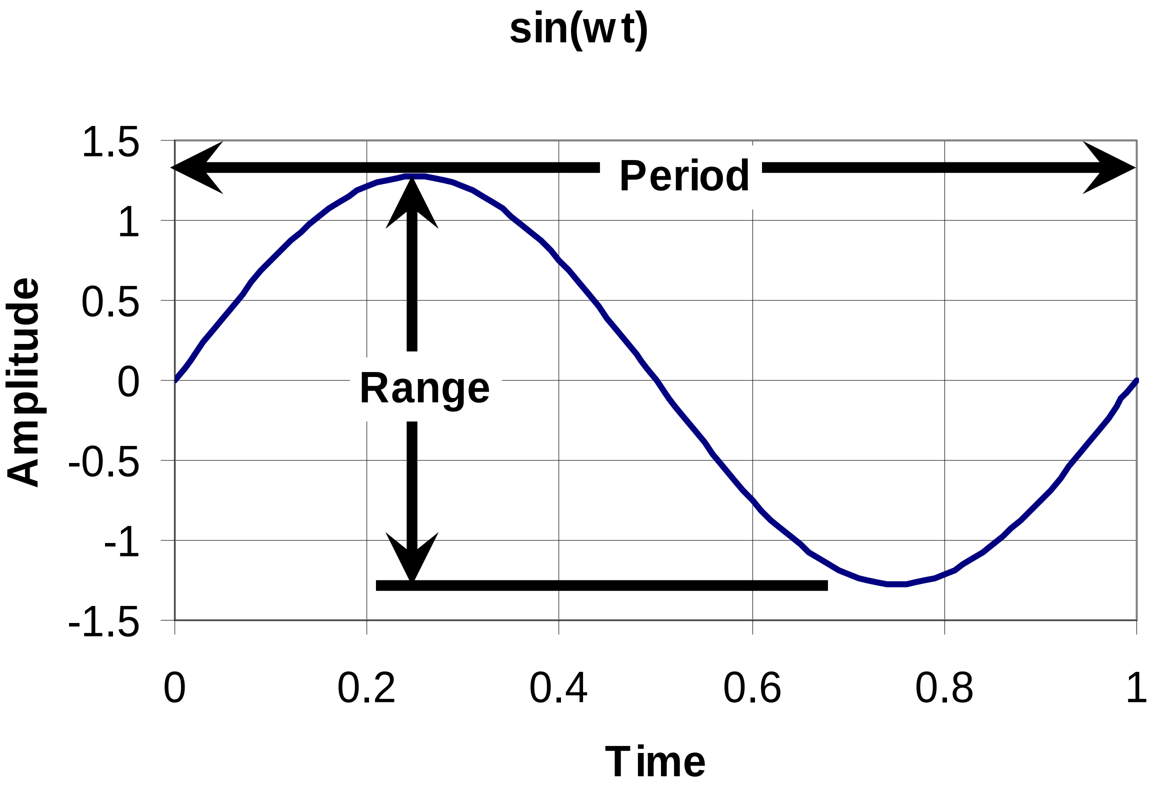
Selection of Measurement System

We need to choose an instrument which has the proper:

