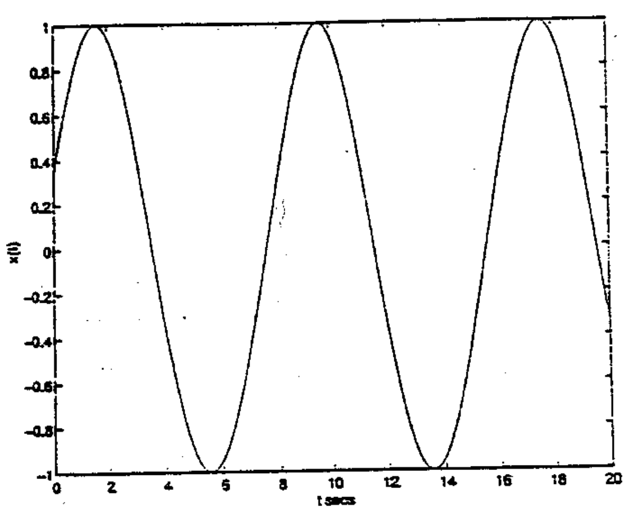
HW#1 Solutions

1.1

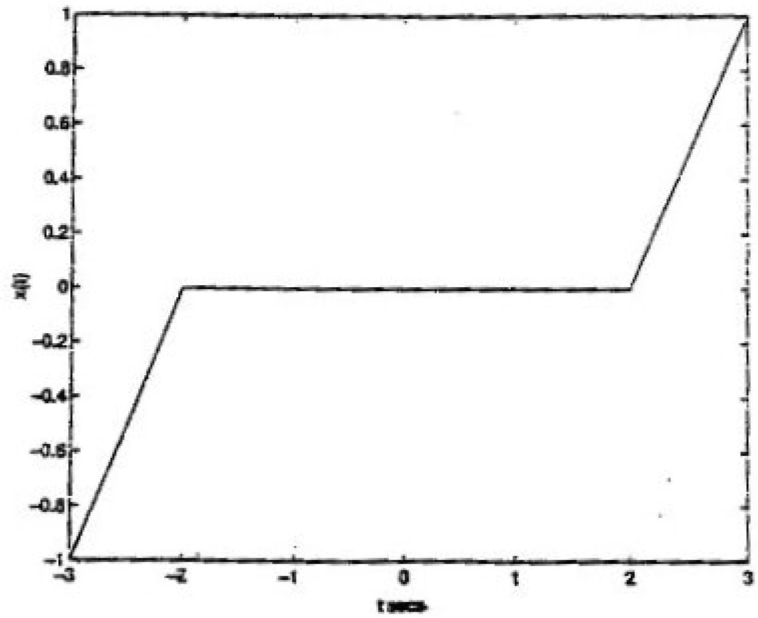
Signal	Period
$\cos(\pi t)$	2
$\sin(2\pi t)$	1
$\cos(3\pi t)$	2/3
$\sin(4\pi t)$	$\frac{1}{2}$
$\cos(\frac{\pi}{2} t)$	4
$\sin(\frac{\pi}{3} t)$	6
$\cos(\frac{5\pi}{2} t)$	$\frac{4}{5}$
$\sin(\frac{4\pi}{3} t)$	$\frac{3}{2}$
$\cos(\frac{\pi}{4} t)$	8
$\sin(\frac{2\pi}{3} t)$	3
$\cos(\frac{3\pi}{5} t)$	$\frac{10}{3}$

1.2



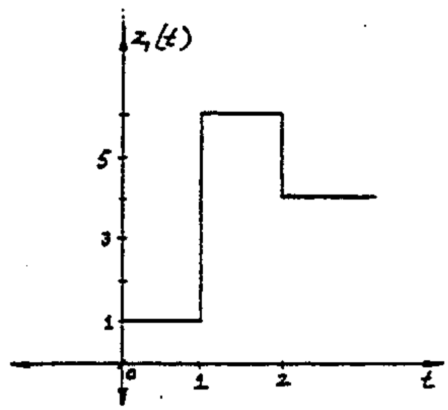
(a) $x(t) = \sin(\frac{\pi}{4} t + 20^\circ)$

1.2 c



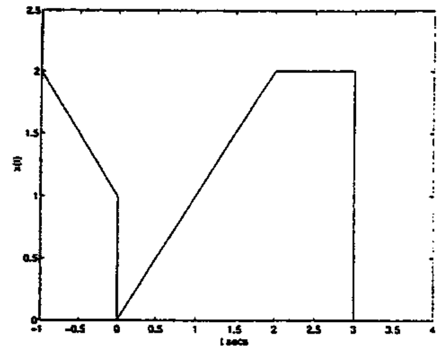
(c) $x(t) = \begin{cases} t+2 & t \leq -2 \\ 0 & -2 \leq t \leq 2 \\ t-2 & 2 \leq t \end{cases}$

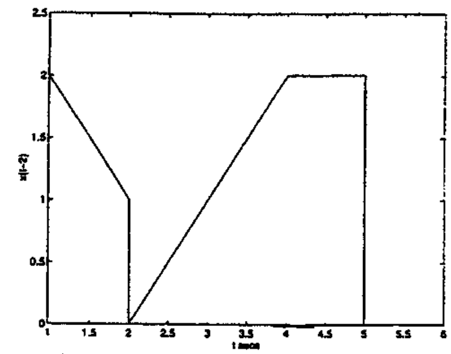
1.13 (a) $x_1(t) = u(t) + 5u(t-1) - 2u(t-2)$



