Probability (continued)

Independent Events:

Two events A and B are said to be independent if and only if $P(AB) = P(A)P(B)$.
Thus $P(A|B) = \dfrac{P(AB)}{P(B)} = P(A)P(B)/P(B) = P(A)$, and similarly $P(B|A) = \dfrac{P(AB)}{P(A)} = P(A)P(B)/P(A) = P(B)$.
So, $P(A|B) = P(A)$, and $P(B|A) = P(B)$.

Examples:

1- An experiment involves choosing an integer N between 0 and 9.
Let $A = \{N \le 5\}$, $B = \{3 \le N \le 7\}$, $C = \{N \text{ is even and } N > 0\}$.

List the points that belong to the events $A \cap B \cap C$, $A \cup (B \cap C^c)$, $(A \cup B) \cap C^c$, $(A \cap B) \cap [(A \cup C)^c]$.

2- Let A, B, and C be *arbitrary* events in the sample space.

Let D_1 = the event that at least two of the events A, B, and C occur,

D_2 = the event that exactly two of the events A, B, and C occur,

D_3 = the event that at least one of the events A, B, and C occur,

D_4 = the event that exactly one of the events A, B, and C occur,

D_5 = the event that not more than two of the events A, B, and C occur.

Express each of the events D_1 through D_5 in terms of A, B, and C using unions, intersections, and complements.

3- A public opinion poll (circa 1850) consisted of the questions: (a) Are you a registered Whig?, (b) Do you approve of President Fillmore’s performance in office? (c) Do you favor the Electoral College system? A group of 1000 people is polled. Assume answers are either “yes” or “no.” It is found that: 550 people answer “yes” to (c) and 450 “no.” 325 people answer “yes” exactly twice; and 100 people answer “yes” to all three questions. 125 registered Whigs approve of President Fillmore’s performance. How many of those who favor the Electoral College system do not approve of Fillmore’s performance, and in addition are not registered Whigs?