(a) range - maximum and minimum
(b) frequency response

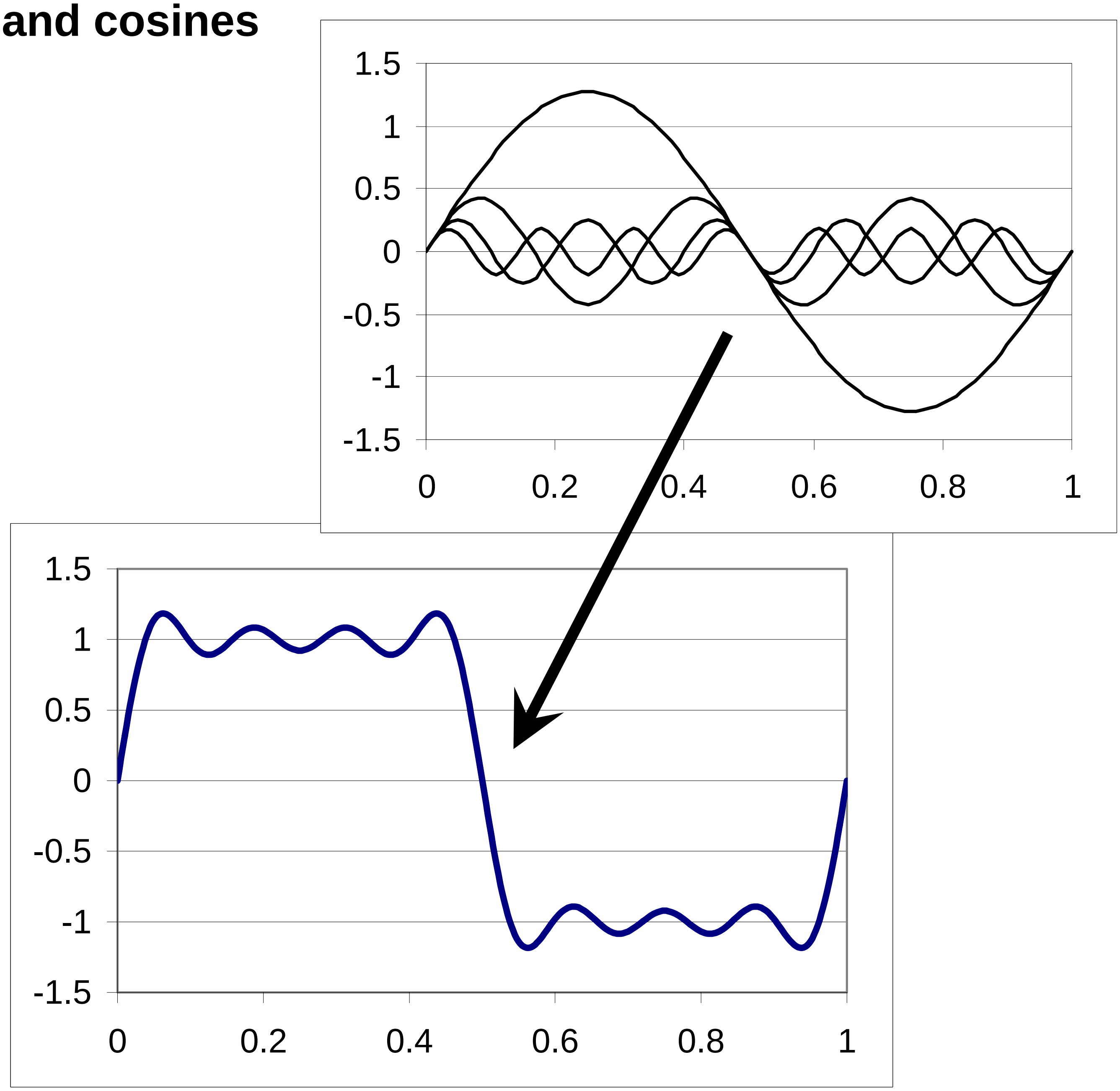
Suppose first we are dealing with a periodic signal



The range is straightforward. How do we determine the required frequency response? We need to determine the "frequency content" of the periodic signal.

Fourier Analysis

If we have a periodic signal which has a finite number of discontinuities (jumps) and has derivatives defined at each point in the interval, we can represent the signal as a sum of sines and cosines



Fourier Analysis - Cont.

Mathematically

$$y(t) = A_o + \sum_{n=1}^{\infty} \left(A_n \cos \frac{2 n \pi t}{T} + B_n \sin \frac{2 n \pi t}{T} \right) \quad (2.15)$$

$$\text{where:} \qquad A_o = \frac{1}{T} \int_{-\pi}^{\pi} y(t)\, dt$$

$$A_n = \frac{2}{T} \int_{-\pi}^{\pi} y(t) \cos \frac{2 n \pi t}{T}\, dt \quad (2.14)$$

$$B_n = \frac{2}{T} \int_{-\pi}^{\pi} y(t) \sin \frac{2 n \pi t}{T}\, dt$$

The result is known as a Fourier Series.

