

Name \_\_\_\_\_

MAE 340: Test III

April 22, 2003

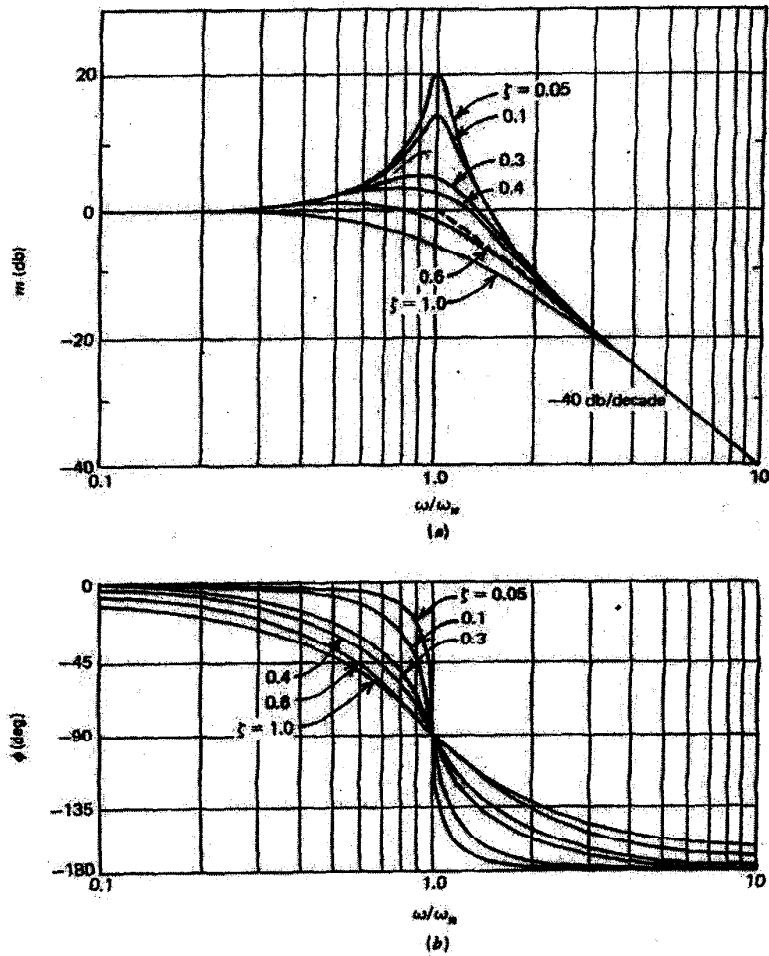
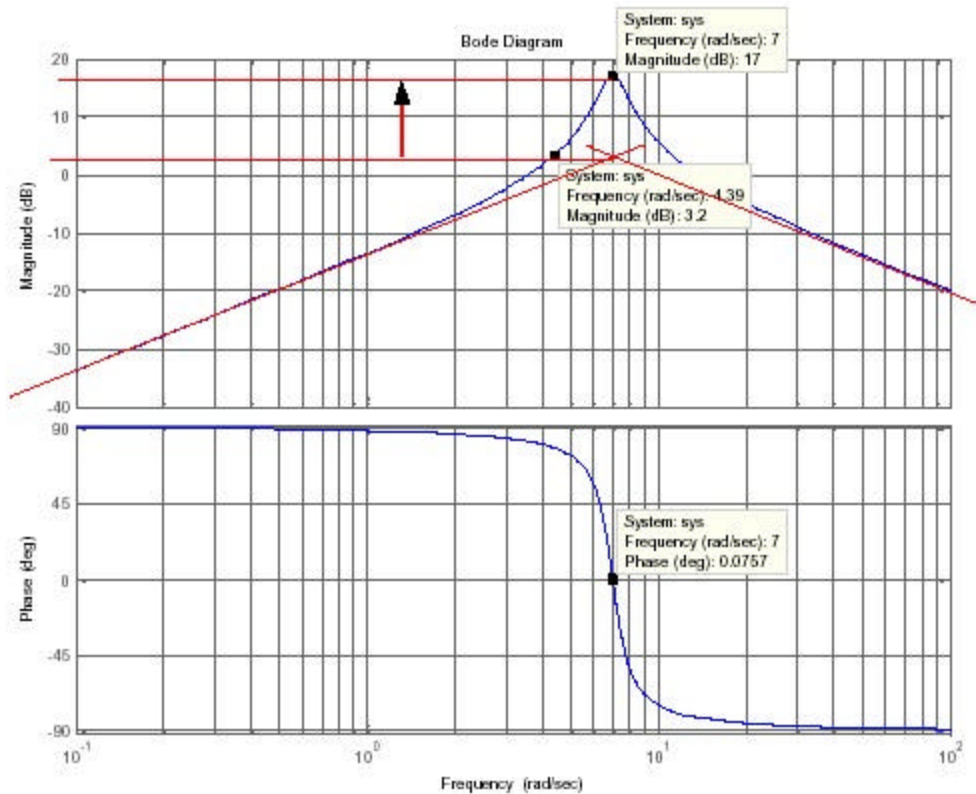


FIGURE 4.20 Frequency response plots for the underdamped system

$$T(s) = \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{2\zeta s}{\omega_n} + 1}$$

Questions 1-10 (worth 7 points each) refer to the Bode Plots on the next page:

- 1) Estimate the system transfer function.



