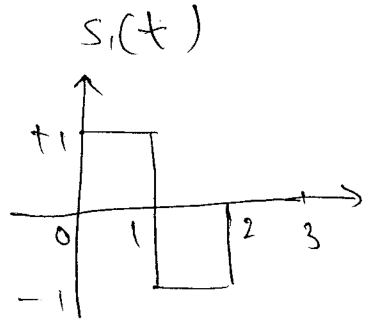
①



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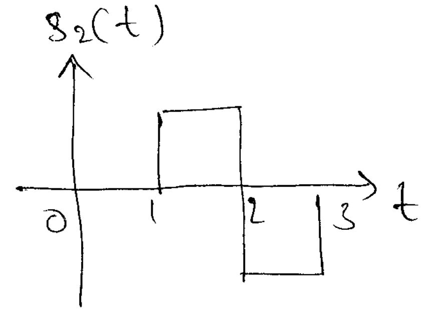
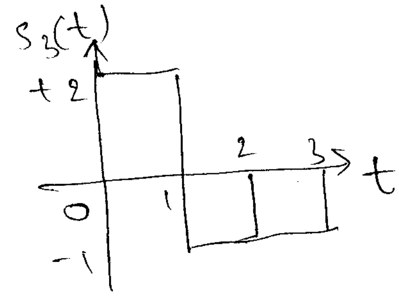




Soln :- step ①: $\psi_1(t) = s_1(t)$ & $\phi_1(t) = \dfrac{\psi_1(t)}{\sqrt{E_{\psi_1}}}$

$$E_{\psi_1} = \int_0^3 s_1^2(t)\,dt$$
$$= 1 + 1 + 0$$
$$= 2$$
$$\therefore \quad \phi_1(t) = \frac{s_1(t)}{\sqrt{2}}$$

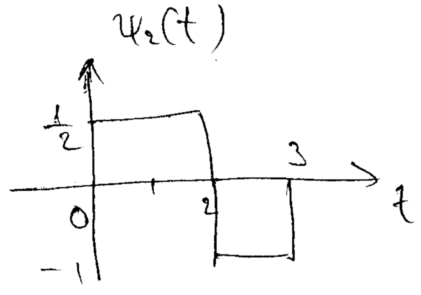
step ②: $\psi_2(t) = s_2(t) - \langle s_2, \phi_1 \rangle \, \phi_1(t)$

$$= s_2(t) - \left\langle s_2, \frac{s_1(t)}{\sqrt{2}} \right\rangle \cdot \frac{s_1(t)}{\sqrt{2}}$$

$$\left\langle s_2, \frac{s_1}{\sqrt{2}} \right\rangle = \int_0^3 s_2(t)\,\frac{s_1(t)}{\sqrt{2}}\,dt$$
$$= \frac{1}{\sqrt{2}}\left[\int_0^1 0 \cdot dt + \int_1^2 (-1)\,dt + \int_2^3 0 \cdot dt\right]$$
$$= \frac{1}{\sqrt{2}}\left[0 - 1 + 0\right] = -\frac{1}{\sqrt{2}}$$

$$\psi_2(t) = s_2(t) - \left(-\frac{1}{\sqrt{2}}\right) \cdot \frac{s_1(t)}{\sqrt{2}} = s_2(t) + \frac{s_1(t)}{2}$$

$$\phi_2(t) = \frac{\psi_2(t)}{\sqrt{E_{\psi_2}}}$$



$$\psi_2(t) = s_2(t) + \frac{1}{2} s_1(t)$$

$$E_{\psi_2} = \int_0^3 |\psi_2(t)|^2 dt$$

$$= \frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2}$$

$$\phi_2(t) = \sqrt{\frac{2}{3}} \left[s_2(t) + \frac{1}{2} s_1(t) \right]$$

(3) $$\psi_3(t) = s_3(t) - \sum_{i=1}^{2} \langle s_3, \phi_i \rangle \phi_i(t)$$

$$= s_3(t) - \underbrace{\langle s_3, \phi_1 \rangle}\phi_1(t) - \underbrace{\langle s_3, \phi_2 \rangle}\phi_2(t)$$

$$= s_3(t) - \frac{3}{\sqrt{2}} \frac{s_1(t)}{\sqrt{2}} - \sqrt{\frac{3}{2}} \left[\sqrt{\frac{2}{3}} \left\{ s_2(t) + \frac{1}{2} s_1(t) \right\} \right]$$

$$= s_3(t) - \frac{3}{2} s_1(t) - s_2(t) - \frac{1}{2} s_1(t)$$

$$= s_3(t) - s_2(t) - 2 s_1(t)$$

$$= 0$$

$$s_3(t) = s_2(t) + \underbrace{2 s_1(t)}$$

$$s_3(t) = \underbrace{\left[s_2(t) + \frac{s_1(t)}{2} \right]} + \frac{3}{2} s_1(t)$$

$$= \psi_2(t) + \frac{3}{2} \psi_1(t)$$

$$s_3(t) = \sqrt{\frac{3}{2}} \phi_2(t) + \frac{3}{2} \sqrt{2} \phi_1(t)$$

$\Rightarrow$ 2 basis functions are sufficient.

