William Singhose
Department of Mechanical Engineering,
Georgia Institute of Technology,
Atlanta, GA 30332-0405
william.singhose@me.gatech.edu

Tarunraj Singh
Department of Mechanical &
Aerospace Engineering,
University at Buffalo,
Buffalo, NY 14260
tsingh@fate.eng.buffalo.edu

Warren Seering
Department of Mechanical Engineering,
Massachusetts Institute of Technology,
Cambridge, MA 02139
seering@mit.edu

On-Off Control With Specified Fuel Usage

A method for generating on-off command profiles for flexible systems is presented. The command profiles move a system without residual vibration while using a specified amount of actuator fuel. Robustness to modeling errors can be incorporated into the design of the command signals. Techniques are presented that facilitate implementation and indicate prudent choices for the amount of fuel to be used. The method is compared to other command generation techniques that balance fuel usage and slew time.

1 Introduction

Slewing flexible systems equipped with on-off actuators presents a challenging control problem. The minimum time required for completing a maneuver is a function of the system parameters and performance criteria. The performance criteria usually include a limit on the residual vibration amplitude. In addition, it may be important to limit the transient deflection, excitation of unmodeled modes, fuel expenditure, and to ensure some robustness to modeling errors.

A bang-bang command profile will yield the fastest rigid-body response given only actuator limits. When a velocity limit exists, the fastest rigid-body motion is accomplished with a bang-off-bang command (trapezoidal velocity command). However, when the system has flexibility, command profiles based on rigid-body dynamics are often unacceptable because large transient and residual oscillations may exist. This necessitates the use of specially shaped command profiles that reduce residual vibration (Thompson et al., 1989; Ben-Asher et al., 1992; Liu and Wie, 1992; Singh and Vadali, 1994; Pao, 1996; Singhose et al., 1996b; Tuttle, 1997) and transient deflection (Banerjee and Singhose, 1995; Singhose et al., 1997).

In addition to exciting elastic modes, command profiles based solely on rigid-body dynamics may use a significant amount of actuator fuel (defined as the amount of time that the actuators are turned on). Including fuel expenditure as a performance criteria may be necessary in applications such as spacecraft control because the fuel is expensive and a limited amount can be carried into space. Previous work has produced methods that generate fuel-minimum command profiles (Meyer and Silverberg, 1996) or fuel/time optimal profiles (VanderVelde and He, 1983; Wie et al., 1993; Singh, 1995). These methods have not attempted to specify the exact amount of fuel used. Other work has generated command profiles that are robust to modeling errors and are inherently fuel efficient (Singhose et al., 1996a). But, once again, the exact amount of fuel used was not a design parameter.

This paper presents a method that generates command profiles that eliminate residual vibration while using a specified amount of actuator fuel. Properties of the command profiles, such as, duration, robustness to modeling errors, and profile complexity are examined. Using these properties a novel design approach is suggested—the fuel usage can be selected to minimize, maximize, or balance the command properties. The relationship between the specified-fuel method and other techniques that balance fuel usage and slew time is examined.

Contributed by the Dynamic Systems and Control Division for publication in the JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL. Manuscript received by the Dynamic Systems and Control Division December 17, 1996; revised manuscript received October 22, 1998. Associate Technical Editor: R. Redfield.

II Statement of Problem

The design of a shaped command profile with specified fuel usage for a flexible linear system can proceed in two ways. The problem can be stated as an optimal control problem in which a cost function is minimized, or it can be formulated as an input shaping problem. The optimal control formulation is useful because it provides a means of checking the validity of numerical solutions. The input shaping formulation provides two advantages. First, it provides a straightforward method for satisfying the state boundary conditions and actuator limits. The time-optimal profiles resulting from this straightforward process can then be verified by the check provided by optimal control theory. Second, input shaping can be used to generate time-optimal profiles subject to auxiliary constraints such as a fixed number of pulses. These profiles are generated easily with input shaping, but would be more difficult to obtain through an optimal control formulation.

For certain ranges of the parameter values, the command profile obtained by the optimal control approach consists of a series of positive pulses followed by a series of negative pulses. In other ranges, the solution is more complicated; it contains intertwined positive and negative pulses. These two types of profiles are illustrated in Fig. 1. Type 2 profiles arise when the fuel usage is very near the amount of fuel used by a time-optimal command generated without regard to fuel usage. When the fuel usage is not constrained at all, the coast periods between the intertwined pulses go to zero and the profile is a multi-switch bang-bang without coast periods.

The variation in the shape of the optimal profile can make implementation difficult. Furthermore, actuator wear can be reduced by keeping the number of command pulses to a minimum. It is in these situations when the input shaping formulation has its advantage because the shape of the command profile can be forced into a simple form with a constant number of pulses. As we will see in a subsequent section, forcing the command profile away from the time-optimal shape incurs only a minor time penalty in most cases.

