Random Variables (Continued)

Continuous R. V.

Define $F_X(x) = P(\{X \le x\})$, and $f_X(x) = \dfrac{d}{dx}F_X(x) =$ **Probability Density Function.**

$$P(\{a < X \le b\}) = F_X(b) - F_X(a) = \int_a^b f_X(x)\ dx.$$

Properties of $f_X(x)$:

1- $f_X(x) \ge 0$

2- $\int_{-\infty}^{\infty} f_X(x) = 1$

Examples of Density Functions

Uniform Probability Density:

$$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & \text{for } a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

Exponential Probability Density:

$$f_T(t) = \begin{cases} ae^{-at}, & \text{for } 0 \le t \\ 0, & \text{otherwise} \end{cases}$$

Normal Probability Density Function:

$$f_X(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$$