Sample Mean

How to estimate from experimental measurements the statistical mean of a R.V.

Example: β-decay of radioactive source. On physical grounds: $p_K(k) = \frac{1}{k!}\lambda T^k e^{-\lambda T}, \quad k = 0, 1, 2, \ldots$, but how to determine λ? $E[K] = \lambda T$, where λ is mean # of carbon atoms that decay per unit time. How to estimate?

Example: A product with a time to failure pdf $\boxed{f_T(t) = \lambda e^{-\lambda t} u(t)}$, how to estimate λ? $E[T] = 1/\lambda$.

Consider a R.V. X. Assume $\overline{X}$ exists, and variance = σ_X^2.

Experiment: make *n* independent measurements or observations of $\overline{X}$. Let X_1 be the first measurement, X_2 the second,..., X_n the nth measurement.

Define *Sample Mean* by $M_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. This is a R.V. Is it a good estimate of $\overline{X}$

?

$E[M_n] = \frac{1}{n}\sum_{i=1}^{n} E[X] = \frac{n\overline{X}}{n} = \overline{X}$. Then M_n is an *unbiased estimate* of $\overline{X}$.

Also $var(M_n) = var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = n\frac{\sigma_X^2}{n^2} = \sigma_X^2/n$. Then $\sigma_{M_n} = \frac{1}{\sqrt{n}}\sigma_X$, and Relative Error $= \frac{\sigma_{M_n}}{\overline{M_n}} = \frac{1}{\sqrt{n}}\frac{\sigma_X}{\overline{X}}$