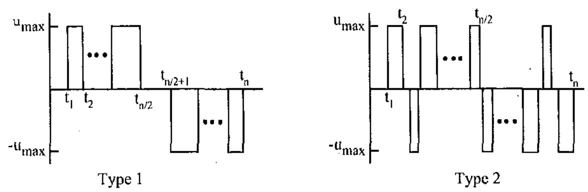

Fig. 1 Two types of time-optimal command profiles

II A Time-Optimal Control Formulation. The time-optimal control formulation can be stated as a minimization of the cost function:

$$J = \int_0^{t_f} dt = t_f \tag{1}$$

where t_f represents the maneuver time. For rest-to-rest slewing of a linear, single-input, time-invariant system described by the equations:

$$\dot{\mathbf{x}}_o = \mathbf{A}_o\mathbf{x}_o + \mathbf{B}_o u \quad \forall t \in [0, t_f] \tag{2}$$

the boundary conditions are:

$$\mathbf{x}_o(0) = [-x_d \quad 0 \quad 0 \quad \ldots \quad 0]^T \quad \text{and} \quad \mathbf{x}_o(t_f) = \mathbf{0} \tag{3}$$

where x_d is the desired slew distance. Actuator constraints can be represented as:

$$-u_{\max} \le u \le u_{\max}. \tag{4}$$

The limit on fuel usage can be expressed as:

$$\int_0^{t_f} |u|\, dt \le U \tag{5}$$

where U represents the fuel available for the slew. When U is less than or equal to the fuel used for the time-optimal command designed without fuel limitations, then the command will use all the available fuel. The constraint given in (5) can be restated by defining a new variable:

$$\dot{\phi} = |u| \tag{6}$$

where ϕ satisfies the boundary conditions:

$$\phi(0) = 0, \quad \phi(t_f) \le U. \tag{7}$$

Augmenting the state equations given in (2) with the fuel usage variable given in (6), and using the combined boundary conditions given in (3) and (7), yields a complete description of the problem under consideration.

A time-optimal solution to the above problem can be obtained by performing a nonlinear numerical optimization. Several methods have been proposed for performing optimizations of this type. These methods include performing an integration of the state equations at each step of the optimization (Banerjee and Singhose, 1995), using the Switching Time Optimization algorithm (Meier and Bryson, 1990; Liu and Singh, 1996), and formulating the state boundary conditions as explicit functions of a parameterized command profile. Because the systems considered here are time-invariant linear systems, the necessary optimizations use parameterized boundary conditions. Relying on Pontryagin's Maximum Principle (PMP) (Pontryagin et al., 1962), the time-optimal command profile can be parameterized by its switch times as shown in Fig. 1. Stating the boundary conditions in terms of the switch times leads to a set of equations that can be satisfied while minimizing the final switch time (the maneuver duration).

A numerical solution of the above problem may not yield the time-optimal solution because there are multiple solutions and nonlinear optimization is susceptible to selection of local minima. Fortunately, the necessary conditions provided by PMP can be used to verify a candidate solution. The necessary conditions of PMP utilize the Hamiltonian which is given by:[1]

$$H = 1 + \psi^T(\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}) = 1 + \lambda^T(\mathbf{A}_o\mathbf{x}_o + \mathbf{B}_o\mathbf{u}_o) + \mu|u| \tag{8}$$

where ψ is the vector of costates. The symbols λ and μ represent subsets of the costates that correspond to the costates of the original system and the fuel usage state, respectively. It can be seen from (8) that the Hamiltonian for the augmented system is identical to the one defined for a fuel/time optimal controller where μ represents the relative weight of the fuel consumed in the cost function (Wie et al., 1993; Singh, 1995). The necessary conditions for optimality require the following equations to be satisfied (Pontryagin et al., 1962; Singh, 1995):

$$\dot{\lambda} = -\mathbf{A}^T\lambda \quad \forall t \in [0, t_f] \tag{9}$$

$$\dot{\mu} = -\frac{\partial H}{\partial \phi} = 0 \rightarrow \mu(t) = \text{constant} \tag{10}$$

$$u = -dez\left(\frac{\mathbf{B}^T\lambda}{\mu}\right) \forall t \in [0, t_f] \tag{11}$$

dez, the deadzone function is defined as:

$$\nu = dez(\alpha) \Rightarrow \begin{cases} \nu = 0 & \text{if} \quad |\alpha| < 1 \\ \nu = \text{sgn}(\alpha) & \text{if} \quad |\alpha| > 1 \\ 0 \le \nu \le 1 & \text{if} \quad \alpha = 1 \\ -1 \le \nu \le 0 & \text{if} \quad \alpha = -1 \end{cases} \tag{12}$$

