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JERK LIMITED TIME OPTIMAL CONTROL OF FLEXIBLE STRUCTURES

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ABSTRACT

This paper deals with the design of jerk-limited time-optimal control sequences for rest-to-rest maneuvers of flexible structures. The resulting jerk profiles will be either bang-bang or bang-off-bang. To ensure quiescent states at the end of the maneuver, a pole cancellation technique will be used. Further constraints account for the geometric boundary conditions. This paper will also investigate the development of the control profile upon variation of the maximum allowable amount of jerk. The last section presents numerical results. The proposed control algorithm is implemented for the Floating Oscillator benchmark problem.

INTRODUCTION

Many researchers have studied the problem of vibration control for slewing flexible structures. Areas of application are as diverse as maneuvering of large space structures (Junkins et al., 1990), flexible arm robots (Ballhaus et al., 1992), computer disk drives (Bhat et al., 1991), and cranes (Singhose et al., 1997). The objective of the time-optimal control problem is to minimize maneuver time while counteracting residual vibration. Further constraints have been added to the control problem formulation such as limits on fuel consumed, maximum deformation permitted, and robustness to modelling uncertainties.

Research in the area of input prefiltering was sparked by Smith's Posicast Control (Smith, 1957). The problem of sensitivity of the Posicast controllers to uncertainties in damping and natural frequency was addressed by Singer and Seering (Singer and Seering, 1990). Singh and Vadali (Singh and Vadali, 1993) used a frequency domain approach to design input prefilters. They showed that a time-delay prefilter which cancels the underdamped poles of the system results in the same control profile as the input shaped controller. An increase in robustness to modeling errors can be achieved by using a series of time-delay filters.

A variety of other methods has been proposed, targeting parametric uncertainties. One approach is to force the derivative of the state sensitivity curve to zero at the nominal points. Another approach is to maximize the uncertain space in which a certain measure of residual vibration is below a pre-specified threshold (Singhose et al., 1995). Recently, Singh (Singh, 2001) proposed a minimax formulation designing a time-delay filter desensitizing the controller with respect to modelling uncertainties.

Jerk is a measure for the time-rate of change of the inertia forces. A change in the inertia forces excites the vibratory modes of the system. Restricting the jerk will therefore result in less excitation of the vibratory modes. Bhat and Miu (Bhat et al., 1991) propose a design method which is based on the minimization of the time integral of the square of the absolute magnitude of jerk. Hindle and Singh (Hindle and Singh, 2000) formulated a control design method which is based on the minimization of a weighted combination of the jerk and the power consumed. They also proposed using the state sensitivity equations to arrive at control profiles which are insensitive to modeling errors. Both of these approaches do not account for finite limits on the control or the jerk.

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This paper is concerned with the problem of designing time-optimal control profiles with constraints on the maximum magnitude and the time rate of change of the control input. This method is based on a pole cancellation approach. A time delay filter augmented by an integrator is used to generate the control profiles, which consist of ramping and coasting periods.

First, the control problem is formulated. Next, a technique for the parameterization of the control profiles is described. The proposed technique is applied to a two-mass/spring system. It will be shown that the control profile has to be point symmetric about the mid-maneuver time for all undamped systems. The spectrum of control profiles includes regions, where the control profile goes into saturation and other regions, where the control profile never reaches the saturation limit. For the former case, the jerk profile is bang-off-bang, whereas for the latter case, the jerk profile is strictly bang-bang and time-optimality can be proven using techniques proposed in the literature (Liu and Wie, 1992; Singh and Vadali, 1994).

The paper concludes with a section on numerical examples, where these techniques are employed for the design of control sequences for the Floating Oscillator benchmark problem. Numerical values for the parameters are given for all described control profiles. Time-optimality of the non-saturating control profiles has been proven using Pontryagin's principle.

PROBLEM FORMULATION

This paper discusses the time optimal control of flexible structures with constraints on both the magnitude as well as the time rate of change of the control input.

