Fast Energy Relaxation of Hot Carriers Near the Dirac Point of Graphene

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ABSTRACT: We investigate energy relaxation of hot carriers in monolayer and bilayer graphene devices, demonstrating that the relaxation rate increases significantly as the Dirac point is approached from either the conduction or valence band. This counterintuitive behavior appears consistent with ideas of charge puddling under disorder, suggesting that it becomes very difficult to excite carriers out of these localized regions. These results therefore demonstrate how the peculiar properties of graphene extend also to the behavior of its nonequilibrium carriers.

KEYWORDS: Graphene, hot carriers, energy relaxation, electron—phonon scattering

The loss of excess energy by “hot” carriers, driven out of equilibrium by optical or electrical excitation, is critical to the operation of modern semiconductor devices. With the emergence of graphene as a promising new material for nanoelectronics, the need to understand the origins of its energy relaxation has stimulated significant theoretical7−14 and experimental15−20 activity. A key aspect of graphene is the large energy of its various optical modes, which ensures that these phonons are ineffective in cooling at all but the highest temperatures. While substrate phonons may provide a path for cooling,21 the cooling of hot carriers is primarily achieved through their interaction with acoustic phonons (especially at the low temperatures of interest here). An important parameter for describing this cooling is the energy-relaxation time (τe), the typical time on which energy is lost by the carriers as a whole.1,2 The variation of this parameter with temperature and density can yield valuable insight into the mechanisms responsible for cooling, an issue that is of particular relevance to graphene for which distinct, disorder-dependent, relaxation pathways have been predicted.7−10,12,13 In this Letter, we therefore determine the energy-relaxation time in this material, utilizing the carrier-heating approach that has previously been applied to conventional semiconductors.30−32

Many of the novel properties of graphene derive from the peculiarities of its bandstructure, whose linear energy bands intersect at so-called Dirac points (DPs) where the density of states also vanishes. In contrast to metals with their large Fermi surfaces, a feature of acoustic-phonon scattering in graphene is that the boundary between its “high-” and “low-” temperature regimes is actually dependent upon carrier density, through the influence of this parameter on the Bloch-Gruneisen temperature (Tbg) (we return to this point below). As the Fermi level is swept toward the DP from either band, the phase space available for scattering shrinks and one might therefore expect hot-carrier cooling to slow significantly. However, recent theories for both clean8 and disordered13 graphene predict exactly the opposite behavior, arguing that the relaxation rate should increase as the DP is approached. Particular attention has focused on ref 13 in which this surprising behavior was obtained by considering how disorder-mediated scattering may relax momentum-conservation constraints, allowing the full thermal-phonon distribution to be utilized in cooling. These “super-collisions” were found to become particularly effective as the DP is approached, where the Bloch-Gruneisen temperature becomes vanishingly small and one effectively enters the regime of high-temperature phonon scattering. Recent experiments performed using pn-junction photocurrents27 and noise thermometry28 appear to provide support for this mechanism.

As has long been known from the study of conventional semiconductors (see refs 30−32, for example), a useful mechanism for investigating hot-carrier cooling involves using a large measurement current to raise the effective carrier...
temperature ($T_F$) above that of the lattice ($T_L$). In this Letter, we make use of this approach to investigate energy relaxation in graphene, working primarily in the high-density limit ($T_{BG} > T_L$) appropriate for many electronic-device applications. By determining the energy-relaxation time from the dependence of $T_F$ on input power, we demonstrate a pronounced increase in the cooling rate (i.e., $\tau_e \to 0$) as the DP is approached from either the electron or hole bands. While this behavior is suggestive of recent theories for energy relaxation in graphene, our observations imply a stronger quantitative variation than that predicted by these models. At the same time, neither the variation of the power-loss rate nor of the relaxation time with temperature is found to be consistent with their predictions. Our results therefore suggest that the understanding of carrier cooling in this material is still incomplete, and we point to the known strong incompressibility of the electron–hole system at the DP as a possible reason for this. The incompressibility implies that excitation of carriers out of electron and hole puddles should be suppressed as the DP is approached, requiring rapid energy relaxation that will have important consequences for the design of future devices that operate far from equilibrium in this material.

