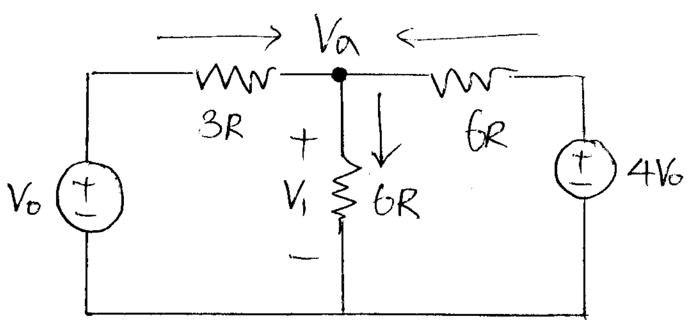
Homework #1 Solution

1.



$$\frac{V_0 - V_a}{3R} + \frac{4V_0 - V_a}{6R} = \frac{V_a}{6R} \quad \cdots\cdots \text{①}$$

$$V_a = V_i \quad \cdots\cdots \text{②}$$

By ① × 6R.

$$2(V_0 - V_a) + 4V_0 - V_a = V_a \quad \cdots\cdots \text{①}'$$

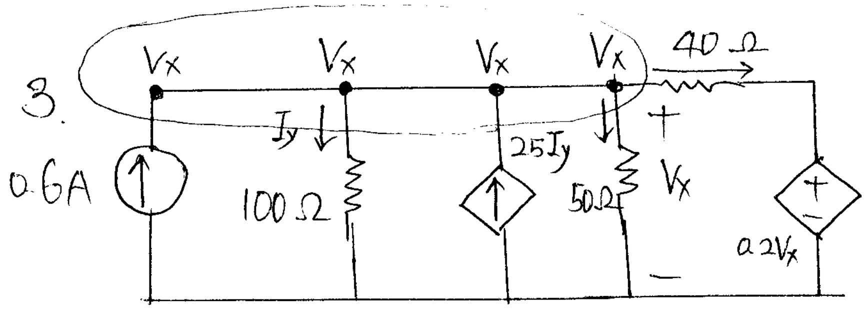
After regrouping terms.

$$4V_a = 6V_0$$

$$V_a = 1.5V_0 = V_i \quad (\because \text{②})$$

$$\therefore \underline{\underline{V_1 = 1.5V_0}}$$

3.



$$0.6 + 25I_y = \frac{V_x}{100} + \frac{V_x}{50} + \frac{V_x - 0.2V_x}{40} \quad \cdots \text{①}$$

$$\frac{V_x}{100} = I_y \quad \cdots\cdots \text{②}$$

By ① × 200,

$$120 + 5000I_y = 2V_x + 4V_x + 4V_x \quad \cdots \text{①}'$$

After regrouping terms,

$$10V_x - 5000I_y = 120 \quad \cdots \text{①}''$$

$$I_y = \frac{V_x}{100} \quad \cdots \text{②}$$

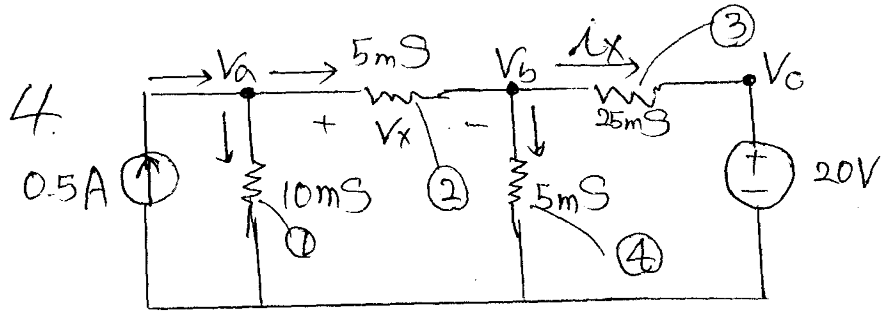
By applying ② to ①″,

$$10V_x - 5000\left(\frac{V_x}{100}\right) = 120$$

$$V_x = -3(V)$$

$$\therefore \; V_x = -3V$$

4.



Reference node d.

