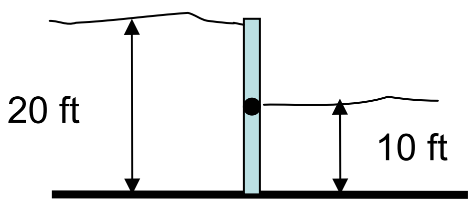
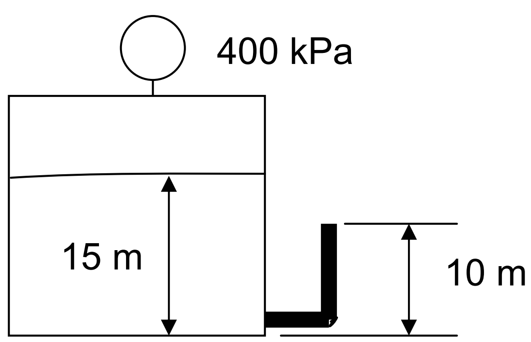
mae 335 FLUID MECHANICS Spring 2007

Name________________

1. A gate 20 ft high in a rectangular constant width channel is pivoted at its center. Water at 40 F on one side of the gate is 20 ft deep. On the other side of the gate water at 40 F is 10 ft deep. What is the magnitude, direction and location of the force required to keep the gate closed?

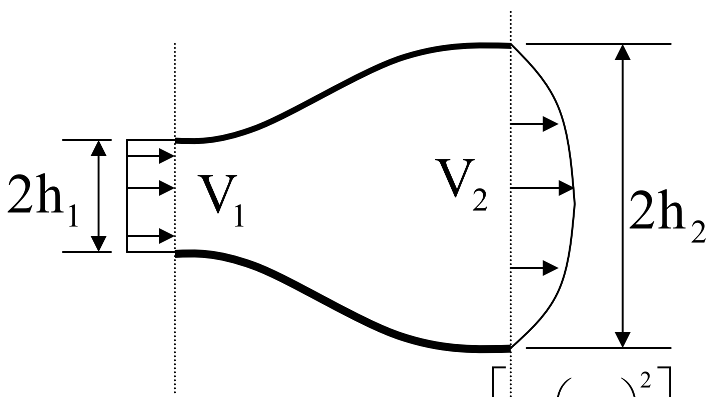


2. A closed rigid tank is filled with 20 C water to a depth of 15 m. A pressure gage in the top of the tank reads 400 kPa. The tank has an exit pipe whose exit is 10 m above the tank bottom. What is the velocity at the pipe exit? How far above the tank bottom would the pipe exit have to be to stop the flow? 1Pa=1 N/m^2, 1N= 1 kg 1 m/sec^2



mae 335 FLUID MECHANICS Spring 2007 Name________________

3. Air at a constant density enters a diffuser of rectangular cross section, $A_1 = w\ h_1$, with a uniform velocity V_1 and pressure p_1. The velocity distribution at the diffuser exit, $A_2 = w\ h_2$, is parabolic with a maximum velocity of V_2 and a pressure p_2. Determine the velocity V_2 in terms of h_1, h_2, and V_1. Derive the integral equation in terms of density, pressures, V_1, and channel heights for determining the force required to keep the diffuser from moving. Do not perform the integration and substitute limits.



$$V = V_2\left[1 - \left(\frac{y}{h_2}\right)^2\right]$$

A_1

p_1

A_2

p_2

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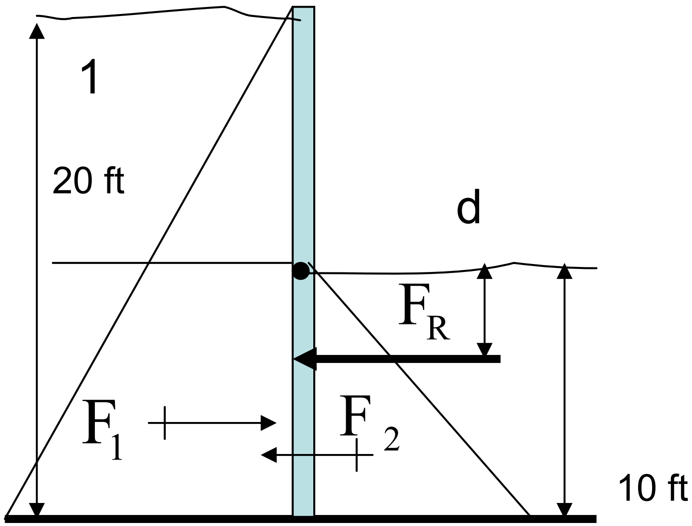