Using (9) and (10), the costates can be represented as:

$$\lambda(t) = \exp(-\mathbf{A}^T t)\lambda(0) \tag{13}$$

and

$$\mu(t) = \text{constant} \tag{14}$$

For optimality, the switching function must satisfy the constraint:

$$\mathbf{B}^T \exp(-\mathbf{A}^T t_s)\lambda(0) = \pm\mu \tag{15}$$

where t_s represents a switch time. We can solve for $\lambda(0)$ and μ from the null space of the matrix $\mathbf{P}$ where:

$$\mathbf{P}\begin{Bmatrix} \lambda(0) \\ \mu \end{Bmatrix} = \begin{bmatrix} \mathbf{B}^T\exp(-\mathbf{A}^T t_1) & \pm 1 \\ \mathbf{B}^T\exp(-\mathbf{A}^T t_2) & \pm 1 \\ \cdots & \pm 1 \\ \cdots & \pm 1 \end{bmatrix} \begin{Bmatrix} \lambda(0) \\ \mu \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \cdots \\ 0 \end{Bmatrix} \tag{16}$$

All of the solutions to (16) are nontrivial assuming $\mu \ne 0$, which is always true because $\mu = 0$ implies there is no control profile. This can be seen by noting that the final column in the above matrix contains values that are ± 1. After performing the matrix multiplication it is obvious that $\lambda(0)$ must be nontrivial to satisfy the condition. If the control profile determined from the equation:

$$u(t) = -dez(\mathbf{B}^T\lambda(t)/\mu) \tag{17}$$

matches the profile resulting from the parameter optimization, then the control is optimal.

II B Input Shaping Formulation. The process of input shaping can be used to generate a command that satisfies the state boundary conditions and actuator limits. Input shaping requires convolving a sequence of impulses with a desired command signal. The result of the convolution is a shaped command

[1] The state space matrices $\mathbf{A}$ and $\mathbf{B}$ represent the system augmented with the fuel usage variable: $\mathbf{A}_o$ and $\mathbf{B}_o$ represent the dynamics of the original system.

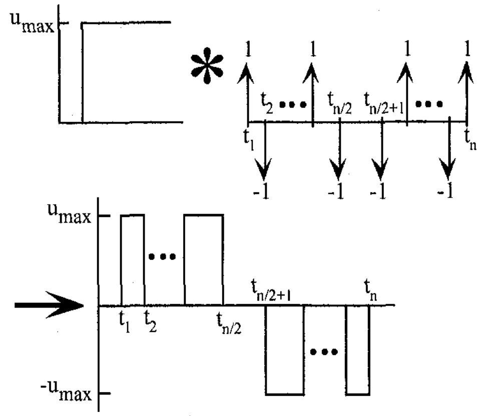

Fig. 2 Using input shaping to generate a Type 1 command profile

profile that is used to drive the system. If the unshaped command signal is a step input whose magnitude equals the actuator limit and the impulse amplitudes are set to specific values, then the resulting shaped command profile will be the time-optimal control profile (Pao and Singhose, 1998). Figure 2 demonstrates that a command profile of type 1 can be obtained by using an input shaper of the form:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & \cdots & -1 & -1 & \cdots & -1 & 1 \\ 0 & t_2 & t_3 & \cdots & t_{n/2} & t_{(n/2+1)} & \cdots & t_{n-1} & t_n \end{bmatrix} \quad (18)$$

where A_i and t_i are the amplitudes and time locations of the impulses, and n is the number of impulses. Note that the time of the first impulse is always zero. The resulting command profile contains $n/2$ pulses, $n/4$ positive and $n/4$ negative pulses. When the impulse amplitudes are fixed as in (18), the problem reduces to finding the unknown impulse time locations $(t_2 - t_n)$. This is the same problem as finding the switch times in the optimal profile when the profile has a type 1 form. To obtain a type 2 profile the shaper must have the form:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & \cdots & -1 & 1 \\ 0 & t_2 & t_3 & t_4 & t_5 & \cdots & t_{n-1} & t_n \end{bmatrix} \quad (19)$$

The requirement of (19) is a result of the profile alternating between positive and negative pulses. Using an input shaper described by either (18) or (19) amounts to enforcing the limit on actuator effort that is given in (4). That is, convolving (18) or (19) with a step function whose magnitude is equal to the maximum actuator effort, results in a command that never exceeds the actuator limit. Furthermore, the shaped profile will always consist of constant amplitude pulses that can be realized by on-off actuators. The number of pulses in the command profile can be set by choosing the value of n. If it is desired that the profile always contain 4 pulses, then n is set equal to 8. Limitations on the number of pulses is an example of a constraint that is straightforward to implement with input shaping, but is much harder to pose in the optimal control formulation.

Note that (19) reduces to (18) if certain impulses occur at the same time. For example, if t_3 and t_4 occur at the same time, then the impulses cancel each other and the negative pulse they form during the convolution is nonexistent. This allows us to use (19) for any parameter values. When the optimal solution in a type 1 profile (the form given in (18)), the appropriate impulses simply align and cancel each other out.

The constraint given in (18) is useful if we want to force the command to be a type 1 profile when otherwise it would be a type 2 profile. Its use facilitates implementation because the command profile will always consist of $n/4$ positive pulses followed by $n/4$ negative pulses. For a small range of parameter values, the shaper described by (18) will result in command profiles that are slightly longer than the time-optimal profiles designed by the optimal control formulation. Note, however, that the input shaping profiles are still time-optimal. They are simply subjected to an additional constraint that is not enforced in the optimal control formulation; they are required to yield a type 1 profile with $n/2$ pulses.

