MAE 552
Heuristic Optimization

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Lecture #16

3/1/02

Taguchi's Orthogonal Arrays

Roulette wheel selection

- Implementation
  - The roulette wheel can be constructed as follows.
    - Calculate the total fitness for the population as the sum of the fitness of each member.

$$F = \sum_{i=1}^{n} f_i$$

Roulette wheel selection

- Implementation
  - The roulette wheel can be constructed as follows.
    - Next, calculate selection probability p_i for each member

$$p_i = \frac{f_i}{F} \qquad i = 1,n$$

Roulette wheel selection

- Implementation
  - The roulette wheel can be constructed as follows.

    - Then, calculate cumulative probability q_k for each member

$$q_i = \sum_{j=1}^{i} p_j \qquad i = 1,n$$

Roulette wheel selection

- The resulting values of q_i will all lie in the range [0, 1]. To actually perform the selection:
  - sort the designs by increasing q_i
  - generate a random number $r = U[0, 1]$
  - select the first design with a q_i higher than r.

Repeat this step until your next population is full.

Background

Taguchi laid the foundation for his Robust Design approach in the 50’s and 60’s.

Since then, the approach has been validated by years of successful application.

What is Robust Design?

Background

Robust Design (as presented here) is an engineering methodology for improving productivity during R & D so that high quality products can be produced quickly and at a low cost.

How does this approach fit into the class of Heuristic Optimization methods?

Background

A focus of this approach is on generating information about how different design parameters affect performance under different usage conditions.

Robust design enables an engineer to generate information necessary for decision-making with less (~half) experimental effort.

Background

The 2 primary tasks performed in Robust Design are:

1. Measurement of Quality during design / development.
   – We want a leading indicator of quality by which we can evaluate the effect of changing a particular design parameter on the products performance.

Background

2. Efficient experimentation to find dependable information about the design parameters.
   - It is essential to obtain dependable information about the design parameters so that design changes during manufacturing and customer use can be avoided. The information should be obtained with minimum time and recources.

Background

So you can tell from the last few slides that we intend to alter some parameters to achieve some goal. This is the very essence of optimization (at least as we know it in this class and in 550).

As indicated in the title of this presentation, we will implement orthogonal matrix experiments in this approach.

Matrix Experiments

A matrix experiment consists of a set of experiments where the settings of various product or process parameters are changed one by one to study their effect.

Conducting matrix experiments using orthogonal arrays allows the effects of several parameters to be determined efficiently.

Orthogonal Arrays

Ok, so what is an orthogonal array?

An orthogonal array has the property that all columns are mutually orthogonal where orthogonality is interpreted in the combinatoric sense. That is, for any pair of columns, all combinations of factor levels occur and they all occur an equal number of times.

Factors and Levels

Ok, so what is a factor level?

A factor is a parameter over which we have some amount of control. In these experiments, all factors are discretized into levels.

So for example, if a factor in our process were temperature, some levels may be 10º, 15º, 20º, etc.

Orthogonal Arrays

Going back to Orthogonal Arrays, an example of an array that accommodates 3 factors with 2 levels each is shown to the right.

L_4 (2^3)	**Factor**		
Exp #	**1**	**2**	**3**
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Orthogonal Arrays

We saw in the previous example that an array with 3 factors at 2 levels each required 4 experiments. In the upper left corner we say the designation of the array written as :

$$\mathbf{L_4\ (2^3)}$$

In general, the designation of an Orthogonal Array is given by:

$$L_{\#\ \text{exps}}\ (\#\ \text{Levels}^{\#\ \text{factors}})$$

Orthogonal Arrays

As previously mentioned, each pair of columns must contain all combination of factors and at an equal frequency, lets see if this is the case for our array.

	1	**2**	**3**
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Orthogonal Arrays

As previously mentioned, each pair of columns must contain all combination of factors and at an equal frequency, lets see if this is the case for our array.

	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Pairing our first and second columns, we see that each possible combination appears once and only once. That is, we have:

1, 1

1, 2

2, 1

2, 2

Orthogonal Arrays

As previously mentioned, each pair of columns must contain all combination of factors and at an equal frequency, lets see if this is the case for our array.

	1	**2**	**3**
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Now, pairing our second and third columns, we see that each possible combination again appears once and only once. That is, we have:

1, 1

2, 2

1, 2

2, 1

Orthogonal Arrays

As previously mentioned, each pair of columns must contain all combination of factors and at an equal frequency, lets see if this is the case for our array.

	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Finally, pairing our first and third columns, we see that each possible combination again appears once and only once. That is, we have:

1, 1

1, 2

2, 2

2, 1

Example

With all these things said, let's get into an example

We are interested in determining the effect of 4 process parameters on the formation of certain surface defects in a chemical vapor deposition (CVD) process. The 4 parameters (factors) are:

A) Temperature

B) Pressure

C) Settling Time

D) Cleaning Method

Example

Each of our factors will have 3 levels as shown in the following table:

Factor	Levels		
	1	2	3
A: Temp	T_0-25	T_0	T_0+25
B: Press	P_0-200	P_0	P_0+200
C: Set Time	t_o	t_o+8	t_o+16
D: Clean	None	CM_2	CM_3

Red indicates the starting level for each factor.

Example

So for a problem with 4 factors at 3 levels each, we need an L (3^4) array. We see from our handout, that such an array does exist and that it is the L_9 array (shown in part below).

Example

	A	B	C	D
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Example

How Do we fill in the matrix?

Each entry in the matrix represents a factor level. Specifically, a level for the factor which heads the column. So we can fill in values from our table of factor levels (a few slides back).