Note: In the 2nd and 3rd edition, equations 2.17 and 2.18 are equivalent, where $\omega = \frac{2\pi}{T}$ **.**

$$y(t) = A_o + \sum_{n=1}^{x} \left(A_n \cos \frac{2n\pi t}{T} + B_n \sin \frac{2n\pi t}{T} \right)$$

may be written as

$$y(t) = A_o + \sum_{n=1}^{x} C_n \cos \left(\frac{2n\pi t}{T} - \phi_n \right) \qquad (2.18)$$

or

$$y(t) = A_o + \sum_{n=1}^{x} C_n \sin \left(\frac{2n\pi t}{T} + \phi_n^* \right) \qquad (2.19)$$

where

$$C_n = \sqrt{A_n^2 + B_n^2}$$

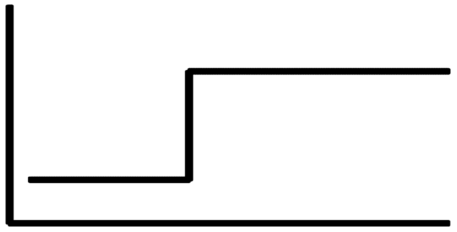
$$\tan \phi_n = \frac{B_n}{A_n} \qquad \tan \phi_n^* = \frac{A_n}{B_n} \qquad (2.20)$$

Note: In the 2nd and 3rd edition, equations 2.19 to 2.21 are equivalent, where $\omega = \frac{2\pi}{T}$**1 .**

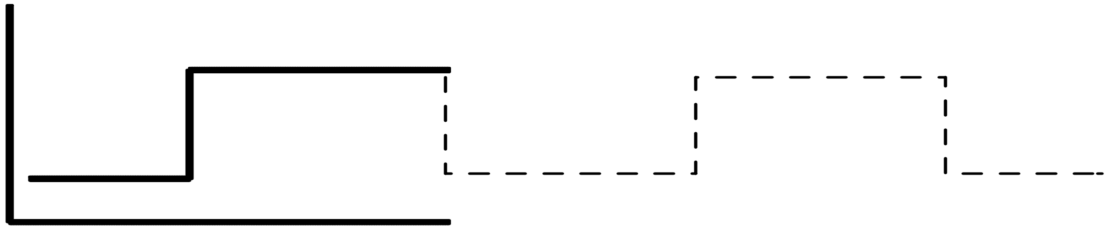
This form has the advantage that it more clearly identifies the magnitude of each frequency component.

Aperiodic Waveforms

What about waveforms which are not periodic, such as a step.



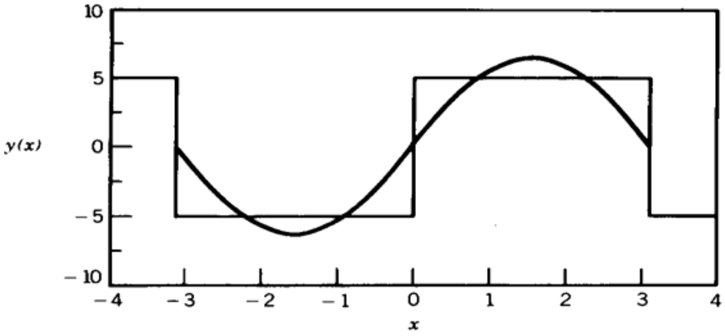
We can extend the waveform so that it becomes periodic:



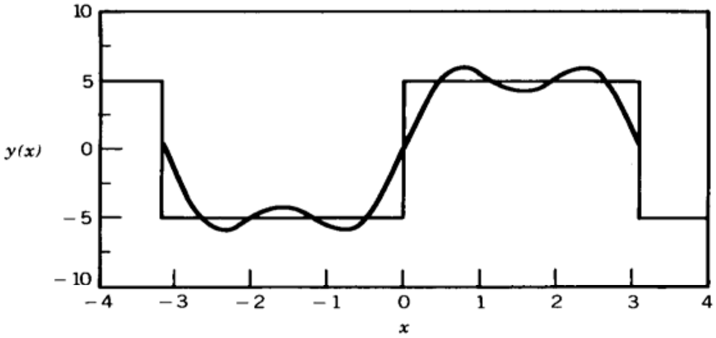
What makes Fourier series useful is that most functions can be closely approximated using a small number of terms. See Example 2.2 and Figure 2.9. (Example 2.3 and Figure 2.16 in the 2nd and 3rd Edition.)

