EE 483 Communications Systems I
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Homework Set 2

1. Prove the following Fourier Transform pairs (from p. 764):

$$\begin{aligned}
F\{\delta(t-t_0)\} &= e^{-j2\pi f t_0} \\
F\{e^{j2\pi f_c t}\} &= \delta(f-f_0) \\
F\{cos(2\pi f_c t)\} &= 0.5\{\delta(f-f_c)+\delta(f+f_c)\} \\
F\{sin(2\pi f_c t)\} &= -0.5j\{\delta(f-f_c)-\delta(f+f_c)\} \\
F\{sgn(t)\} &= \tfrac{1}{j\pi f} \\
F\{u(t)\} &= 0.5\delta(f)+\tfrac{1}{j2\pi f}
\end{aligned}$$

2. A signal $x(t)$ of finite energy is applied to a square-law device whose output $y(t)$ is defined by

$$y(t) = x^2(t)$$

The spectrum of $x(t)$ is limited to the frequency interval $-W \leq f \leq W$. Hence, show that the spectrum of $y(t)$ is limited to $-2W \leq f \leq 2W$. *Hint:* Express $y(t)$ as $x(t)$ multiplied by itself.

3. Prove that if $g(t)$ is a real-valued function of time t, then $G^*(f) = G(-f)$. *Hint*: Use the definition for $G(f)$ and the Euler equation.

3. The Fourier Transform $G(f)$ of a signal $g(t)$ is defined by

$$G(f) = \left\{ \begin{array}{ll} 1, & f > 0 \\ \frac{1}{2}, & f = 0 \\ 0, & f < 0 \end{array} \right\}.$$

Determine the signal $g(t)$. *Hint*: Use the Fourier transform of $u(t)$ and the duality property.