The behavior of flexible structures is governed by the set of differential equations

$$M\ddot{y}(t) + Ky(t) = Du(t). \tag{1}$$

M and K denote the mass and stiffness matrices. This paper focuses on time optimal control sequences, i. e. control sequences which minimize the cost function

$$F = \frac{1}{2}T_{final}^2, \tag{2}$$

where T_{final} designates the maneuver time. Constraints on the the control amplitude,

$$-1 \leq u(t) \leq 1 \;\forall\, t \in [0, T_{final}], \tag{3}$$

and the maximum allowable amount of jerk,

$$-J \leq \dot{u}(t) \leq J \;\forall\, t \in [0, T_{final}], \tag{4}$$

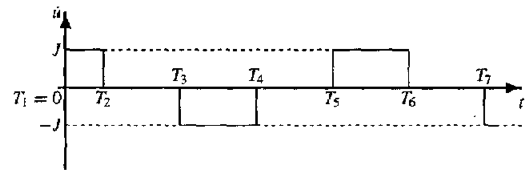

Figure 1. PARAMETERIZATION OF JERK PROFILE

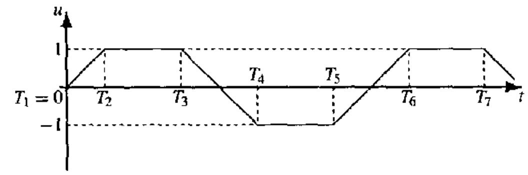

Figure 2. CONTROL PROFILE CORRESPONDING TO JERK PROFILE SHOWN IN FIG. 1

are imposed. In these constraint equations, J denotes the magnitude of permissible jerk. The boundary conditions for a rest-to-rest maneuver of unit displacement are given as

$$\begin{array}{ll} y(0) = 0 & y(T_{final}) = 1 \\ \dot{y}(0) = 0 & \dot{y}(T_{final}) = 0 \\ \ddot{y}(0) = 0 & \ddot{y}(T_{final}) = 0. \end{array} \tag{5}$$

PARAMETERIZATION OF JERK PROFILES

The optimal jerk-constrained control profile will consist of ramping periods and intervals, where the control profile is in saturation. This kind of profile can be realized by the summation of a number of time-delayed ramp functions with different slopes. Time optimality requires that $u(t)$ is in saturation for as much time as possible during the maneuver. It is assumed that a symmetric actuator is used, i. e. that the upper and lower limits of saturation are of the same magnitude, $u_{max} = -u_{min} = 1$. As was already mentioned in the problem formulation, the time rate of change of the control is limited by $|\dot{u}(t)| \leq J$. The parameterization of the jerk profile is shown in Fig. 1, resulting in a control profile as graphed in Fig. 2.

$\dot{u}(t)$ will be written as

$$\dot{u}(t) = J\left(\sum_{i=1}^{n} A_i \mathcal{H}(t - T_i)\right), \tag{6}$$

where $\mathcal{H}$ denotes the Heavyside function and A_i is restricted to

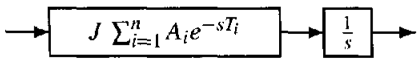

Figure 3. TIME DELAY FILTER STRUCTURE

be

$$A_i \in \{-2,\ -1,\ 1,\ 2\}\ \forall\, i. \tag{7}$$

For the first switch, $i = 1$, $A_1 = 1$ and $T_1 = 0$. Similarly, for the last switch, $i = n$, $A_n \in \{-1,\ 1\}$ and $T_n = T_{final}$.

The output of the time delay filter is integrated over time, thus the input $u(t)$ will consist of a sum of time-delayed ramp signals,

$$u(t) = J\left(\sum_{i=1}^{n} A_i \langle t - T_i\rangle\right), \tag{8}$$

where $\langle x\rangle = x\mathcal{H}(x)$. This profile can be realized by means of a controller structure as shown in Fig. 3, which is composed of a time-delay filter and an integrator.

The structure of the jerk profiles will now be investigated. Constraint equations will be derived which permit solving for the control profile parameters. The whole development is carried out for undamped systems, but can easily be applied to damped systems as well (Muenchhof, 2000).

