This problem deals with experiments performed in a laboratory on adsorption of water from air in a bed filled with silica gel (length $L = 60$ cm). Measured concentration profiles are shown in Fig. 4 for various times after the initial time $t = 0$, with x = distance along bed. You may assume that $c(L,t) = 0$ for $t \leq 2$h. Details of the experiment are as follows:

u_o = superficial gas velocity = 13.1 cm/s;

y_o = water mole fraction in entering gas = 5210×10^{-6};

T = 20°C;

P = 3.0 atm;

ρ_{bed} = 0.30 g silica gel/cm^3 bed volume

The break-point time t_b is defined by the criterion $c(L,t_b) = 0.10\ c_o$.

The experiment was interrupted at five hours and therefore there is insufficient data to determine the saturation capacity of the bed by time integration. Figure 5 will allow you to determine w_{sat} by alternate means.

(a) (10 points) What is the saturation capacity of the bed W_{sat} in g water/g silica gel?

(b) (30 points) What is the unused capacity of the silica gel (in terms of equivalent bed length) at the break-point?

(c) (10 points) Estimate the break-point time for a longer bed length of 100 cm, all other conditions being equal.

Data: vapor pressure of water

T(°C)	$P^{sat}_{H_2O}$ (mm Hg)
20	19.8
30	31.8
40	55.3

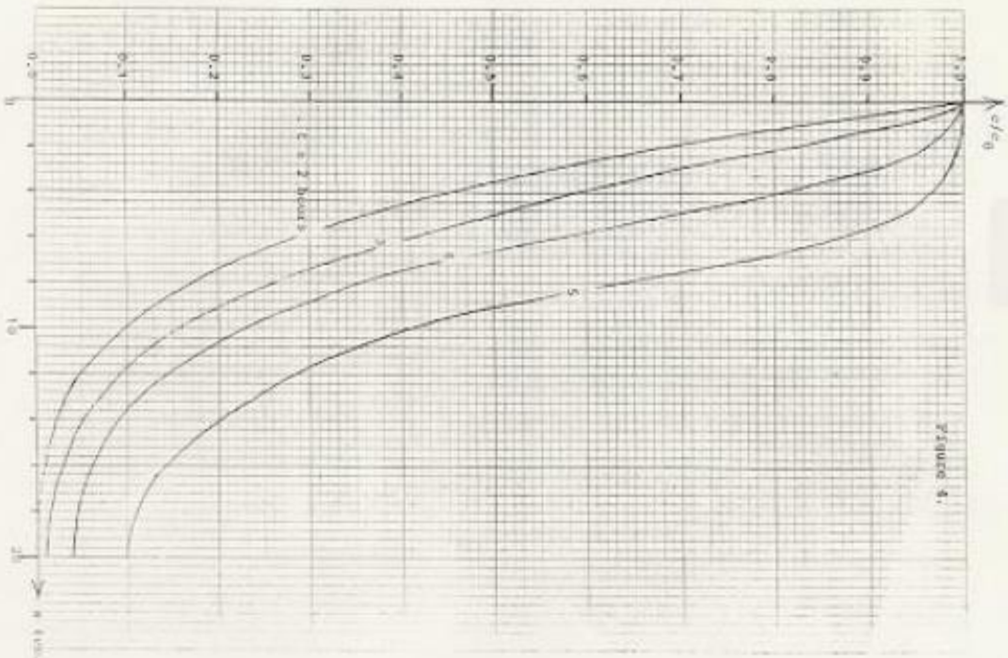

Figure 5.

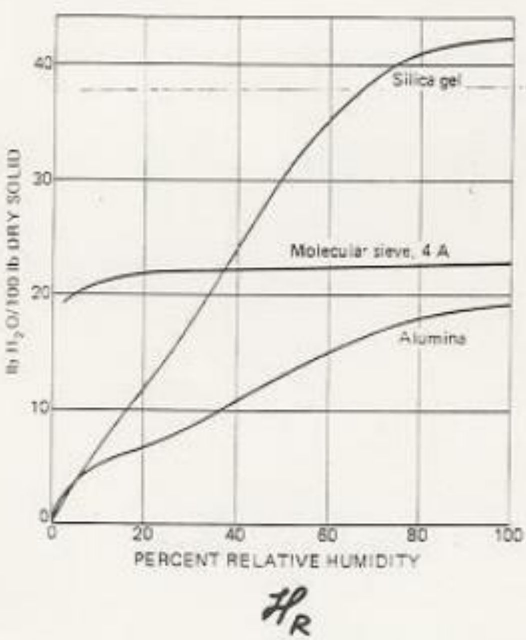

Figure 24-3 Adsorption isotherms for water in air at 20 to 50°C.

(a) Incoming air: $p_{H_2O} = y_{H_2O} P$

$$= (5210 \cdot 10^{-6})(3 \text{ atm} \times \frac{760 \text{ mm Hg}}{1 \text{ atm}})$$

$$= 11.88 \text{ mm Hg}$$

at 0°C $p_{H_2O}^{sat} = 19.8$ mm Hg. Then

$$h_R = (100\%)\frac{p_{H_2O}}{p_{H_2O}^{sat}} = (100\%)\frac{11.88}{19.8} = 60.0\%$$

From Fig. 5, $W_{sat} = \frac{35.2 \text{ lb } H_2O}{100 \text{ lb silica gel}} = \boxed{0.352 \frac{g\ H_2O}{g \text{ silica gel}}}$

W_{sat}

(b) Solute in:

$$(F_A)_{in} = \left(\frac{\text{mass}}{\text{area} \cdot \text{time}}\right) \text{ solute in}$$

total cross-sectional area

$$= u_0 \cdot (c_{tot}\, y_0) \cdot M_A$$

c_0

$$\left(\frac{\text{volume}}{\text{area} \cdot \text{time}}\right)\left(\frac{\text{moles solute}}{\text{volume}}\right)\frac{g \text{ solute}}{\text{mol solute}}$$

