Binary Arithmetic

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 1 = 0$ carry 1 to next higher bit

Ex1:

$$\begin{array}{r} (110)_2 \\ +(111)_2 \\ \hline (1101)_2 \end{array} \qquad \begin{array}{r} (6)_{10} \\ (7)_{10} \\ \hline (13)_{10} \end{array}$$

$$(1101)_2 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3$$

$$= 1 + 0 + 4 + 8 = (13)_{10}$$

Complement of a Binary Number

Twos Complement

$$N_2^* = 2^n - N$$

n - number of bits in binary integer

Ex2: $N = (1101)_2$ $\quad n = 4$

$$N_2^* = 2^4 - (1101)_2$$

$$= (10000)_2 - (1101)_2 = ?$$

This can be calculated by hand, but it's easy to make a mistake. There is an easier way.

Ones Complement

$$N_1^* = N_2^* - 1$$

The ones complement of a binary number is found by switching all the ones and zeros to zeros and ones, respectively

Ex: $N = (1101)_2$

$$N_1^* = (0010)_2$$

The Two Complement is given by

$$N_2^* = N_1^* + 1 = (0010)_2 + (0001)_2$$

$$= (0011)_2$$

Subtraction is carried out in the computer b using the twos complement

$$M - N = M + N_2^*$$

where M and N are binary numbers

Ex 3: Suppose $M = (01101)_2$ and $N = (01010)_2$ and the computer word length is 5 bits

Find the ones complement of N

$$N_1^* = (10101)_2$$

Find the twos complement of N

$$N_2^* = N_1^* + 1 = (10101)_2 + (00001)_2$$
$$= (10110)_2$$

Calculate the difference

$$M - N = M + N_2^* = (01101)_2 + (10110)_2$$
$$= (100011)_2$$

since computer can handle only 5 bits

drop the leftmost bit

$M - N = (00011)_2$

Verify result with decimal calculation

$M = (01101)_2 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 0 \times 2$

$= (13)_{10}$

$N = (01010)_2 = 0 \times 2^0 + 1 \times 2^1 + 0 \times 2^2 + 1 \times 2^3 + 0 \times 2^4$

$= (10)_{10}$

$M - N = (13)_{10} - (10)_{10} = (3)_{10}$

From the binary calculation

$M - N = M + N_2^* = (00011)_2$

$= 1 \times 2^0 + 1 \times 2^1 + 0 \times 2^2 + 0 \times 2^3 + 0 \times 2$

$= 1 + 2 = (3)_{10}$

Table 7.1 Binary Codes (Example: 4-Bit Words)

Bits	Straight	Offset	Twos Complement	Ones Complement	AVPS
0000	0	−7	+0	+0	−0
0001	1	−6	+1	+1	−1
0010	2	−5	+2	+2	−2
0011	3	−4	+3	+3	−3
0100	4	−3	+4	+4	−4
0101	5	−2	+5	+5	−5
0110	6	−1	+6	+6	−6
0111	7	−0	+7	+7	−7
1000	8	+0	−8	−7	+0
1001	9	+1	−7	−6	+1
1010	10	+2	−6	−5	+2
1011	11	+3	−5	−4	+3
1100	12	+4	−4	−3	+4
1101	13	+5	−3	−2	+5
1110	14	+6	−2	−1	+6
1111	15	+7	−1	−0	+7