Grade	No
100	1
90	5
80	3
70	15
60	13
50	9
40	
30	3
20	1
ave	69

FORCES

$$F = \int p\, dA$$

$$F = (\rho g h_{\substack{\text{average}\\ \text{depth}}})\, w\, h_{\text{area}} = (\rho\, g\, w) h_{\substack{\text{avweage}\\ \text{depth}}} \times h_{\text{area}}$$

$$F_1 = (\rho\, g\, w)\frac{h_1}{2} \times h_1 = \frac{h_1^2}{2}(\rho\, g\, w)$$

$$F_1 = (\rho\, g\, w)\frac{h_2}{2} \times h_2 = \frac{h_2^2}{2}(\rho\, g\, w)$$

$$\sum F = F_{\text{reaction}} + F_1 - F_{23}$$

$$F_{\text{reaction}} = \rho\, g\, w\left(\frac{h_1^2}{2} - \frac{h_2^2}{2}\right) = 1.94 \times 32.2 \times w(200 - 50)$$

$$F_{\text{reaction}} = 9370\ w\ lb_f = 9370\ lb_f/f\ \text{tof width}$$

MOMENTS (+ assumed clockwise)

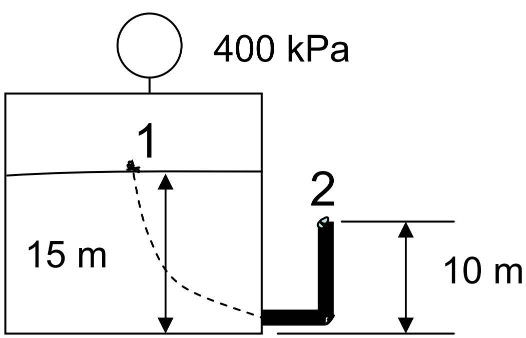
$$F_1\left(\frac{2h_1}{3} - \frac{h_1}{2}\right) - F_2\left(\frac{2h_2}{3}\right) = (\rho\, g\, w)F_{\text{Reaction}} \times d$$

$$\frac{h_1^2}{2}(\rho\, g\, w) \times \frac{h_1}{6} - \frac{h_2^2}{2}(\rho\, g\, w) \times \frac{2h_2}{3} = (\rho\, g\, w)F_{\text{Reaction}} \times d$$

$$d = \frac{666.66 - 333.33}{6} = 2.22 \text{ ft below the pivot}$$

2

A closed rigid tank is filled with 20 C water to a depth of 15 m. A pressure gage in the top of the tank reads 400 kPa. The tank has an exit pipe whose exit is 10 m above the tank bottom. What is the velocity at the pipe exit? How far above the tank bottom would the pipe exit have to be to stop the flow? 1Pa=1 N/m^2, 1N= 1 kg   1 m/sec^2



VELOCITY

$$\frac{p_1}{\rho}+\frac{V_1^2}{2}+g\,h_1=\frac{p_2}{\rho}+\frac{V_2^2}{2}+g\,h_2$$

$$p_1 = 400\text{ kPa gage, } (400+p_{atm})\text{ kPa absolute}$$

$$p_1 = 0\text{ kPa gage, } p_{atm}\text{ kPa absolute}$$

$$\frac{400{,}000\text{ N/m}^2+p_a}{998.2\text{ kg/m}^3}+0+15\,g=\frac{p_a\text{ N/m}^2}{998.2\text{ kg/m}^3}+\frac{V_2^2}{2}+10\,g$$

