HW 12 Solutions

P 8.2 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -250$

$$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -1000$$

Adding the above equations, $\quad -2\alpha = -1250$

$$\alpha = 625 \text{ rad/s}$$

$$\frac{1}{2RC} = \frac{1}{2R(0.1 \times 10^{-6})} = 625$$

$$R = 8\,\text{k}\Omega$$

$$2\sqrt{\alpha^2 - \omega_o^2} = 750$$

$$4(\alpha^2 - \omega_o^2) = 562{,}500$$

$$\therefore \quad \omega_o = 500 \text{ rad/s}$$

$$\omega_o^2 = 25 \times 10^4 = \frac{1}{LC}$$

$$\therefore \quad L = \frac{1}{(25 \times 10^4)(0.1 \times 10^{-6})} = 40\,\text{H}$$

[b] $i_\text{R} = \dfrac{v(t)}{R} = -1e^{-250t} + 4e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$

$$i_\text{C} = C\frac{dv(t)}{dt} = 0.2e^{-250t} - 3.2e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_\text{L} = -(i_\text{R} + i_\text{C}) = 0.8e^{-250t} - 0.8e^{-1000t}\,\text{mA}, \qquad t \geq 0$$

P 8.3 **[a]** $i_R(0) = \frac{15}{200} = 75\text{mA}$

$i_L(0) = -45\text{mA}$

$i_C(0) = -i_L(0) - i_R(0) = 45 - 75 = -30\,\text{mA}$

[b] $\alpha = \frac{1}{2RC} = \frac{1}{2(200)(0.2 \times 10^{-6})} = 12{,}500$

$\omega_o^2 = \frac{1}{LC} = \frac{1}{(50 \times 10^{-3})(0.2 \times 10^{-6})} = 10^8$

$s_{1,2} = -12{,}500 \pm \sqrt{1.5625 \times 10^8 - 10^8} = -12{,}500 \pm 7500$

$s_1 = -5000$ rad/s; $\quad s_2 = -20{,}000$ rad/s

$v = A_1 e^{-5000t} + A_2 e^{-20{,}000t}$

$v(0) = A_1 + A_2 = 15$

$\frac{dv}{dt}(0) = -5000A_1 - 20{,}000A_2 = \frac{-30 \times 10^{-3}}{0.2 \times 10^{-6}} = -15 \times 10^4\text{V/s}$

Solving, $\quad A_1 = 10; \quad A_2 = 5$

$v = 10e^{-5000t} + 5e^{-20{,}000t}\,\text{V}, \qquad t \geq 0$

[c] $i_C = C\frac{dv}{dt}$

$= 0.2 \times 10^{-6}[-50{,}000e^{-5000t} - 100{,}000e^{-20{,}000t}]$

$= -10e^{-5000t} - 20e^{-20{,}000t}\,\text{mA}$

$i_R = 50e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}$

$i_L = -i_C - i_R = -40e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \quad t \geq 0$

P 8.4 $$\frac{1}{2RC} = \frac{1}{2(312.5)(0.2 \times 10^{-6})} = 8000$$

$$\frac{1}{LC} = \frac{1}{(50 \times 10^{-3})(0.2 \times 10^{-6})} = 10^8$$

$$s_{1,2} = -8000 \pm \sqrt{8000^2 - 10^8} = -8000 \pm j6000 \text{ rad/s}$$

$\therefore$ response is underdamped

$$v(t) = B_1 e^{-8000t} \cos 6000t + B_2 e^{-8000t} \sin 6000t$$

$$v(0^+) = 15\text{ V} = B_1; \qquad i_\mathrm{R}(0^+) = \frac{15}{312.5} = 48\text{ mA}$$

$$i_\mathrm{C}(0^+) = [-i_\mathrm{L}(0^+) + i_\mathrm{R}(0^+)] = -[-45 + 48] = -3\text{ mA}$$

$$\frac{dv(0^+)}{dt} = \frac{-3 \times 10^{-3}}{0.2 \times 10^{-6}} = -15{,}000 \text{ V/s}$$

$$\frac{dv(0)}{dt} = -8000B_1 + 6000B_2 = -15{,}000$$

$$6000B_2 = 8000(15) - 15{,}000; \qquad \therefore \quad B_2 = 17.5\text{ V}$$

$$v(t) = 15e^{-8000t} \cos 6000t + 17.5e^{-8000t} \sin 6000t \text{ V}, \qquad t \geq 0$$

.7 **[a]** $\alpha = 20{,}000; \qquad \omega_d = 15{,}000$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 225 \times 10^6 + 400 \times 10^6 = 625 \times 10^6$$

$$\frac{1}{LC} = 625 \times 10^6$$

$$L = \frac{1}{(625 \times 10^6)(40 \times 10^{-9})} = 40\,\text{mH}$$

[b] $\alpha = \dfrac{1}{2RC}$

$$\therefore \quad R = \frac{1}{2\alpha C} = \frac{1}{2(20{,}000)(40 \times 10^{-9})} = 625\,\Omega$$

[c] $V_o = v(0) = 100\,\text{V}$

[d] $I_o = i_{\text{L}}(0) = -i_{\text{R}}(0) - i_{\text{C}}(0)$

$$i_{\text{R}}(0) = \frac{V_o}{R} = \frac{100}{625} = 160\,\text{mA}$$

$$i_{\text{C}}(0) = C\frac{dv}{dt}(0)$$

$$\frac{dv}{dt} = 100\{e^{-20{,}000t}[-15{,}000 \sin 15{,}000t - 30{,}000 \cos 15{,}000t] -$$

$$20{,}000e^{-20{,}000t}[\cos 15{,}000t - 2 \sin 15{,}000t]$$

$$\frac{dv}{dt}(0) = 100\{1(-30{,}000) - 20{,}000\} = -500 \times 10^4$$

$$C\frac{dv}{dt}(0) = -500 \times 10^4 (40 \times 10^{-9}) = -200\,\text{mA}$$

$$\therefore \quad I_o = 200 - 160 = 40\,\text{mA}$$

[e]
$$\frac{dv}{dt} = 100e^{-20{,}000t}[25{,}000 \sin 15{,}000t - 50{,}000 \cos 15{,}000t]$$

$$= 25 \times 10^5 e^{-20{,}000t}[\sin 15{,}000t - 2 \cos 15{,}000t]$$

$$C\frac{dv}{dt} = 0.1e^{-20{,}000t}(\sin 15{,}000t - 2 \cos 15{,}000t)$$

$$i_{\text{C}}(t) = 0.1e^{-20{,}000t}(\sin 15{,}000t - 2 \cos 15{,}000t)\,\text{A}$$

$$i_{\text{R}}(t) = 0.16e^{-20{,}000t}(\cos 15{,}000t - 2 \sin 15{,}000t)\,\text{A}$$

$$i_{\text{L}}(t) = -i_{\text{R}}(t) - i_{\text{C}}(t)$$

$$= e^{-20{,}000t}(40 \cos 15{,}000t + 220 \sin 15{,}000t)\,\text{mA}, \quad t \geq 0$$