$\left(\frac{J}{m^3}\right)$

$$= (13.1 \text{ cm/s})\left[\frac{(3 \times 101325 \text{ Pa})}{(8.314 \frac{J}{mol\,K})(293.15 K)} \times \frac{1 m^3}{10^6 cm^3}\right]\left(\frac{5210}{10^6}\right) \times \frac{18.015 \text{ g solute}}{\text{mol solute}} \times \frac{3600 s}{1 h}$$

$$= 0.552 \frac{g \text{ solute}}{cm^2 \cdot h}$$

Solute out:

$c(L,t)$

$$(F_A)_{out} = u_0\, c\, M_A = (u_0\, c_0\, M_A)(c/c_0) = (F_A)_{in}(c/c_0)$$

Solute accumulated: Solute accumulated in bed / area in time interval $(t, t+dt)$ is $[(F_A)_{in} - (F_A)_{out}]\,dt$

$$= (F_A)_{in}\left[1 - \frac{c(L,t)}{c_0}\right]dt$$

Solute accumulated/area from time 0 to t is given by

$$\text{solute adsorbed/area} = (F_A)_{in} \int_0^t \left[1 - \frac{c(L,t')}{c_0}\right] dt'$$

Read off value of $c(L,t)$ from graph. Prepare table. Break-point time t_b is time at which $c(L,t_b) = 0.10\, c_0$, so $t_b = 5\,h$.

$t(h)$	$\frac{c(L,t)}{c_0}$	$1-\frac{c(L,t)}{c_0}$	Contrib. to integral† $\int_0^t (1-\frac{c}{c_0})dt'$ (h)
0	0	1	
			2.000
2	0	1	
			0.995
3	0.01	0.99	
			0.975
4	0.04	0.96	
			0.930
$t_b \longrightarrow$ 5	0.10	0.90	
			4.900

So: $\int_0^{t_b}\left[1 - \frac{c(L,t)}{c_0}\right] dt = 4.900\text{ h}$,

and

$$\left(\frac{\text{mass solute}}{\text{area}}\right) \text{adsorbed upto } t_b = (F_A)_{in}\,(4.900\text{h})$$

To get capacity in mass solute adsorbed/mass silica gel, use the fact that

$$\left(\frac{\text{mass silica gel}}{\text{area}}\right) = \frac{\text{bed vol.}}{\text{bed area}} \cdot \frac{\text{mass silica gel}}{\text{bed vol.}}$$

$$= L\,\rho_{bed} = (60\text{ cm})(0.30\ \tfrac{\text{g silica gel}}{\text{cm}^3})$$

$$= 18.0\ \frac{\text{g silica gel}}{\text{cm}^2}$$

Then

$$W_b = \frac{\text{mass adsorbed upto } t_b}{\text{mass silica gel}}$$

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† Use trapezoid rule, $\int_a^b f(x)dx \cong \frac{b-a}{2}[f(a)+f(b)]$, for each interval

$$= \frac{\text{mass adsorbed up to } t_b / \text{area}}{\text{mass silica gel / area}}$$

$$W_b = \frac{(F_A)_{in}\ (4.900\ h)}{(L \rho_{bed})} = \frac{\left(0.352 \frac{g\ solute}{cm^2 \cdot h}\right)(4.900\ h)}{18.0\ \frac{g\ silica\ gel}{cm^2}}$$

$$= 0.150 \frac{g\ solute}{g\ silica\ gel}$$

Fraction of bed capacity used at break-point

$$= W_b / W_{sat} = 0.150 / 0.352 = 0.426$$

for 60 cm bed. Used length $= (0.426)(60\ cm)$
$= 25.6\ cm$

Unused length $= (60 - 25.6)\ cm = 34.4\ cm$

Unused capacity of bed (in terms of equivalent bed length) at break-point is $\boxed{\ell_{unused} = 34.4\ cm}$

(c) Scale up principle — ℓ_{unused} at t_b independent of bed length. For new longer bed,

$$\ell_{used} = L - \ell_{unused} = 100\ cm - 34.4\ cm$$
$$= 65.6\ cm$$

and $$\frac{W_b}{W_{sat}} = \frac{\ell_{used}}{L} = \frac{65.6}{100}$$

Mass balance:

cross-sectional area of bed

$$(F_A)_{in} A_c t_b - 0 = \underbrace{W_b}_{\left(\frac{\text{mass solute}}{\text{mass silica gel}}\right)} \cdot \underbrace{L A_c \rho_{bed}}_{\text{mass silica gel}}$$

(mass in up to t_b)

neglect solute out (every small and still only $0.1 c_0$ at t_b).

Solve for t_b:

$$t_b = \frac{W_b \, L \, \cancel{A_c} \, \rho_{bed}}{(F_A)_{in} \, \cancel{A_c}}$$

actually $\frac{W_b}{W_{sat}} \cdot L = l_{used} = 65.6 \text{ cm}$

$$= \frac{\left(\frac{W_b}{W_{sat}}\right) W_{sat} \, L \, \rho_{bed}}{(F_A)_{in}}$$

$$= \frac{\left(\frac{65.6}{100}\right)\left(0.352 \frac{\text{g solute}}{\text{g silica gel}}\right)(100 \text{ cm})\left(0.70 \frac{\text{g silica gel}}{\text{cm}^3}\right)}{0.552 \frac{\text{g solute}}{\text{cm}^2 \cdot \text{h}}}$$

$$\boxed{t_b = 12.5 \text{ h}}$$ predicted break-point time for 100 cm bed