Example

	A	B	C	D
1	T_0-25	P_0-200	t_o	None
2	T_0-25	P_0	t_o+8	CM_2
3	T_0-25	P_0+200	t_o+16	CM_3
4	T_0	P_0-200	t_o+8	CM_3
5	T_0	P_0	t_o+16	None
6	T_0	P_0+200	t_o	CM_2
7	T_0+25	P_0-200	t_o+16	CM_2
8	T_0+25	P_0	t_o	CM_3
9	T_0+25	P_0+200	t_o+8	None

Example

So we can see that each row of the matrix defines an entire configuration of our product or process.

Now, we want to use this matrix experiment in some way to determine the best setting for each parameter such that our surface defects are minimized.

The first thing we need to do is determine how we will compute our observation value (this is akin to determination of our fitness in a GA).

Example

In our example, we are concerned with defects on the surface of a silicon wafer. So in order to determine the relative performance of each of our configurations, we will do the following.

- Set up the CVD equipment according to the parameters
- Create a bunch of chips
- Count the defects in 3 areas on each of 3 of our produced chips for a total of 9 counts per experiment.

Example

We can then define a summary statistic, η_i, that is computed as follows (for experiment i):

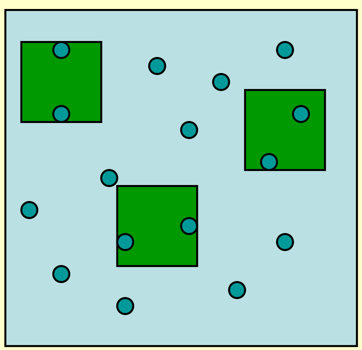
$$\eta_i = -10 \log_{10} (\text{mean square defect count}_i)$$

The mean square defect count is the average of the squared-counts in each of the 9 areas for each experiment. We will then call eta our observation value. Does anyone recognize this formulation?

It is the signal to noise (S/N) ratio.

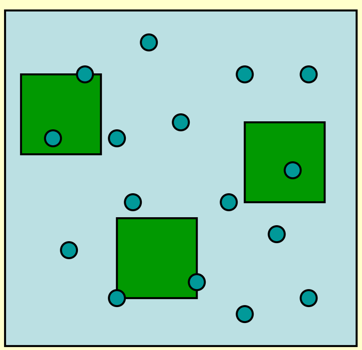
Example

So say for example that in one of our experiments, we had the following 3 chips:



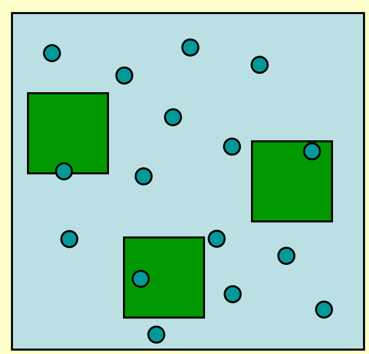
Counts:

1, 1, 1



Counts:

2, 2, 2



Counts:

2, 2, 1

Example

We can calculate η for this experiment as follows:

Mean squared defect count:

$$(2^2 + 2^2 + 2^2 + 1^2 + 1^2 + 1^2 + 2^2 + 2^2 + 1^2) / 9 = 3.11$$

The observation value is then:

$$\eta = -10 * \mathrm{Log}_{10}(3.11) = -4.93$$

Clearly, minimizing our surface defects becomes a job of maximizing our observation value.

Example

	A	B	C	D	η
1	T_0-25	P_0-200	t_o	None	**-20**
2	T_0-25	P_0	t_o+8	CM_2	**-10**
3	T_0-25	P_0+200	t_o+16	CM_3	**-30**
4	T_0	P_0-200	t_o+8	CM_3	**-25**
5	T_0	P_0	t_o+16	None	**-45**
6	T_0	P_0+200	t_o	CM_2	**-65**
7	T_0+25	P_0-200	t_o+16	CM_2	**-45**
8	T_0+25	P_0	t_o	CM_3	**-65**
9	T_0+25	P_0+200	t_o+8	None	**-70**

Example

Clearly we are not done, simply conducting the experiments and selecting the best one would make this a somewhat trivial approach.

What we have to do now is attempt to determine the effect that each factor has on the observation value based on what we observed in our matrix experiment.

Example

The first step will be to find the overall mean of the observation values *m*.

$$m = \frac{1}{9}\sum_{i=1}^{9}\eta_i = -41.67$$

Since each of our factor levels was represented evenly in our matrix experiment, m in this case is referred to as a balanced overall mean.

Example

The effect of a factor level is defined as the deviation it causes from the overall mean.

For example, we may wish to evaluate what effect temperature at level 3 has on the process.

To do this, we must calculate mean of the observation values in which temperature was at level 3. Note that there were 3 such experiments in our matrix experiment.

Example

The 3 experiments containing temperature at level 3 were #'s 7, 8, and 9. So:

$$m_{A3} = 1/3\ (\eta_7 + \eta_8 + \eta_9) = -60$$

So the deviation from the overall mean caused by temperature at level 3 is:

$$(m_{A3} – m) = -60 – (-41.67) = -18.3$$

Example

Notice that in each of experiments 7, 8, and 9, pressure, settling time, and cleaning method all take on levels of 1, 2, and 3 (in different orders).

Therefore, m_{A3} represents an average η when the temperature is at level 3 where the averaging is done in a balanced manner over all levels of each of the other 3 factors.

The remaining factor level effects can be computed in the exact same fashion.