- Final slop is -20 (db/dec), the difference of orders (Den.-Num.) is 1.
- Phase angles: +90 → 0 → -90. We have 1 “zeros” 2 “ploes”. Besides, the starting angle is +90, therefore, the “zeros” has “0” real part.
- From the peak and the arrow, we know the damping ratio is less than 0.707 and it is actually  $\alpha \cong 0.1$  (by comparing the arrow to the given reference plot).
- From 0 degree crossing (“+90” + “-90” crossing), we know  $\omega_n \cong 7$  (rad/sec).

- The Transfer Function should have the form as: 
$$\frac{K S}{\omega_n^2 \left[ \frac{S^2}{\omega_n^2} + \frac{2\alpha S}{\omega_n} + 1 \right]}$$

- For  $\omega = 0.1 \ll \omega_n$ ,

$$20 \log |FRF| \cong 20 \log \left( \frac{K}{7^2} \right) + 20 \log(\omega) - 20 \log \sqrt{\left[ 1 - \left( \frac{\omega}{7} \right)^2 \right]^2 + \left[ \frac{0.2\omega}{7} \right]^2}$$

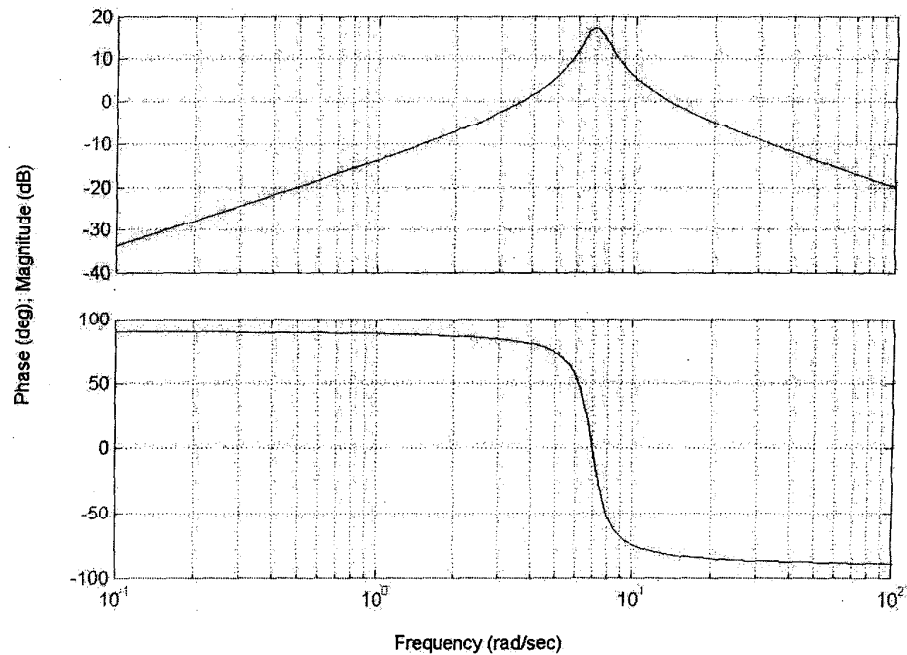
$$\cong 20 \log \left( \frac{K}{7^2} \right) + 20 \log(0.1) - 20 \log(1) \cong 20 \log \left( \frac{K}{7^2} \right) - 20 \cong -34 \text{ (db)}$$