Our graphene devices were fabricated by exfoliating Kish graphite onto a doped Si substrate with a 300 nm SiO$_2$ cap layer. Further details are provided in ref 35 (where we investigated mesoscopic conductance fluctuations (CF) in monolayer and bilayer flakes) and in the Supporting Information where we demonstrate the basic characterization of our devices. Here, we explore nonlinear transport in three new devices, one of which (M1) was realized from a monolayer flake, while the other two (B1 and B2) were formed in bilayer graphene. (Assignments made on the basis of optical microscopy and Raman spectroscopy.) The devices were contacted with Cr/Au electrodes as indicated in Figure 1a (B1 and B2 had a similar geometry). Key transport parameters for the three devices are summarized in Table 1. Carrier density ($n$) was varied by means of the voltage ($V_g$) applied to the conductive Si substrate, and in device M1 Hall measurements were used to reliably determine its value down to a level of $\sim 4 \times 10^{11}$ cm$^{-2}$ (see Supporting Information). A simple parallel-plate capacitor expression was found to yield values within 10% of those obtained from the Hall studies, giving us confidence in applying this formula to devices B1 and B2. In the figures that follow, error bars in the density primarily reflect the difference between these two approaches. All conductance measurements were made by low-frequency (13 Hz) lock-in detection, using a steady ac current whose RMS value was varied from 100 nA to 20 $\mu$A.

Previously, we showed that the linear conductance of our graphene devices exhibits mesoscopic fluctuations at low temperatures, whose amplitude is suppressed with increasing $T_L$ [35,36] (also see the Supporting Information, Figures S1–S3). This is also illustrated in Figure 1b in which we show fluctuations at a number of different lattice temperatures (with fixed current of 100 nA). In contrast to this, in Figure 1c we show corresponding features as a function of current (at $T_L = 1.8$ K). For the purpose of CF thermometry, we determine the RMS fluctuation ($\delta g_{rms}$ in units of e$/h$) for each data set by averaging over a small range of $V_g$ that nonetheless contains several fluctuations. By plotting $\delta g_{rms}(T_L)$ and $\delta g_{rms}(I)$ on a common graph, we assign an effective $T_F$ to each value of the current [36] (see Supporting Information). From the average resistance over the same range of $V_g$, we then determine the input power required to achieve a given $T_F$, comparing the form of this with theoretical predictions. For an independent estimate of $T_F(I)$, we were also able to make use of the temperature dependence of the background conductance (which can be seen in the raw data shown in the Supporting Information), which provided values for $T_F$ in close agreement with those determined from the CF. In the figures that follow, error bars in $T_F$ predominantly reflect the difference between these two approaches.

In discussions of conventional semiconductors, information on energy relaxation is typically obtained from the energy-loss rate ($P_e$), the average rate of energy loss per carrier [37] (we give a review of this in the Supporting Information). An important factor in this discussion is the $T_{BG}$, the value of $T_F$ at which the energy of the largest-wavevector phonon is comparable to the carrier temperature.
Since the largest-momentum phonon that can be emitted by the carriers is equal to \(2k_F\), this temperature is defined as \(T_{BG} = 2\hbar s k_F/k_B\) where \(s\) is the sound velocity and \(k_F\) is the Fermi radius. Taking \(s = 2.1 \times 10^4\, \text{ms}^{-1}\) for graphene, \(T_{BG} \sim 54 n^{-1/2}\) K (with \(n\) in units of \(10^{12}\, \text{cm}^{-2}\)). As will be seen below, this implies that our experiments are performed exclusively in the low-temperature (\(T_e < T_{BG}\)) limit, where \(P_e\) usually is related to the electron and lattice temperatures as

\[
P_e = A(T_e^p - T_L^p)
\]

with constants \(A\) and \(p\) dependent upon the specific scattering mechanisms that govern energy relaxation. In the inset to Figure 2a, we plot the variation of \(P_e\) for device B1 as a function of \(T_e\) for various gate voltages. The dotted lines in this log-log plot indicate different power-law variations of \(T_e\), namely \(p = 2\) (predicted for cooling via out-diffusion\(^2\)), 3, and 4 (for acoustic phonon processes\(^7\,\,^{10}\)). Clearly, the data follow neither a \(T^2\) nor \(T^3\) variation, appearing close instead to a \(1/\ln(n)\) function (indicated) and at \(T_e = 1.8\) K. The dotted line corresponds to a linear relation between the axes. Inset: same data as in the main panel, replotted with \(T_e\) as the abscissa. Dotted lines indicates various power-law relations. (b) Carrier power loss \(P_e\) as a function of \((T_e^3 - T_L^3)\) for all three devices at an electron density of \(4 \times 10^{11}\, \text{cm}^{-2}\) and \(T_L = 1.8\) K. (c) Variation of carrier temperature as a function of supplied power for device B1 at various gate voltages (indicated).