In order to determine the impulse time locations (switch times), the state boundary conditions must be satisfied. These boundary conditions are satisfied in a straightforward manner given the input shaping formulation. If we consider the simple model shown in Fig. 3, then the rigid-body boundary conditions for rest-to-rest slewing can be expressed as:

$$v_d = \int_0^{t_n} \frac{u(t)}{M}\, dt = 0 \quad (20)$$

and,

$$x_d = \int_0^{t_n} \int_0^{t_n} \frac{u(t)}{M}\, dt^2 \quad (21)$$

where v_d is the desired terminal velocity (zero), x_d is the desired slew distance, and M is the total system mass. With an input shaper of the form given in (18) these boundary conditions can be stated algebraically as:

$$0 = (t_2 - t_3 + t_4 - \ldots + t_{n/2} + t_{n/2+1} - t_{n/2+2} + \ldots - t_n)/M \quad (22)$$

$$x_d = \frac{\begin{pmatrix} -t_2^2 + t_3^2 - \ldots - t_{n/2}^2 - t_{n/2+1}^2 + \ldots - t_n^2 + 2t_2t_n - 2t_3t_n \\ + \ldots + 2t_{n/2}t_n + 2t_{n/2+1}t_n - \ldots - 2t_{n-2}t_n + t_{n-1}t_n \end{pmatrix}}{M}. \quad (23)$$

For more complicated models, such as flexible rotational systems whose inertia is time-varying, the rigid-body boundary conditions may need to be expressed in a more general form than that shown in (20) and (21).

To specify the amount of fuel used (in units of time) the summation of the pulse widths is set less than or equal to the available fuel. For profiles described by (18), this constraint is:

$$U \geq \sum_{i=1}^{n} (-1)^i t_i \quad (24)$$

If a particular flexible mode is described by its natural frequency, ω, and its damping ratio, ζ, then the boundary conditions of zero residual vibration is given by (Singer and Seering, 1990):

$$0 = e^{-\zeta\omega t_n} \sqrt{[C(\omega, \zeta)]^2 + [S(\omega, \zeta)]^2} \quad (25)$$

where,

$$C(\omega, \zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega t_i} \cos(\omega\sqrt{1 - \zeta^2}\, t_i) \quad (26)$$

$$S(\omega, \zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega t_i} \sin(\omega\sqrt{1 - \zeta^2}\, t_i). \quad (27)$$

Multi-mode systems require that multiple versions of (25) be

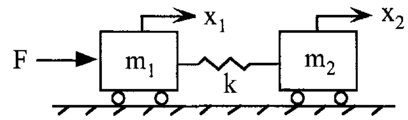

Fig. 3 Simple model of a flexible system

used, each with different values for ω and ζ. Unfortunately, if modeling errors exist, then large amplitude residual vibration may occur. One method for obtaining robustness to modeling errors is to set the derivative of the residual vibration with respect to vibration frequency equal to zero (Singer and Seering, 1990). In equation form, this constraint is:

$$0 = \frac{d}{d\omega}(e^{-\zeta\omega t_n}\sqrt{[C(\omega,\zeta)]^2 + [S(\omega,\zeta)]^2}). \quad (28)$$

Note that an equivalent form of robustness can be obtained through the optimal control formulation by using an augmented system of state equations that have two poles at each of the locations of the original systems flexible poles (Bhat and Miu, 1990; Liu and Wie, 1992; Singh and Vadali, 1994; Pao and Singhose, 1998).

Enforcement of the state boundary conditions for rest-to-rest slewing and actuator limits is accomplished by satisfying (18) or (19) and (22)–(25). To obtain robust slewing, (28) is added to the problem formulation. A numerical optimization can be performed to satisfy the above constraints while minimizing the slew duration (the time of the final impulse). The resulting impulse time locations determine the optimal command profile.

III Specified-Fuel Commands

Command profiles that use a specified amount of fuel can be generated using a variety of system models. If the system has little flexibility, then it can be modeled as a rigid body. Flexibility requires the inclusion of elastic modes in the state equations. Uncertainty in the values of the elastic modes require a robust formulation. The following three types of specified fuel (SF) commands are discussed here:

1) Rigid body (RB)
2) Zero residual Vibration (ZV)
3) Zero Vibration and Derivative (ZVD)

The RB SF profile is obtained by specifying the desired fuel usage and satisfying the rigid-body constraints, such as (22) and (23). For the system shown in Fig. 3 this profile is simply a positive pulse, followed by a region of coasting, followed by a negative pulse that is equal in duration to the positive pulse. The coasting period may be absent in some special cases. The ZV SF profile is obtained by adding the zero residual vibration constraint (25) to the equations used to generate the RB SF. Finally, the ZVD SF profile is obtained by adding the zero derivative constraint (28) to the equations used to generate the ZV SF command. There are several important qualities of the command profiles that will be investigated: a) slew duration, b) robustness to modeling errors, and c) profile complexity.

