controlTime]
            puts $remainTime
                algorithm Newton
                integrator Newmark 0.50 0.25
                set ok [analyze 100 [expr $dt_analysis/2.0]]
                MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
                if  {$CollapseFlag == "YES"} {
                        set ok 0
                }
        }
                if {$ok != 0} {
                puts "!!!!!Run Newton with 1/10 of step.."
                test EnergyIncr 1.0e-3 100   0
                set controlTime [getTime]
                set remainTime [expr $TmaxAnalysis - $controlTime]
            puts $remainTime
                algorithm Newton
                integrator Newmark 0.50 0.25
                set ok [analyze 100 [expr $dt_analysis/10.0]]
                MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
                if  {$CollapseFlag == "YES"} {
                        set ok 0
                }
        }
                if {$ok != 0} {
                puts "!!!!!Run Newton with 1/100 of step.."
                test EnergyIncr 1.0e-3 100   0
                set controlTime [getTime]
                set remainTime [expr $TmaxAnalysis - $controlTime]
            puts $remainTime
                algorithm Newton
                integrator Newmark 0.50 0.25
                set ok [analyze 100 [expr $dt_analysis/100.0]]
                MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
                if  {$CollapseFlag == "YES"} {
                        set ok 0
                }
```

```
		}



		if {$ok != 0 } {
			puts "!!!!!Run Newton with Initial Tangent with 1/10 of original step.."
			test EnergyIncr 1.0e-2 100 0
			set controlTime [getTime]
			set remainTime [expr $TmaxAnalysis - $controlTime]
			algorithm Newton -initial
			set ok [analyze 100 [expr $dt_analysis/10.0]]
			MaxDriftTester $numStories $DriftLimit $FloorNodes  $h1 $htyp
			if  {$CollapseFlag == "YES"} {
				set ok 0
			}
		}

		if {$ok != 0 } {
			puts "!!!!!Newton with line Search and 1/10 of the original step .."
			test EnergyIncr 1.0e-2 100 0
			algorithm NewtonLineSearch 0.50
			set ok [analyze 100 [expr $dt_analysis/10.0]]
			MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
			if  {$CollapseFlag == "YES"} {
				set ok 0
			}
		}

		if {$ok != 0 } {
			puts "!!!!!Newton Initial with 1/10 of step and Displacement Control 
Convergence.."
			test NormDispIncr 1.0e-2 100  0
			algorithm Newton -initial
			set ok [analyze 100 [expr $dt_analysis/10.0]]
			MaxDriftTester $numStories $DriftLimit $FloorNodes  $h1 $htyp
			if  {$CollapseFlag == "YES"} {
				set ok 0
			}
		}

		if {$ok != 0 } {
			puts "!!!!!Newton Initial with 1/10 of step energy Control Convergence and 
HTTP Inegrator.."
			test EnergyIncr 1.0e-2 100  0
			algorithm Newton -initial
			integrator HHTHSIncrReduct 0.5 0.95
			set ok [analyze 100 [expr $dt_analysis/10.0]]
			MaxDriftTester $numStories $DriftLimit $FloorNodes  $h1 $htyp
			if  {$CollapseFlag == "YES"} {
```

```
                    set ok 0
                }
            }
            if {$ok != 0 } {
                puts "Go Back to Newton with tangent Tangent and original step.."
                test EnergyIncr 1.0e-2 50   0
                set controlTime [getTime]
                set remainTime [expr $TmaxAnalysis - $controlTime]
                set NewRemainSteps [expr round(($remainTime)/($dt_analysis))]
                algorithm Newton
                integrator Newmark 0.50 0.25
                set ok [analyze $NewRemainSteps [expr $dt_analysis]]
                MaxDriftTester $numStories $DriftLimit $FloorNodes  $h1 $htyp
                if  {$CollapseFlag == "YES"} {
                    set ok 0
                }
            }

            if {$ok != 0 } {
                puts "Newton with 1/1000 of step and do only 1 step.."
                test NormDispIncr 1.0e-1 100  0
                set controlTime [getTime]
                set remainTime [expr $TmaxAnalysis - $controlTime]
                set NewRemainSteps [expr round(($remainTime)/($dt_analysis/1000))]
                algorithm Newton -initial
                set ok [analyze 10 [expr $dt_analysis/1000.0]]
                MaxDriftTester $numStories $DriftLimit $FloorNodes $h1 $htyp
                if  {$CollapseFlag == "YES"} {
                    set ok 0
                }
            }
            set controlTime [getTime]
            set endTime [getTime];
            set passTime [expr $endTime-$startTime];
            puts "PassTime is equal to $passTime"
            if { $passTime < 0.0000001 } {
            set k [expr $k+1];
            puts " K is equal to $k"
            } else {
            set k 1;
            }

            if {$k > 20} {
            set CollapseFlag "YES"
            set Diverge "YES"
            set ok 0; break;
            }
            }
}
```

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