(a) A voltage Source ties node c to the reference node. Hence the node voltage V_c is fixed at 20V.

By applying KCL to node a.b.

$$0.5 = 0.01\,V_a + 0.005\,(V_a - V_b) \quad \text{———} \quad ①$$

$$0.005\,(V_a - V_b) = 0.005\,V_b + 0.025\,(V_b - 20) \quad \text{———} \quad ②$$

After regrouping terms,

$$3V_a - V_b = 100 \quad \text{———} \quad ①'$$

$$V_a - 7V_b = -100 \quad \text{———} \quad ②'$$

Therefore,
$$\begin{pmatrix} 3 & -1 \\ 1 & -7 \end{pmatrix}\begin{pmatrix} V_a \\ V_b \end{pmatrix} = \begin{pmatrix} 100 \\ -100 \end{pmatrix},$$

(b)
$$\begin{pmatrix} 3 & -1 \\ 1 & -7 \end{pmatrix}^{-1} = \frac{1}{3\times(-7)-(-1)\cdot 1}\begin{pmatrix} -7 & 1 \\ -1 & 3 \end{pmatrix} = -\frac{1}{20}\begin{pmatrix} -7 & 1 \\ -1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 \\ 1 & -7 \end{pmatrix}^{-1}\begin{pmatrix} 3 & -1 \\ 1 & -7 \end{pmatrix}\begin{pmatrix} V_a \\ V_b \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 1 & -7 \end{pmatrix}^{-1}\begin{pmatrix} 100 \\ -100 \end{pmatrix}$$

$$\begin{pmatrix} V_a \\ V_b \end{pmatrix} = -\frac{1}{20}\begin{pmatrix} -7 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 100 \\ -100 \end{pmatrix}$$

$$= -\frac{1}{20}\begin{pmatrix} -7\times 100 + 1\times(-100) \\ -1\times 100 + 3\times(-100) \end{pmatrix} = \begin{pmatrix} 40 \\ 20 \end{pmatrix} = \begin{pmatrix} V_a \\ V_b \end{pmatrix}$$

$\therefore V_a = 40V.\ V_b = 20V$

4. (Continued)

(c) $V_X = V_a - V_b = 40 - 20 = 20(V)$

$V_{da} = V_d - V_a = 0 - 40 = -40(V)$

$V_{db} = V_d - V_b = 0 - 20 = -20(V)$

(d) $i_x = (\cancel{V_b} - V_c)\, 0.025 = 0$

Therefore, the power delivered by voltage source is zero.

On the other hand, the power delivered by current source is as follows

$P_{①} = IV_a = (-0.5) \times 40 = -20(W)$

The power absorbed by each resistor is as follows.

$P_{①} = GV_a^2 = 0.01 \times 40^2 = 16(W)$

$P_{②} = GV_x^2 = 0.005 \times 20^2 = 2(W)$

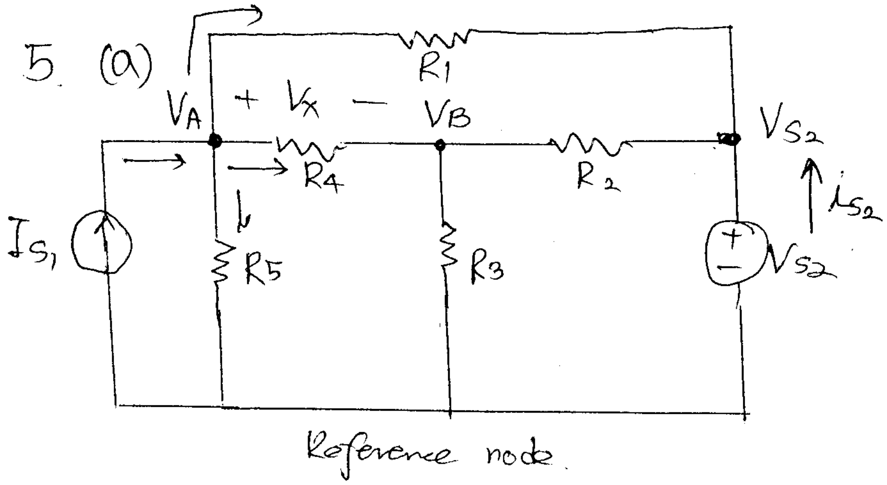
$P_{③} = GV_b^2 = 0.005 \times 20^2 = 2(W)$

$P_{④} = 0.$

Total power absorbed by each resister is $16+2+2 = 20(W)$

Therefore, the principle of conservation of power is verified.