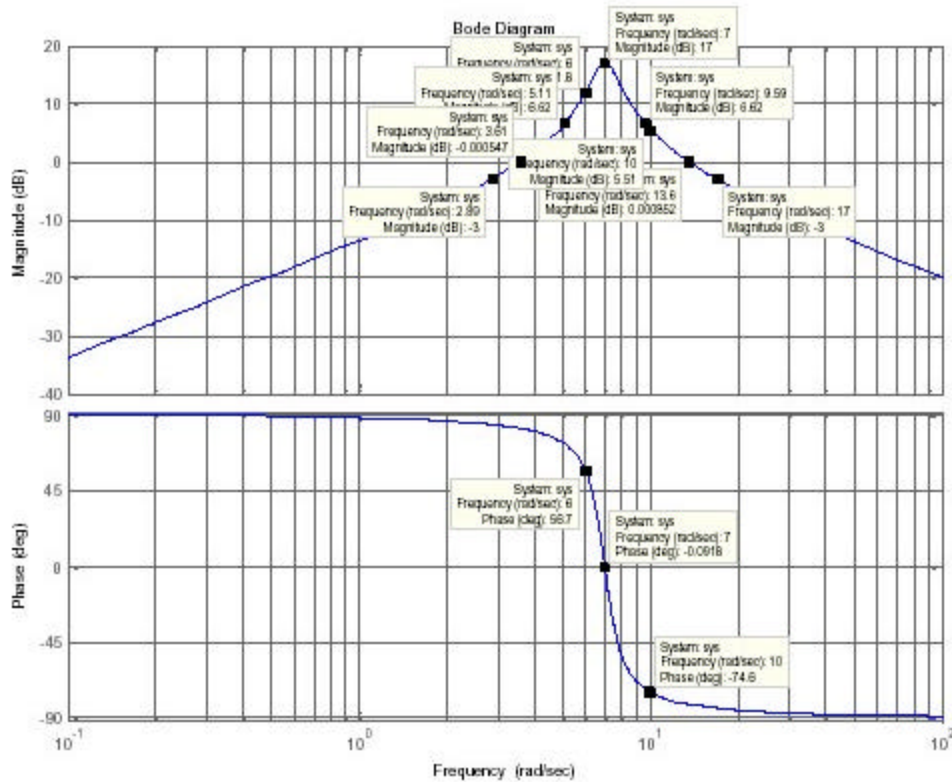
$$20 \log \left( \frac{K}{7^2} \right) = -14 \text{ (db)}, \Rightarrow K \cong 9.8. \Rightarrow T.F. \cong \frac{9.8 S}{S^2 + 1.4 S + 49}$$

- Note: The actual plot was generated by  $T.F. \cong \frac{10 S}{S^2 + 1.4 S + 49}$ .

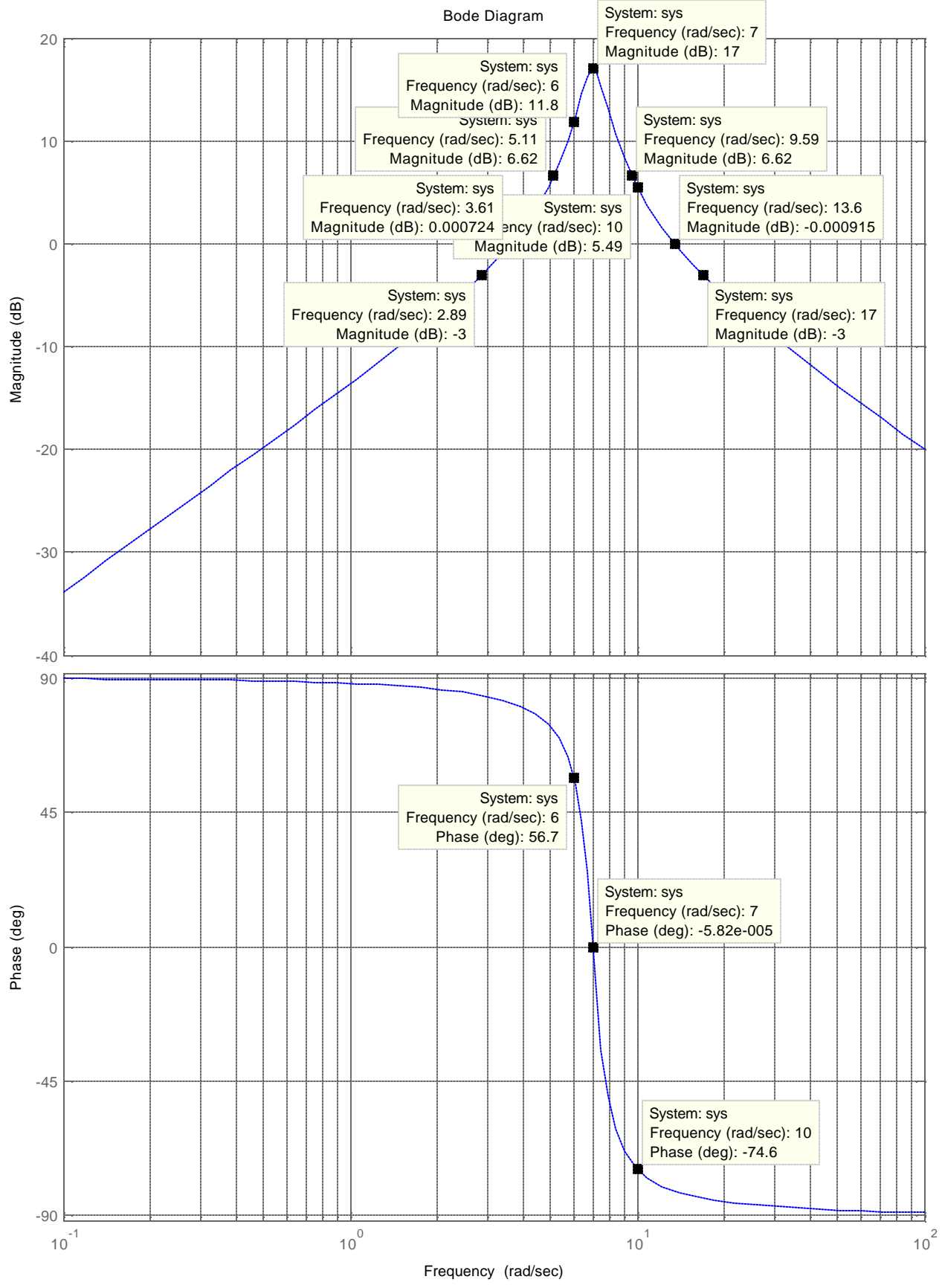
- 2) What is the output if the input is  $u = 3 * \sin(6t)$ ?
- 3) What input will produce an output of  $y = 10 * \sin(10t)$ ?
- 4) At what frequency(ies) (if any) is the output magnitude equal to the input magnitude?
- 5) Assume that the input to the system is  $u = 7 * \sin(w*t)$  , and  $w$  can vary over the range shown in the Bode plot. At what frequency (if any) is the magnitude of the output the greatest?
- 6) Assume that the input to the system is  $u = 7 * \sin(w*t)$  , and  $w$  can vary over the range shown in the Bode plot. At what frequency (if any) is the magnitude of the output equal to 15?
- 7) Assume that the input to the system is  $u = 24 * \sin(w*t)$  , and  $w$  can vary over the range shown in the Bode plot. At what frequency (if any) is the output magnitude exactly one-tenth the size of the greatest output magnitude?
- 8) At what frequency (if any) is the output exactly out of phase with the input?
- 9) At what frequency (if any) is the output exactly in phase with the input?
- 10) What is the approximate time constant of the system?