Singh and Vadali (Singh and Vadali, 1993) proposed a technique for the time-optimal control of flexible structures, which requires cancellation of the poles of the system by the zeros of the time delay filter. In the following, this technique is applied to the problem of designing jerk limited control sequences for the Floating Oscillator problem. As was mentioned previously, the time delay filter is acting upon an integrator, which will now be treated as being part of the plant. Thus, the plant has an undamped oscillatory pair of poles and a total of three poles at the origin of the complex plane, two stemming from the rigid body mode dynamics and one from the integrator.

Constraint equations must be formulated which ensure that a given number of poles is placed at certain points in the s-domain. These constraints can be formulated by exploiting the fact that a transfer function has at least n zeros at s_0, if the function evaluated at s_0 and its first $n-1$ derivatives with respect to s, evaluated at s_0, are zero. This can be stated as

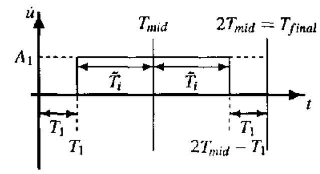

Figure 4. RELATION BETWEEN SWITCHING TIMES FOR AN AXIS-SYMMETRIC CONTROL PROFILE

$$G(s) = (s-s_0)^n R(s) \ \longleftrightarrow\ G(s)|_{s=s_0} = 0 \quad \text{and} \quad \left.\frac{\partial^i G(s)}{\partial s^i}\right|_{s=s_0} = 0\ \forall\ i = 1\ldots(n-1). \tag{9}$$

Thus, the requirement to cancel three poles at the origin of the s-plane results in the set of constraints

$$\begin{aligned} G(s)|_{s=0} &= 0 \\ \left.\frac{\partial G(s)}{\partial s}\right|_{s=0} &= 0 \\ \left.\frac{\partial^2 G(s)}{\partial s^2}\right|_{s=0} &= 0. \end{aligned} \tag{10}$$

Cancellation of the undamped conjugate complex pair of poles at $s = \pm j\omega$ requires that

$$G(s)|_{s=\pm j\omega} = 0. \tag{11}$$

In order to reduce the number of constraints, some general properties of point symmetric control profiles will be examined and subsequently employed. The curve of the time rate of change of a point symmetric control profile is axis symmetric to the mid-maneuver time. For a switch that takes place a time interval $\tilde{T}_i$ before the mid-maneuver time, there must be a switch at the time $\tilde{T}_i$ after the mid-maneuver time. The second switch must be of the same magnitude but opposite in sign to the first switch. These relations are illustrated in Fig. 4. It must be emphasized that this figure is only meant to illustrate the basic idea of parameterizing the control profile.

The transfer function of a time delay filter realizing an axis-symmetric profile can be written as

$$G(s) = \sum_{i=1}^{n}\left(A_i e^{-sT_i} - A_i e^{-s(2T_{mid}-T_i)}\right), \tag{12}$$

where n must be even, A_i denotes an arbitrary amplitude, and T_i denotes the distance of switch i from $T = 0$, and — due to the symmetry of the profile — the distance of switch $2n+1-i$ from $T_{end} = 2T_{mid}$.

Equation 10 evaluates to

$$G(s)|_{s=0} = \left(\sum_{i=1}^{n} \left(A_i e^{-sT_i} - A_i e^{-s(2T_{mid}-T_i)} \right) \right) \bigg|_{s=0} = 0, \quad (13)$$

$$\begin{aligned} \frac{\partial G(s)}{\partial s}\bigg|_{s=0} &= \frac{\partial}{\partial s} \left(\sum_{i=1}^{n} \left(A_i e^{-sT_i} - A_i e^{-s(2T_{mid}-T_i)} \right) \right) \bigg|_{s=0} \\ &= 2 \left(-\sum_{i=1}^{n} A_i\, T_i + \sum_{i=1}^{n} A_i\, T_{mid} \right) = 0. \end{aligned} \quad (14)$$

and

$$\begin{aligned} \frac{\partial^2 G(s)}{\partial s^2}\bigg|_{s=0} &= \frac{\partial^2}{\partial s^2} \left(\sum_{i=1}^{n} \left(A_i e^{-sT_i} - A_i e^{-s(2T_{mid}-T_i)} \right) \right) \bigg|_{s=0} \\ &= -4T_{mid} \left(-\sum_{i=1}^{n} A_i\, T_i + \sum_{i=1}^{n} A_i\, T_{mid} \right) = 0. \end{aligned} \quad (15)$$