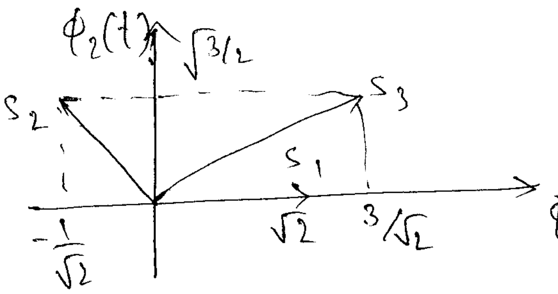
$$s_1(t) = \sqrt{2} \phi_1(t)$$

$$s_2(t) = -\frac{1}{\sqrt{2}} \phi_1(t) + \sqrt{\frac{3}{2}} \phi_2(t)$$

$$s_3(t) = \sqrt{\frac{3}{2}} \phi_1(t) + \sqrt{\frac{3}{2}} \phi_2(t)$$

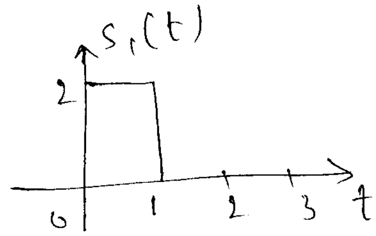
$\underline{s_i(t)} = \underline{A}\ \underline{\phi}(t)$

$$\underline{A} = \begin{bmatrix} \sqrt{2} & 0 \\ -1/\sqrt{2} & \sqrt{3}/2 \\ 3/\sqrt{2} & \sqrt{3}/2 \end{bmatrix}$$



← Signal constellation

Q(2)



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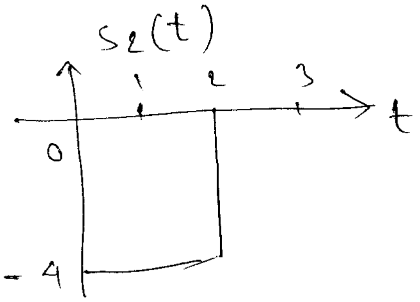
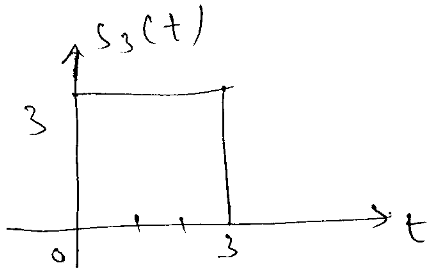




(a) Using the Gram-Schmidt orthogonalization procedure, find a set of orthonormal basis function for the given set.

(b) Express the signals in terms of the set of basis function.

Soln: Step 1: $\psi_1(t) = s_1(t)$

$$E_{\psi_1} = E_{s_1} = \int_0^3 s_1^2(t)\,dt = 4$$

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_\psi}} = \frac{1}{2} s_1(t).$$

<u>step 2:</u> $\psi_2(t) = s_2(t) - \langle s_2, \phi_1 \rangle \phi_1(t)$

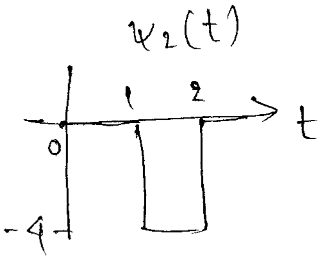
$$\langle s_2, \phi_1 \rangle = \int_0^3 s_2(t)\,\phi_1(t)\,dt$$

$$= \frac{1}{2}\int_0^1 s_2(t)\,s_1(t)\,dt$$

$$= -4$$

$$\therefore\ \psi_2(t) = s_2(t) + \frac{4}{2}s_1(t)$$

$$= s_2(t) + 2s_1(t)$$



$$E\psi_2 = \int_0^3 |\psi_2(t)|^2\,dt$$

$$= 0 + 16 + 0 = 16$$

$$\phi_2(t) = \frac{1}{\sqrt{E_{\psi_2}}}\psi_2(t) = \frac{1}{4}s_2(t) + \frac{s_1(t)}{2}$$

<u>step 3:</u> $\psi_3(t) = s_3(t) - \underbrace{s_{31}}_{\langle s_3, \phi_1 \rangle}\phi_1(t) - \underbrace{s_{32}}_{\langle s_3, \phi_2 \rangle}\phi_2(t)$

$$s_{31} = \int_0^3 s_3(t)\,\phi_1(t)\,dt$$

$$= \frac{1}{2}\int_0^3 s_3(t)\,s_1(t)\,dt$$

$$\underline{s_{31} = 3}$$

$$s_{32} = \int_0^3 s_3(t)\,\phi_2(t)\,dt$$

$$= \frac{1}{4}\int_0^3 s_3(t)\,[\underbrace{s_2(t) + 2s_1(t)}]\,dt$$

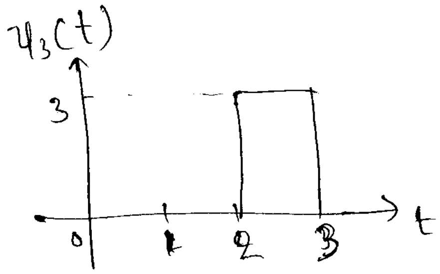
$$= \frac{1}{4}\int_0^3 s_3(t)\,\psi_2(t)\,dt$$

$$\underline{s_{32} = -3}$$

$$\psi_3(t) = s_3(t) - 3\phi_1(t) + 3\phi_2(t)$$

$$= s_3(t) - \frac{3}{2}s_1(t) + \frac{3}{4}s_2(t) + \frac{3}{2}s_1(t)$$

$$\psi_3(t) = s_3(t) + \frac{3}{4}s_2(t)$$



$$E_{\psi_3} = \int_0^3 |\psi_3(t)|^2\,dt$$

$$= 9$$

$$\phi_3(t) = \frac{s_3(t)}{3} + \frac{3}{4}\cdot\frac{s_2(t)}{3}$$

$$= \frac{s_3(t)}{3} + \frac{1}{4}s_2(t)$$

(b)

$$s_1(t) = 2\phi_1(t)$$

$$s_2(t) = 4\left[\phi_2(t) - \phi_1(t)\right]$$

$$s_3(t) = 3\left[\phi_3(t) - \phi_2(t) + \phi_1(t)\right]$$