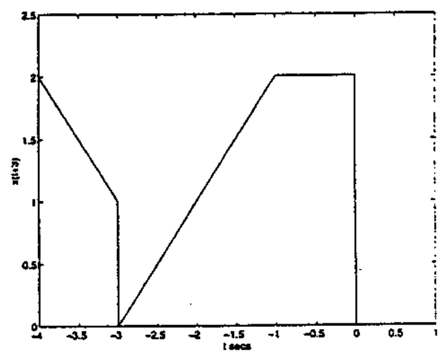
1.11

١.١١

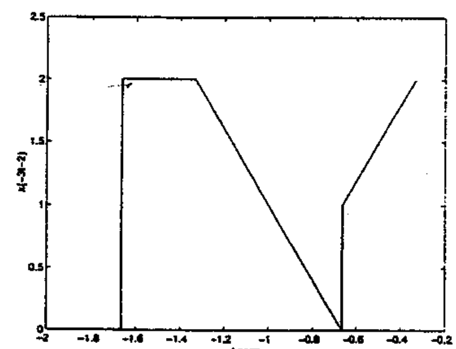




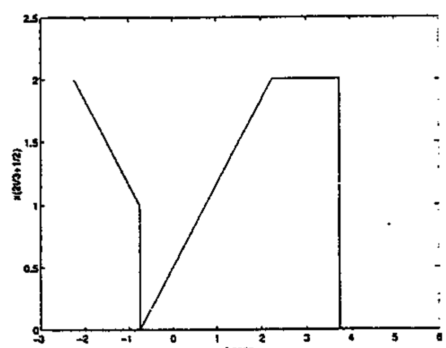
$$x(t-2)=\begin{cases} -t+3 & 1\leq t<2 \\ t-2 & 2\leq t<2 \\ 2 & 4\leq t<5 \\ 0 & \text{otherwise} \end{cases}$$



$$x(t+3)=\begin{cases} -t-3 & -4\leq t<-3 \\ t+3 & -3\leq t<-1 \\ 2 & -1\leq t<0 \\ 0 & \text{otherwise} \end{cases}$$



$$x(-3t-2)=\begin{cases} 3t+3 & -\frac{2}{3}<t\leq-\frac{1}{3} \\ -3t-2 & -\frac{4}{3}<t\leq-\frac{2}{3} \\ 2 & \\ 0 & -\frac{5}{3}<t\leq-\frac{4}{3} \\ & \text{otherwise} \end{cases}$$



$$x(\frac{2}{3}t+\frac{1}{2})=\begin{cases} \frac{2}{3}t+\frac{1}{2} & -\frac{9}{4}\leq t<-\frac{3}{4} \\ \frac{2}{3}t+\frac{1}{2} & -\frac{3}{4}\leq t<\frac{9}{4} \\ 2 & \frac{9}{4}\leq t<\frac{15}{4} \\ 0 & \text{otherwise} \end{cases}$$

2.1 (a) Nonlinear, causal, time-invariant, memoryless.

(b) Nonlinear, causal, time-invariant, memoryless.

(c) Linear, causal, time-invariant, memoryless.

(d) Linear, causal, time-varying, memoryless.

(e) Linear, causal, memoryless.

To check for time-invariance, let $x(t) = u(t)$ and let $x_1(t) = x(t+1)$. Then

$$y(t) = \begin{cases} 1 & t \ge 0 \\ 0 & t < 0 \end{cases}$$

$$y_1(t) = \begin{cases} x(t+1) & t \ge 0 \\ -x(t+1) & t < 0 \end{cases}$$

$$= \begin{cases} 1 & t \ge 0 \\ -1 & -1 \le t < 0 \end{cases} \ne y(t+1)$$

so that the system is time-varying.

(f) Linear, causal, with memory.

To check for time-invariance, let $x_1(t) = x(t - t_0)$. Then

$$y_1(t) = \int_{-\infty}^{t} x_1(\tau)d\tau = \int_{-\infty}^{t} x(\tau - t_0)d\tau$$

$$= \int_{-\infty}^{t-t_0} x(\tau)d\tau = y(t - t_0)$$

so that the system is time-invariant.

(j) Nonlinear, time-invariant, causal, with memory.

(k) Linear, non-causal, time-varying, with memory.

(l) Linear if zero initial conditions, time-invariant, causal, with memory.

2.6 (a) $x(t) * y(-t) = \int_{-\infty}^{\infty} x(\tau)\, y(-(t-\tau))\, d\tau$

$$= \int_{-\infty}^{\infty} x(\tau)\, y(\tau - t)\, d\tau = R_{xy}(t)$$

(b) $R_{yx}(t) = \int_{-\infty}^{\infty} y(\tau)\, x(\tau - t)\, d\tau$

$$= \int_{-\infty}^{\infty} x(\tau')\, y(\tau' + t)\, d\tau' \neq R_{xy}(t)$$

(c) $R_{yx}(-t) = \int_{-\infty}^{\infty} x(\tau')\, y(\tau' - t)\, d\tau' = R_{xy}(t)$