Figure 2. (a) The main panel plots the carrier power loss \(P_e\) as a function of \((T_e^p - T_L^p)\) for device B1 at different gate voltages (indicated) and at \(T_e = 1.8\) K. The dotted line corresponds to a linear relation between the axes. Inset: same data as in the main panel, replotted with \(T_e\) as the abscissa. Dotted lines indicates various power-law relations. (b) Carrier power loss \(P_e\) as a function of \((T_e^3 - T_L^3)\) for all three devices at an electron density of \(4 \times 10^{11}\, \text{cm}^{-2}\) and \(T_L = 1.8\) K. (c) Variation of carrier temperature as a function of supplied power for device B1 at various gate voltages (indicated).

Figure 3. (a) Computed energy relaxation time for device B2 at \(T_e = 1.8\) K, at various measurement currents (indicated). (b) Comparison of the density-dependent energy relaxation time for the three different devices. The data in this figure were obtained for a fixed current density of \(2\, \mu\text{A/μm}\) per graphene layer, for which \(T_e \sim 11, 33,\) and \(30\) K for devices M1, B1, and B2, respectively. (c) Comparison of the energy-relaxation rate (top) and supercollision enhancement factor \(F\), this temperature is defined as \(T_{BG} = 2\hbar s k_F/k_B\) where \(s\) is the sound velocity and \(k_F\) is the Fermi radius. Taking \(s = 2.1 \times 10^4\, \text{ms}^{-1}\) for graphene, \(T_{BG} \sim 54 n^{-1/2}\) K (with \(n\) in units of \(10^{12}\, \text{cm}^{-2}\)). As will be seen below, this implies that our experiments are performed exclusively in the low-temperature (\(T_e < T_{BG}\)) limit, where \(P_e\) usually is related to the electron and lattice temperatures as

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function of density, for several measurement currents. (The shading denotes the region near the DP where the value of \(n\) cannot be verified directly from Hall measurements.) While results are shown for device B2, the variations in Figure 3a are quite generic to all three devices as can be seen from the comparison in Figure 3b. While it is tempting to conclude from this figure that the relaxation rate is typically faster in bilayer samples, caution is required here since these data were obtained for fixed current density and so do not correspond to equivalent \(T_e\) (see caption). In fact, we have thus far not been able to infer any conclusive dependence of the relaxation rate on layer structure. Further studies of larger ensembles of samples are likely required to resolve this issue, and for now the main feature that we emphasize is that energy relaxation in both

\[
P_e = \frac{I^2 R}{nWL} = \frac{k_B(T_e - T_L)}{\tau_e}
\]

where \(W\) and \(L\) are the width and length of the graphene flake, respectively. In Figure 3a, we plot the variation of \(\tau_e\) as a function of density, for several measurement currents. (The shading denotes the region near the DP where the value of \(n\) cannot be verified directly from Hall measurements.) While results are shown for device B2, the variations in Figure 3a are quite generic to all three devices as can be seen from the comparison in Figure 3b. While it is tempting to conclude from this figure that the relaxation rate is typically faster in bilayer samples, caution is required here since these data were obtained for fixed current density and so do not correspond to equivalent \(T_e\) (see caption). In fact, we have thus far not been able to infer any conclusive dependence of the relaxation rate on layer structure. Further studies of larger ensembles of samples are likely required to resolve this issue, and for now the main feature that we emphasize is that energy relaxation in both

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monolayer and bilayer graphene becomes faster as the DP is approached.

Focusing on the behavior at any fixed \( n \) in Figure 3a, \( \tau_e \) shows a tendency to decrease with increasing current. That is, cooling occurs more quickly at higher currents, which can be understood by considering the temperature dependence of \( \tau_e \) shown in Figure 4. In Figure 4a, we plot \( \tau_e(T_e) \) for device B2 on other. An alternative way of representing this is presented in the inset to Figure 4b, which was obtained from DP data by determining for each device the value of \( \tau_e \) and \( T_e \) at a measurement current of 1 \( \mu A \). Plotting these data on a common graph, we once again obtain a variation close to \( \tau_e \propto T^{−3/2} \).