$$\frac{V_2^2}{2}=\frac{400{,}000}{998.2}+5\times 9.81$$

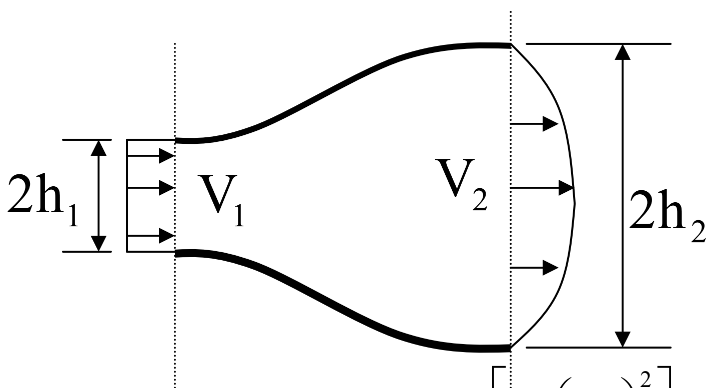
$$V = 29.9\text{ m/sec}$$

HEIGHT

$$\frac{400{,}000\text{ N/m}^2}{998.2\text{ kg/m}^3}+0+15g=h\times g$$

$$h=\frac{400{,}000}{9.81\times 998.2}+15=55.89\text{ m}$$

3. Air at a constant density enters a diffuser of rectangular cross section, $A_1 = w\ 2h_1$, with a uniform velocity V_1 and pressure p_1. The velocity distribution at the diffuser exit, $A_2 = w\ 2h_2$, is parabolic with a maximum velocity of V_2 and a pressure p_2. Determine the velocity V_2 in terms of h_1, h_2, and V_1. Derive the integral equation in terms of density, pressures, V_1, and channel heights for determining the force required to keep the diffuser from moving. Do not perform the integration and substitute limits.



$$V = V_2\left[1-\left(\frac{y}{h_2}\right)^2\right]$$

A_1

p_1

A_2

p_2

CONTINUITY

$$\rho A_1 V_1 = \int \rho V dA$$

$$\rho A_1 V_1 = 2\int_0^{h_2} \rho V\ w dy$$

$$= 2w\rho V_2 \int \left(1-\frac{y^2}{h_2^2}\right) dy$$

$$= 2w\rho V_2 \left(y-\frac{y^3}{3h_1^2}\right)_0^{h_2}$$

$$\rho w 2h_1 V_1 = 2w\rho V_2 \left(h_2-\frac{h_2^3}{3h_2^2}\right)$$

$$\rho w 2h_1 V_1 = 2w\rho V_2 \left(h_2-\frac{h_2}{3}\right)$$

$$V_2 = \frac{3}{2} V_1 \frac{h_1}{h_2}$$

$$\text{Momentum Flux}_1 = (\rho A_1 V_1)V_1 = (\rho w 2h_1)V_1^2$$

$$\text{Momntum Flux}_2 = 2\int_0^h (\rho V)Vw\,dy$$

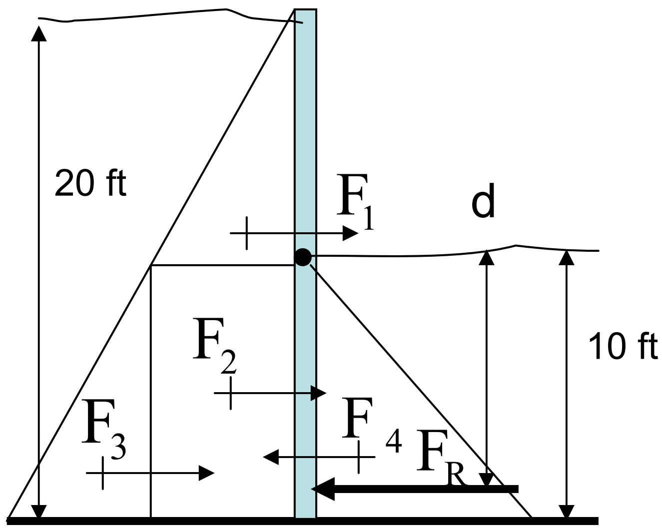
$$= 2\rho w V_2^2 \int_0^h \left(1 - \frac{y^2}{h_2^2}\right)^2 dy$$

$$= 2\rho w V_2^2 \int_0^h \left(1 - \frac{2y^2}{h_2^2} + \frac{y^4}{h_2^4}\right) dy$$

$$F = (p_2 w 2h_2 - p_1 w 2h_1) + (\rho w 2h_1)V_1^2 - 2\rho w V_2^2 \int_0^h \left(1 - \frac{2y^2}{h_2^2} + \frac{y^4}{h_2^4}\right) dy$$

$$F = (p_2 w 2h_2 - p_1 w 2h_1) + (\rho w 2h_1)V_1^2 - 2\rho w \frac{9}{4} V_2^2 \frac{h_1}{h_2} \int_0^h \left(1 - \frac{2y^2}{h_2^2} + \frac{y^4}{h_2^4}\right) dy$$

