PROJECT for EE 483

COMMUNICATIONS SYSTEMS I - Fall 2004

Computer Assignment 6: Random Variables

Exercise 1: ***Random Numbers***

MATLAB provides two functions for the generation of random numbers. The function `randn` returns random numbers chosen from a normal (Gaussian) distribution with mean zero and variance 1, and arranges them in matrix form. For example,

```
randn(2,10)
```

will return a 2×10 matrix with entries chosen from the aforementioned distribution, while `randn(1,10)` will return a vector with 10 elements chosen from the Gaussian distribution. The second function, `rand`, returns random numbers chosen from a uniform distribution on the interval $[0, 1]$. Similarly to `randn`,

```
rand(2,10)
```

will return a 2×10 matrix with entries chosen from the uniform distribution, while `rand(1,10)` will return a vector with 10 elements chosen from the uniform distribution.
In addition, MATLAB provides the function `mean` for the calculation of the mean value of a set of random numbers. So, if `x` is a *vector* containing random numbers, we can calculate their mean by typing

```
mean(x)
```

Create an M-file to:

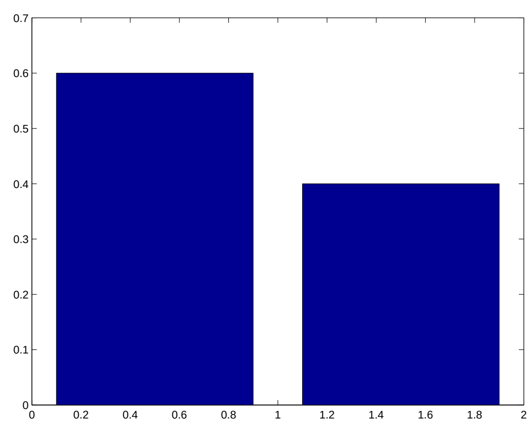
(a) Plot a vector containing 1000 random numbers chosen from a normal distribution with mean zero and variance one.

(b) Plot a vector containing 1000 random numbers chosen from a normal distribution with mean one and variance 2.

(c) Plot a vector containing 1000 random numbers chosen from a uniform distribution on $[0, 1]$.

Exercise 2: ***Histograms***

A *histogram* is a special kind of plot. Suppose that $\mathbf{x}$ is a vector whose largest element is x_{max} and smallest element is x_{min}. If we divide the range $[x_{min}, x_{max}]$ into N intervals (called *bins*) of the same length $(x_{max} - x_{min})/N$ then the histogram shows the fraction of the total number of elements of $\mathbf{x}$ that fall inside each bin. For example, the histogram of the vector $[0, 0.5, 0.75, 1.5, 2]$ using 2 bins is:



This histogram shows that 60% of the elements of $\mathbf{x}$ fall inside the first bin (i.e. they fall in the interval $[0, 1]$) and 40% fall in the second bin (i.e. in the interval $[1, 2]$). If $\mathbf{x}$ contains random numbers chosen from a specific distribution then its histogram will approximate the probability density function (pdf) of that distribution, provided that the number of bins is large and the total area under the histogram is adjusted (normalized) so that it is equal to unity.
Create an M-file to:

(a) Plot the histogram of a vector containing 100,000 random numbers chosen from a normal distribution with mean zero and variance one.

Use 40 bins.

(b) Plot the histogram of a vector containing 100,000 random numbers chosen from a uniform distribution on $[0, 1]$. Use 40 bins.

You may use the function `histogram` included in the appendix that automatically adjusts the area under the histogram to unity.

Exercise 3: *Functions of Random Variables*

Let X be a random variable (RV) with pdf $p_X(x)$. Suppose that we want to plot the pdf $p_Y(y)$ of a random variable Y defined by $Y = f(X)$. If $\mathbf{x} = [x_1, \ldots, x_N]^T$ is a vector containing N instances of the random variable X (i.e. N random numbers chosen from a distribution with pdf $p_X(x)$) then $p_Y(y)$ is approximated by the histogram of the vector $\mathbf{y} = [f(x_1), \ldots, f(x_N)]^T$. The mean value of Y is approximately equal to the mean value of the elements of $\mathbf{y}$. For example, to plot the pdf of $Y = \cos(2\pi X)$ where X is a Gaussian random variable with mean 0 and variance 1 we can do the following:

```
N=10000;
x=randn(1,N);
y=cos(2*pi*x);
histogram(y,40);
```

In addition, the mean value of Y can be found by typing `mean(y)` .
Create an M-file to:

(a) Plot the pdf and calculate the mean value of the RV Y defined by

$$Y = X + 10,$$

where X is a Gaussian random variable with mean 0 and variance 1. Use 100,000 random numbers chosen from a normal distribution and 40 bins.How does this plot compare to the plot obtained in 2(a)?

(b) Plot the pdf and calculate the mean value of the RV Y defined by

$$Y = X/2,$$

where X is a Gaussian random variable with mean 0 and variance 1. Use 100,000 random numbers chosen from a normal distribution and 40 bins.How does this plot compare to the plot obtained in 2(a)?

(c) Plot the pdf and calculate the mean value of the RV Z defined by

$$Z = \sqrt{X^2 + Y^2},$$

where X and Y are Gaussian random variables with mean 0 and variance 1. Use 100,000 random numbers chosen from a normal distribution and 40 bins.How does this plot compare to the plot obtained in 2(a)? (Note: The distribution of Z defined as $\sqrt{X_1^2 + \ldots + X_n^2}$ where the $X_i, i = 1, \ldots n$ are Gaussian RVs is called *chi* distribution with n degrees of freedom. For the special case of n=2 degrees of freedom the *chi* distribution is also known as *Rayleigh* distribution.)

Appendix

In Exercise 2 and 3 you will need the function `histogram` given below that plots the histogram of a vector `rv` using `n` bins.

```
function histogram(rv,n)
[y,x]=hist(rv,n);
binwidth=(max(rv)-min(rv))/n;
y=y/(binwidth*length(rv));
bar(x,y);
```

Note

Your report should include all plots and M-files you are asked to create in Exercises 1, 2 and 3.