To demonstrate the procedure for constructing specified fuel profiles and investigate their properties, profiles will be designed for the single-mode flexible system represented by the model in Fig. 3. The force acting on mass m_1 is restricted to $-u_{\max} \le F(t) \le u_{\max}$. If all system parameters (m_1, m_2, k, $u_{\max}$) are set equal to 1, then the system has a natural frequency of $\sqrt{2}$ radians/sec (0.2251 Hz). The model shown in Fig. 3 may appear overly simplified; however, shaped command profiles based on this model have been shown to be very effective on more complicated systems (Banerjee and Singhose, 1995; Singhose et al., 1996b). Furthermore, it has been used as a benchmark case for flexible-body control techniques (Liu and Wie, 1992; Wie and Bernstein, 1992; Wie and Liu, 1992; Singh and Vadali, 1994; Pao, 1996; Singhose et al., 1996b).

III A Slew Duration. Figure 4 compares the move duration as a function of the amount of fuel used for the baseline system when the slew distance is 5 units. The amount of fuel used is measured by the duration of time during which the actuators are turned on. The RB SF data was obtained by satisfying (19) and (22)–(24), while minimizing the time of the final impulse. As mentioned previously, for low values of fuel usage, the intertwining positive and negative pulses cancel out and (19) effectively becomes the same as (18). The ZV SF data was obtained by satisfying (19) and (22)–(25). The ZVD SF data was obtained by satisfying (19), (22)–(25), and (28).

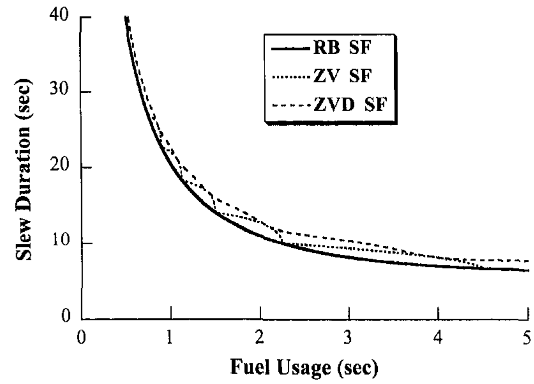

Fig. 4 Slew duration as a function of fuel usage

When the fuel usage is high, the fuel consumption can be reduced considerably with very little time penalty. However, attempting to save fuel when the fuel usage is low results in a large time penalty. Considering all three types of commands, the average slew duration increases 1.53 sec. when the fuel is reduced from 4 s to 3 s. On the other hand, the average slew duration is increased 9.55 s, when the fuel is reduced from 2 s to 1 s.

The RB SF is, of course, the shortest command profile, while the ZVD SF is the longest. The duration of the ZV SF ranges between the other two, equaling one or the other at certain levels of fuel usage. Note that only a small increase in slew duration accompanies the ZVD constraints. The ZVD profile is, at most, 27 percent longer than the RB profile, while typical increases are on the order of 15–20 percent. The benefits obtained by sacrificing this time are documented in the following sections.

III B Robustness to Modeling Errors. The model of any real system will not be perfect; therefore, the shaped profiles will not yield exactly zero residual vibration. To explore this effect we can plot the amount of residual vibration as a function of the actual system frequency. Figure 5 shows two such curves for the ZV SF command profile when the fuel usage is 3.5 s and 4 s. The data shown in the remainder of this paper will be based on a five unit slew. When the system model is exact (0.2251 Hz), both commands yield exactly zero residual vibration. Note that the residual vibration increases rapidly with mod-