1. A gate 20 ft high in a rectangular constant width channel is pivoted at its center. Water at 40 F on one side of the gate is 20 ft deep. On the other side of the gate water at 40 F is 10 ft deep. What is the magnitude, direction and location of the force required to keep the gate closed?



FORCES

$$F = \int p\, dA$$

$$F = (\rho\rho g_{\substack{\text{average}\\\text{depth}}})\ w\ h_{\text{area}} = (\rho\ g\ w) h_{\substack{\text{avweage}\\\text{depth}}} \times h_{\text{area}}$$

$$F_1 = (\rho\ g\ w) 5 \times 10 = 50(\rho\ g\ w)$$

$$F_2 = (\rho\ g\ w) 10 \times 10 = 100(\rho\ g\ w)$$

$$F_3 = (\rho\ g\ w) 5 \times 10 = 50(\rho\ g\ w)$$

$$F_4 = (\rho\ g w) 5 \times 10 = 50(\rho\ g\ w)$$

$$\sum F = F_{\text{reaction}} + F_4 - F_1 - F_2 - F_3$$

$$F_{\text{reaction}} = \rho\ g\ w(-50 + 50 + 100 + 50)$$

$$F_{\text{reaction}} = 150 \rho\ g\ w$$

MOMENTS (+ assumed clockwise)

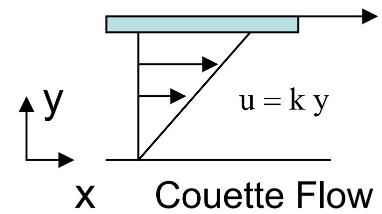
$$\left(F_1 \times \frac{10}{3} + F_4 \frac{2 \times 10}{3} + F_{\text{reaction}} \times d - F_2 \times 5 - F_3 \frac{2 \times 10}{3} \right) = 0$$

$$\frac{500}{3} + \frac{1000}{3} + 150d - 500 - \frac{1000}{3} = 0$$

$$d = \frac{333.33}{150} = 2.22 \text{ ft below the pivot}$$

mae 335 FLUID MECHANICS Exam 2 Spring 2007

1. The velocity distribution in Couette Flow is u = k y. Show that this velocity distribution is an exact solution to the Navier Stokes Equations beginning with equation (6.18) and to the Continuity Equation beginning with equation (6.8)



$$\nabla \bullet \vec{V} = -\frac{1}{\rho}\frac{D\rho}{Dt} \quad (6.8)$$

$$\rho\frac{D\vec{V}}{Dt} = -\nabla p + \rho g + \mu\nabla^2 \vec{V} \quad (6.18)$$

2. A 2 dimensional source flow of strength, $q = 6\ m^3/sec$ in placed in a uniform flow with a velocity of $U_\infty = 3$ m/sec. The pressure and temperature of the uniform flow are 100 kPa and 20 C at a point far from the location of the source.

a) Specify the stagnation pressure increase over the pressure of the uniform flow and the location of the stagnation point relative to the center of the source flow.

b) Specify the location on the surface where the static pressure is the static pressure in the uniform flow plus ½ the stagnation pressure increase,.

3. The following forward velocity V, and thrust T, data were measured in a wind tunnel at sea level where the, air density is $1.2\ kg/m^3$ with an .8 m diameter propeller operating at 2000 rpm. Assume that the thrust T, is a function of forward velocity, density, diameter and rotational speed only.

