Probability

Probability Measure

Quantitative way of characterizing “our” uncertainty about the outcome of a random experiment. To each event A, we assign a real number $P(A)$, which is a measure of its relative “likelihood” of occurring. It is required that:

0.1: $0 \le P(A) \le 1$

0.2: $P(s) = 1$

0.3: *If* $A_1, A_2, A_3, \ldots$ *are mutually exclusive; i.e* $A_i \cap A_j = \varnothing, i \neq j$, *then* $P(A_1 \cup A_2 \cup A_3 \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$

As corollary of above: $P(\varnothing) = 0$.

Probability Assignment and Examples:

Derived Relations

1- $P(S) = 1 = P(A) + P(A^c)$. Results from $S = A + A^c$ and 0.3 above.

2- $P(A \cap B^c)$: $A = AB^c + AB$, then $P(A) = P(AB^c) + P(AB)$, then $P(AB^c) = P(A) - P(AB)$.

3- $P(A + B) = P(AB^c + BA^c + AB) = P(A) - P(AB) + P(B) - P(AB) + P(AB) = P(A) + P(B) - P(AB$

4- $P(A + B + C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$

Conditional Probability

Definition: $P(A|B) = \dfrac{P(AB)}{P(B)}$, $P(B) \neq 0$

Let $A_i, i = 1, 2, \ldots, n$ partition S, then $XS = X(A_1 + A_2 + \ldots + A_n) = XA_1 + XA_2 + \ldots + XA_n$,then

$$P(X) = \sum_i P(XA_i) = \sum_i P(X|A_i)P(A_i)$$

Bayes Formula

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(A_i|X) = \frac{P(X|A_i)P(A_i)}{\sum P(X|A_i)P(A_i)}$$