It follows that all constraints (Eq. 13, Eq. 14, and 15) will be satisfied simultaneously, if $u(t)$ is both zero at the mid-maneuver time and point-symmetric to T_{mid}. $u(t)$ is given by

$$u(t) = \left(\sum_{i=1}^{n} \left(A_i \langle t - T_i \rangle - A_i \langle t - (2T_{mid} - T_i) \rangle \right) \right). \quad (16)$$

The parameterization of the jerk profile already guarantees point-symmetry of the control profile with respect to the mid-maneuver time. Now, it must be ensured, that $u(t)$ is zero at the mid-maneuver time, which can be written as

$$\begin{aligned} u(t)|_{t=T_{mid}} &= \sum_{i=1}^{n} A_i \left(T_{mid} - T_i \right) \\ &= -\sum_{i=1}^{n} A_i\, T_i + \sum_{i=1}^{n} A_i\, T_{mid} = 0. \end{aligned} \quad (17)$$

Equation 17 is satisfied for all point-symmetric control profiles which are zero at t_{mid}. For this class of profiles, Eq. 13, Eq. 14, and Eq. 15 can be taken out of the problem formulation.

The set of constraints can be reduced even further. For any $s = \pm j\omega$, $\omega \in (0, \infty)$, the requirements $\Re\{G(j\omega)\} = 0$ and $\Im\{G(j\omega)\} = 0$ result in the same constraint. For the following proof, T_i will be substituted by

$$T_i = T_{mid} - \tilde{T}_i \;\forall\, i \in 1 \ldots n. \quad (18)$$

The relation is illustrated in Fig. 4. The real part of $G(j\omega)$ is given by

$$\begin{aligned} \Re\{G(j\omega)\} &= \Re\left\{ \sum_{i=1}^{n} \left(A_i e^{-j\omega T_i} - A_i e^{-j\omega(2T_{mid}-T_i)} \right) \right\} \\ &= \Re\left\{ \sum_{i=1}^{n} \left(A_i e^{-j\omega(T_{mid}-\tilde{T}_i)} - A_i e^{-j\omega(T_{mid}+\tilde{T}_i)} \right) \right\} \\ &= 2\sin(\omega T_{mid}) \sum_{i=1}^{n} A_i \sin(\omega \tilde{T}_i), \end{aligned} \quad (19)$$

whereas the imaginary part of $G(j\omega)$ is given by

$$\begin{aligned} \Im\{G(j\omega)\} &= \Im\left\{ \sum_{i=1}^{n} \left(A_i e^{-j\omega T_i} - A_i e^{-j\omega(2T_{mid}-T_i)} \right) \right\} \\ &= \Im\left\{ \sum_{i=1}^{n} \left(A_i e^{-j\omega(T_{mid}-\tilde{T}_i)} - A_i e^{-j\omega(T_{mid}+\tilde{T}_i)} \right) \right\} \\ &= 2\cos(\omega T_{mid}) \sum_{i=1}^{n} A_i \sin(\omega \tilde{T}_i). \end{aligned} \quad (20)$$

Equating the real and imaginary part of $G(j\omega)$ to zero results in the requirement

$$\sum_{i=1}^{n} A_i \sin(\omega \tilde{T}_i) = \sum_{i=1}^{n} A_i \sin\left(\omega (T_{mid} - T_i)\right) = 0. \quad (21)$$

The constraint equations developed so far only cover the pole cancellation requirements. But geometric boundary conditions had also been imposed, which will now be taken into consideration.