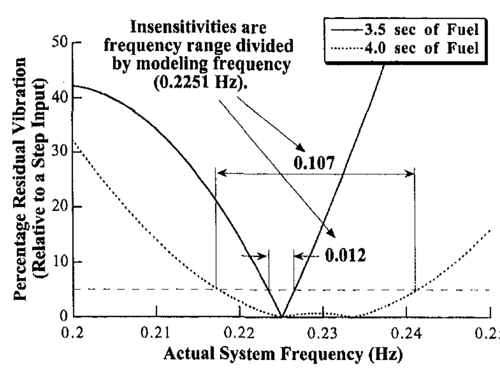

Fig. 5 Sensitivity curves for two ZV SF profiles

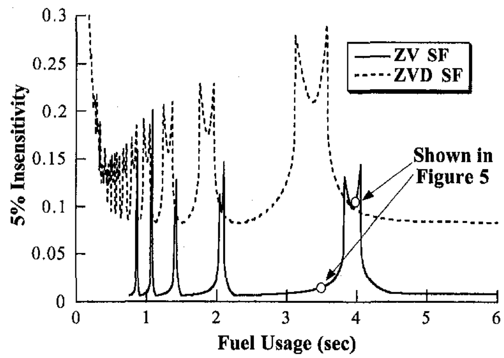

Fig. 6 Insensitivity as a function of fuel usage

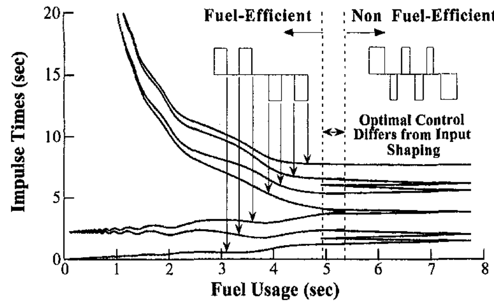

Fig. 8 Impulse times (switch times) for the ZVD SF profile

eling errors when the command uses 3.5 s of fuel. When 4 s of fuel is used, the vibration stays at a low level over a much wider frequency range. That is, the ZV SF command that uses 4 s of fuel is more robust to modeling errors than the command that uses 3.5 s of fuel. To quantify the relative robustness, we can measure the width of the two curves at some acceptable level of residual vibration. The normalized widths (frequency range divided by modeling frequency) of the curves at 5 percent have been labeled on Fig. 5. These widths are referred to as the 5 percent Insensitivities (I). The 5 percent I for the profile using 3.5 s of fuel is only 0.012, while the 5 percent I for the 4 sec. profile is 0.107.

To understand how the robustness (or lack there of) changes, the 5 percent I can be plotted as a function of the fuel usage. Figure 6 shows these curves for both the ZV SF and the ZVD SF commands; the RB SF is not shown because it does not attempt to eliminate vibration and, hence, robustness is poorly defined. The robustness of the ZV profile can vary by an order of magnitude, but it is usually very small. The robustness of the ZVD profile varies by a factor of 3 and it is almost always much greater than for the ZV profile. The advantage of the derivative robustness constraint is clearly visible from Fig. 6. Furthermore, Fig. 6 suggests a novel idea: the fuel usage can be used to effect the robustness of the profile. For example, when designing a ZV profile, there is a huge benefit from using 4 sec. of fuel instead of 3.5 s as was demonstrated in Fig. 5. The exact opposite is true for the ZVD profile. Figure 6 shows that a 3.5 s ZVD command is much more robust than a 4 s ZVD command. For either type of profile, small changes in fuel usage can cause large changes in the robustness.

III C Profile Complexity. The ease of implementation is not a straightforward quantity to be measured. However, the number of pulses in a profile and the ease of which they can be described will certainly effect implementation. Figure 7 shows the impulse times for the ZV SF profile as a function of the fuel usage. Two main regions of the solution space are evident. For high levels of fuel usage, the profile consists of

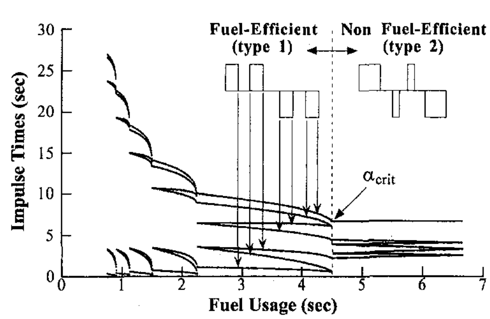

Fig. 7 Impulse times (switch times) for the ZV SF profiles

alternating positive and negative pulses (a type 2 profile). This region has been labeled Non Fuel-Efficient because increasing the fuel usage in this region yields no meaningful decrease in move duration. (The time of the final impulse is nearly constant.) For example, when the fuel usage is increased from 4.5 s to 6.7 s, the move time is decreased only 0.03 s. At lower levels of fuel usage the profile consists of positive pulses followed by negative pulses; positive and negative pulses are not intertwined (type 1). This region is labeled Fuel-Efficient because increasing fuel usage results in a noticeable decrease in slew duration.

Note that there are points where small changes in the fuel usage cause large changes in move duration (4.5 s, 2.2 s, etc.). These points occur when the profile collapses from 4 pulses down to just 2 pulses, one positive and one negative. The first of these points marks the boundary between the Fuel-Efficient and Non Fuel-Efficient regions. When considering the trade-off between fuel and time, these points represent good choices.

Control profiles based on the transition point between type 1 and type 2 profiles have been proposed as an attractive operating point that yields fast, but fuel-efficient control (Singhose et al., 1996a). These command profiles are generated by first requiring that the commands be of type 1 and then minimizing the move duration. No explicit limit is placed on the fuel usage. The resulting command corresponds to the profile at the first node of the fuel/duration curve (the curve formed by the final impulse in Fig. 7). This point has been labeled α_{crit} for reasons to be discussed in Section IV.

Figure 8 shows the impulse times for the ZVD SF profile. In this case, the optimal control formulation switches from type 1 to type 2 at a fuel usage of just less than 5 s. Increasing fuel consumption above 5.0 s yields no noticeable decrease in move time. Although the ZVD SF profile does not have distinct points where decreasing fuel leads to rapid increases in slew duration, there are still regions where this effect occurs (at approximately 4 s and 2 s of fuel). These regions are prudent choices when considering both fuel and time.