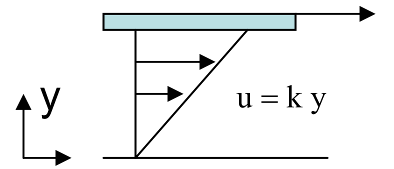
V, forward velocity, m/sec	0	10	20	30
T, Thrust, Newtons	200	278	211	100

a) What are the dimensionless numbers which govern the propeller operation?

b) Using the experimental wind tunnel data determine the thrust generated by a geometrically similar propeller operating at 1500 rpm and a forward speed of 45 m/sec at an altitude where the density of air is half that at sea level.

mae 335 FLUID MECHANICS Exam 2 Spring 2007

1.



Couette Flow

steady
1 dimensional
viscous
constant density

CONTINUITY

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=-\frac{1}{\rho}\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}\right)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial (u=ky)}{\partial x}+\frac{\partial (v=0)}{\partial x}=0$$

$$0+0=0$$

MOMENTUM

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \qquad x \;\; \text{direction}$$

$$\frac{\partial u}{\partial t}=0\,\text{steady} \qquad \frac{\partial u}{\partial x}=\frac{\partial (ky)}{\partial x}0, \qquad v=0, \qquad w=0$$

$$\frac{\partial p}{\partial x}=0, \quad \frac{\partial^2 (ky)}{\partial x^2}=0, \qquad \frac{\partial^2 u}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial (ky)}{\partial y}\right)=\frac{\partial k}{\partial y}=0$$

$$\rho(0+0+0+0)=0+\mu(0+0+0)$$

Grade	No
90	3
80	10
70	9
60	4
50	8
40	7
30	4
20	1

ave 64

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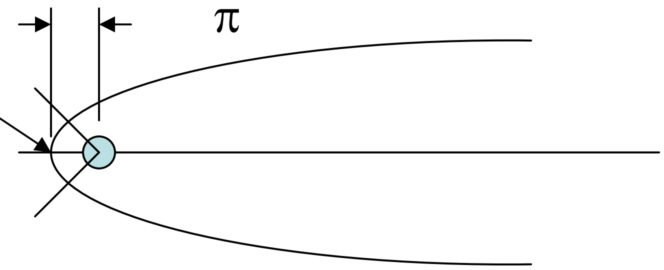
2.



a) $\Psi = \Psi_1 + \Psi_2$

$$\Psi = U_\infty r \sin\theta + \frac{q}{2\pi}\theta$$

$$u_r = \frac{1}{r}\frac{\partial \Psi}{\partial r} = U_\infty \cos\theta + \frac{q}{2\pi r}$$

$$u_\theta = -\frac{\partial \Psi}{\partial r} = -U_\infty \sin\theta$$

stagnation point where, $u_r = 0, u_\theta = 0$

$u_\theta = 0$ when $\sin\theta = 0, \theta = 0, \pi$

$u_r = 0$ when $r = -\dfrac{q}{2\pi U_\infty \cos\theta}$

$$\text{stagnationpo int}, \theta = \pi, r = -\frac{q}{2\pi U_\infty} = \frac{6}{2\pi 3} = -\frac{1}{\pi}$$

$$p_O - p_\infty = \frac{\rho U_\infty^2}{2} = \frac{1}{2} \times 1.2 \times 3^2 = 5.4 \text{ Pa}$$

b) $$p_\infty + \frac{\rho U_\infty^2}{2} = p + \frac{\rho (U_\infty \sin\theta)^2}{2}$$

$$p - p_\infty = \frac{\rho U_\infty^2}{2} - \frac{\rho (U_\infty \sin\theta)^2}{2}$$

$$\sin\theta^2 = 1 - \frac{2(p - p_\infty)}{\rho U_\infty^2} = 1 - \frac{2 \times 2.7}{1.2 \times 9}$$

$$\sin\theta^2 = .5$$

$$\sin\theta \ = .707 \Rightarrow \theta = 45 \text{ and } 135$$

$$\theta = \pm 45 \text{ from stagnation point}$$

3.