The rigid body mode, which describes the dynamics of the C of G of the system obeys Newton's second law given as

$$\begin{aligned} a_r(t)\, m_r &= u(t) \\ \leftrightarrow a_r(t) &= \frac{u(t)}{m_r}, \end{aligned} \quad (22)$$

where the subscript r denotes the rigid body mode. The mass of the rigid body mode, m_r, is the total mass of the structure. The

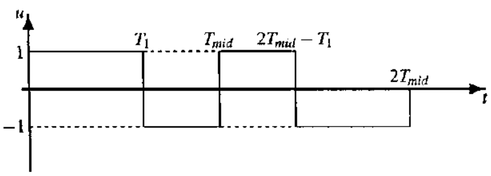

Figure 5. INFINITE JERK CONTROL PROFILE

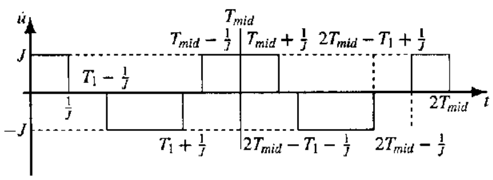

Figure 6. UNCOLLAPSED JERK PROFILE

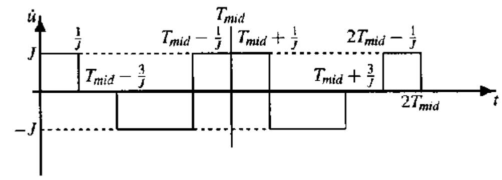

Figure 7. PARTIALLY COLLAPSED JERK PROFILE (CF. 1)

displacement of the C of G at time t is given as

$$y_r(t) = \int_0^t \left(\int_0^{\tau_2} \frac{u(\tau_1)}{m_r} d\tau_1 \right) d\tau_2. \tag{23}$$

Incorporating the geometric boundary conditions yields

$$y_r(T_{mid}) = \int_0^{T_{mid}} \left(\int_0^{\tau_2} \frac{u(\tau_1)}{m_r} d\tau_1 \right) d\tau_2 = \frac{1}{2}. \tag{24}$$

VARIATION OF JERK PROFILES

In this section, the entire spectrum of possible jerk profiles from $J \to \infty$ to $J \to 0$ is studied.

The control profiles have been named according to their characteristics. This will be commented on in the individual sections.

Infinite Jerk Profile

The first control profile described in this section is for the case $J \to \infty$. This control profile is identical to the solution of the jerk unconstrained time optimal control problem, which has already been the subject of extensive studies. The structure of this profile is shown in Fig. 5. The calculation of this control profile has been described by Singh and Vadali (Singh and Vadali, 1994).

Uncollapsed Jerk Profile

For this class of control profiles, the control $u(t)$ will go to its upper and lower bounds alternately. However, since a slight constraint has been imposed on the time rate of change of $u(t)$, the control cannot go from one bound to the other instantaneously.

The time derivative of the control profile is shown in Fig. 6 and the corresponding time-delay filter is given by

$$\begin{aligned} G_F(s) = J \Big(& 1 - e^{-s\frac{1}{J}} - e^{-s\left(T_1 - \frac{1}{J}\right)} + e^{-s\left(T_1 + \frac{1}{J}\right)} \\ & + e^{-s\left(T_{mid} - \frac{1}{J}\right)} - e^{-s\left(T_{mid} + \frac{1}{J}\right)} \\ & - e^{-s\left(2T_{mid} - T_1 - \frac{1}{J}\right)} + e^{-s\left(2T_{mid} - T_1 + \frac{1}{J}\right)} \\ & + e^{-s\left(2T_{mid} - \frac{1}{J}\right)} - e^{-s2T_{mid}} \Big). \end{aligned} \tag{25}$$

As will be seen, some of the switches will coincide if the limit on the maximum allowable amount of jerk is reduced. Two switches of the same magnitude at the same time can then be combined into one switch with twice the magnitude. The process will be termed "collapse of switches". In this terminology, the control profiles will be classified as Uncollapsed, Partially Collapsed and Totally Collapsed.

Partially Collapsed Jerk Profile

Upon reduction of the maximum allowable amount of jerk J, the aforementioned collapse of switches will take place. The profile under consideration in this section is termed "Partially Collapsed", because some of the switches have already been combined. Different configurations can arise depending on which switches will combine first.

