Fluid Flow in Rivers

Outline
1. Flow uniformity and steadiness
2. Newtonian fluids
3. Laminar and turbulent flow
4. Mixing-length concept
5. Turbulent boundary layer
6. Mean boundary shear stress
7. Velocity distribution and the “Law of the Wall”
8. Depth-averaged velocity
Flow in Rivers

- Rivers are **nonuniform, unsteady, Newtonian, hydraulically rough turbulent flows**
Open Channel Flow

Uniform Flow: \[
\frac{\partial (U \cdot A)}{\partial x} = 0
\]

Nonuniform Flow: \[
\frac{\partial (U \cdot A)}{\partial x} \neq 0
\]

Fig. 2.1 Definition of uniform and non-uniform flow.

- $Q$ – discharge
- $w$ – width
- $d$ – depth
- $v, U$ – velocity
- $A$ – area
- $x$ – downstream distance

$Q = w \times d \times v$
Open Channel Flow

Steady Flow: \[ \frac{\partial(U \cdot A)}{\partial t} = 0 \]

Unsteady Flow: \[ \frac{\partial(U \cdot A)}{\partial t} \neq 0 \]

\( t \) – time

Definitions of terms used to describe hyetographs and response hydrographs. See Table 9-1.
Fig. 2.5 Definition of fluid viscosity. Fluid adjacent to the upper, moving plate is moving at the same speed as the plate, $\delta u$. Fluid adjacent to the stationary plate is not moving. Therefore, shear stresses, $\tau$, exist within the fluid because of its viscosity.

(Bridge, 2003)
Types of fluids

Molecular viscosity is independent of the magnitude of shear once yield strength is exceeded (mud and lava flows)

Molecular viscosity is dependent on shear and it has a yield strength (paint)

Molecular viscosity is independent of the magnitude of shear (rivers)

Fig. 2.6 Common types of fluid defined by their rheology.
Laminar flow, Re < 2000 (viscous forces dominate)

Turbulent flow, Re > 4000 (turbulent forces dominate)

Re = \frac{turbulent \ forces}{viscous \ forces} = \frac{U\rho}{\nu}

Large masses of fluid “eddies” are being transported

ρ - fluid density
ν – kinematic fluid viscosity
Re – Boundary Reynolds number
Laminar and Turbulent Flow

- For 1 m deep flow:
  \[ \text{Re} = 300, \ U \sim 0.0003 \ \text{m/s} \]
  \[ \text{Re} = 3000, \ U \sim 0.003 \ \text{m/s} \]
- Susquehanna River near Waverly, PA
  \[ d \sim 2 \ \text{m}, \ U \sim 1 \ \text{m/s}, \ \text{Re} \sim 2,000,000 \]

\[
\text{Re} = \frac{\text{turbulent forces}}{\text{viscous forces}} = \frac{Ud\rho}{\nu}
\]
Turbulent velocity time series

Fig. 2.7 Definition of turbulent fluctuation in flow velocity.

- $u'$ – instantaneous fluctuation in $u$ velocity
- Overbar – timed average velocity
- $T,t$ – total time, time

(Bridge, 2003)
Turbulent Velocities

\[
\bar{u} = \frac{1}{T} \int_0^T u dt = \frac{1}{n} \sum_{i=1}^{n} u_i
\]

\[u' = u_i - \bar{u}\]

\[\bar{u}' = \frac{1}{n} \sum_{i=1}^{n} (u_i - \bar{u}) = 0\]

\[u_{rms} = \sqrt{\bar{u}'} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (u_i - \bar{u})^2} \neq 0\]

\(u_i\) – instantaneous velocity
\(n\) – number of measurements
\(rms\) – root-mean-square
Mixing length concept

$l$: the vertical distance over which the momentum of a fluid parcel is changed (mixing length)

Reynolds Stress

$$\tau = -\rho u'v' = \rho l^2 \left( \frac{\partial u}{\partial y} \right)^2$$

Fig. 2.8 Definition of turbulent motions in a boundary layer.

(Bridge, 2003)
Turbulent boundary layer

U* – shear velocity
y – height above the boundary

fully turbulent

OUTER ZONE

INNER ZONE

buffer layer
(turbulence generation region)

viscous sublayer

Lower part of turbulent boundary layer

(Bridge, 2003)
Viscous sublayer streaks

Vortex structure within viscous sublayer

(Bridge, 2003)
Turbulent “bursting” process

~70% of all turbulence in open channel flows is due to the bursting process

(Allen, 1985; Bridge, 2003)
Grain roughness effects on turbulent boundary layer

Figure 6.2. Hydraulically smooth and rough boundaries

\[ \text{Re}_g = \frac{U_* D}{v} \]