FIGURE 2.9 First four partial sums of the Fourier series $(20/\pi)\,(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots)$ in comparison with the exact waveform.

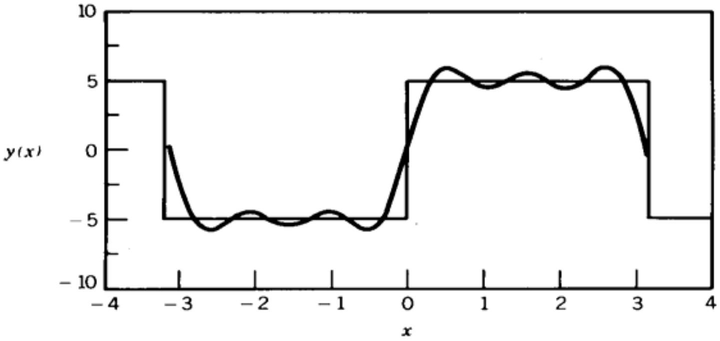
(*a*) First partial sum



(*b*) Second partial sum



(*c*) Third partial sum



(*d*) Fourth partial sum

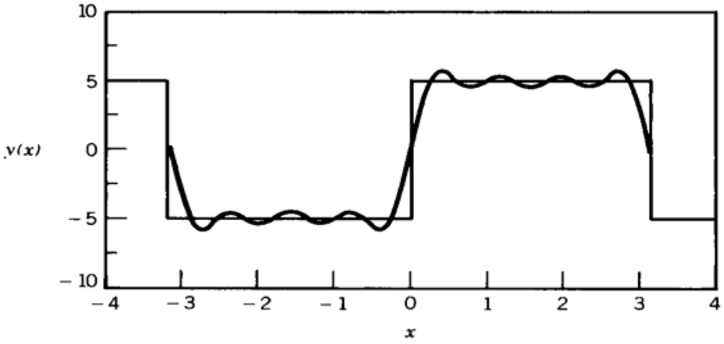


Figure 2.16 in 2^{nd} and 3^{rd} Edition

Suppose the equation describing the waveform is not known so I am not able to perform the integrations analytically. I can then resort to numerical integration, using an appropriate numerical integration algorithm. If the number of points is large, a simple summation is probably sufficient:

$$\int_{a}^{b} f(t)\sin(nt)\,dt \_$$

$$\sum_{i=0}^{k} f(t_i)\sin(nt_i)\,\Delta t$$

$$where\ t_0 = a, \Delta t = a - b, t_i = i\,\Delta t$$

Laboratory 5 - Fourier analysis of low pass filter and waveforms.

The equations from the 2nd edition of the text are the easiest to work with:

$$y(t) = A_0 + \sum_{n=1}^{\infty} (A_n \cos n\,\omega\, t$$

$$+ B_n \sin n\,\omega\, t\ ) \quad (2.18)$$

where

$$A_0 = \frac{1}{T} \int_{-T/2}^{T/2} y(t)\, dt$$

$$A_n = \frac{2}{T} \int_{-T/2}^{T/2} y(t) \cos n\,\omega\, t\ dt$$

$$B_n = \frac{2}{T} \int_{-T/2}^{T/2} y(t) \sin n\,\omega\, t \qquad (2.17)$$

Since you don't have analytical expressions for y(t), you will have to use numerical integration. You have a large number of data points, so Simpson's rule is probably adequate.

In order to assess the importance of the higher harmonics, it is easiest to convert Equation 2.18 to the form

$$y(t) = A_0 + \sum_{n=1}^{\infty} (C_n \cos n\omega t - \phi_n) \quad (2.19)$$

$$\text{where } C_n = \sqrt{A_n^2 + B_n^2}$$

$$\tan \phi_n = \frac{B_n}{A_n} \quad (2.21)$$