Configuration 1 presents the case, where the second pair of switches collapses. It was numbered "configuration 1", because it represents the time optimal solution of the Floating Oscillator problem. The jerk profile is graphed in Fig. 7. It corresponds to the step response of a time delay filter with the transfer function

$$\begin{aligned} G_F(s) = J \Big(& 1 - e^{-s\frac{1}{J}} - e^{-s\left(T_{mid} - \frac{3}{J}\right)} + 2e^{-s\left(T_{mid} - \frac{1}{J}\right)} \\ & - 2e^{-s\left(T_{mid} + \frac{1}{J}\right)} + e^{-s\left(T_{mid} + \frac{3}{J}\right)} \\ & + e^{-s\left(2T_{mid} - \frac{1}{J}\right)} - e^{-s2T_{mid}} \Big). \end{aligned} \tag{26}$$

The second configuration, shown in Fig. 8, represents the case, where the first pair of switches collapsed. Calculation of this control profile configuration is very similar to calculation of configuration 1. Thus, it will not be treated in detail here.

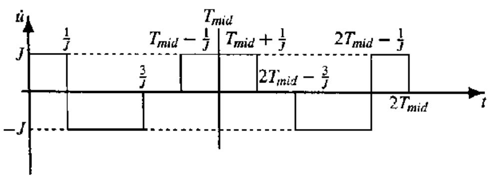

Figure 8. PARTIALLY COLLAPSED JERK PROFILE (CF. 2)

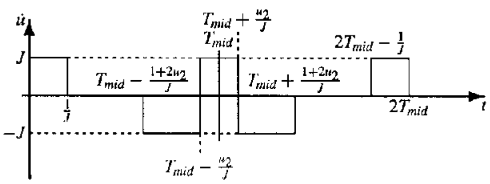

Figure 9. INTERMEDIATE JERK PROFILE (CF. 1)

Intermediate Jerk Profile

This control profile is termed "Intermediate Control Profile" characterizing it by the fact that it stands between the Uncollapsed and the Totally Collapsed Control Profiles or, to be more precise, it stands between the respective limiting cases designated as Partially Collapsed Profile and Transition Profile. As the constraint on J is tightened further, the switches are not far enough apart for $u(t)$ to get saturated at the end of each ramping interval. Due to this, the maximum respectively minimum values of $u(t)$ in some intervals become variables to be optimized for. For configuration 1, this will be $u_{min} = -u_2$ in the first half and $u_{max} = u_2$ in the second half of the control profile. The shape of the Intermediate Jerk Profile is shown in Fig. 9. The control profile parameters are the mid-time T_{mid} and the amplitude u_2. The transfer function of the time-delay filter is given as

$$
\begin{aligned}
G_F(s) = J\Big(1 - e^{-s\frac{1}{J}} - e^{-s\left(T_{mid} - \frac{1+2u_2}{J}\right)} + 2e^{-s\left(T_{mid} - \frac{u_2}{J}\right)} \\
- 2e^{-s\left(T_{mid} + \frac{u_2}{J}\right)} + e^{-s\left(T_{mid} + \frac{1+2u_2}{J}\right)} \\
+ e^{-s\left(2T_{mid} - \frac{1}{J}\right)} - e^{-s2T_{mid}}\Big).
\end{aligned}
\tag{27}
$$

The calculation of the Intermediate Jerk Profile configuration 2 follows the same procedure.

Transition Jerk Profile

The name of this class of profiles alludes to the transition to a totally collapsed control profile, which does not consist of any coasting periods, i. e. where the control input $u(t)$ never reaches the saturation level. Because there are two different configurations for the Intermediate Jerk Profile, there are also two different configurations for the Transition Jerk Profile. Both profiles have in common that the control input $u(t)$ will touch its limits $|u(t)| = 1$ only at two distinct points during the entire maneuver. Since the control will never be outside the imposed bounds, the constraint on $u(t)$ can be taken out of the problem formulation for this and also all following control profiles. Time optimality of all non-saturating control profiles can be verified by the application of Pontryagin's principle.