By far the most striking feature of our data in all three devices and at all measurement currents is the decrease of the relaxation time by more than an order of magnitude that occurs as the DP is approached from either side (Figures 3a,b). Such behavior is actually reminiscent of recent theories for energy relaxation due to acoustic-phonon scattering, which yield a relaxation time that varies close to \( \tau_e \propto n^{-1/2} \) in both clean and disordered graphene. In ref 8, this conclusion was reached after applying a low-temperature approximation to the Fermi distribution and calculating carrier cooling via unscreened acoustic deformation-potential coupling. In the limit of vanishing transferred phonon momentum, the energy-loss rate was found to vary as \( \sim E_F/n \propto n^{-1/2} \) (where \( E_F \) is the Fermi energy), diverging as the DP is approached. This surprising result was explained by noting that the relaxation rate \( \tau_e^{-1} \approx (1/C_e)(\partial P/\partial T_e) \), where \( C_e \) is the specific heat and is therefore proportional to the density of states. Since the latter vanishes as the DP is approached in graphene, the relaxation rate should therefore simultaneously diverge (i.e., \( \tau_e \rightarrow 0 \)). A similar conclusion was reached, albeit within a very different model in ref 13 where impurity-mediated “supercollisions” were also predicted to yield an energy-loss rate \( \tau_e \propto T^{−3/2} \), reminiscent of the behavior in Figure 2. Despite this, there are a number of other factors that suggest that the variations of \( \tau_e \) that we observe are inconsistent with the theories of either refs 8 or 13. First, the quantitative scaling of \( \tau_e(n) \) that we find near the DP is stronger than the \( n^{1/2} \) dependence predicted in refs 8 and 13 (see Supporting Information for more details). Second, while supercollisions are predicted to dominate at high temperatures (\( T_L > T_{BG} \)), our experiments are actually performed in the opposite regime (\( T_L < T_{BG} \)) where they should not be dominant. To emphasize this point, we determine the enhancement factor (\( F \))

\[
F = \frac{0.77 (T_L^2 + T_L T_{BG} + T_{BG}^2)}{k_B l / T_{BG}}
\]

where \( l \) is the mean-free path. In Figure 3c, we plot the energy-relaxation rate \( (\tau_e^{-1}) \) along with the variation of \( F \) determined for our data. Results are plotted for two measurement currents and, while the qualitative variations of \( \tau_e^{-1} \) follow those of \( F \) closely, the value of \( F \) is always less than one. In other words, for the range of densities that we consider supercollisions would only be expected to lead to no more than a doubling of the relaxation rate, a change that is too small to account for the order-of-magnitude variations of \( \tau_e \) in Figure 3. Consistent with this, we also note that the power-law decay of \( \tau_e \) \( (\alpha T^{−3/2}) \) in Figure 4 is distinct to that \( (\alpha T^{−1}) \) reported for supercollisions.

Having suggested that our results are inconsistent with existing theories for phonon-mediated energy relaxation, we note that the presence of the CF we use as a thermometer points to the disordered nature of the graphene. In the presence of this, the system is known to break up into puddles of localized electrons and holes, characterized by strong incompressibility at and around the

![Figure 4](image-url)
DP. The incompressibility indicates that the puddle state becomes almost “frozen” into the system, much as the vapor pressure is reduced over a frozen liquid. Since the power-per-carrier increases as the DP is approached, the suggestion is that the associated decrease of $\tau_0$ is somehow a self-consistent consequence of the incompressibility. The essential point is that the decrease of $\tau_0$ avoids a divergence of $T_\text{e}$ (see eq 2), which would correspond to strong delocalization of carriers out of their puddles, violating the notion of their incompressibility.

In conclusion, we have studied current-induced heating of carriers in graphene, determining the energy-relaxation time on both the electron and hole sides of the Dirac point. While the power law behavior of the power input appears suggestive of a connection to supercollision scattering, the size of the effect and the dependence of the relaxation time on temperature and density do not support this process. The dramatic collapse of the relaxation time near the Dirac point may be consistent with ideas of charge puddling, by suggesting that it becomes very difficult to excite carriers out of these localized regions. While we are unable to comment at this stage on the incompressibility of their puddles, violating the notion of their incompressibility.


**REFERENCES**

(34) Fuhrer, M. S. *Physics 2010*, 3, 106.