Bode Diagrams

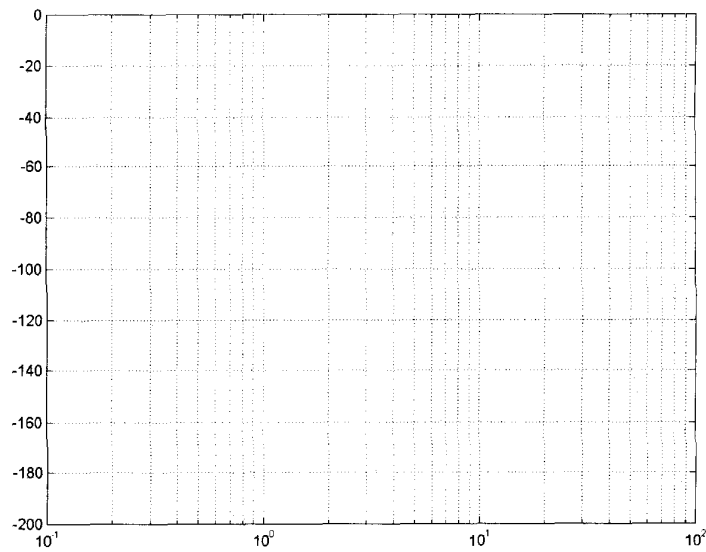
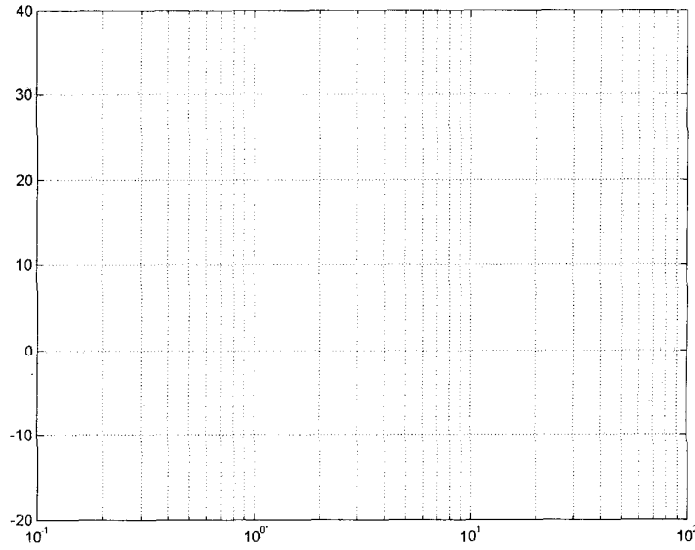