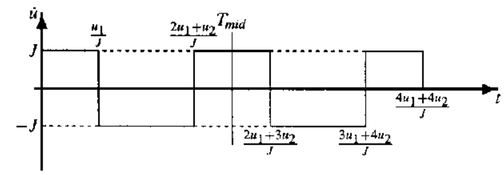

Figure 10. TOTALLY COLLAPSED JERK PROFILE

For configuration 1, the second pair of switches has already been combined into one switch. Now, the first pair of switches will be combined, too. A time-delay filter generating this profile is given by

$$
\begin{aligned}
G_F(s) = J\Big(1 - 2e^{-s\frac{1}{J}} + 2e^{-s\frac{2+u_2}{J}} - 2e^{-s\frac{2+3u_2}{J}} \\
+ 2e^{-s\frac{3+4u_2}{J}} - e^{-s\frac{4+4u_2}{J}}\Big).
\end{aligned}
\tag{28}
$$

No separate figure has been added for this profile since it is very similar to the Totally Collapsed Jerk Profile (Fig. 10).

Totally Collapsed Jerk Profile

The Totally Collapsed Jerk Profile is characterized by the fact that both pairs of switches have collapsed. This jerk profile can be generated by a time delay filter, whose transfer function

$$
\begin{aligned}
G_F(s) = J\Big(1 - 2e^{-s\frac{u_1}{J}} + 2e^{-s\frac{2u_1+u_2}{J}} - 2e^{-s\frac{2u_1+3u_2}{J}} \\
+ 2e^{-s\frac{3u_1+4u_2}{J}} - e^{-s\frac{4u_1+4u_2}{J}}\Big)
\end{aligned}
\tag{29}
$$

is parameterized by the maximum and minimum amplitudes of $u(t)$ for both time-intervals $[0, T_{mid}]$ and $(T_{mid}, 2T_{mid}]$ of the control profile. The profile is shown in Fig. 10. Pontryagin's principle can be used to test time optimality of this non-saturating control profile.

Triangular Control Profile

The Triangular Control Profiles marks another transition. The name for this profile stems from the distinct shape of the

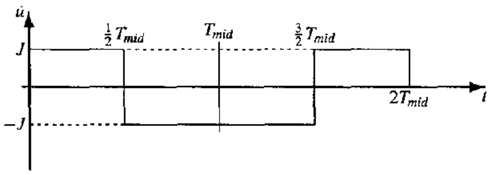

Figure 11. TIME DERIVATIVE OF THE TRIANGULAR CONTROL PROFILE

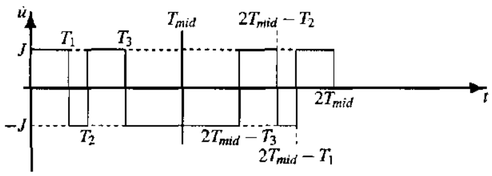

Figure 12. VERY SMALL JERK PROFILE

control profile, which is triangular in nature. The jerk profile is shown in Fig. 11. The transfer function of the time delay filter generating this input is given as

$$G_F(s) = J\left(1 - 2e^{-s\frac{T_{mid}}{2}} + 2e^{-s\frac{3T_{mid}}{2}} - e^{-s2T_{mid}}\right). \tag{30}$$

Very Small Jerk Profile

The last jerk profile under consideration in this paper is the Very Small Jerk Profile, which is graphed in Fig. 12. The transfer function

$$\begin{aligned} G_F(s) = J\Big(&1 - 2e^{-sT_1} + 2e^{-sT_2} - 2e^{-sT_3} + 2e^{-s(2T_{mid}-T_3)} \\ &- 2e^{-s(2T_{mid}-T_2)} + 2e^{-s(2T_{mid}-T_1)} - e^{-s2T_{mid}}\Big). \end{aligned} \tag{31}$$

reveals that additional switches had to be inserted in order to suppress oscillatory movement at the end of the maneuver even under such a stringent jerk limitation.

The mid-time has to be substituted by

$$T_{mid} = 2T_3 - 2T_2 + 2T_1 \tag{32}$$

in order to ensure that $u(t)$ is zero at T_{mid}. The remainder of the constraints are developed using the technique proposed earlier. The name of this profile stems from the fact, that the time rate of change of this control profile is very small.