By using the input shaping formulation, the profile can be held in the type 1 configuration until a fuel usage of approximately 5.4 s. The system cannot be made to slew faster by increasing the fuel usage above this amount. Both the optimal control and input shaping solutions are shown in the region between 5.0 and 5.4 s of fuel. Differences in the intermediate switch times are evident, but there is no discernible difference in slew time. This indicates that there is essentially no time penalty for using the input shaping formulation to hold the profile in the type 1 configuration. Outside the 5.0 to 5.4 s interval the input shaping and time-optimal solutions are identical.

IV Determination of ZV Command Transitions

It can be seen from Fig. 7 that the second and third and the sixth and seventh impulse times of the ZV SF profile tend to approach each other as fuel usage increases. To see this, start at the left hand side and move to the right. The impulse time locations approach each other and eventually equal the same value. After this point the impulses separate in a discontinuous jump and again start to approach each other with increasing fuel usage. When the impulses come together, the result is a two-pulse profile as shown in Fig. 9. (Following the discontinuity there is the appearance of two new pulses.) It was noted earlier that these points of discontinuity are good choices when there is no fixed limit on fuel usage. In this section we solve for the fuel usage levels that correspond to these transitions in the ZV SF profile.

In order for the profile shown in Fig. 9 to yield zero residual vibration, the second (negative) pulse must start at an integral multiple of the vibration period. That is,

$$t_3 = n\frac{2\pi}{\omega}. \tag{29}$$

The constraint on fuel usage is:

$$t_2 = \frac{U}{2}. \tag{30}$$

From the rigid-body boundary conditions we obtain:

$$x_d = \frac{u_{\max}}{M} t_2 t_3 \tag{31}$$

Substituting (29) and (30) into (31) yields:

$$x_d = n\frac{U\pi u_{\max}}{M\omega} \tag{32}$$

We can now solve for the fuel usage points that correspond to the transitions in the ZV SF profile:

$$U = \frac{M\omega x_d}{nU\pi u_{\max}} \tag{33}$$

The fuel-efficient commands previously proposed (Singhose et al., 1996a) correspond to $n = 1$. If it is desired to use even less fuel, then $n = 2$ represents a prudent choice of fuel usage. Higher values of n place greater importance on fuel usage. Note that the command profiles corresponding to the nodes of the fuel/duration curve can also be obtained using a fuel/time optimal problem formulation, where the cost function is:

$$J = \int_0^{t_f} (1 + \alpha|u|)dt \tag{34}$$

The cost function is minimized subject (2) and (3). α_{crit} is a special value of the weighting factor that produces the command at the division between fuel-efficient and non-fuel-efficient commands. This command represents a good tradeoff between fuel usage and move duration. We can calculate this value and then use α_{crit} in (34) (Singh, 1995). Figure 10 shows that α_{crit} is a function of the desired move distance, just as the fuel usage level at the nodes (33) depends on move distance.

If it is desired to use command profiles corresponding to the nodes of the fuel/duration curves, then the easiest procedure is to use the fuel-efficient formulation previously proposed (Singhose et al., 1996a), as it is valid for any robustness criteria

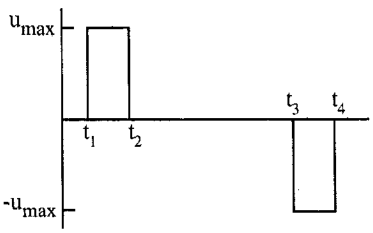

Fig. 9 Structure of two-pulse ZV SF profile

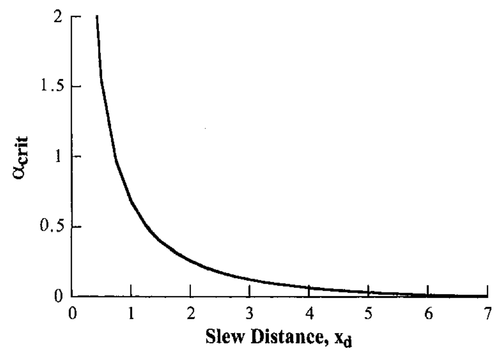

Fig. 10 α_{crit} as a function of move distance

including both ZV and ZVD constraints. Use of (33) with the specified-fuel formulation described above is also straightforward when ZV commands are used. The calculation of α_{crit} and then using a fuel/time optimization is a slightly more difficult process to obtain the same result.

V Conclusions

A method was presented for designing on-off command profiles for flexible systems that use a specified amount of actuator fuel. Three types of specified-fuel commands were discussed, those based on rigid-body dynamics, flexible-body dynamics, and robust flexible-body dynamics. Properties of the command profiles were compared as a function of the fuel usage. Plots of the slew duration versus fuel usage show that the fuel consumption can be significantly reduced (as compared to a purely time-optimal command), with very little increase in slew time. However, very low levels of fuel usage require very long slew durations. The robust formulation of the problem accommodates large modeling uncertainty at the cost of a small increase in slew duration. The fuel usage can be selected to minimize, maximize, or balance the command properties.

Locations where the command profile based on flexible dynamics transitions from 4 pulses to 2 pulses are good choices and the corresponding fuel usage was determined in closed form. Finally, the input shaping formulation was shown to be advantageous because it can be used to force the command profile into a consistent form.