$$T = f(D, N, V, \rho)$$

$$T = f\,(D^a, N^b, V^c, \rho^d)$$

unit exponents of parameters

	T	D	N	V	ρ
M	1	0	0	0	1
L	1	1	0	1	-3
T	-2	0	-1	-1	0

5 parameters - 3 units

expect 2 dimensionless parameters

solve for a, c, d, in terms of b,

$$\sum \text{exponents} = 0$$

$$\text{find}: \begin{pmatrix}\text{first}\\ \text{dimensionless}\\ \text{number}\end{pmatrix}^{\text{integer}}, \begin{pmatrix}\text{second}\\ \text{dimensionless}\\ \text{number}\end{pmatrix}^{b}$$

$$\text{M units}: \quad 1 = d$$

$$\text{L units}: \quad 1 = a + c - 3d$$

$$\text{T units}: \quad -2 = -b - c$$

$$\text{from T}, \quad c = 2 - b$$

$$\text{from L}, \quad 1 = a + 2 - b - 3$$

$$a = 2 + b$$

$$T = f\left((D)^a (N)^b (V)^c (\rho)^d\right)$$

$$T = f(\;)^{\text{integer}}(\;)^b$$

$$T = f\left((D)^{a=(2+b)} (N)^{b=b} (V)^{c=2-b} (\rho)^{d=1}\right)$$

$$T = f(\;)^{\text{integer}}(\;)^b$$

$$T = \left(D^2 V^2 \rho\right)\left(\frac{DN}{V}\right)^b$$

$$\frac{T}{D^2 V^2 \rho} = f\left(\frac{DN}{V}\right)$$

3.

$$\left(\frac{\omega D}{V}\right)_{prototype} = \left(\frac{\omega D}{V}\right)_{test} \qquad \left(\frac{T}{\rho V^2 D^2}\right)_{prototpe} = \left(\frac{T}{\rho V^2 D^2}\right)_{test}$$

$$D_{prototype} = \left(\frac{V}{\omega}\right) \times \left(\frac{\omega D}{V}\right)_{test} \qquad T_{prototype} = \left(\frac{V^2 D^2}{\rho}\right) \times \left(\frac{T}{\rho V^2 D^2}\right)_{test}$$

Test	V	T	$\left(\frac{T}{\rho V^2 D^2}\right)_{test}$	$\left(\frac{\omega D}{V}\right)_{test}$	$D_{prototype}$ $\left(\frac{45}{1500}\right) \times \left(\frac{\omega D}{V}\right)_{test}$	$T_{prototype}$ $\left(\frac{45^2 D^2}{1.2}\right) \times \left(\frac{T}{\rho V^2 D^2}\right)_{test}$
1	0	300	∞	∞		
2	10	278	3.62	160	4.8	35,491. newtons
3	20	211	.687	80	2.4	7,494. newtons
4	30	100	.145	53	1.59	1,582. newtons

mae 335 FLUID MECHANICS Spring 2007 Final Exam

1. Water at 60 F is pumped from a ground level tank to an elevated tank. The liquid level in the ground level tank is 20 ft above the pump inlet. The pipe entrance from the ground level tank is square edged. The pipe run is 250 ft of 4 in ID commercial steel pipe with a roughness of .0004, two standard 90 degree elbows and a gate valve. The water dischargers abruptly into an elevated tank open to the atmosphere at a elevation 60 ft above the pump inlet. What power is required to operate the pump at 300 gpm if it is 50 % efficient?

2. Air leaves a diverging nozzle having an exit area of .2 m^2, at 100 kpa, 300 K and 200 m/sec. a) What is the stagnation pressure and stagnation temperature required to produce this flow? b) What stagnation pressure and stagnation temperature are required to achieve the 100 kpa and 300 K exit conditions at an exit Mach Number of 1. c) With an exit Mach Number of 1 what can be done to increase the mass flow though the .2 m^2 nozzle exit?

3. A golf ball manufacturer wants to study the dimple size on performance of a golf ball. A model four times the size of a regular ball is installed in a wind tunnel. Performance is dependent on the variables of velocity, dimple depth, air viscosity, diameter, air density, and spin rotational rate.

a) What dimensionless parameters must be controlled to model the golf ball performance?
b) What should be the speed of the wind tunnel to simulate a ball in flight at 200 ft/sec
c) What rotational speed must be used to simulate the flight if the regular ball rotates at 60 revolutions per second?

4. For a boundary layer velocity profile given by , $\dfrac{u}{U_\infty} = \dfrac{y}{\delta}$

find the wall shear stress and the displacement thickness in terms of the boundary layer thickness, δ .