NUMERICAL EXAMPLE

The structures of the different control profiles are shown for the benchmark Floating Oscillator problem, whose setup is

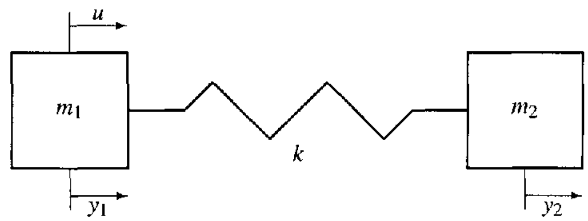

Figure 13. FLOATING OSCILLATOR

shown in Fig. 13.

Tables 1 and 2 list numerical values of the parameters of control profiles for different amounts of maximum permissible jerk. The control profiles are parameterized as

$$u(t) = \sum_i A_i < t - T_i > . \tag{33}$$

Table 1 lists the coefficients of all saturating control profiles, whereas Table 2 contains the coefficients for all non-saturating control profiles.

Table 1. COEFFICIENTS OF THE CONTROL SEQUENCES (SATURATING PROFILES)

J	2	1.7540	1.6
A_1	2.0000	1.7540	1.6000
T_1	0.0000	0.0000	0.0000
A_2	-2.0000	-1.7540	-1.6000
T_2	0.5000	0.5701	0.6250
A_3	-2.0000	-1.7540	-1.6000
T_3	0.7687	0.7384	0.7162
A_4	2.0000	3.5080	3.2000
T_4	1.7687	1.8787	1.9140
A_5	2.0000	-3.5080	-3.2000
T_5	1.9008	3.0189	3.0596
A_6	-2.0000	1.7540	1.6000
T_6	2.9008	4.1592	4.2575
A_7	-2.0000	1.7540	1.6000
T_7	3.0330	4.3274	4.3486
A_8	2.0000	-1.7540	-1.6000
T_8	4.0330	4.8976	4.9736
A_9	2.0000		
T_9	4.3017		
A_{10}	-2.0000		
T_{10}	4.8017		

Table 2. COEFFICIENTS OF THE CONTROL SEQUENCES (NON-SATURATING PROFILES)

J	1.4451	1.2	0.3	0.0912	0.05	0.02
A_1	1.4451	1.2000	0.3000	0.0912	0.0500	0.0200
T_1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_2	-2.8901	-2.4000	-0.6000	-0.1824	-0.1000	-0.0400
T_2	0.6920	0.7366	1.2604	2.2214	1.8046	3.4994
A_3	2.8901	2.4000	0.6000	0.1824	0.1000	0.0400
T_3	1.9584	2.0475	2.9679	6.6643	2.5064	4.5117
A_4	-2.8901	-2.4000	-0.6000	-0.0912	-0.1000	-0.0400
T_4	3.1071	3.1963	3.8621	8.8858	3.6217	4.7325
A_5	2.8901	2.4000	0.6000		0.1000	0.0400
T_5	4.3735	4.5073	5.5696		8.0579	10.1482
A_6	-1.4451	-1.2000	-0.3000		-0.1000	-0.0400
T_6	5.0655	5.2438	6.8299		9.1732	10.3690
A_7					0.1000	0.0400
T_7					9.8750	11.3813
A_8					-0.0500	-0.0200
T_8					11.6796	14.8807

CONCLUSIONS

This paper presented a technique for the design of time-optimal jerk limited control sequences. The shape of the resulting control profiles has been analyzed. It has been shown, how the range of jerk from $J \to 0$ to $J \to \infty$ can be divided into different regions by the shape of the control profile. The development of the jerk limited time optimal controller was carried out for undamped flexible structures but can easily be applied to damped structures by eliminating the requirement that the jerk profile has to be point-symmetric about the mid-maneuver time. Numerical results have been obtained for the standard Floating Oscillator benchmark problem. They show that by accepting a small increase in the final time, the jerk can be reduced considerably. All non-saturating control profiles have been verified to be time-optimal.

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