Acknowledgments

Support for this work was provided by the Massachusetts Space Grant Fellowship Program.

References

Banerjee, A. K., and Singhose, W. E., 1995, "Minimum Time Fuel Efficient Maneuver of Flexible Spacecraft with Vibration Amplitude Constraint," *AAS Astrodynamics Specialist Conf.*, Halifax, Nova Scotia, Vol. AAS 95-318.

Ben-Asher, J., Burns, J. A., and Cliff, E. M., 1992, "Time-Optimal Slewing of Flexible Spacecraft," *J. of Guidance, Control, and Dynamics*, Vol. 5(2), pp. 360–367.

Bhat, S. P., and Miu, D. K., 1990, "Precise Point-to-Point Positioning Control of Flexible Structures," ASME Journal of Dynamic Systems, Measurement, and Control, Vol. 112(4), pp. 667–674.

Liu, Q., and Wie, B., 1992, "Robust Time-Optimal Control of Uncertain Flexible Spacecraft," *J. of Guidance, Control, and Dynamics*, Vol. 15(3), pp. 597–604.

Liu, S.-W., and Singh, T., 1996, "Robust Time-Optimal Control of Nonlinear Structures with Parameter Uncertainties," *AIAA Guidance, Nav., and Cont. Conf.*, San Diego, CA.

Meier, E.-B., and Bryson, A. E., 1990, "Efficient Algorithm for Time-Optimal Control of a Two-Link Manipulator," *J. of Guidance, Control, and Dynamics*, Vol. 13(5), pp. 859–866.

Meyer, J. L., and Silverberg, L., 1996, "Fuel Optimal Propulsive Maneuver of an Experimental Structure Exhibiting Spacelike Dynamics," *J. of Guidance, Control, and Dynamics*, Vol. 19(1), pp. 141–9.

Pao, L. Y., 1996, "Minimum-Time Control Characteristics of Flexible Structures," *J. of Guidance, Control, and Dynamics*, Vol. 19(1), pp. 123–9.

Pao, L. Y., and Singhose, W. E., 1998, "Robust Minimum Time Control of Flexible Structures," *Automatica*, Vol. 34(2), pp. 229–236.

Pontryagin, L. S., Boltyanskii, V. G., Gamkrelidze, R. V., and Mishchenko, E. F., 1962, *The Mathematical Theory of Optimal Processes*, Wiley, New York, pp. 360.

Singer, N. C. and Seering, W. P., 1990, "Preshaping Command Inputs to Reduce System Vibration," ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Vol. 112, Mar., pp. 76–82.

Singh, T., 1995, "Fuel/Time Optimal Control of the Benchmark Problem," *Journal of Guidance, Control, and Dynamics*, Vol. 18(6), pp. 1225–31.

Singh, T., and Vadali, S. R., 1994, "Robust Time-Optimal Control: A Frequency Domain Approach," *J. of Guidance, Control and Dynamics*, Vol. 17(2), pp. 346–353.

Singhose, W., Banerjee, A., and Seering, W., 1997, "Slewing Flexible Spacecraft with Deflection-Limiting Input Shaping," *J. of Guidance, Control, and Dynamics*, Vol. 20(2), pp. 291–298.

Singhose, W., Bohlke, K., and Seering, W., 1996a, "Fuel-Efficient Pulse Command Profiles for Flexible Spacecraft," *J. of Guidance, Control, and Dyn.*, Vol. 19(4), pp. 954–960.

Singhose, W., Derezinski, S., and Singer, N., 1996b, "Extra-Insensitive Input Shapers for Controlling Flexible Spacecraft," *J. of Guidance, Control, and Dynamics*, Vol. 19(2), pp. 385–91.

Thompson, R. C., Junkins, J. L., and Vadali, S. R., 1989, "Near-Minimum Time Open-Loop Slewing of Flexible Vehicles," *J. of Guidance, Control, and Dynamics*, Vol. 12(1), pp. 82–88.

Tuttle, T., 1997, Creating Time-Optimal Commands for Linear Systems, Ph.D., M.I.T.

VanderVelde, W., and He, J., 1983, "Design of Space Structure Control Systems Using On-Off Thrusters," *J. Guidance, Control and Dynamics*, Vol. 6(1), pp. 53–60.

Wie, B., and Bernstein, D. S., 1992, "Benchmark Problems for Robust Control Design," *J. of Guidance, Control, and Dynamics*, Vol. 15(5), pp. 1057–9.

Wie, B., and Liu, Q., 1992, "Comparison Between Robustified Feedforward and Feedback for Achieving Parameter Robustness," *Journal of Guidance, Control, and Dynamics*, Vol. 15(4), pp. 935–943.

Wie, B., Sinha, R., Sunkel, J., and Cox, K., 1993, "Robust Fuel- and Time-Optimal Control of Uncertain Flexible Space Structures," *AIAA Guidance, Navigation, and Control Conf.*, Monterey, CA, Vol. AIAA-93-3804-CP, pp. 939–948.