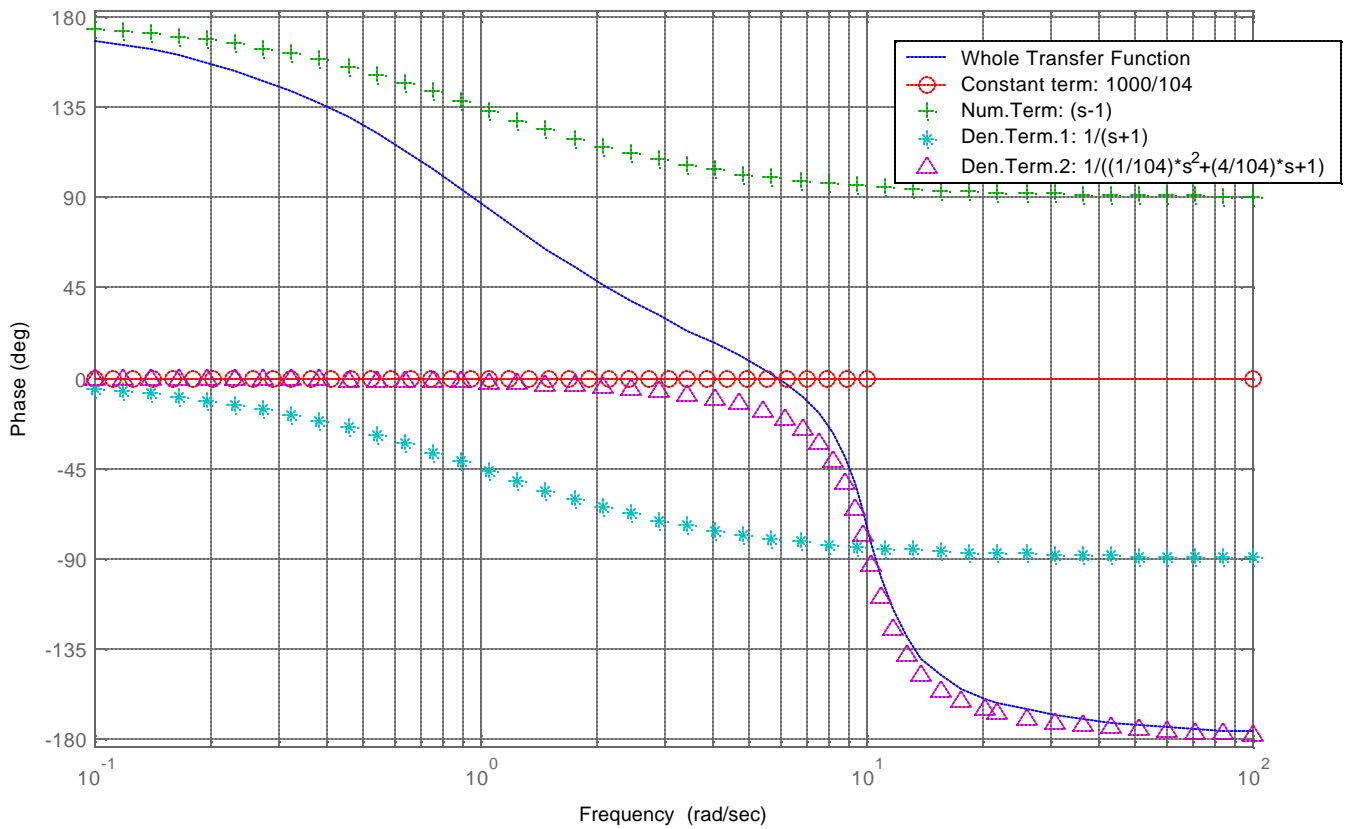
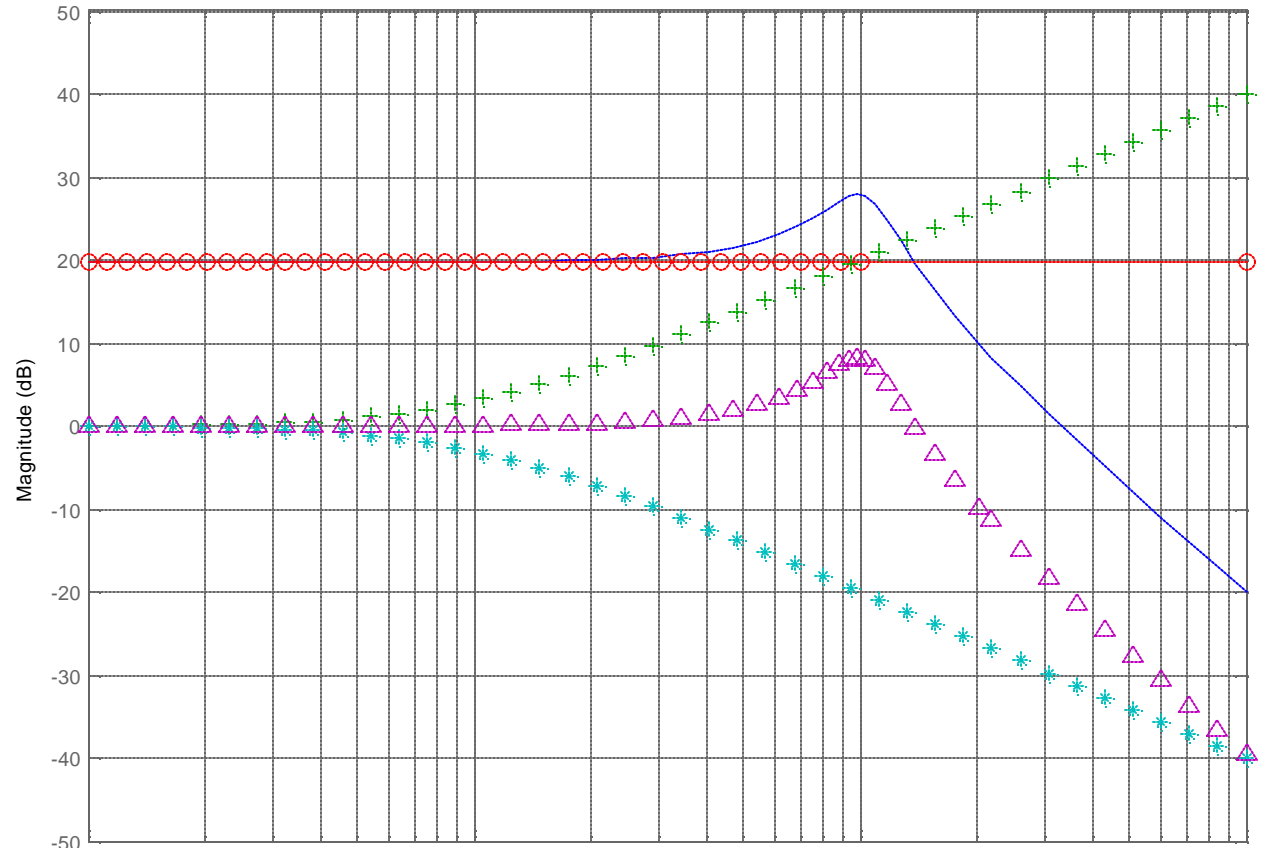
- ❖  $Output = 3 * 10^{\frac{11.8}{20}} \sin(6t + 56.7^\circ) \cong 11.67 \sin(6t + 56.7^\circ)$
- ❖  $Input = 10^{\frac{5.49}{20}} \sin(10t + 74.6^\circ) \cong 5.315 \sin(10t + 74.6^\circ)$
- ❖ Find 0 (db),  $\omega \cong 3.6$  &  $13.6$  (rad / sec)
- ❖ Peak happen @  $\omega \cong 7$  (rad / sec)
- ❖  $20 \log\left(\frac{15}{7}\right) \cong 6.62$  (db),  $\omega \cong 5.1$  &  $9.6$  (rad / sec)
- ❖  $-20$  (db) from the peak,  $\Rightarrow 17 - 20 = -3$  (db),  $\omega \cong 2.9$  &  $17$  (rad / sec)
- ❖ From the plot, there are NO exactly out of phase (180 degrees difference)
- ❖ Exactly in phase (zero degree difference),  $\omega \cong 7$  (rad / sec)
- ❖  $TimeCons \tan t \ t = \frac{1}{\omega \omega_n} = \frac{1}{0.1 \times 7} \cong 1.43$  (sec)