1.

assume 60^{o} F water

$$\rho = 1.938\frac{\text{slugs}}{\text{ft}^3}\ ,\upsilon = .0000121\frac{\text{f}^2\text{t}}{\text{sec}}$$

$$m = \frac{300\text{ gpm} \times 1.938 \times 32.2}{7.48\text{ ft}^3/\text{gal} \times\ 60\text{ sec/hr}} = 41.69\text{ lb/sec}$$

$$A = \frac{\pi D}{4} = \frac{3.1416}{4} \times \left(\frac{4.026}{12}\right)^2 = .0884\text{ ft}^2$$

$$V = \frac{Q}{A} = \frac{300\text{ gpm}}{7.48 \times 60\text{ sec/hr} \times .0884\text{ ft}^2} = 7.56\text{ ft/sec}$$

$$N_{RE} = \frac{\rho VD}{\mu} = \frac{62.37\text{ lb/ft}^3 \times 7.56\text{ ft/sec} \times \left(\frac{4.026}{12}\right)}{2.713\text{ lb}_m\text{/ft hr/3600 sec/hr}}$$

$$N_{RE} = 209{,}915.$$

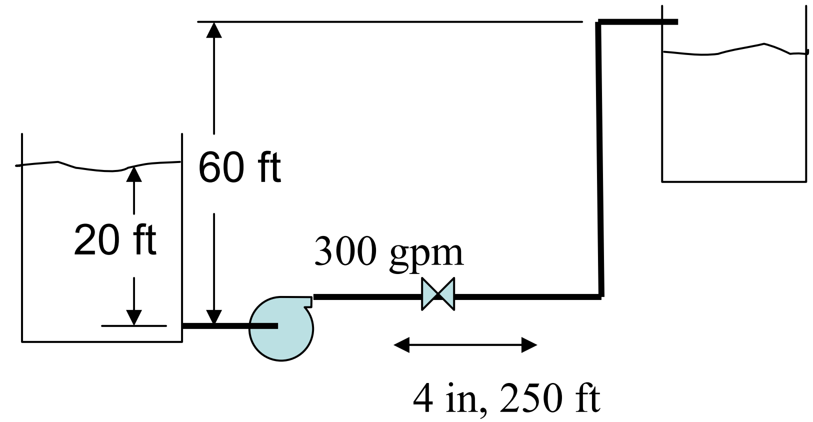
$$@\ \ R_e\text{and}\frac{\varepsilon}{D} = .0004, \quad f = .0172$$

$$K_{entrance} = .5, K_{exit} = 1., K_{\substack{gate\\valve}} = .2, K_{elbow} = .75$$

$$l_f = \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K\right)$$

$$l_f = \frac{7.56^2}{2 \times 32.2}\left(.0172 \times \left(\frac{300\text{ft}}{4.026/12}\right) + (.5 + 2 \times .76 + .2 + 1)\right)$$

$$l_f = 14.21\text{ ft}$$



$$\frac{p_1}{\rho_1} + \frac{V_1^2}{2g_c} + z_1\frac{g}{g_c} = \frac{p_2}{\rho_2} + \frac{V_2^2}{2g_c} + z_2\frac{g}{g_c} + \frac{W}{m} + \frac{g}{g_c}l_f$$

$$20 = 60 + w + 14.21$$

$$w = 54.21\text{ ft lb}_f/\text{lb}_m$$

$$W = m \times w = 54.21 \times 41.45 = 2247.\text{ ft lb/sec}$$

$$W_{ideal} = \frac{2247.\text{ ftlb/sec}}{550\text{ ft lb/ sec/ HP}} = 4.085\text{ HP}$$

$$W_{actual} = 4.085\text{ HP/50\%} = 8.17\text{ HP}$$

2.