\text{Re}_g \text{ – Grain Reynolds number}
D – grain size

(Julien, 1998)
Large-scale Turbulent Motions

(Falco, 1977)

(Nakagawa & Nezu, 1981)

(Ferguson et al., 1996)

(Belanger et al., 2000)
Boundary Shear Stress

\[ \tau_0 = \rho g dS \]
\[ \tau = \tau_0 (1 - y/d) \]

Conservation of downstream momentum: Impelling force (downstream component of weight of water) = resistive force

Parameters
- Flow depths: \(d_{\text{up}}, d_{\text{down}}\)
- Cross sectional areas of flow: \(a_{\text{up}}, a_{\text{down}}\)
- Mean hydrostatic pressures: \(P_{\text{up}}, P_{\text{down}}\)
- Boundary friction: \(F = \tau_0 a\)
- Bed shear stress: \(\tau_0\)
- Cross-sectional area of bed: \(a\)
- Gravity force: \(G\)
- Bed slope: \(S = \sin \alpha\)

Uniform flow
- \(d_{\text{up}} = d_{\text{down}}\)
- \(a_{\text{up}} = a_{\text{down}}\)
- \(P_{\text{up}} = P_{\text{down}}\)
- \(U_{\text{up}} = U_{\text{down}}\)
- \(F = G \sin \alpha\)

Non-uniform flow
- \(d_{\text{up}} \neq d_{\text{down}}\)
- \(a_{\text{up}} \neq a_{\text{down}}\)
- \(P_{\text{up}} \neq P_{\text{down}}\)
- \(U_{\text{up}} \neq U_{\text{down}}\)
- \(F \neq G \sin \alpha\)

Fig. 2.13 Time- and depth-averaged forces in straight uniform and non-uniform flows.

(Bridge, 2003)
“Law of the Wall”

\[
\frac{\partial u}{\partial y} = \frac{u_*}{\nu y}; \quad \frac{u}{\nu} = \frac{1}{\kappa} \ln \left( \frac{y}{y_0} \right)
\]

\[
u = \sqrt{\frac{\tau_0}{\rho}}
\]

\[
|u'| \approx |w'| \approx L \frac{du}{dy}
\]

\[
\frac{\partial \tau}{\partial y} = \frac{\partial}{\partial y} \left( \frac{\tau_{\text{vis}} - \rho u'w'}{\nu} \right) = \frac{\partial p}{\partial x} - \gamma \sin \alpha
\]

- Derived from Reynolds-averaged Navier-Stokes equations (Gomez, 2006)
  - Nearly universal use
- Important assumptions:
  - Prandtl’s mixing length theory
  - Zero slope (?!)
  - Constant shear stress (no vertical gradient; ?!)

\[\kappa\] – von Karman’s coefficient
\[\gamma\] – unit weight of water
\[p\] – pressure
\[L\] – mixing length

(Taylor, 1921; Prandtl, 1925, 1926)
Utility of the “Law of the Wall”

Application of the "Law of the wall"

\[
\frac{u}{u_*} = \frac{1}{\kappa} \ln \left( \frac{y}{y_0} \right)
\]

From regression data:

\[ u_* = \text{slope} \times \kappa \]

\[
\ln(y_0) = -\frac{\text{intercept}}{\text{slope}}
\]

\[ \tau_0 = \rho u_*^2 \]

\[ y_0 = \text{zero - velocity roughness height} \]

\[ k_s \approx 30.2 y_0; \text{equivalent sand roughness height} \]
For a mobile upper stage plane bed (mm-scale bedwaves), the boundary is essentially flat. Thus, the mean shear stress determined using a Reynolds stress projection, the law of the wall, and the depth-slope product (relative water surface elevation; WSE) are nearly identical (from Bennett et al., 1998).
Reynolds Stress vs. Velocity Gradient Shear Stress over a Fixed Dune

Near-bed Reynolds stress

Law of the wall

Dune Profile

Crest

Trough

Distance from Reattachment (m)
Depth Averaged Velocity

Depth-integrated flow velocity:

General form: \[ U = \frac{1}{d} \int_{y=0}^{d} u \, dy \]

Specific form:

\[ \Delta y_i = y_i - y_{i-1} \]
\[ \bar{u}_i = (u_i + u_{i-1})/2 \]

\[ U = \frac{1}{d} \sum_{i=1}^{n+1} \bar{u}_i \times \Delta y_i \]

where: \( y_0, u_0 = 0; y_{n+1} = d; u_{n+1} = u_n \)

\( n \) is the number of measurements
Conclusions

1. River flow is unsteady, non-uniform, turbulent, and hydraulically rough
2. River flow can be treated as a boundary layer, and its distribution of velocity can be determined
3. Turbulence is derived from bursting process
4. Bed shear stress can be determined from bulk hydraulic parameters and velocity profiles