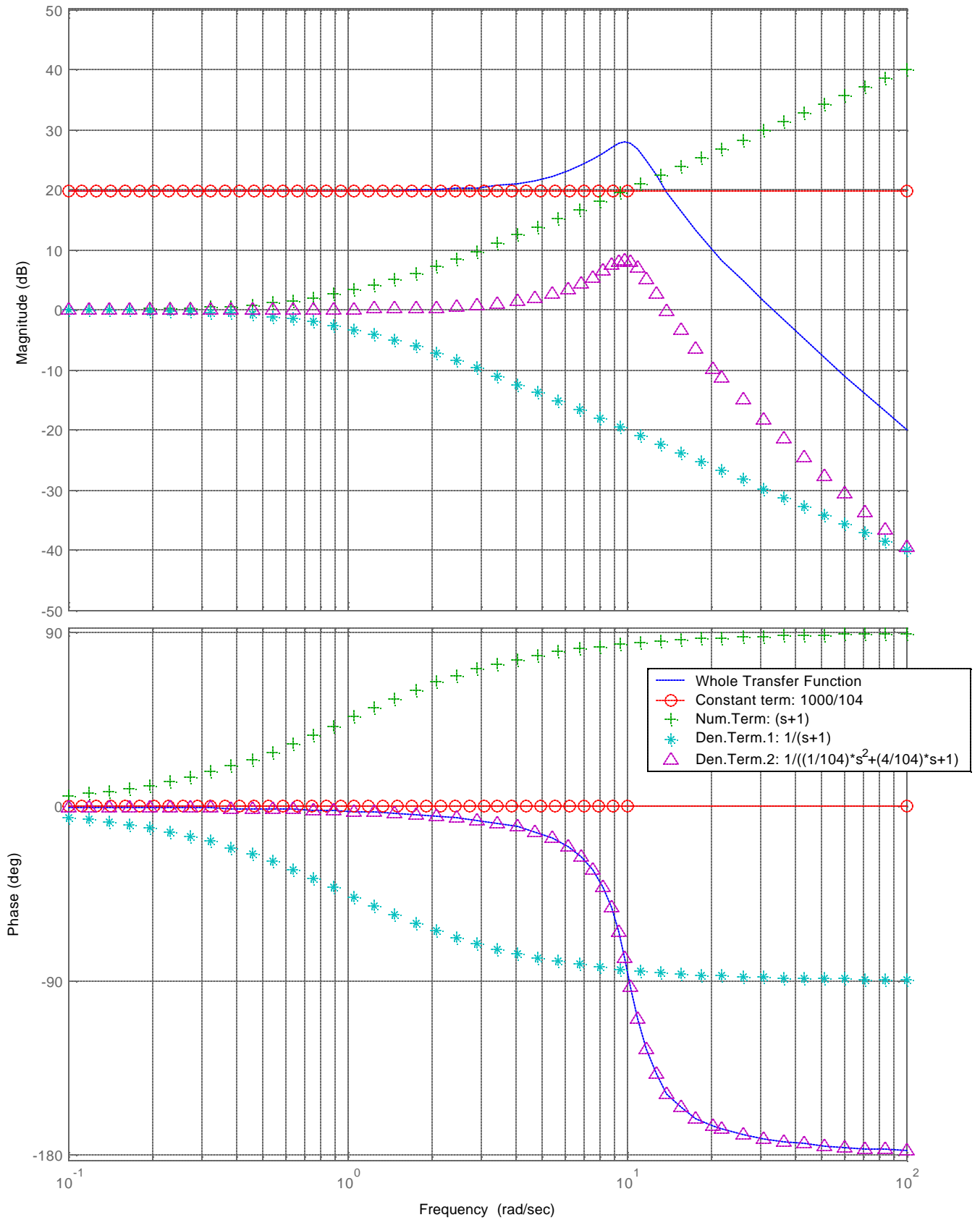
30 points: Plot (by hand) the Bode magnitude and phase angle plot for the system with a zero at  $s = 1$  and poles at  $s = -1, -2 \pm i*10$ , all multiplied by a constant of 1000.