5. (a)



$I_{S1} = 4mA$

$V_{S2} = 20V$

$R_1 = 2.5k\Omega$

$R_2 = R_3 = R_4 = 10k\Omega$, $R_5 = 5k\Omega$

By applying KCL to node A and B,

$$I_{S1} = \frac{V_A}{R_5} + \frac{V_A - V_B}{R_4} + \frac{V_A - V_{S2}}{R_1} \quad \text{----} \quad ①$$

$$\frac{V_B - V_A}{R_4} + \frac{V_B}{R_3} + \frac{V_B - V_{S2}}{R_2} = 0 \quad \text{------} \quad ②$$

After regrouping terms,

$$7V_A - V_B = 120 \quad \text{----} \quad ①'$$

$$-V_A + 3V_B = 20 \quad \text{----} \quad ②'$$

$$\begin{pmatrix} 7 & -1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} 120 \\ 20 \end{pmatrix} \Rightarrow \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} 7 & -1 \\ -1 & 3 \end{pmatrix}^{-1}\begin{pmatrix} 120 \\ 20 \end{pmatrix}$$

$$= \begin{pmatrix} 19 \\ 13 \end{pmatrix}$$

$\therefore$ $V_A = 19V$. $V_B = 13V$

$V_x = V_A - V_B = 6V$

$$P_{R4} = \frac{V^2}{R} = \frac{V_x^2}{R_4} = 3.6mW$$

5. (b)

By applying KCL to node A and B.

$$2 = \frac{V_A}{12.5} + \frac{V_A - V_B}{10} + \frac{V_A - 100}{200} \quad \cdots\cdots \quad ①$$

$$\frac{V_B - V_A}{10} + \frac{V_B}{50} + \frac{V_B - 100}{40} = 0 \quad \cdots\cdots \quad ②$$

After regrouping terms.

$$37V_A - 20V_B = 500 \quad \cdots\cdots ①'$$

$$-20V_A + 29V_B = 500 \quad \cdots\cdots ②'$$

So,
$$\begin{pmatrix} 37 & -20 \\ -20 & 29 \end{pmatrix} \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} 500 \\ 500 \end{pmatrix} \Rightarrow \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} 37 & -20 \\ -20 & 29 \end{pmatrix}^{-1} \begin{pmatrix} 500 \\ 500 \end{pmatrix}$$

$$\therefore V_A \doteqdot 36.4(V) \qquad V_B \simeq 42.3(V)$$

And, the power delivered by voltage source is,

$$P_v = (-i_{s_2}) \cdot V_{s_2} \qquad \left(\because i_{s_2} = \frac{V_{s_2} - V_B}{R_2} + \frac{V_{s_2} - V_A}{R_1} \approx 1.7(A) \right)$$

$$= (-1.7)\, 100 = -170(W)$$

the Power delivered by current source is.

$$P_c = (-I_{s_2}) \cdot V_A = (-2) \cdot 36.4 = -72.8(W)$$

The Powers absorbed by each resistor are as follows.

$$P_{R1} = \frac{(V_{s_2} - V_A)^2}{R_1} \approx 14.4(W), \quad P_{R2} = \frac{(V_{s_2} - V_B)^2}{R_2} \approx 83.2(W)$$

$$P_{R3} = \frac{V_B^2}{R_3} \approx 35.8(W) \quad P_{R4} = \frac{V_X^2}{R_4} \approx 3.5(W) \quad P_{R5} = \frac{V_A^2}{R_5} \approx 106(W)$$

$$\therefore \underbrace{(P_v + P_c)}_{\text{Power delivered}} + \underbrace{(P_{R1} + P_{R2} + P_{R3} + P_{R4} + P_{R5})}_{\text{Power absorbed}} \simeq 0$$