

**CE427 - CHEMICAL ENGINEERING LABORATORY III
FALL 2004**

CONTRACTION COEFFICIENTS AND TANK DRAINING

Skill Outcomes:

In completing this particular experiment, the student should develop or improve mastery of the following skills:

- Ability to safely operate the “Tank Draining” apparatus located in 121 Jarvis
- Ability to determine the Fanning friction factor for smooth pipe.
- Ability to apply a mechanical energy balance to the draining of liquid from a tank.
- Definition of a contraction coefficient and ability to estimate coefficients from empirical formulas
- Understand the difference between a sharp-edged and a rounded contraction.
- Ability to determine uncertainty limits for physical measurements and to use error propagation to determine uncertainty limits for calculated values
- Ability to use uncertainty limits to draw conclusions from data

Objectives:

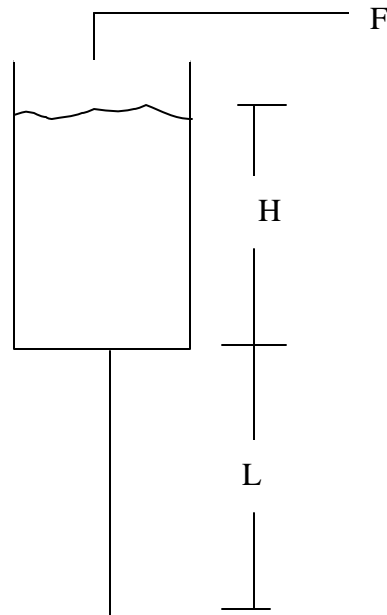
- 1.) Estimate the contraction coefficient at the exit of the tank, for both the large and small tank located in 121 Jarvis.
- 2.) Compare predicted drain times to measured drain times.
- 3.) Determine the effect, if any, of the length of drain pipe on the draining time of a tank.

System:

Two tanks located in 121 Jarvis have diameters of 8.375 inches and 4.0 inches. The two tanks have exit lines that are replaceable with 0.49 inch diameter tubes having different lengths. The lengths available are approximately 6, 12, 24, 36, 48, 60, and 72 inches long. There are two types of contraction assemblies that can be placed on either of the tanks. One is sharp-edged and the other is rounded. The tanks can be filled with water using a pump and plugs for the drain lines are available. A pressure transducer, located at the tank exit, transmits data to a PC running a LabView program which records the water level as a function of time as the tank drains.

Theory:

If the tank system is operated as shown in the figure below, the flow in, F , is sufficient to maintain the level in the tank at H inches while the tank is being drained through a pipe of length L .



In this steady state situation, Bernoulli's equation with friction takes the form:

$$L + H = K_c \frac{V^2}{2g} + K_e \frac{V^2}{2g} + \frac{4fLV^2}{2gd} + \frac{V^2}{2g} \quad (1)$$

Where L = length of drain pipe

H = liquid level above the bottom of the tank

K_c = Contraction loss coefficient

K_e = Expansion loss coefficient (at exit)

V = velocity in drain tube

g = acceleration of free fall
 f = Fanning friction factor
 d = drain pipe diameter

The loss coefficients and the Fanning friction factor are discussed in Chapter 5 of McCabe, Smith, and Harriott (M,S,&H) and it is expected that the experimenter will consult the text for further information. For the system in this assignment, the fluid leaving the pipe is somewhat confined since it is a free jet (it does not spray out as soon as it leaves the tube exit). Therefore, K_e is small and can be neglected.

Equation (1) can be more compactly written as:

$$L + H = \left[1 + K_c + \frac{4fL}{d}\right] \frac{V^2}{2g} \quad (2)$$

If the tank is draining without an inlet flow, then an unsteady state material balance on the tank and exit pipe gives:

$$S_b V = -S_a \frac{dH}{dt} \quad (3)$$

(note that the minus sign is needed because the derivative is negative) S_a and S_b are the cross-sectional areas of the tank and the drain pipe, respectively and it is sometimes convenient to define the parameter, b , as follows:

$$b = \frac{S_b}{S_a} = \frac{d^2}{D^2} \text{ where } D = \text{Tank diameter.} \quad (4)$$

Substitution for V from Eqn.(2) into Eqn. (3), rearrangement, and integration yield:

$$\int_0^{t_e} dt = -\frac{1}{b} \int_{H_i}^{H_e} \sqrt{\frac{a}{H+L}} dH \quad (5)$$

Where t_e = drain time

H_i = initial liquid level

H_e = final liquid depth

$$a = \left(1 + K_c + \frac{4fL}{d}\right) \left(\frac{1}{2g}\right)$$

After integration, we obtain:

$$t_e = \frac{2\sqrt{a}}{b} \left\{ \sqrt{H_i + L} - \sqrt{H_e + L} \right\} \quad (6)$$

(In the integration of eqn (5) it was assumed that the friction factor, f , is a constant throughout the draining process. Explain why this is valid)

Comments and Items to Consider

Determination of Contraction Coefficients:

It can be seen from Eqn (1) that if there is no drain pipe, L is zero, the friction factor term in the pipe is zero, and K_c is the one remaining unknown. Thus, in a steady state run, with water flowing into and out of the tank at the same rate, a steady state level, H , can be measured. The flow rate can be measured by collecting water in a bucket over a timed interval. Hence the contraction coefficient can be calculated.

However, due to considerable spraying of water in the actual experimental system when no drain pipe is used, it is not feasible to calculate the contraction coefficient in this manner. One method (but not the only method) to overcome this complication is to perform a series of tests with the two shortest exit pipes. By assuming a linear relationship between flow rate and pipe length near zero pipe length, the data can be extrapolated to determine the flow rate that would exist when $L=0$.

1. Determine the contraction coefficient for both tanks. Remember, the type of contraction is different for each tank.
2. Use a bucket and stop watch to make the flow rate measurements.
3. Take at least three readings at each flow rate using both the 6 inch and 12 inch drain pipes. Try to maintain the steady state water level the same for all runs.
4. Compare the K_c 's you obtained to literature values for sharp-edge and rounded contractions. Provide possible explanations for your results. Include uncertainty limits for the K_c 's you determined. (Hint: while an empirical formula for sharp-edged contraction is given in MS&H, a contraction loss coefficient for a rounded exit must be found in another source)

Drain Time Measurements:

For measurements of drain time, initially fill the tanks to the overflow line while plugging the drain tube. Then remove the plug while simultaneously starting the data acquisition system to record the drain time.

- 1) Use both tanks and at least 4 drain tubes. Obtain at least three readings for each tube.
- 2) Because it is difficult to determine exactly when the tank is empty, record the time to drain to a consistent depth (1 or 2 inches) above the bottom of the tank.

- 3) Obtain predicted drain times using Eqn (6). Note that you will need to estimate the friction factor. To do this, you will need to calculate the “average” velocity of the water leaving the tube based on the volume of water in the tank at the beginning and how long it took for it to reach the bottom.
- 4) How do the measured values compare to those predicted? Include uncertainty limits for both your measured and predicted drain times.
- 5) Does changing the pipe length “significantly” alter the flow rate measured in the range of your experiments?

References

1. McCabe, Smith, and Harriott, *Unit Operations of Chemical Engineering*, 6th Edition, McGraw-Hill, 2001.
2. Perry, R.H. and D.W. Green (eds.), *Chemical Engineer’s Handbook*, 7th ed., McGraw-Hill, 1997.