This deals with an experimental set up studying adsorption of n-hexanol from air in a 10 cm long carbon filled bed. Measure concentration profiles are show in Figure 1 for various times after the initial time t – 0, with x being the distance along the bed.

Details of the experiment are as follows:

u_o = superficial gas velocity = 72 cm / s
c_o = 250 ppm hexanol (ppm = mole fraction x 10^6)
T^o = 50 °C
P = 690 mm Hg
ρ_{bed} = 0.45 g carbon / cm^3 bed volume

You may assume that $c(L,t) = 0$ for $t \leqslant 2$ h and $c(L,t) = c_o$ for for $t \geqslant 9$ h.

(a) Prepare in either graphical or numerical form a representation of the breakthrough curve.

(b) Calculate the saturation capacity of the carbon and the fraction of the bed used (in terms of equivalent bed length) at the beakpoint if this is defined according to the criterion $c(L,t_b) = 0.10\ c_o$.

(c) Assuming that the unused length of bed is independent of total bed length, estimate the break-point time for a 20 cm long bed.

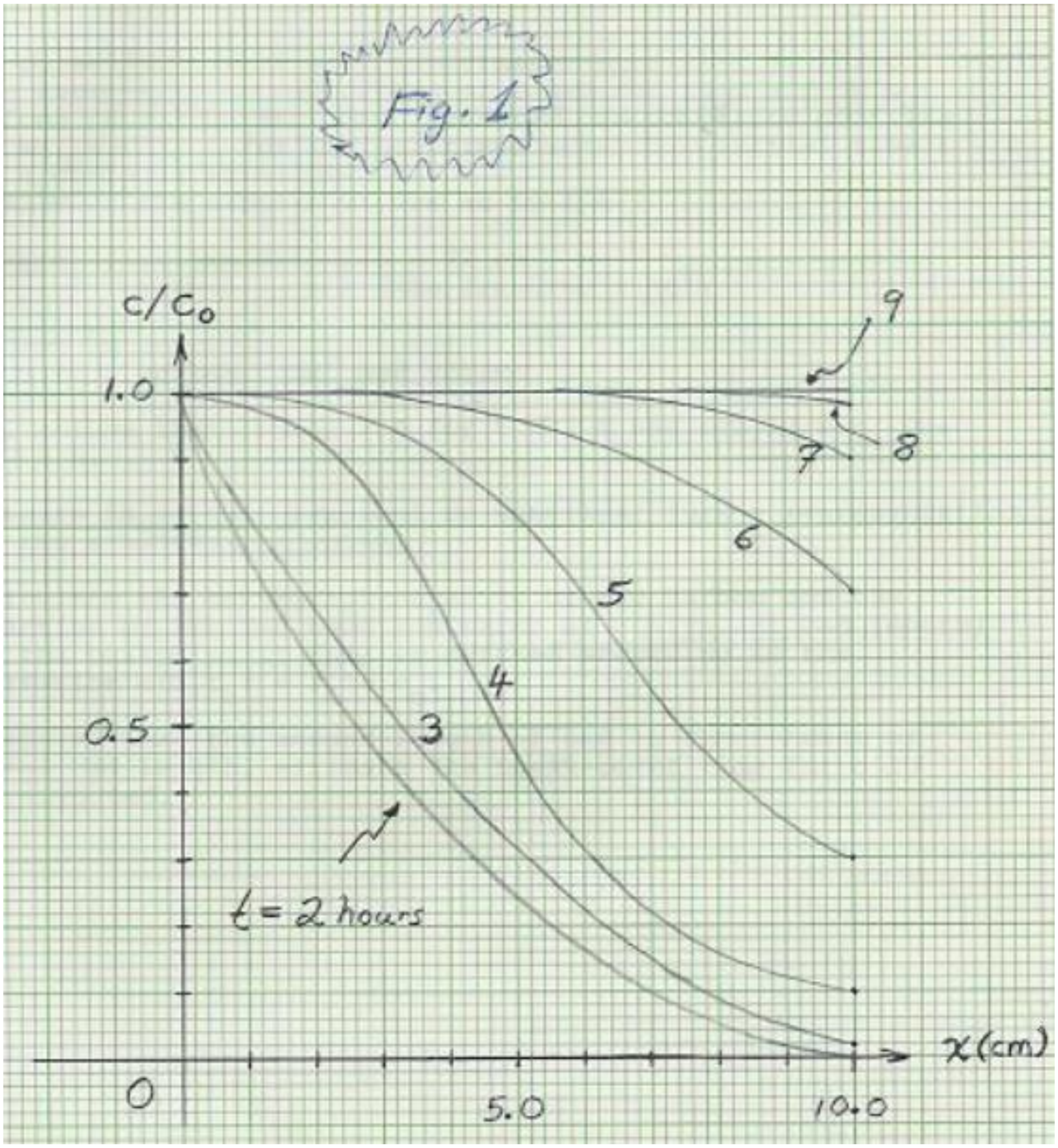


Fig. 1
c/c₀
1.0
0.5
0
5.0
10.0
x (cm)
t = 2 hours
3
4
5
6
7
8
9

(total bed length, 10 cm)

(a) Breakthrough curve is $c(L,t)/c_0$ plotted as function of time. From graph provided (Fig. 1), construct table.

t (h)	$c(L,t)/c_0$
0	0
2	0
3	0.02
4	0.10
5	0.30
6	0.70
7	0.90
8	0.98
9	1
(all later times	1)

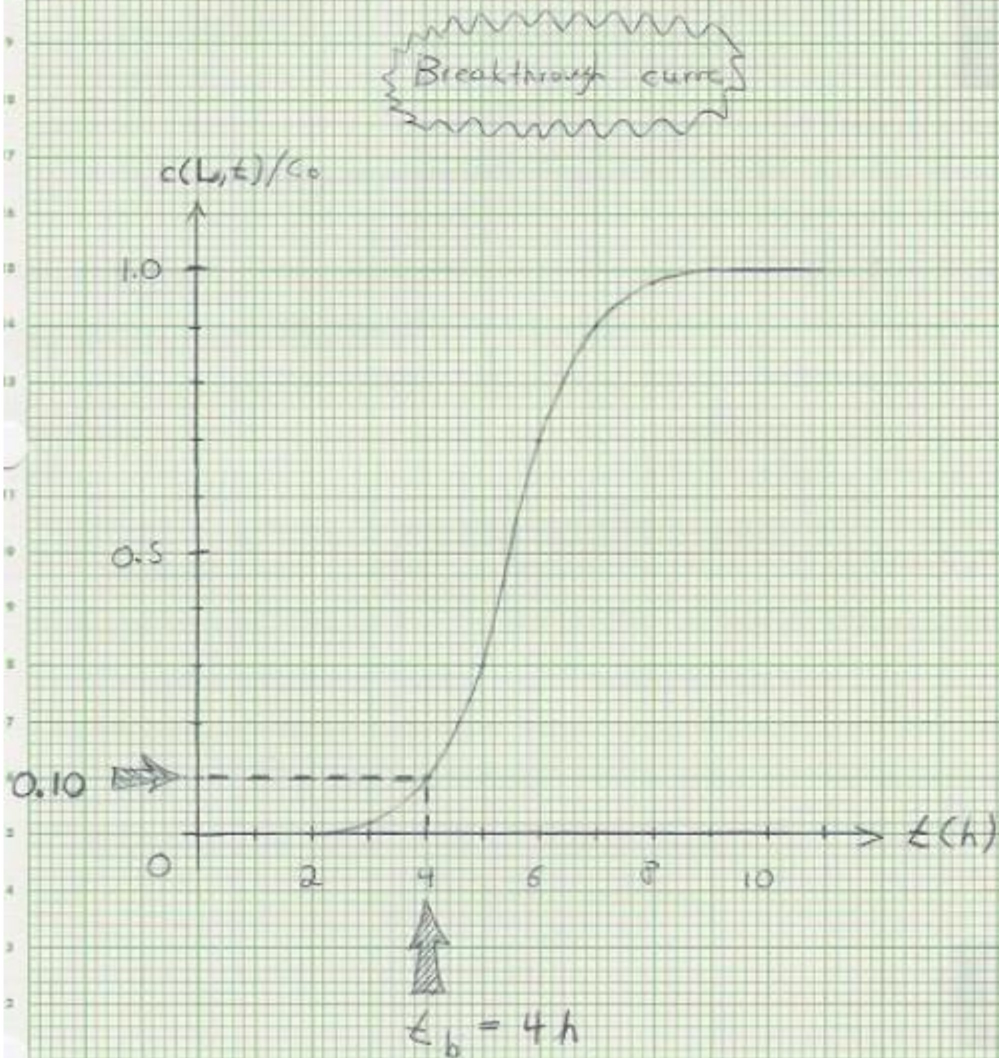
Note: breakpoint (time @ which $c(L,t)/c_0 = 0.10$) is $t_b = 4$ h.