Bode Diagram

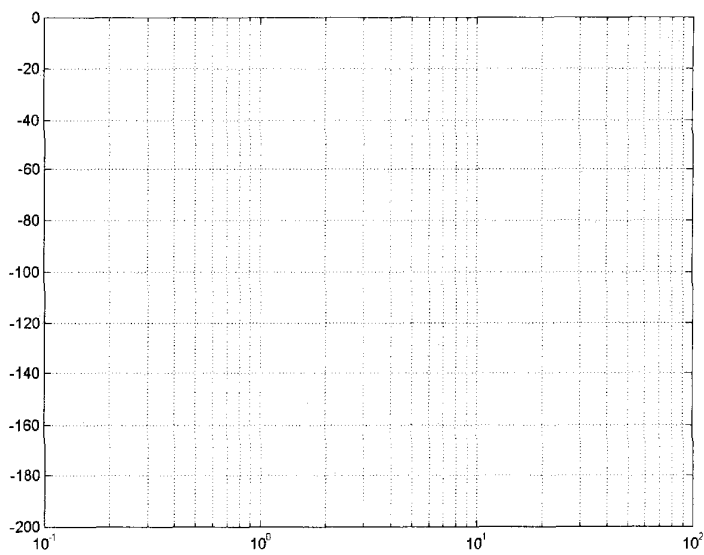
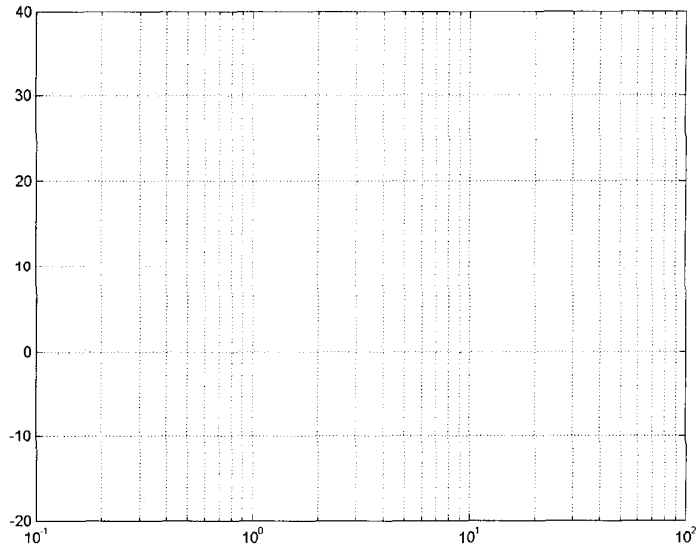