$$a = \sqrt{\gamma\, R\, T} = \sqrt{1.4 \times 287 \times 300} = 347.2 \text{ m/sec}$$

$$M = \frac{200 \text{ m/sec}}{347.2} = .576$$

Table C.10 @ M. = .58

a) $\dfrac{T}{T_O} = .937, \qquad T_O = \dfrac{300}{.937} = 320.2 \text{ K}$

$\dfrac{p}{p_O} = .7962, \qquad p_O = \dfrac{100}{.7962} = 125.6 \text{ kPa}$

b) @ $M = 1$,

$\dfrac{T}{T_O} = .833, \qquad T_O = \dfrac{300}{.833} = 360\text{K}$

$\dfrac{p}{p_O} = .5283, p_O = \dfrac{100}{.7962} = 189.3\text{kPa}$

c) increase the density at the throat

$m = \rho \times g \times A \times a$

decrease the stagnation temperature at the nozzle inlet

Grade	No
90	5
80	11
70	9
60	14
50	5
40	1
30	
ave	70.8

3.

$$\text{Force} = f(D, k, V, \mu, \rho, \omega)$$

	Drag	D^a	k^b	V^c	μ^d	ρ^e	ω^f
M	1	0	0	0	1	1	0
L	1	1	1	1	-1	-3	0
T	-2	0	0	-1	-1	0	-1

$$1 = d + e$$

$$1 = a + b + c - d - 3e$$

$$-2 = -c - d - f$$

solve for b, c, d

$$d = 1 - e$$

$$c = 2 - d - f$$

$$c = 2 - 1 + e - f$$

$$c = 1 + e - f$$

$$1 = a + b + (1 + e - f) - (1 - e) - 3e$$

$$1 = a + b - e - f$$

$$b = 1 - a + e + f$$

$$\text{Drag} \quad D^a \quad k^b \quad V^c \quad \mu^d \quad \rho^e \quad \omega^f$$

$$\text{Drag} \quad D^a \quad k^{(1-a+e+f)} \quad V^{(1+e-f)} \quad \mu^{(1-e)} \quad \rho^e \quad \omega^f$$

$$\text{Drag} = f(\ )^{\text{number}}(\ )^a(\ )^e(\ )^f$$

$$\text{Drag} = f(kV\mu)\left(\frac{D}{k}\right)^a\left(\frac{kV\rho}{\mu}\right)^e\left(\frac{\omega k}{V}\right)^f$$

$$(kV\mu)\left[\frac{kV\rho}{\mu}\right]\left[\frac{D^2}{k^2}\right] = \rho V^2 D^2$$

$$\left(\frac{D}{k}\right)^a\left[\frac{k^2}{D^2}\right] = \frac{k}{D}$$

$$\left(\frac{kV\rho}{\mu}\right)^e\left[\frac{D}{k}\right] = \frac{\rho DV}{\mu}$$

$$\left(\frac{\omega k}{V}\right)^f\left[\frac{D}{k}\right] = \frac{\omega D}{k}$$

$$\frac{\text{Drag}}{\rho V^2 D^2} = f\left(\frac{k}{D}, \frac{\rho DV}{\mu}, \frac{\omega D}{k}\right)$$

b) $$\left(\frac{\rho DV}{\mu}\right)_{\text{model}} = \left(\frac{\rho DV}{\mu}\right)_{\text{prototype}}$$

$$\rho/\mu = \text{constant}$$

$$V_{\text{model}} = \frac{D_{\text{prototype}}}{D_{\text{model}}} \times V_{\text{prototype}} = \frac{1}{4} \times 200 = 50 \text{ ft/sec}$$

c) $$\left(\frac{\omega D}{k}\right)_{\text{model}} = \left(\frac{\omega D}{k}\right)_{\text{prototype}}$$

$$\omega_{\text{model}} = \left(\frac{D_{\text{prototype}}}{D_{\text{model}}}\right)\left(\frac{V_{\text{model}}}{V_{\text{prototype}}}\right)\omega_{\text{prototype}}$$

$$\omega_{\text{model}} = \frac{1}{4} \times \frac{1}{4} \times 50 = 3.75 \text{ rps}$$

4.

$$\frac{u}{U_\infty} = \frac{y}{\delta}$$

a) $$\tau_{wall} = \mu \left.\frac{du}{dy}\right|_{y=0} = \mu \frac{U_\infty}{\delta}$$

b) $$\delta^* = \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy$$

$$\delta^* = \left(y - \frac{y^2}{2\delta}\right)_0^\delta = \left(\delta - \frac{\delta^2}{2\delta}\right) = \frac{\delta}{2}$$