Graph next page.

(b) First calculate $(F_A)_{in} = \left(\frac{\text{mass}}{\text{area}\cdot\text{time}}\right)$ solute in. (← "adsorbate")

$$(F_A)_{in} = u_0 \, C_0 \, M_A = u_0 \, C_{tot} \, y_0 \, M_A$$

- C_0: inlet; $\frac{\text{moles solute}}{\text{volume}}$
- C_{tot}: total concentration
- y_0: mole fr. at inlet
- solute = "A"
- u_0: superficial velocity $= \frac{\text{volume}}{\text{area}\cdot\text{time}}$ (area = total cross-sectional area of bed.)
- M_A: mol. wt. $= \frac{\text{mass solute}}{\text{mole solute}}$


Breakthrough curve
$c(L,t)/c_0$
1.0
0.5
0.10
0
2
4
6
8
10
$t\,(h)$
$t_b = 4\ h$

$$= \left(72\ \frac{cm}{s}\right)\left(\frac{1\ mol}{22400\ cm^3}\right) \times \frac{273\ K}{323\ K} \times \frac{690\ mmHg}{760\ mmHg}$$

(molar density @ 273 K and 1 atm) (correct for 50°C) (correct for 690 mmHg)

$$\times \underbrace{(250 \times 10^{-6})}_{\text{mole fraction hexanol} = \frac{\text{moles hexanol}}{\text{mole}}} \underbrace{\left(102.18\ \frac{g\ hexanol}{mol\ hexanol}\right)}_{\text{mol. wt.}}$$

$$= \left(6.301 \times 10^{-5}\ \frac{g}{cm^2 \cdot s}\ \text{hexanol}\right)\left(\frac{3600\,s}{h}\right)$$

$$(F_A)_{in} = 0.227\ \frac{g\ solute}{cm^2 \cdot h}$$

(here c means $c(L,t)$, i.e., solute conc. at exit.)

For review, $(F_A)_{out} = u_0 c M_A = (u_0 c_0 M_A)\left(\frac{c}{c_0}\right)$
$= (F_A)_{in} \frac{c}{c_0}$

Solute accumulated in bed in time interval $(t, t+dt)$ is $[(F_A)_{in} - (F_A)_{out}]\,dt = (F_A)_{in}\left(1 - \frac{c}{c_0}\right)dt$. ← $\left(\frac{mass}{area}\right)$

Total solute accumulated by bed from time zero to time t is $(F_A)_{in}\int_0^t \left(1 - \frac{c}{c_0}\right)dt$. Note: what we here call $(F_A)_{in}$ book calls F_A.

$$\left(\frac{mass}{area}\right)_{\text{adsorbed upto } t_b} = (F_A)_{in} \int_0^{t_b} \left(1 - \frac{c}{c_0}\right)dt \quad (t_b = 4h)$$

$$\left(\frac{mass}{area}\right)_{\text{adsorbed upto } t \to \infty} = (F_A)_{in} \int_0^{\infty} \left(1 - \frac{c}{c_0}\right)dt = \text{capacity of bed}$$

(can stop at 9 h because $1 - \frac{c}{c_0}$ is essentially zero for all later times.)

Prepare table for numerical integration (next page).