Bode Diagram

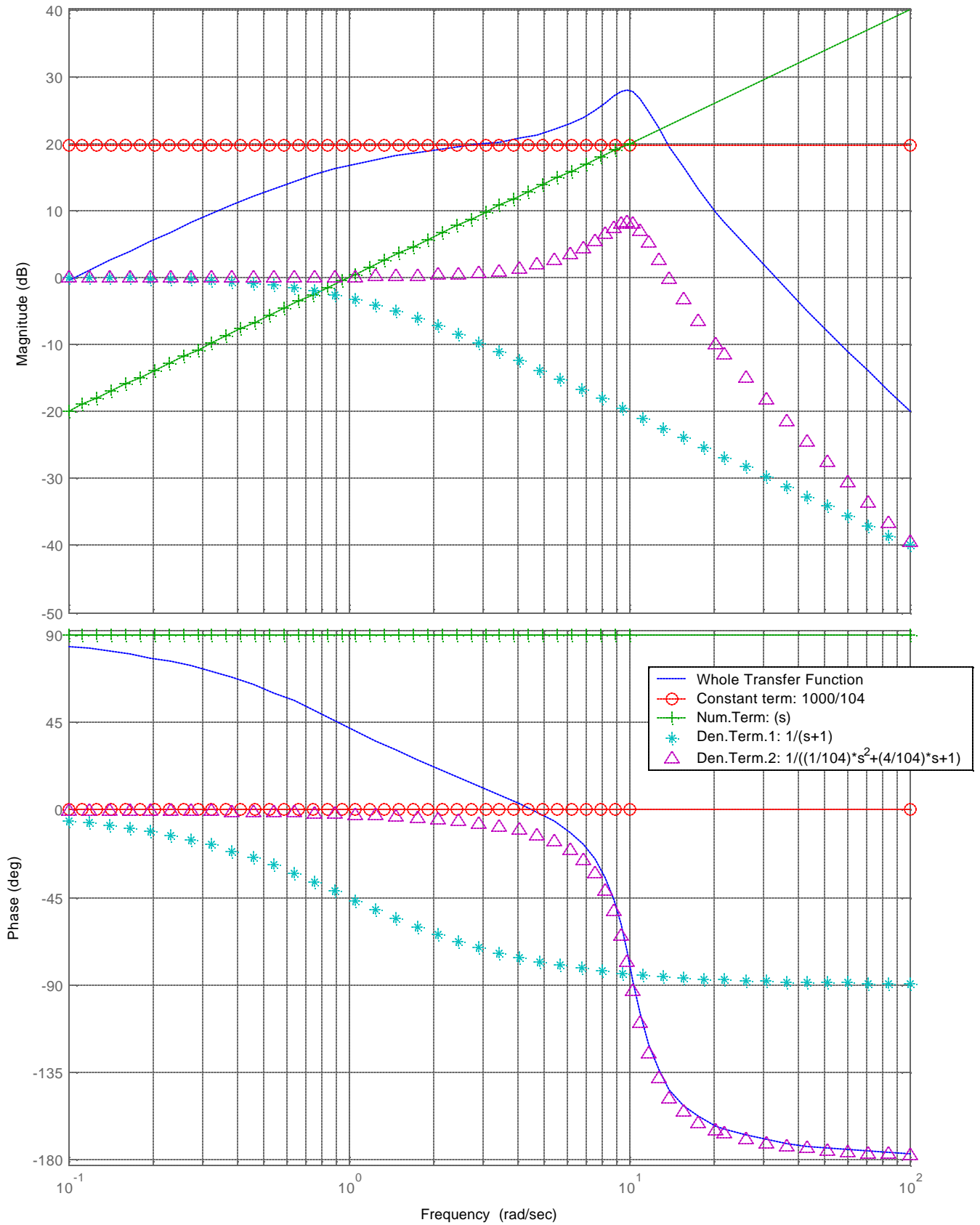


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30 points. Plot (by hand) the Bode magnitude and phase angle plot for the system with a zero at  $s = 0$  and poles at  $s = -1$ ,  $-2 \pm i*10$ , all multiplied by a constant of 1000.



Bode Diagram





Bode Diagram

