6-19 (a) $\quad \underset{\sim}{z} = \underset{\sim}{x}/\underset{\sim}{y} \qquad \underset{\sim}{w} = \underset{\sim}{y} \qquad J = 1/y$

$$f_z(z) = \int_{-\infty}^{\infty} |w| f_x(zw) f_y(w)\,dw \qquad z > 0$$

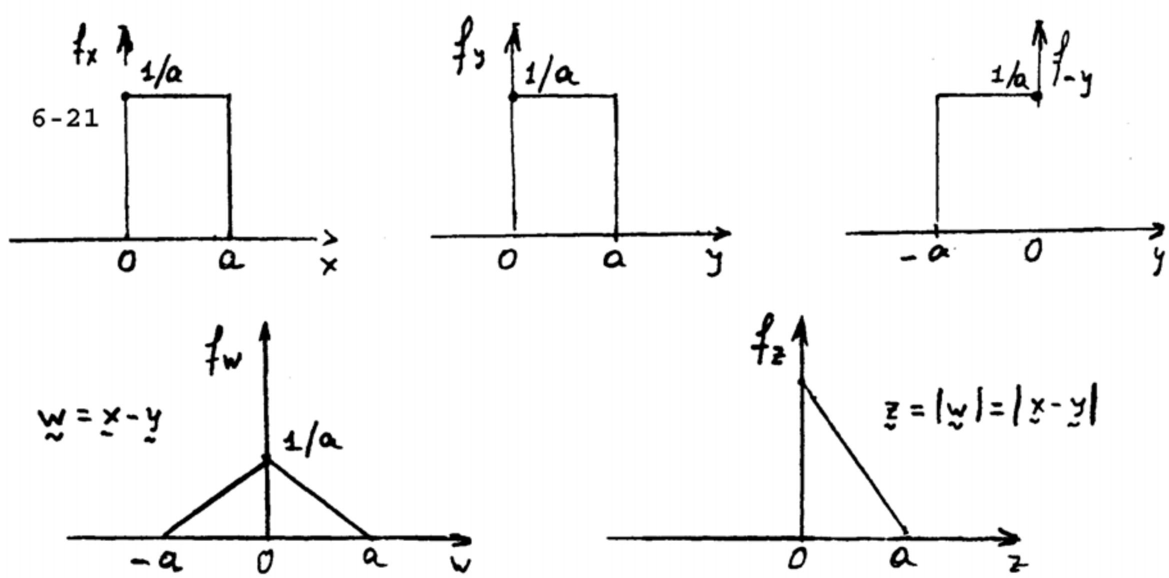
$$= \frac{z}{\alpha^2\beta^2} \int_0^{\infty} w^3\, e^{-cw^2} dw = \frac{z}{2\alpha^2\beta^2c^2} \qquad c = \frac{z^2}{2\alpha^2} + \frac{1}{2\beta^2}$$

$$= \frac{2\alpha^2}{\beta^2} \frac{z}{(z^2 + \alpha^2/\beta^2)^2} \qquad \text{for } z > 0 \text{ and zero otherwise}$$

(b) $$F_z(z) = \int_0^z \frac{2\alpha^2 z\, dz}{\beta^2(z^2 + \alpha^2/\beta^2)^2} = \frac{\alpha^2}{\beta^2} \int_{\alpha^2/\beta^2}^{z^2 + \alpha^2/\beta^2} \frac{dt}{t^2}$$

$$= \frac{z^2}{z^2 + \alpha^2/\beta^2} = P\{\underset{\sim}{z} \le z\} = P\{\underset{\sim}{x} \le zy\}$$

6-21



6-38 $\qquad \underset{\sim}{z} = \underset{\sim}{x}\,\underset{\sim}{y} \qquad \underset{\sim}{y} = \cos(\omega t + \underset{\sim}{\theta})$

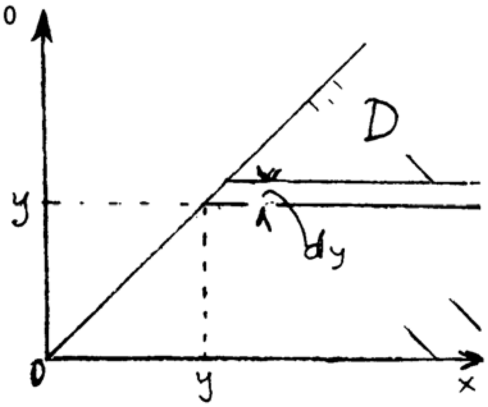
$$\underset{\sim}{w} = \underset{\sim}{y} \qquad J = |y| \qquad f_y(y) = \begin{cases} \dfrac{1}{\pi\sqrt{1-y^2}} & |y| < 1 \\ 0 & |y| > 1 \end{cases}$$

The RVs $\underset{\sim}{x}$ and $\underset{\sim}{y}$ are independent. Hence,

$$f_{zw}(z,w) = \frac{1}{|w|} f_x\left(\frac{z}{w}\right) f_y(w)$$

$$f_z(z) = \frac{1}{\pi}\int_{-1}^{1} \frac{f_x(z/w)}{|w|\sqrt{1-w^2}}\,dw = \frac{1}{\pi}\int_{|x|>z} \frac{f_x(x)}{\sqrt{x^2-z^2}}\,dx$$

6-50



$$E\{\underset{\sim}{z}\} = \iint_D (x-y)f(x,y)\,dxdy$$

$$= \int_0^{\infty}\int_y^{\infty} (x-y)e^{-x}\,e^{-y}\,dxdy = \frac{1}{2}$$

6-71 The mass density in the square $|\underset{\sim}{x}|\le 1$, $|\underset{\sim}{y}|\le 1$ of the xy plane equals 1/4; hence, $P\{\underset{\sim}{r}\le 1\}=\pi/4$ and $P\{\underset{\sim}{r}\le r\} = \pi r^2/4$ for $r<1$. This yields

$$P\{r\le r, r\le 1\} = \begin{cases} P\{r\le r\} = \pi r^2/4 & r\le 1 \\ P\{r\le 1\} = \pi/4 & r>1 \end{cases}$$

$$F_r(r|M) = \frac{P\{r\le r, M\}}{P(M)} = \begin{cases} r^2 & r\le 1 \\ 1 & r>1 \end{cases} \qquad f_r(r|m) = \begin{cases} 2r, & r<+1 \\ 0 & \text{otherwise} \end{cases}$$