$t(h)$	$1-\frac{c}{c_0}$ (c = c(L, t))	contribution to integral $\int_0^t (1-\frac{c}{c_0})dt$ † (h)
0	1	
		2
2	1	
		0.99
3	0.98	
		0.94
4	0.90	
		0.80
5	0.70	
		0.50
6	0.30	
		0.20
7	0.10	
		0.06
8	0.02	
		0.01
9	0	

(2, 0.99, 0.94: upto breakpoint)

$$\int_0^{t_b (4h)} (1-\frac{c}{c_0})\,dt = (2+0.99+0.94)h = 3.93\ h$$

$$\int_0^{\infty} (1-\frac{c}{c_0})\,dt = (3.93+0.80+0.50+0.20+0.06+0.01)h = 5.50\ h$$

$(\frac{\text{mass}}{\text{area}})$ adsorbed upto t_b = $(0.227 \frac{g}{cm^2 h})(3.93 h)$ solute

$(\frac{\text{mass}}{\text{area}})$ adsorbed upto $t\to\infty$ = $(0.227 \frac{g}{cm^2 h})(5.50 h)$ solute

Capacity generally expressed in terms of W ($\frac{\text{mass adsorbed}}{\text{mass adsorbent}}$).

Know $\frac{\text{mass bed}}{\text{area}} = \frac{\text{vol. bed}}{\text{area}} \cdot \frac{\text{mass bed}}{\text{volume}} = L\,\rho_{bed}$

$= (10\ cm)(\frac{0.45\ g}{cm^3}) = 4.5\ \frac{g\ \text{carbon}}{cm^2}$

† Use trapezoid rule, $\int_a^b f(x)dx \cong \frac{b-a}{2}[f(a)+f(b)]$, for each interval.

adsorbed up to breakpoint

$$W_b = \frac{(0.227\ \frac{g}{cm^2 h})(3.93h)\ \text{solute}}{4.5\ \frac{g}{cm^2}\ \text{carbon}} = 0.198\ \frac{g\ \text{solute}}{g\ \text{carbon}}$$

$$W_{sat} = \frac{(0.227\ \frac{g}{cm^2 h})(5.50h)\ \text{solute}}{4.5\ \frac{g}{cm^2}\ \text{carbon}} = \boxed{0.277\ \frac{g\ \text{solute}}{g\ \text{carbon}}}$$

bed completely saturated

saturation capacity of the carbon

Fraction of bed capacity utilised at breakpoint

$$= W_b / W_{sat} = \frac{0.198}{0.277} = 0.715$$

fr. bed capacity used at t_b

for 10 cm bed. Used length = (0.715)(10 cm) = 7.15 cm

Unused length = 10 − 7.15 = 2.85 cm

equivalent unused length

equivalent used length

(c) By assumption, unused length (same) = 2.85 cm.

Then utilized length = 20 cm − 2.85 cm = 17.15 cm

$$\text{Fraction bed capacity utilized} = \frac{17.15\ \text{cm}}{20\ \text{cm}} = 0.858 = \frac{W_b}{W_{sat}}$$

new bed length L = 20 cm

new value for longer bed.

Now

$$W_b = (F_A)_{in}(t_b) \times \left(\frac{1}{L\rho_{bed}}\right),$$

$(F_A)_{in}$: $\left(\frac{\text{mass}}{\text{area}\cdot\text{time}}\right)$ solute in; t_b: (break pt time); together: mass solute/area at break pt.

$\frac{1}{L\rho_{bed}}$: $\frac{\text{area}}{\text{mass carbon}}$ ← see p. 4

so $$t_b = \frac{W_b\, L\, \rho_{bed}}{(F_A)_{in}} = \frac{(W_b/W_{sat})\, L\, \rho_{bed}\, W_{sat}}{(F_A)_{in}}$$

$$= \frac{(0.858)(20\text{ cm})(0.45\text{ g carbon/cm}^3)\left(\frac{0.277\text{ g solute}}{\text{g carbon}}\right)}{0.227\ \frac{\text{g solute}}{\text{cm}^2\cdot\text{h}}}$$

$$= \boxed{9.42\text{ h}}$$

estimated breakpoint time for 20 cm long bed